Relay-Assisted Mixed FSO/RF Systems Over Málaga-$\mathcal{M}$ and κ–μ Shadowed Fading Channels

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***Abstract*—This letter presents a unified analytical framework for the computation of the ergodic capacity and the outage probability of relay-assisted mixed FSO/RF transmission. In addition to accounting for different FSO detection techniques, the mathematical model offers a twofold unification of mixed FSO/RF systems by considering mixed Málaga-$\mathcal{M}$/κ-μ shadowed fading, which includes as special cases nearly all linear turbulence/fading models adopted in the open literature.**

***Index Terms*—Amplify-and-forward (AF), ergodic capacity, free-space optics (FSO), κ-μ shadowed fading, Málaga-$\mathcal{M}$ distribution, outage probability, pointing errors.**

I. Introduction

RECENTLY, free-space optical (FSO) communications have gained a significant attention due to their advantages of higher bandwidth in unlicensed spectrum and higher throughput compared to their RF counterparts [1]. Hence, the gathering of both FSO and RF technologies arises as a promising solution for securing connectivity between the RF access and the fiber-optic-based backbone networks. As such, there has been prominent interest in mixed FSO/RF systems where RF transmission is used at one hop and FSO transmission at the other [2]–[4]. Most contributions within this research line consider restrictive irradiance and channel gain probability density function (PDF) models for the FSO and RF links, respectively. The most commonly utilized models for the irradiance in FSO links are the lognormal and the Gamma-Gamma ($\mathcal{G}$-$\mathcal{G}$) ([3], [4] and references therein). Recently, a new generalized statistical model, the Málaga-$\mathcal{M}$ distribution, was proposed in [5] to model the irradiance fluctuation of an unbounded optical wavefront propagating through a turbulent medium under all irradiance conditions. Characterized in [6] as a mixture of Generalized-$\mathcal{K}$ and discrete Binomial distributions, the Malága-$\mathcal{M}$ distribution unifies most statistical models exploited so far and is able to better reflect a wider range of turbulence conditions [5], [6]. On the RF side, previous works typically assume either Rayleigh or Nakagami-m fading [3], [4], thereby lacking the flexibility to account for disparate signal propagation mechanisms as those characterized in 5G communications which will accommodate a wide range of usage scenarios with diverse link requirements. To bridge this gap in the literature, the κ-μ shadowed fading model, recently derived in [7], is an attractive proposition. In addition to offering an excellent fit to the fading observed in a range of real-world applications (e.g., device-to-device, and body-centric fading channels [8]), the κ-μ shadowed fading encompasses several RF channel models such as Nakagami-m, Rayleigh, Rice, κ-μ and shadowed Rician fading distributions. This new channel fading model offers far better and much more flexible representations of practical fading LOS (line of sight), NLOS (non-LOS), and shadowed channels than the Rayleigh and Nakagami-m distributions. Under the assumption of AF relaying and taking into account the effect of pointing errors while considering both heterodyne and intensity modulation/direct (IM/DD) detection techniques, we derive closed-form expressions for the ergodic capacity and outage probability of dual-hop FSO/RF systems over Málaga-$\mathcal{M}$/κ-μ shadowed channels. To the best of the authors' knowledge, for mixed FSO/RF AF systems, only [9] has considered the Málaga-$\mathcal{M}$ for the FSO link, let alone the κ-μ shadowed case.

II. Channel and System Models

We consider a relay-assisted mixed FSO/RF transmission composed of both Málaga-$\mathcal{M}$ with pointing errors and κ-μ shadowed fading environments. The source communicates with the destination through an intermediate relay, able to activate both heterodyne and IM/DD detection techniques at the reception of the optical beam.

The FSO (*S*-*R*) link irradiance is assumed to follow a Málaga-$\mathcal{M}$ distribution with pointing errors impairments for which the PDF of the irradiance, I, is given by [5, eq. (5)]

$$f_I(x) = \frac{\xi^2 A}{x\Gamma(\alpha)} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \mathrm{G}_{1,3}^{3,0}\left[\frac{\alpha\beta}{g\beta+\Omega}\frac{x}{A_0}\middle| \begin{matrix} \xi^2+1 \\ \xi^2, \alpha, k \end{matrix}\right], \quad (1)$$

where ξ is the ratio between the equivalent beam radius and the pointing error displacement standard deviation (i.e., jitter) at the relay (for negligible pointing errors $\xi \to \infty$), A_0 defines the pointing loss [1], $A = \alpha^{\frac{\alpha}{2}}[g\beta/(g\beta+\Omega)]^{\beta+\frac{\alpha}{2}} g^{-1-\frac{\alpha}{2}}$ and $b_k = \binom{\beta-1}{k-1}(g\beta+\Omega)^{1-\frac{k}{2}}[(g\beta+\Omega)/\alpha\beta]^{\frac{\alpha+k}{2}}(\Omega/g)^{k-1}(\alpha/\beta)^{\frac{k}{2}}$, where α, β, g, and Ω are the fading parameters related to the atmospheric turbulence conditions [5]. Moreover in (1), $\mathrm{G}_{p,q}^{m,n}[\cdot]$ and $\Gamma(\cdot)$ stand for the Meijer-G [10, eq. (9.301)] and the incomplete gamma [10, eq. (8.310.1)] functions, respectively. It is worth highlighting that the $\mathcal{M}$ distribution unifies most of the proposed statistical models characterizing the optical irradiance in homogeneous and isotropic turbulence [5]. Hence both $\mathcal{G}$-$\mathcal{G}$ and $\mathcal{K}$ models are special cases of the Málaga-$\mathcal{M}$ distribution, as they mathematically derive from (1) by setting ($g = 0$, $\Omega = 1$) and ($g \neq 0$, $\Omega = 0$ or $\beta = 1$), respectively [5].

The RF (*R*-*D*) link, experiences the κ-μ shadowed fading with non-negative real shape parameters κ, μ and m, for which the PDF of instantaneous SNR, γ_2, is given by [7, eq. (4)]

$$f_{\gamma_2}(x) = \frac{\mu^\mu m^m (1+\kappa)^\mu}{\Gamma(\mu)\bar{\gamma}_2(\mu\kappa+m)^m}\left(\frac{x}{\bar{\gamma}_2}\right)^{\mu-1} e^{-\frac{\mu(1+\kappa)x}{\bar{\gamma}_2}} \times {}_1F_1\left(m, \mu; \frac{\mu^2\kappa(1+\kappa)}{\mu\kappa+m}\frac{x}{\bar{\gamma}_2}\right), \quad (2)$$

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where ${}_1F_1(\cdot)$ is the confluent hypergeometric function [10, eq. (9.210.1)] and $\bar{\gamma}_2 = \mathbb{E}[\gamma_2]$. This fading model jointly includes large-scale and small-scale propagation effects, by considering that only the dominant components (DSCs) are affected by Nakagami-m distributed shadowing [7]. The shadowed κ-μ distribution is an extremely versatile fading model that includes as special cases nearly all linear fading models pertaining to LOS and NLOS scenarios, such as κ-μ ($m \to \infty$), Nakagami-m ($\mu = m$ and $\kappa \to 0$), Rayleigh ($\mu = m = 1$ and $\kappa \to 0$), and Rice ($\mu = 1, \kappa = K$ and $m \to \infty$), to name a few [7, Table I].

Assuming AF relaying with channel state information (CSI), then the end-to end SNR can be expressed as [4, eq. (7)]

$$\gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}, \tag{3}$$

where $\gamma_1 = (A_0 h(g + \Omega))^{-r} \mu_r I^r$ is the instantaneous SNR of the FSO link (S–R) with r being the parameter that describes the detection technique at the relay (i.e., $r = 1$ is associated with heterodyne detection and $r = 2$ is associated with IM/DD) and, μ_r refers to the electrical SNR of the FSO hop [5] and $h = \xi^2/(\xi^2 + 1)$. In particular, for $r = 1$, $\mu_1 = \mu_{\text{heterodyne}} = \mathbb{E}[\gamma_1] = \bar{\gamma}_1$ and for $r = 2$, $\mu_2 = \mu_{\text{IM/DD}} = \mu_1 \alpha \xi^2 (\xi^2 + 1)^{-2} (\xi^2 + 2)(g + \Omega)/((\alpha + 1)[2g(g + 2\Omega) + \Omega^2(1 + 1/\beta)])$ [5, eq. (8)].

III. Exact Performance Analysis

In this section, a new mathematical framework investigating the average capacity and the outage probability of the mixed FSO/RF transmission composed of both Málaga-$\mathcal{M}$ with pointing errors and shadowed κ-μ fading environments and accounting for both detection techniques is presented. To the best of the author's knowledge, there are few works that consider these metrics of mixed FSO/RF systems, yet mostly considering the mixed $\mathcal{G}$-$\mathcal{G}$/Nakagami-m fading ([3], [4] and references therein). This letter completes and extends the efforts of [3] and [4] by unifying the ergodic capacity and the outage probability analysis for any turbulence/fading model under both types of detection techniques.

A. Ergodic Capacity

Hereafter, we provide capacity formulas for the considered system by using the complementary moment generation function CMGF-based approach [11] as

$$C \stackrel{\Delta}{=} \frac{\mathbb{E}[\ln(1+\gamma)]}{2\ln(2)} = \frac{1}{2\ln(2)} \int_0^\infty s e^{-s} M_{\gamma_1}^{(c)}(s) M_{\gamma_2}^{(c)}(s) ds, \tag{4}$$

where $M_X^{(c)}(s) = \int_0^\infty e^{-sx} F_X^{(c)}(x) dx$ stands for the CMGF with $F_X^{(c)}(x)$ denoting the complementary cumulative distribution function (CCDF) of X.

The ergodic capacity of mixed Málaga-$\mathcal{M}$/κ-μ shadowed fading FSO transmission system under heterodyne and IM/DD detection techniques with pointing errors taken into account is given for

- *Integer m, μ, with $m \geq \mu$* as

$$C = \frac{\xi^2 A r \mu_r B^{-r}}{2\ln(2)\Gamma(\alpha)} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \sum_{l=1}^{m} \frac{\chi_l}{\Gamma(m)} T(\theta_2, l, m), \tag{5}$$

where $B = \alpha\beta h(g + \Omega)/[(g\beta + \Omega)]$, and,

$$\chi_l = \begin{cases} \binom{m}{l}\theta_2^l - \binom{m-\mu}{l}\theta_1^l & \text{for } 1 \leq l \leq m - \mu, \\ \binom{m}{l}\theta_2^l & \text{for } l > m - \mu, \end{cases}$$

with $\theta_1 = \frac{\bar{\gamma}_2}{\mu(1+\kappa)}$, and, $\theta_2 = \frac{\bar{\gamma}_2(\mu\kappa+m)}{\mu m(1+\kappa)}$. Moreover in (5),

$$T(x, y, z) = \mathrm{H}_{1,0:4,3:1,1}^{0,1:1,4:1,1}\left[\begin{matrix} (-y,1,1) \\ - \end{matrix} \middle| \begin{matrix} (\sigma, \Sigma) \\ (\phi, \Phi) \end{matrix} \middle| \begin{matrix} (1-z,1) \\ (0,1) \end{matrix} \middle| \frac{\mu_r}{B^r}; x \right], \tag{6}$$

where $\mathrm{H}_{p_1,q_1:p_2,q_2:p_3,q_3}^{m_1,n_1:m_2,n_2:m_3,n_3}[\cdot]$ denotes the Fox-H function (FHF) of two variables [15, eq. (1.1)] also known as the bivariate FHF whose Mathematica implementation may be found in [12, Table I], whereby $(\sigma, \Sigma) = (1-r, r), (1-\xi^2-r, r), (1-\alpha-r, r), (1-k-r, r)$ and $(\phi, \Phi) = (0, 1), (-\xi^2 - r, r), (-r, r)$. Moreover, it becomes for

- *Integer m, μ, with $m < \mu$* as

$$C = \frac{\xi^2 A r \mu_r}{2\ln(2)\Gamma(\alpha)B^r} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \sum_{\substack{p=0 \\ (p,q)\neq(0,0)}}^{m} \sum_{q=0}^{\mu-m} \binom{m}{p}\binom{\mu-m}{q}$$
$$\theta_2^p \theta_1^q \left(\sum_{i=1}^{\mu-m} \frac{\Delta_{1i}}{\Gamma(\mu-m-i+1)} T(\theta_1, q+p, \mu-m-i+1) - \sum_{i=1}^{m} \frac{\Delta_{2i}}{\Gamma(m-i+1)} T(\theta_2, q+p, m-i+1) \right), \tag{7}$$

where $\Delta_{1i} = (-1)^m \binom{m+i-2}{i-1} \left(\frac{m}{\mu\kappa+m}\right)^m \left(\frac{\mu\kappa}{\mu\kappa+m}\right)^{-m-i+1}$ and $\Delta_{2i} = (-1)^{i-1} \binom{\mu-m+i-2}{i-1} \left(\frac{m}{\mu\kappa+m}\right)^{i-1} \left(\frac{\mu\kappa}{\mu\kappa+m}\right)^{m-\mu-i+1}$.

Proof: Capitalizing on (4) and recalling the fact that the FSO link's CMGF $M_{\gamma_1}^{(c)}(s) = \mathcal{L}(F_{\gamma_1}^{(c)}(x))$ where $\mathcal{L}$ denotes the Laplace transform operator and the FSO link's CCDF is obtained as

$$F_{\gamma_1}^{(c)}(x) = F_I^{(c)}\left(A_0 h(g+\Omega) \left(\frac{x}{\mu_r}\right)^{\frac{1}{r}} \right)$$
$$\stackrel{(a)}{=} \frac{\xi^2 A}{\Gamma(\alpha)} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \mathrm{G}_{2,4}^{4,0}\left[B\left(\frac{x}{\mu_r}\right)^{\frac{1}{r}} \middle| \begin{matrix} \xi^2+1, 1 \\ 0, \xi^2, \alpha, k \end{matrix} \right]. \tag{8}$$

where (a) follows from integrating (1) using [10]. Then, expressing the Meijer-G function in (8) in terms of Fox-H function by means of [13, eq. (1.111)] and resorting to [13, eq. (2.19)] with some additional manipulations using [13, eqs. (1.58), (1.59), and (1.60)] yield

$$M_{\gamma_1}^{(c)}(s) = \frac{\xi^2 A r \mu_r}{\Gamma(\alpha)B^r} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \mathrm{H}_{4,3}^{1,4}\left[\frac{\mu_r}{B^r} s \middle| \begin{matrix} (\sigma, \Sigma) \\ (\phi, \Phi) \end{matrix} \right], \tag{9}$$

where $\mathrm{H}_{p,q}^{m,n}[\cdot]$ is the Fox-H function [13, eq. (1.2)].

On the RF side, the CMGF of γ_2 under shadowed κ-μ fading is given by

$$M_{\gamma_2}^{(c)}(s) = \frac{1 - M_{\gamma_2}(s)}{s} \stackrel{(a)}{=} \frac{1 - \frac{(\theta_1 s+1)^{m-\mu}}{(\theta_2 s+1)^m}}{s}, \tag{10}$$

where (a) follows from the recent result in [7, eq. (5)]. By assuming integer-valued m and μ, the RF link's CMGF can be rewritten after resorting to the transformation $\Gamma(\alpha)(1+z)^{-\alpha} = \mathrm{H}_{1,1}^{1,1}[z|\begin{smallmatrix}(1-\alpha,1)\\(0,1)\end{smallmatrix}]$ in [13, eq. (1.43)] as

$$M_{\gamma_2}^{(c)}(s) \underset{\mu \leq m}{\stackrel{(a)}{=}} \sum_{l=1}^{m} \frac{\chi_l s^{l-1}}{\Gamma(m)} \mathrm{H}_{1,1}^{1,1}\left[\theta_2 s \middle| \begin{matrix} (1-m, 1) \\ (0, 1) \end{matrix} \right], \tag{11}$$

and

$$M_{\gamma_2}^{(c)}(s) \underset{\mu > m}{\stackrel{(b)}{=}} \sum_{\substack{p=0 \\ (p,q)\neq(0,0)}}^{m} \sum_{q=0}^{\mu-m} \binom{m}{p}\binom{\mu-m}{q} \theta_2^p \theta_1^q s^{p+q-1}$$

$$\left(\sum_{i=1}^{\mu-m} \frac{\Delta_{1i}\mathrm{H}_{1,1}^{1,1}\left[\theta_1 s \middle| \begin{smallmatrix}(m+i-\mu,1)\\(0,1)\end{smallmatrix}\right]}{\Gamma(\mu-m-i+1)} - \sum_{i=1}^{m} \frac{\Delta_{2i}\mathrm{H}_{1,1}^{1,1}\left[\theta_2 s \middle| \begin{smallmatrix}(i-m,1)\\(0,1)\end{smallmatrix}\right]}{\Gamma(m-i+1)}\right), \tag{12}$$

where (a) and (b) follow after applying the binomial expansion and the partial fraction decomposition [14, eq. (27)], respectively. Plugging (9), (11) and (12) into (4) and resorting to [15, eq. (2.2)] complete the proof. ■

B. Outage Probability

The quality of service (QoS) of the considered mixed FSO/RF system is ensured by keeping the instantaneous end-to-end SNR, γ, above a threshold γ_{th}. The probability of outage in the mixed FSO/RF relaying setup is expressed as

$$P_{\mathrm{out}} = \Pr[\gamma < \gamma_{th}] = \Pr\left[\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1} < \gamma_{th}\right]. \tag{13}$$

Marginalization over γ_1, and letting $u=1+\gamma/\gamma_{th}$ in (13) yield

$$P_{\mathrm{out}}(\gamma_{th}) = 1-\gamma_{th}\int_1^{\infty} F_{\gamma_2}^{(c)}\left(\gamma_{th}+\frac{1+\gamma_{th}}{u-1}\right) f_{\gamma_1}(u\gamma_{th})du, \tag{14}$$

where $F_{\gamma_2}^{(c)}$ is the CCDF of γ_2 and f_{γ_1} is the PDF of the first-link SNR obtained from deriving (8) with respect to x as

$$f_{\gamma_1}(x) = \frac{\xi^2 AB^r}{\Gamma(\alpha)\mu_r}\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)}\mathrm{H}_{1,3}^{3,0}\left[\frac{B^r x}{\mu_r}\middle| \begin{matrix}(\xi^2+1-r,\,r)\\(\xi^2-r,r),\,(\alpha-r,r),(k-r,r)\end{matrix}\right]. \tag{15}$$

Plugging (15) and the RF link's CCDF expression recently derived in [14, eq. (10)] for integer m, μ with $m \geq \mu$ into the above integral and making a Taylor expansion of exponential and power terms, we infer that

$$P_{\mathrm{out}}(\gamma_{th}) = 1 - \frac{\xi^2 AB^r\gamma_{th}e^{-\frac{\gamma_{th}}{\theta_2}}}{\Gamma(\alpha)\mu_r}\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)} \times \sum_{i=0}^{m-\mu}\sum_{j=0}^{m-i-1}\sum_{p=0}^{j}\frac{\binom{j}{p}\Upsilon_i}{j!\theta_2^j}\gamma_{th}^{j-p}(\gamma_{th}+1)^p \times I, \tag{16}$$

with $\Upsilon_i = \binom{m-\mu}{i}\left(\frac{m}{\mu\kappa+m}\right)^i\left(\frac{\mu\kappa}{\mu\kappa+m}\right)^{m-\mu-i}$, and I given by

$$I = \sum_{q=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^q(p+q)_l}{q!l!\theta_2^q}(\gamma_{th}+1)^q\int_1^{\infty}u^{-p-q-l} \mathrm{H}_{1,3}^{3,0}\left[\frac{B^r\gamma_{th}}{\mu_r}u\middle| \begin{matrix}(\xi^2+1-r,\,r)\\(\xi^2-r,\,r),\,(\alpha-r,\,r),\,(k-r,\,r)\end{matrix}\right]du. \tag{17}$$

Substituting (3) into (16) after resorting to [13, eq. (2.54)] yields the outage probability of mixed FSO/RF in Málaga-$\mathcal{M}/\kappa$-μ shadowed fading ($\mu \leq m$) environments with pointing errors under both detection techniques as

$$P_{\mathrm{out}}(\gamma_{th}) = 1 - \frac{\xi^2 AB^r\gamma_{th}}{e^{\frac{\gamma_{th}}{\theta_2}}\Gamma(\alpha)\mu_r}\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)}\sum_{i=0}^{m-\mu}\sum_{j=0}^{m-i-1}\frac{\Upsilon_i}{\theta_2^j}\,\Xi(\theta_2), \tag{18}$$

where

$$\Xi(x) = \sum_{p=0}^{j}\sum_{q=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^q(p+q)_l\binom{j}{p}\gamma_{th}^{j-p}}{j!q!l!x^q(\gamma_{th}+1)^{-p-q}}\mathrm{H}_{2,4}^{4,0}\left[\frac{B^r\gamma_{th}}{\mu_r}\middle| \begin{matrix}(\sigma_1,\,\Sigma_1)\\(\phi_1,\,\Phi_1)\end{matrix}\right], \tag{19}$$

with $(a)_n$ standing for the Pochhammer symbol [10], $(\sigma_1, \Sigma_1) = (\xi^2+1-r, r), (l+p+q, 1)$, and $(\phi_1, \Phi_1) = (l+p+q-1, 1), (\xi^2-r, r), (\alpha-r, r), (k-r, r)$.

Similar to (18) and using [14, eq. (9)], the outage probability of mixed FSO Málaga-$\mathcal{M}$/RF shadowed κ-μ $(m < \mu)$ is

$$P_{\mathrm{out}}(\gamma_{th}) = 1 - \frac{\xi^2 AB^r\gamma_{th}}{\Gamma(\alpha)\mu_r}\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)} \left(e^{-\frac{\gamma_{th}}{\theta_1}}\sum_{i=1}^{\mu-m}\sum_{j=0}^{\mu-m-i}\frac{\Delta_{1i}}{\theta_1^j}\,\Xi(\theta_1) + e^{-\frac{\gamma_{th}}{\theta_2}}\sum_{i=1}^{m}\sum_{j=0}^{m-i}\frac{\Delta_{2i}}{\theta_2^j}\,\Xi(\theta_2)\right). \tag{20}$$

IV. Asymptotic Analysis

To gain more insights into the effect of turbulence/fading parameters on both the ergodic capacity and the outage probability, we study hereafter their asymptotic behaviors. To this end, we invoke the asymptotic expansions of the Fox-H function [16, Ths. 1.7 and 1.11] and the Mellin-Barnes integrals involving the bivariate Fox-H function [13, eq. (2.56)].

A. Asymptotic Ergodic Capacity

We assume that the average SNR of the RF link $\bar{\gamma}_2$ goes to infinity for a fixed and finite valued average SNR in the FSO link. Then, resorting to the Mellin-Barnes representation of the bivariate FHF [13, eq. (2.56)] in (5), and evaluating the residue at the poles $\{-m, -1-l\}$ yield the asymptotic capacity, when $m \geq \mu$ as

$$C^{\infty} = \frac{\xi^2 Ar\mu_r}{2\ln(2)\Gamma(\alpha)B^r}\sum_{k=1}^{\beta}\sum_{l=1}^{m}\frac{b_k\chi_l}{\Gamma(k)}\left(\frac{\mathrm{H}_{5,4}^{2,5}\left[\frac{\mu_r}{B^r\theta_2}\middle| \begin{smallmatrix}(-l,1),(\sigma,\Sigma)\\(m-1-l,1),(\phi,\Phi)\end{smallmatrix}\right]}{\theta_2^{1+l}\Gamma(m)} + \theta_2^{-m}\mathrm{H}_{5,3}^{1,5}\left[\frac{\mu_r}{B^r}\middle| \begin{matrix}(m-l,\,1),\,(\sigma,\,\Sigma)\\(\phi,\,\Phi)\end{matrix}\right]\right). \tag{21}$$

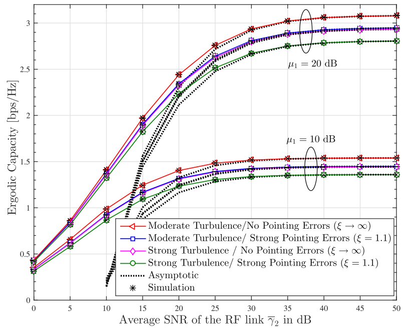
It is worth noting that (21) is much easier and faster to calculate than the exact capacity in (5). Moreover, C^{∞} when $m < \mu$ follows in the same line of (21) while considering (7).

B. Asymptotic Outage Probability

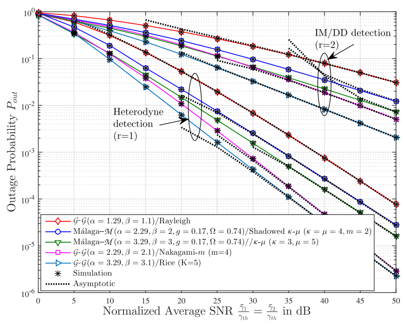
At high SNR values, the outage probability of the mixed FSO/RF relaying system can be expressed as $P_{\mathrm{out}} \simeq (G_c\mathrm{SNR})^{-G_d}$, where G_c and G_d denote the coding gain and the diversity order of the system, respectively. Hence, as $\mu_r \to \infty$ while keeping the low-order terms in (18), i.e., $q+l < 1$, and then applying [16, eq. (1.8.5)] yield the asymptotic CDF when $m \geq \mu$ as

$$P_{\mathrm{out}}^{\infty} = 1 - \frac{\xi^2 Ae^{-\frac{\gamma_{th}}{\theta_2}}}{\Gamma(\alpha)}\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)}\sum_{i=0}^{m-\mu}\sum_{j=0}^{m-i-1}\sum_{p=0}^{j}\frac{\binom{j}{p}\Upsilon_i\gamma_{th}^{j-p}\theta_2^{-j}}{j!(\gamma_{th}+1)^{-p}} \sum_{t=1}^{4}\frac{1}{\Phi_{1t}}\frac{\prod_{s=1, s\neq t}^{4}\Gamma\left(\phi_{1s}-\phi_{1t}\frac{\Phi_{1s}}{\Phi_{1t}}\right)}{\prod_{s=1}^{2}\Gamma\left(\sigma_{1s}-\phi_{1t}\frac{\Sigma_{1s}}{\Phi_{1t}}\right)}\left(\frac{B^r\gamma_{th}}{\mu_r}\right)^{\frac{\phi_{1t}}{\Phi_{1t}}+1}. \tag{22}$$

Compared to (18) which is expressed in terms of Fox-H function, (22) includes only finite summations of elementary functions. The diversity gain of the studied system over atmospheric turbulence conditions is inferred after applying $e^{-\frac{\gamma_{th}}{\theta_2}} \underset{\bar{\gamma}_2\gg 1}{\approx} 1-\frac{\gamma_{th}}{\theta_2}$ to (22) as $G_d = \min\{\mu, \frac{\xi^2}{r}, \frac{\alpha}{r}, \frac{\beta}{r}\}$.



(a)



(b)

Fig. 1. Performance of relay-assisted mixed Málaga/κ-μ shadowed fading.

For $\mu = m$, $\kappa \to 0$, $g = 0$ and $\Omega = 1$, the CDF in (22) reduces to P_{out}^{∞} for $\mathcal{G}$-$\mathcal{G}$/Nakagami-m fading channels as

$$P_{\text{out}}^{\infty} = 1 - \frac{\xi^2 e^{-\frac{m\gamma_{th}}{\bar{\gamma}_2}}}{\Gamma(\alpha)\Gamma(\beta)} \sum_{j=0}^{m-1} \sum_{p=0}^{j} \sum_{t=1}^{4} \frac{\binom{j}{p} m^j \gamma_{th}^{-p} (\alpha\beta h)^{r\left(\frac{\phi_{1t}}{\Phi_{1t}}+1\right)}}{j!\,\Phi_{1t}(\gamma_{th}+1)^{-p}} \frac{\prod_{s=1, s\neq t}^{4} \Gamma\left(\phi_{1s} - \phi_{1t}\frac{\Phi_{1s}}{\Phi_{1t}}\right)}{\prod_{s=1}^{2} \Gamma\left(\sigma_{1s} - \phi_{1t}\frac{\Sigma_{1s}}{\Phi_{1t}}\right)} \left(\frac{\gamma_{th}}{\mu_r}\right)^{\frac{\phi_{1t}}{\Phi_{1t}}+1} \left(\frac{\gamma_{th}}{\bar{\gamma}_2}\right)^{j}, \tag{23}$$

thereby inferring [4, eq. (29)], i.e., $G_d = \min\{m, \frac{\xi^2}{r}, \frac{\alpha}{r}, \frac{\beta}{r}\}$. Similar to (22) while considering (20), the asymptotic outage probability can be derived in closed form when $m < \mu$. However, the derived expression is omitted due to space limitations.

V. Numerical Results

Fig. 1(a) investigates the impacts of the turbulence-induced fading and pointing errors on the system performance when the RF link is subject to Rician shadowed fading distribution ($\kappa = 5, \mu = 1, m = 2$). As expected, the ergodic capacity deteriorates by decreasing the pointing error displacement standard deviation, i.e., for smaller ξ, or decreasing the turbulence fading parameter, i.e., smaller α and β, where we associate the strong turbulence to $(\alpha, \beta) = (2.29, 2)$ and the moderate turbulence to $(\alpha, \beta) = (4.2, 3)$. At high SNR, the asymptotic expansion in (21) matches very well its exact counterpart, which confirms the validity of our mathematical analysis for different parameter settings.

Fig. 1(b) depicts the outage probability of mixed FSO/RF relay systems in Málaga-$\mathcal{M}$ and shadowed κ-μ fading channels for both heterodyne and IM/DD detection at the relay. Throughout our numerical experiments, we found out that regardless of the average SNRs and turbulence/fading settings, accurate analytical curves can be obtained by truncating the infinite sums at $q = 10$ and $l = 5$ terms. In the legend, please note that we have identified some particular turbulence and fading distribution cases that simply stem from the general Málaga and κ-μ shadowed fading scenarios, respectively. The exact match with Monte-Carlo simulation results confirms the precision of the theoretical analysis of Section III-B. Moreover, we notice that the exact and asymptotic expansion in (22) agree very well at high SNRs.

VI. Conclusion

We have presented a unified analytical framework for relay-assisted mixed FSO/RF systems that remarkably accommodates generic turbulence/fading models including Málaga-$\mathcal{M}$ with pointing errors and shadowed κ-μ distribution that account for shadowed LOS and NLOS scenarios. The results demonstrate the unification of various FSO turbulent/RF fading scenarios into a single closed-form expression for the ergodic capacity and the outage probability while accounting for both IM/DD and heterodyne detection techniques at the relay.

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