On the Ergodic Capacity of Amplify-and-Forward Relay Channels with Interference in Nakagami-m Fading

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***Abstract*—Integrating relaying techniques into cellular communications sheds new light on higher capacity and broader coverage. However, applying relaying techniques in practice has to take into account important issues such as co-channel interference (CCI). In this work, a generalized framework for the ergodic capacity analysis of dual-hop fixed-gain amplify and forward (AF) relaying systems in the presence of interference is presented. New expressions for the ergodic capacity are derived considering transmissions over independent but not necessarily identically distributed Nakagami-m fading channels in the presence of a finite number of co-channel interferers. Our results establish that the ergodic capacity is dominated by the source-relay interference power and that it improves slowly with the average signal-to-noise ratio (SNR) increasing. It slightly deteriorates, however, with a larger Nakagami-m fading parameter for interference channels. Furthermore, our results offer an analytical insight into the key impact of relay placement on performance. Our new ergodic capacity expressions could therefore provide a very practical/low-cost performance optimization tool for relayed-communication system designers.**

***Index Terms*—Amplify and forward, ergodic capacity, fixed-gain relaying, Nakagami-m fading, co-channel interference.**

I. Introduction

RECENT proposals for new wireless standards such as IEEE 802.16j and 3GPP-LTE have adopted two-hop relay transmission for cellular communications [1]. The key advantage of relaying is to enable high capacity where traditional architectures are unsatisfactory due to location constraints (cell-edge, shadowing, indoor), leading to a more homogenous user experience. Depending on the nature and complexity of the relaying technique, relay nodes can be broadly categorized as either amplify and forward (AF) or decode and forward (DF) [2]. While AF relays act as repeaters, DF relays decode and recode the received signal prior to forwarding it to the receiver, thereby implying a larger delay than with a simple repeater.

In the open literature, several works investigating relaying communications exist (cf. [2]-[15] and references therein). Despite their importance, many of the existing results have been based on the assumption that the system is thermal-noise limited. However, relaying-access capacity is also affected by strong co-channel interference due to the aggressive frequency reuse in cellular networks. Cochannel interference, which is an essential feature of wireless networks, can cause more severe performance degradation than thermal noise in many wireless networks. In a variable-gain relay scenario, existing contributions range from the analysis of interference-limited relay [3], [5] or destination [4], to multiple interferers at both sides [6]. Apart from the very recent work of [5] and [6], existing works that have studied the effects of interference on the performance of variable-gain AF relaying have mainly considered Rayleigh fading channels. By deriving expressions for the outage probability and the average bit error rate (BER), the performance of variable-gain AF relaying over Nakagami-m channels with interference has been investigated in [5].

Other works have considered fixed-gain relaying [7]-[10]. In [7], the outage probability of a fixed-gain AF relay system with interference-limited destination has been derived assuming Rayleigh fading channels. The analysis has been later extended in [8] and [9] to the case of multiple interferers at both relay and destination. Recently, the outage probability of fixed-gain AF relaying systems with a single interferer were analyzed by Surawera et al. [10], where both the desired signal have an integer Nakagami-m distribution. Although being a long-standing open problem, the capacity of multihop relaying systems has gained less attention over the last years. Only few contributions have been, so far, proposed, notably [12] and [13]. In [12], Y. Chen et al. provided an upper bound on the transmission capacity defined as the number of successful transmissions that can occur per unit area. Previously, the authors of [13] proposed some throughput scaling laws of multihop systems over generalized fading.

From the aforementioned up-to-date technical literature, there appear to be no analytical ergodic capacity results which apply for fixed-gain dual-hop systems with arbitrary channel configurations.

This paper goes toward filling this gap by deriving new exact analytical expressions for the ergodic capacity of fixed-gain AF single-relay systems in interference-impaired channels. In contrast to previous results, our expressions apply for any finite number of interferers at the relay and the destination and for arbitrary Nakagami-m fading on the desired and interfering links.

Based on our analytical expressions, we investigate the effect of different system and channel parameters on the ergodic

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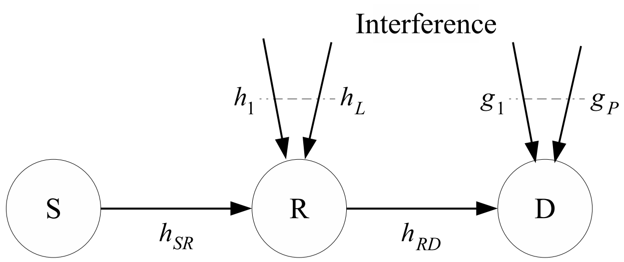

Fig. 1. Single-relay transmission system with cochannel interference.

capacity. For example, we show that the ergodic capacity is dominated by the source-relay interference constraint and that it improves slowly with the average SNR increasing. Furthermore, a larger Nakagami-m fading parameter for interference channels slightly deteriorates the system's performance. Moreover, in a distance relay-dependent link layout, our results show that the new ergodic capacity expressions provide a very practical tool for the optimization of relay location to significantly improve the system's capacity.

The remainder of this paper is organized as follows: In Section II, the basic definitions and background related to the dual-hop fixed-gain relaying suffering interference are provided. Section III presents our new integral relations of the ergodic capacity evaluation. These integrals are subsequently used to derive more compact forms for the ergodic capacity under integer Nakagami-m fading in Sections IV. Section V assesses the performance analysis by numerical examples, and Section IV concludes the paper. All of the main mathematical proofs have been placed in the Appendices.

II. Interference-Limited Relaying: System Model

Fig. 1 illustrates a single-relay system. The fading gains from the source-to-relay and relay-to-destination are denoted by h_{SR} and h_{RD}, respectively. These channel gains are assumed to be independently and identically distributed (i.i.d.) Nakagami-m fading. In the first time slot, the relay node receives a faded noisy signal from the source and a finite number of faded cochannel interfering signals from L external interferers. Thus, the signal received at the relay node is given by

$$y_{SR} = \sqrt{P_s} h_{SR} s_0 + \sum_{l=1}^{L} \sqrt{\alpha_l} h_l s_l + n_r, \tag{1}$$

where s_0 denotes the unit-energy signal transmitted from the source; and P_s indicates the transmit energy from the said node. In the second term of the right-hand-side (RHS) of (1), s_l is the l-th cochannel interferer's signal affecting the relay with energy equal to α_l, L is the total number of interferers that affect the relay, and h_l is the flat Nakagami-m fading coefficient of the l-th interference channel. Finally, the third term of the RHS of (1), i.e., n_r, represents the additive white Gaussian noise (AWGN) term at the relay, with zero mean and variance σ_R^2. In the second time slot, the relay forwards a scaled version of the received signal y_{SR} to the destination

$$y_{RD} = b_F h_{RD} y_{SR} + \sum_{p=1}^{P} \sqrt{\beta_p} g_p c_p + n_d, \tag{2}$$

where b_F is the amplification coefficient aiming to guarantee that the average transmitted power does not exceed the power budget available at the relay node. As such, let P_r be the transmission power available at the relay, then the amplification coefficient is chosen as

$$b_F = \sqrt{\mathrm{E}\left[Pr/(Ps|h_{SR}|^2 + \sum_{l=1}^{L} \alpha_l |h_l|^2 + \sigma_R^2)\right]}. \tag{3}$$

In the R.H.S of (2), c_p is the p-th cochannel interferer's signal affecting the destination with energy equal to β_p, P is the total number of interferers that affect the destination, and g_p is the flat Nakagami-m fading coefficient of the p-th interference channel. Finally, n_d represents the AWGN term at the destination, with zero mean and variance σ_D^2. By assuming mutual independency between the different links, the end-to-end signal-to-interference-plus-noise ratio (SINR) at the destination can be obtained as

$$\gamma = \frac{b_F^2 P_s |h_{SR}|^2 |h_{RD}|^2}{b_F^2 |h_{RD}|^2 \sigma_R^2 + b_F^2 |h_{RD}|^2 \sum_{l=1}^{L} \alpha_l |h_l|^2 + \sum_{p=1}^{P} \beta_p |g_p|^2 + \sigma_D^2}. \tag{4}$$

After some manipulations, (4) can be further simplified to

$$\begin{aligned} \gamma &= \frac{\gamma_1 \gamma_2}{\gamma_2 \left(1 + \sum_{l=1}^{L} Y_l\right) + \frac{P_s}{\sigma_R^2 b_F^2} \left(1 + \sum_{p=1}^{P} Z_p\right)} \\ &= \frac{\gamma_1 \gamma_2}{\gamma_2 (1 + \lambda) + G_F (1 + \chi)}, \end{aligned} \tag{5}$$

where $\gamma_1 = P_s |h_{SR}|^2 / \sigma_R^2$ and $\gamma_2 = P_r |h_{RD}|^2 / \sigma_D^2$ indicate the instantaneous SNR of the source-to-relay and the relay-to-destination links, respectively. Likewise, $\lambda = \sum_{l=1}^{L} Y_l$ and $\chi = \sum_{p=1}^{P} Z_p$ denote the total interference-to-noise ratios (INRs) at the relay and the destination, respectively, whereby $Y_l = \alpha_l |h_l|^2 / \sigma_R^2$ and $Z_p = \beta_p |g_p|^2 / \sigma_D^2$. Finally, $G_F = P_r / \sigma_R^2 b_F^2$.

III. Ergodic Capacity Analysis: Integral Form

Capacity analysis is of extreme importance in the design of wireless systems since it determines the maximum achievable rates in the network. A reliable capacity performance study has to take into account important issues such as co-channel interference. This motivates us to introduce the following theorem:

Theorem 1: The ergodic capacity[1] (bit/s/Hz) of dual-hop fixed gain relaying systems suffering interference is given by

$$\begin{aligned} C_E &= \frac{1}{2\ln(2)} \left\{ \int_0^\infty E_0(\xi) M_\lambda(\xi) M_{\frac{1+\chi}{\gamma_2}}(G_F \xi)\, d\xi - \right. \\ &\qquad \left. \int_0^\infty E_0(\xi) M_{\gamma_1}(\xi) M_\lambda(\xi) M_{\frac{1+\chi}{\gamma_2}}(G_F \xi)\, d\xi \right\}, \end{aligned} \tag{6}$$

[1] In this paper, the ergodic capacity is a measure that corresponds to the long-term average achievable rate over all states of the time-varying fading channel [17].

where $M_z(\cdot)$ stands for the moment generating function (MGF) of z and $E_\nu(\cdot)$ denotes the exponential integral function of order ν [16, Eq. (8.485)].

Proof: : See Appendix A.

The result in (6) offers a flexible and simple MGF-based approach for the computation of the ergodic capacity of fixed-gain relaying systems suffering interference. (6) relies on the knowledge of the MGFs of the first-hop SNR and INR γ_1 and λ, as well as the second-hop inverse SINR $(1+\chi)/\gamma_2$. To the best of our knowledge, closed-form and exact expressions for these MGFs exist for most fading models.

Most importantly, with respect to the MGF-based approach for the ergodic capacity computation in [11], the final result in (6) is much more easier to compute given that: i) [11, Eq. (7)] employs the end-to-end MGF expression which turns out to be untractable for fixed-gain relaying systems suffering interference, and ii) [11, Eq. (7)] requires the first and the second derivatives of the MGF, a fact that can highly increase the computational burden of the ergodic capacity calculation, notably when the MGF is expressed as a product of more than two functions. Therefore, though considered as a prominent contribution to the ergodic capacity analysis, the unified approach proposed in [11] cannot be applied here.

Hereafter, we will restrict the scope of (6) to the yet challenging scenario of non-identically distributed Nakagami-m fading channels. In this context, new expressions for the ergodic capacity are presented.

Corollary 1: The ergodic capacity of an interference-limited multi-hop AF relaying system in arbitrary Nakagami-m fading is given by

$$C_E=\sum_{p=1}^{P}\sum_{k=1}^{m_{Z_p}}\sum_{n=0}^{k-1}\frac{G_F^{m_2}A_{p,k,n}}{2\ln(2)}\int_0^\infty \xi^{m_2}E_0(\xi)\prod_{l=1}^{L}\left(1+\frac{\bar{Y}_l}{m_{Y_l}}\xi\right)^{-m_{Y_l}} \left(1-\left(1+\frac{\bar{\gamma}_1}{m_1}\xi\right)^{-m_1}\right)\Psi\left(m_2+n+1;m_2+1;\frac{G_F m_2\bar{Z}_p}{m_{Z_p}\bar{\gamma}_2}\xi\right)d\xi. \tag{7}$$

where $\Psi(a;b;z)$ is the Triconomi confluent hypergeometric function [16, Eq. (9.211.4)].

Proof: Under Nakagami-m fading, the MGFs of the SNR γ_1 (or γ_2) and individual INR Y_l (or Z_p) are given by

$$\begin{aligned} M_{\gamma_1}(s) &= \left(1+\tfrac{\bar{\gamma}_1}{m_1}s\right)^{-m_1}, \\ M_{Y_l}(s) &= \left(1+\tfrac{\bar{Y}_l}{m_{Y_l}}s\right)^{-m_{Y_l}}, \end{aligned} \tag{8}$$

where m_1 (or m_2) and $\bar{\gamma}_1$ (or $\bar{\gamma}_2$) are, respectively, the Nakagami fading parameter and the average power of the desired signal. Similarly, m_{Y_l} (or m_{Z_p}) and $\bar{Y}_l$ (or $\bar{Z}_p$) are the Nakagami fading parameter and the average power of the l-th interfering signal to the relay (or the p-th interfering signal to the destination), respectively. On the other hand, the MGF of the combined INR λ (or χ), which is the sum of independent Gamma-distributed random variables Y_l (or Z_p), is equal to the product of the individual MGFs, yielding

$$M_\lambda(s)=\prod_{l=1}^{L}M_{Y_l}(s)=\prod_{l=1}^{L}\left(1+\frac{\bar{Y}_l}{m_{Y_l}}s\right)^{-m_{Y_l}}. \tag{9}$$

When the Nakagami shape factor is assumed to be an integer value, the product of the MGFs of P Gamma-distributed random variables can be further derived using [18] as

$$\prod_{p=1}^{P}(1+\alpha_p z)^{-m_{Z_p}}=\sum_{p=1}^{P}\sum_{k=1}^{m_{Z_p}}\frac{\beta_{p,k}}{(\alpha_p z+1)^k}, \tag{10}$$

where

$$\beta_{p,k}=\alpha_p^{k-m_{Z_p}}\sum_{\tau(p,k)}\sum_{i=1,i\neq p}^{P}\binom{m_{Z_i}+q_i-1}{q_i}\frac{(-\alpha_i)^{q_i}}{\left(1-\frac{\alpha_i}{\alpha_l}\right)^{m_{Z_i}+q_i}}, \tag{11}$$

with $\alpha_i=\bar{Z}_i/m_{Z_i}$ and $\tau(p,k)$ denotes a set of P-tuples such that $\tau(i,j)=\{(q_1,...,q_P):q_i=0,\sum_{k=1}^{P}q_k=(m_{Z_i}-j)\}$ where q_is are non-negative integers.

To solve (6), the MGF of the ratio $\frac{1+\chi}{\gamma_2}$ is required. For convenience, let $U=\frac{1+\chi}{\gamma_2}$, then the MGF of U is shown to be given by

$$M_U(s)=\sum_{p=1}^{P}\sum_{k=1}^{m_{Z_p}}\sum_{n=0}^{k-1}A_{p,k,n}s^{m_2}\Psi\left(m_2+n+1;m_2+1;\frac{m_2\bar{Z}_p}{m_{Z_p}\bar{\gamma}_2}s\right), \tag{12}$$

where

$$A_{p,k,n}=\frac{(-1)^{n+1}\exp\left(\frac{m_{Z_p}}{\bar{Z}_p}\right)\left(\frac{m_2\bar{Z}_p}{m_{Z_p}\bar{\gamma}_2}\right)^{m_2}\beta_{p,k}\Gamma(m_2+n+1)}{\left(\frac{m_{Z_p}}{\bar{Z}_p}\right)^{n+1}\Gamma(m_2)\Gamma(k-n)}, \tag{13}$$

Proof: See appendix B.

Hence, substituting (8) and (12) into (6) yields the end-to-end ergodic capacity expression under Nakagami-m fading as given in (7). The latter holds for arbitrary Nakagami-m factors m_1, m_2 and $m_{Y_l}, l=1,...,L$. Nevertheless, $m_{Z_p}, p=1,...,P$, are constrained to have non-negative integer values. To overcome this shortcoming, we consider the special case of Nakagami-m fading channel with P independent identically distributed (i.i.d.) interfering signals at the destination where AWGN is neglected and, hence, the second-hop SIR is given by $U=\chi/\gamma_2$. It is noteworthy that this case is often encountered in real-world applications especially when the destination is located at the cell-edge where the received interference is dominant.

Corollary 2: The ergodic capacity of an interference-limited AF relaying system with i.i.d interferers at the destination in Nakagami-m fading is given by

$$C_E = \frac{\Gamma(m_2+Pm_I)}{2\ln(2)\Gamma(m_2)}\int_0^\infty E_0(\xi)\prod_{l=1}^{L}\left(1+\frac{\bar{Y}_l}{m_{Y_l}}\xi\right)^{-m_{Y_l}} \left(1-\left(1+\frac{\bar{\gamma}_1}{m_1}\xi\right)^{-m_1}\right)\Psi\left(Pm_I;1-m_2;\frac{G_F m_2\bar{Z}_I}{m_I\bar{\gamma}_2}\xi\right)d\xi, \tag{14}$$

Proof: By following the same rationale to obtain (12), the MGF of $U=\chi/\gamma_2$ can be readily derived as

$$M_U(s) = \frac{\Gamma(m_2+Pm_I)\left(\frac{m_2\bar{Z}_I s}{m_I\bar{\gamma}_2}\right)^{m_2}}{\Gamma(m_2)} \Psi\left(m_2+Pm_I;m_2+1;\frac{m_2\bar{Z}_I}{m_I\bar{\gamma}_2}s\right), \tag{15}$$

where $m_{Z_1} = ... = m_{Z_P} = m_I$ and $\bar{Z}_1 = ... = \bar{Z}_P = \bar{Z}_I$. Substituting (15) leads to the desired result in (14).

Here, we would like to emphasize the fact that the scenario pertaining to $L = P = 1$ in (14) has been recently addressed in [3], where the authors have provided a closed-form expression for the probability density function (pdf) of the output SINR. The latter can therefore be exploited to derive the ergodic capacity which is expected to have an infinite integral form similar to (14). Unfortunately, this pdf-based approach owes its tractability to the assumption of integer Nakagami-m fading on the first hop [3]. Unlike [3], the result in (14) avoids this limitation and is therefore more general.

Also of interest is the case of dual-hop relaying in interference-free fading. Such a useful result stands as a benchmark that can highlight the effect of interference on the system's performance. Therefore, substituting $\lambda = \chi = 0$ in (6) yields the ergodic capacity in the absence of interference given by

$$C_E = \frac{1}{2\ln(2)}\left\{ \int_0^{\infty} E_0(\xi)\, M_{\gamma_2^{-1}}(G_F\xi)\, d\xi - \int_0^{\infty} E_0(\xi)\, M_{\gamma_1}(\xi) M_{\gamma_2^{-1}}(G_F\xi)\, d\xi \right\}. \tag{16}$$

Interference-free relaying has been extensively studied in the literature, particulary under Nakagami-m fading (c.f., [15] and references therein). However, to the best of our knowledge, there appears to be no analytical ergodic capacity expression which applies for arbitrary Nakagami-m fading. While the MGF of the first-hop SNR γ_1 is given in (8), the MGF of the second-hop inverse SNR γ_2^{-1} is given by [15] as

$$M_{\gamma_2^{-1}}(s) = 2\frac{(\frac{m_2}{\bar{\gamma}_2})^{\frac{m_2}{2}}}{\Gamma(m_2)} s^{\frac{m_2}{2}} K_{m_2}\left(2\sqrt{\frac{s m_2}{\bar{\gamma}_2}}\right), \tag{17}$$

where $K_\lambda(\cdot)$ is the modified Bessel function of the second kind and order λ [16, Eq. (8.432.1)]. Accordingly, the ergodic capacity of a single-relay system in Nakagami-m is given by

$$C_E = \frac{(\frac{G_F m_2}{\bar{\gamma}_2})^{\frac{m_2}{2}}}{\ln(2)\Gamma(m_2)} \int_0^{\infty} \xi^{\frac{m_2}{2}} E_0(\xi)\left(1 - \left(1+\frac{\bar{\gamma}_1}{m_1}\xi\right)^{-m_1}\right) K_{m_2}\left(2\sqrt{\frac{G_F \xi m_2}{\bar{\gamma}_2}}\right) d\xi. \tag{18}$$

Note that, although closed-form expressions for (7), (14) and (18) cannot be obtained in general, they can be easily evaluated numerically since their integrands involve only elementary functions and hypergeometric and exponential integral built-in functions available in most popular mathematical software. As far as the numerical evaluation of (7) and (14) is concerned, it can be seen that their integrands are continuous and possess all derivatives for $\xi > 0$. Moreover, it can be easily shown that the integrands are bounded and non negative (and therefore have no singular points) in the range of integration. Nevertheless, to compute (7), (14) and (18), an analytical expression of G_F is required. The latter is shown to be given by

$$G_F^{-1} = \frac{m_1}{\bar{\gamma}_1}\Phi_2\left(1; m_{Y_1}, ..., m_{Y_L}; m_1; \frac{\bar{Y}_1 m_1}{m_{Y_1}\bar{\gamma}_1}, ..., \frac{\bar{Y}_L m_1}{m_{Y_L}\bar{\gamma}_1}, \frac{m_1}{\bar{\gamma}_1}\right), \tag{19}$$

where $\Phi_2(a; b_1, ..., b_K; z; x_1, ..., x_K, y)$ is the second-kind confluent hypergeometric function of multiple variables given by [19]

$$\Phi_2(a; b_1, ..., b_K; z; x_1, ..., x_K, y) = \frac{1}{\Gamma(a)}\int_0^{\infty} \exp(-yt)t^{a-1}(1+t)^{a-z-1}\prod_{k=1}^{K}(1+x_i t)^{-b_i} dt. \tag{20}$$

Proof: Recalling the definition of G_F in section II, this constant can be calculated in view of [15] as

$$G_F^{-1} = \mathrm{E}\left\{\frac{1}{\gamma_1+\lambda+1}\right\} = \int_0^{\infty} \exp(-\xi) M_{\gamma_1}(\xi) M_\lambda(\xi) d\xi, = \int_0^{\infty} \exp(-\xi)\left(1+\frac{\bar{\gamma}_1}{m_1}\xi\right)^{-m_1}\prod_{l=1}^{L}\left(1+\frac{\bar{Y}_l}{m_{Y_l}}\xi\right)^{-m_{Y_l}} d\xi. \tag{21}$$

By performing the change of variable $z = \frac{\bar{\gamma}_1}{m_1}\xi$ in (21), one can recognize that the latter can be expressed in terms of $\Phi_2(a; b_1, ..., b_K; z; x_1, ..., x_K, y)$ as shown in (19). Note that in the absence of interference, G_F reduces to the already known expression in the literature [15], given by

$$G_F = \left[\frac{m_1}{\bar{\gamma}_1}\Psi\left(1; 2-m_1; \frac{m_1}{\bar{\gamma}_1}\right)\right]^{-1}. \tag{22}$$

IV. Ergodic Capacity Analysis: Compact Form

Due to the high degree of difficulty in resolving the problem it raises with respect to the current state of the art and given the complicated structure of the integral-based ergodic capacity expressions obtained in the previous section, the authors are unaware of any closed-form solution to these integrals. However, under some special circumstances, compact forms for the ergodic capacity in terms of a special function, namely the Meijer's G-function of two variables, is possible. The Meijer's G-function of two variables is a non-elementary or a built-in function about which a significant literature has developed because of its importance in either mathematical theory or in practice [20], [21]. Recently, an excellent algorithm for the computation of the bivariate Meijer's G-function has been developed in [22]. In what follows let

$$G^{p,q,k,r,l}_{A,[C,E],B,[D,F]}\left(z_1, z_2 \,\middle|\, \begin{matrix}\alpha_1, ..., \alpha_A; \gamma_1, ..., \gamma_C; \epsilon_1, ..., \epsilon_E\\ \beta_1, ..., \beta_B; \delta_1, ..., \delta_D; \phi_1, ..., \phi_F\end{matrix}\right), \tag{23}$$

denote the Meijer's G-function of two variables z_1 and z_2 defined in [21, Eq. (2.3)].

Corollary 3: The ergodic capacity of fixed-gain relaying in an interference-limited destination scenario with an integer Nakagami-m fading parameter of the useful/interfering information-bearing first link is expressed as

$$C_E = \frac{1}{2\ln(2)\Gamma(m_2)\Gamma(Pm_I)}\sum_{k=1}^{m_1}\binom{m_1}{k}\left(\frac{\bar{\gamma}_1}{m_1}\right)^k\sum_{l=1}^{L+1}\sum_{p=1}^{M_l}\Upsilon_{l,p}\, \mathrm{G}^{2,1,1,1,2}_{2,[1:1],1,[1:2]}\left(\frac{G_F m_2 \bar{Z}_I}{m_{Z_I}\bar{\gamma}_2}, \Delta_l \,\middle|\, \begin{matrix}1-k, -k; p, Pm_I\\ k+1; 0; 0, m_2\end{matrix}\right). \tag{24}$$

Note that in an interference-limited destination scenario, the relay experiences interference and AWGN, while the destination is AWGN-free and subject to interference only. This case amounts to a downlink communication, in which the destination is close to the cell boundary.

Proof: Under the assumption that the Nakagami-m factors m_1 and $m_{Y_l}, l=1,...,L$ are integer values, we have

$$\prod_{l=1}^{L}\left(1+\frac{\bar{Y}_l}{m_{Y_l}}\xi\right)^{-m_{Y_l}}\left(1-\left(1+\frac{\bar{\gamma}_1}{m_1}\xi\right)^{-m_1}\right) = \sum_{k=1}^{m_1}\binom{m_1}{k}\left(\frac{\bar{\gamma}_1}{m_1}\right)^{k}\sum_{l=1}^{L+1}\sum_{p=1}^{M_l}\frac{\Upsilon_{l,p}\xi^k}{(\Delta_l\xi+1)^p}, \quad (25)$$

whereby $M = \{m_1, m_{Y_1}, m_{Y_2}, ..., m_{Y_L}\}$, $\Delta = \{\bar{\gamma}_1/m_1, \bar{Y}_1/m_{Y_1}, ..., \bar{Y}_L/m_{Y_L}\}$, and

$$\Upsilon_{l,k} = \Delta_l^{k-M_l}\sum_{\tau(l,k)}\sum_{i=1,i\neq l}^{L+1}\binom{M_i+q_i-1}{q_i}\frac{(-\Delta_i)^{q_i}}{\left(1-\frac{\Delta_i}{\Delta_l}\right)^{M_i+q_i}}, \quad (26)$$

where, to obtain (25), we exploited (10) and the binomial expansion theorem in [16, Eq. (1.111)]. Hence, upon substitution of (25) into (7), it follows that the ergodic capacity can be expressed as

$$C_E = \frac{\Gamma(m_2+Pm_I)}{2\ln(2)\Gamma(m_2)}\sum_{k=1}^{m_1}\binom{m_1}{k}\left(\frac{\bar{\gamma}_1}{m_1}\right)^{k}\sum_{l=1}^{L+1}\sum_{p=1}^{M_l}\Upsilon_{l,p} \underbrace{\int_0^\infty \frac{\xi^k E_0(\xi)}{(\Delta_l\xi+1)^p}\Psi\left(Pm_I; 1-m_2; \frac{G_F m_2 \bar{Z}_I}{m_{Z_I}\bar{\gamma}_2}\xi\right)d\xi}_{\Phi}. \quad (27)$$

We now have to solve the integral Φ in order to derive the ergodic capacity for the relaying system under consideration. For the purpose of further analytical evaluation of Φ, we represent the functions constituting the integrands as Meijer's G-functions. Therefore, using [23, Eqs. (8.4.3.1) and (8.5.2.5)], we have

$$E_0(x) = \mathrm{G}_{1,2}^{2,0}\left(z\,\middle|\,\begin{matrix}0\\-1,0\end{matrix}\right) \quad \text{and}$$
$$(1+z)^{-\nu} = \frac{1}{\Gamma(\nu)}\mathrm{G}_{1,1}^{1,1}\left(z\,\middle|\,\begin{matrix}1-\nu\\0\end{matrix}\right), \nu\geq 0 \quad (28)$$

and, using [23, Eq. (8.4.46.1)], we get

$$\Psi(a,b;z) = \frac{1}{\Gamma(a)\Gamma(a-b+1)}\mathrm{G}_{1,2}^{2,1}\left(z\,\middle|\,\begin{matrix}1-a\\0,1-b\end{matrix}\right). \quad (29)$$

Upon substitution of (28) and (29) into Φ, we obtain

$$\Phi = A\int_0^\infty \xi^k \mathrm{G}_{1,2}^{2,0}\left(\xi\,\middle|\,\begin{matrix}0\\-1,0\end{matrix}\right)\mathrm{G}_{1,1}^{1,1}\left(\Delta_l\xi\,\middle|\,\begin{matrix}1-p\\0\end{matrix}\right) \mathrm{G}_{1,2}^{2,1}\left(\frac{G_F m_2\bar{Z}_I}{m_{Z_I}\bar{\gamma}_2}\xi\,\middle|\,\begin{matrix}1-Pm_I\\0,m_2\end{matrix}\right)d\xi, \quad (30)$$

where $A = 1/\Gamma(Pm_I)\Gamma(Pm_I+m_2)$. The second and third Meijer's G-functions in the R.H.S of (30) are transformed into a Meijer's G-function of two variables according to the functional relation

$$\mathrm{G}_{C,D}^{r,q}\left(z_1\,\middle|\,\begin{matrix}\gamma_1,...,\gamma_c\\ \delta_1,...,\delta_D\end{matrix}\right)\mathrm{G}_{E,F}^{l,k}\left(z_2\,\middle|\,\begin{matrix}\epsilon_1,...,\epsilon_E\\ \phi_1,...,\phi_F\end{matrix}\right) = \mathrm{G}_{0,[C,E],0,[D,F]}^{0qkrl}\left(z_1,z_2\,\middle|\,\begin{matrix}...;1-\gamma_1,...,1-\gamma_C;1-\epsilon_1,...,1-\epsilon_E\\ ....;\delta_1,...,\delta_D;\phi_1,...,\phi_F\end{matrix}\right). \quad (31)$$

Then, by exploiting [21, Eq. (3.2)], we obtain

$$\Phi = A\mathrm{G}_{2,[1:1],1,[1:2]}^{2,1,1,1,2}\left(\frac{G_F m_2\bar{Z}_I}{m_{Z_I}\bar{\gamma}_2},\Delta_l\,\middle|\,\begin{matrix}1-k,-k;p,Pm_I\\k+1;0;0,m_2\end{matrix}\right). \quad (32)$$

Finally, substituting (32) into (27) yields the desired result.

Corollary 4: Under an interference-free destination scenario, the relay experiences interference and AWGN while the destination is interference-free and subject to AWGN only. This case amounts to an uplink communication in which the destination communicates with the source using a relay, which is located in the proximity of the cell edge. Thus the ergodic capacity becomes

$$C_E = \frac{(G_F\frac{m_2}{\bar{\gamma}_2})^{\frac{m_2}{2}}}{\ln(2)\Gamma(m_2)}\sum_{k=1}^{m_1}\binom{m_1}{k}\left(\frac{\bar{\gamma}_1}{m_1}\right)^{k}\sum_{l=1}^{L+1}\sum_{p=1}^{M_l}\Upsilon_{l,p} \mathrm{G}_{2,[1:0],1,[1:2]}^{2,1,0,1,2}\left(\frac{G_F m_2}{\bar{\gamma}_2},\Delta_l\,\middle|\,\begin{matrix}1-k-\frac{m_2}{2},-\frac{m_2}{2}-k;p;...\\ \frac{m_2}{2}+k+1;0;\frac{m_2}{2},-\frac{m_2}{2}\end{matrix}\right). \quad (33)$$

Proof: When $\chi = 0$, the ergodic capacity can be expressed by inserting (8), (9) and (17) into (6). Then, (33) is obtained by following the same rationale to obtain (24) upon representing the Bessel K function using the Meijer's G-function as [23, Eq. (8.4.23.1)]

$$K_\lambda\left(\sqrt{\beta}x\right) = \frac{1}{4}\mathrm{G}_{2,0}^{0,2}\left(\frac{\beta x^2}{4}\,\middle|\,\begin{matrix}-\\ \frac{\lambda}{2},\frac{-\lambda}{2}\end{matrix}\right). \quad (34)$$

V. Numerical Examples and Discussions

The aim of this section is to illustrate the expressions derived in Sections III and IV using numerical examples and examine the effect of interference on the system's capacity.

In Figs. 2-4, to exclude the effect of the relay's placement, we assume a symmetric network such that the relay sits halfway between the source and the destination, so that $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}$. The interference is assumed to be i.i.d on each link with mean power $\bar{Y}_I$ and $\bar{Z}_I$. To exclude the effect of the interference positions, we also assume that $\bar{Y}_I = \bar{Z}_I = \bar{\gamma}_I$. The effect of varying the relative placement of the relay and interference will be treated later in Figs. 5 and 6.

Fig. 2 shows the ergodic capacity performance when the number of interferers increases from 1 to 4, and the average strength of the interfering links varies between 10 and 20 dB lower than that of the useful link so that $\bar{\gamma}_I = \bar{\gamma} - 10$ dB and $\bar{\gamma}_I = \bar{\gamma} - 20$ dB. We can see that as the number of interferers increases, the ergodic capacity relatively improves very slowly with increasing SNR. Moreover, the ergodic capacity attains a saturation level which is more noticeable when the difference between the received powers of the useful and interfering signals decreases. Hence, interference imposes severe constraints on the system's capacity, thereby highlighting the significance of using interference cancelation techniques in interference-impaired relay systems in order to attain the beneficial effects of relaying.

Notice that the Meijer's G-function of two variables in (23) converges if the following conditions are satisfied [20]: $A+D+B+C < 2(r+q+p)$ and $A+F+B+E < 2(l+k+p)$. It is straightforward to show that the parameters of the Meijer's G-function in (24) and (33) satisfy these sufficient conditions, and therefore the Meijer's G-functions converge.

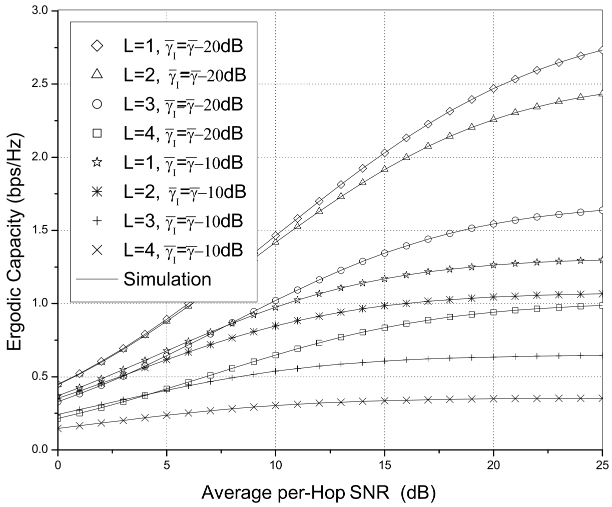

Fig. 2. Ergodic capacity for different numbers of interferers at the relay and the destination when $P = L$, $m_1 = m_2 = 1.5$, $m_I = 1$, and $\bar{\gamma}_I = \bar{\gamma} - \{10, 20\}$ dB.

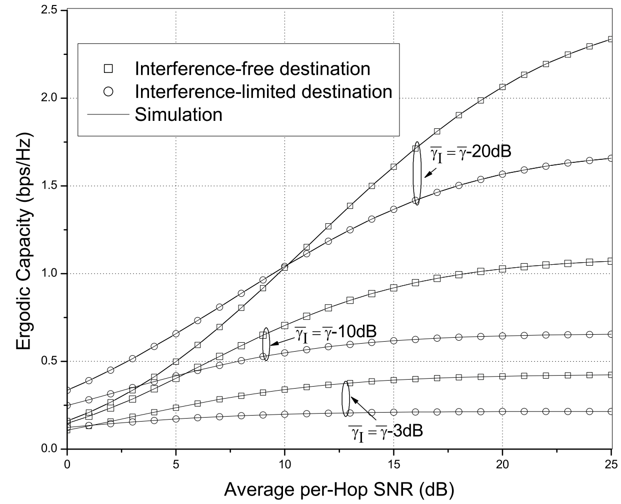

Fig. 3. Ergodic capacity of both interference-free ($L = 3$, $P = 0$) and interference-limited ($L = 3$, $P = 3$) destination scenarios when $m_1 = 1$, $m_2 = 1.5$, and $m_I = 1$.

Fig. 3 deals with fixed-gain relaying under interference-free and interference-limited destination scenarios and illustrates the ergodic capacity performance obtained from *Corollaries* 3 and 4. The average strength of the interference link was set in both cases to $3, 10$ and 20 dB lower than that of the useful link. Here, we observe cross-over points in the ergodic capacity curves, in the low-to-moderate SNR region, where the interference-limited destination scenario performs better than the interference-free destination scenario, especially in the presence of weak interference. This observation can be explained by the fact that in the low-to-moderate SNR region, the noise-dominant configuration shows the worst ergodic capacity performance for all channel conditions. Therefore, a low interference power, $\bar{\gamma}_I = \bar{\gamma} - 20$ dB at the destination can lead to a higher ergodic capacity in the case of interference-limited reception, as compared to high AWGN power at the destination in case of an interference-free destination scenario. As the SNR increases, the performance gaps among the considered interference scenarios become larger and the interference-dominant configuration has the worst performance in the high-SNR region. Furthermore, since in the interference-limited destination scenario both the relay and the destination experience cochannel interference, the saturation level on the ergodic capacity performance is reached at lower SNR values (i.e., 5 dB), compared to the interference-free destination scenario (i.e., 15 dB). Hence, it is shown that the downlink is more vulnerable to the presence of interference than the uplink.

When mapped into a link-level study, the observations made above can lead to useful decisions about relay placement/selection options to improve performance on both downlink and uplink in different regions of the cell experiencing different interference levels.

From this figure, a good accuracy is retained by the analytical expressions, which turn out to be not only very accurate but also numerically stable.

Fig. 4 plots the ergodic capacity of a dual-hop fixed-gain relaying system under interference-free Nakagami-m fading, as computed with (18), and compares it with Monte Carlo simulations. As can be seen from this figure, a very good match is retained over the SNR range of interest. To highlight the importance of the proposed framework, we also report in Fig. 4 the ergodic capacity bounds obtained in [14, Eq. (8)]. As can be seen, in the SNR range of interest, these bounds are not sufficiently tight to be considered for predicting the ergodic capacity of dual-hop fixed gain relaying. Although the tightness of the proposed bounds improves with m, it is more likely to decrease for the new communication systems, such as LTE-Advanced [1], where the fading is more pronounced (i.e., lower values of m) due to the terminal's mobility and the high level of shadowing.

Fig. 5 investigates the impact of the relay location on the AF system's capacity with a single interferer at both the relay and the destination. In this figure, it is assumed that the relay is arbitrarily located between the source and the destination. To compare the effect of network geometry fairly and generally, the distance between nodes can be normalized by the length of the source-to-destination link. Therefore, the normalized local mean SNR of the source-to-relay and the relay-to-destination links can be, respectively, expressed by

$$\bar{\gamma}_1 = \left(\frac{d_{\mathbf{S-D}}}{d_{\mathbf{S-R}}}\right)^{\alpha}\Omega, \quad \bar{\gamma}_2 = \left(\frac{d_{\mathbf{S-D}}}{d_{\mathbf{R-D}}}\right)^{\alpha}\Omega, \tag{35}$$

where α is the pathloss exponent and Ω is the transmitter's SNR. An interferer is said to be far from the relay (or the destination) if $\bar{Y}_I(\text{or } \bar{Z}_I) = \Omega - 30$ dB and close to the relay (or the destination) if $\bar{Y}_I(\text{or } \bar{Z}_I) = \Omega - 5$ dB. The obtained curves illustrate the ergodic capacity against the normalized source-relay distance i.e., $d_{\mathbf{S-R}}/d_{\mathbf{S-D}}$ with $\alpha = 4$, as the relay moves on the **S-D** line (a serial configuration is assumed).

It can be seen from Fig. 5 that as the interference at the

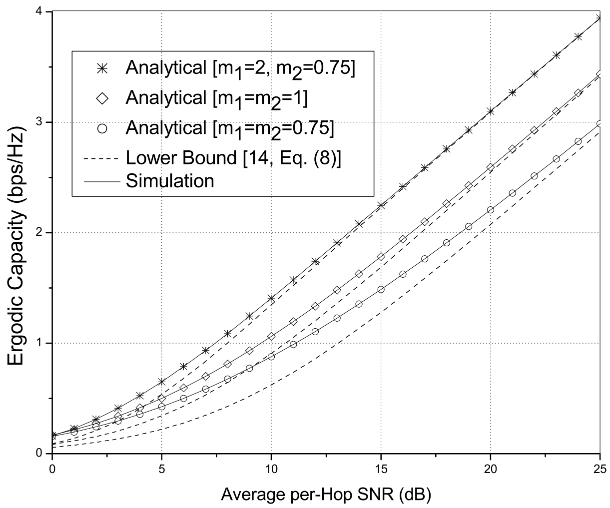

Fig. 4. Ergodic capacity of fixed-gain AF dual-hop systems in Nakagami-m fading channels: simulation and analytical/exact results and lower bound. $\bar{\gamma}_1 = \bar{\gamma}_2$.

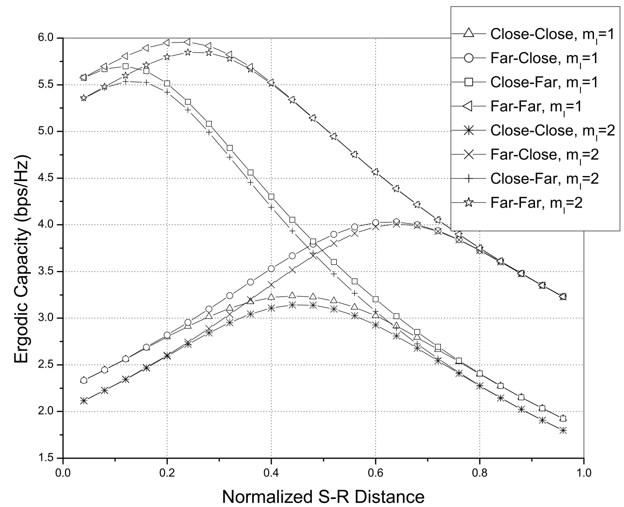

Fig. 5. Ergodic capacity versus the normalized source-to-relay distance for various interference configurations when $m_1 = m_2 = 2.5$. Far-Far: $\bar{Y}_I = \bar{Z}_I = \Omega - 30$ dB; Close-Close: $\bar{Y}_I = \bar{Z}_I = \Omega - 5$ dB.

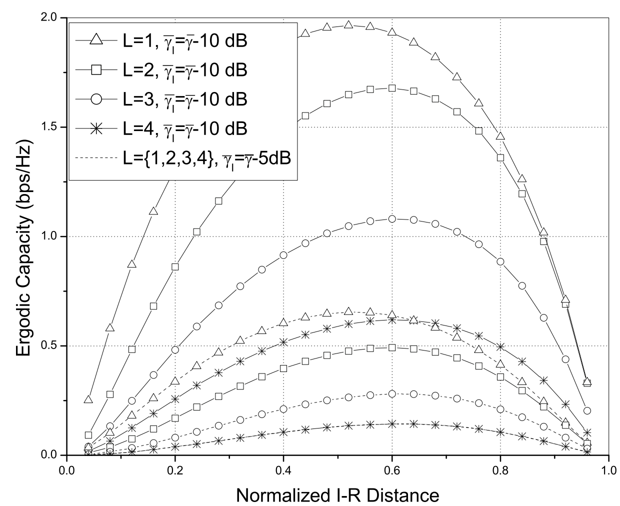

Fig. 6. Ergodic capacity versus normalized interference-to-relay distance for various numbers of interferers when $m_1 = m_2 = 2.5, m_I = 2, \bar{\gamma}_I = \bar{\gamma} - 10$ dB and $\bar{\gamma}_I = \bar{\gamma} - 5$ dB.

relay becomes stringent, the optimal position for the relay will be closer to the source, and vice-versa. This behavior can be expected since as the source-relay link gets worse, the relay needs to be closer to the source so as to compensate for its low quality. However, we observe that the relay has to approach the source more than the destination. The reason is that the relay requires higher SINR than the destination as the interference increases, since the relay is more vulnerable to interference than the destination. It can also be seen that under a strong interference regime and for identical interference channels, i.e., when $\bar{Y}_I = \bar{Z}_I = \Omega - 5$ dB, the midpoint relay position is almost the optimal solution with a slight advantage for the source direction. Nevertheless, for identical weak interference channels, the optimal position for the relay will be closer to the source. From Fig. 5, it can be seen that the ergodic capacity decreases very quickly as the relay moves form his best position. Therefore, choosing the optimal relay location significantly improves the system's capacity. On the other hand, for a given interference power limit, it is seen that the ergodic capacity varies very slightly with the fading parameters of the interference channels. Moreover, when the interference power is important, the scenario where interference channels are subject to Rayleigh fading performs slightly better than the scenario where interference channels are subject to Nakagami-m fading.

Fig. 6 shows the effect of imbalanced interference powers at the relay and the destination on the ergodic capacity of the AF relaying system. In this figure, it is assumed that a cluster of L interferers is located between the relay and the destination and the interference power is $\bar{\gamma}_I$. Thus, the local mean of the INRs of the interference-to-relay and the interference-to-destination can be expressed as

$$\bar{Y}_I = \left(\frac{d_{\mathbf{S-D}}}{d_{\mathbf{I-R}}}\right)^{\alpha} \bar{\gamma}_I, \quad \bar{Z}_I = \left(\frac{d_{\mathbf{S-D}}}{d_{\mathbf{I-D}}}\right)^{\alpha} \bar{\gamma}_I, \qquad (36)$$

respectively. The obtained curves illustrate the ergodic capacity against the normalized interference-relay distance, i.e., $d_{\mathbf{I-R}}/d_{\mathbf{S-D}}$ with $\alpha = 4$. As can be seen, the best position of the interference moves from the relay to the destination as L increases. We therefore observe the same behavior previously reported from Fig. 5 in that the relay is more susceptible to interference.

Combining all the observations from Figs. 2 to 6, we conclude that our new analytical expressions provide an invaluable analytical insight on: 1) how the ergodic capacity is dominated by the average interference power and how it improves slowly with the average SNR increasing, thereby inducing saturation levels; 2) how a larger Nakagami-m fading parameter on interference channels slightly deteriorates the ergodic capacity;

3) how instantly identifying the optimal relay location owing to our analytical/exact results (i.e., without heavy simulations) can quickly and simply help significantly increase the system's capacity.

VI. Conclusion

In this paper we presented a comprehensive framework for the ergodic capacity evaluation of dual-hop fixed-gain relaying systems in interference-limited channels. The obtained results are useful to understand how fading and interference at the relay and/or the destination can degrade the ergodic capacity performance of the considered system. Our results show that the ergodic capacity is dominated by the average interference power, especially at the relay. Moreover, it slowly improves with the average SNR increasing while it slightly deteriorates with larger values of the Nakagami-m fading parameters pertaining to the interference channels. Furthermore, our results offer an analytical insight into the key impact of relay placement on performance. Our new ergodic capacity expressions could therefore provide a very practical/low-cost performance optimization tool for relayed-communication system designers.

VII. Appendix A: Proof of Theorem 1

The fixed-gain single relay ergodic capacity in interference-limited fading can be developed as

$$C_E = \frac{1}{2}\mathrm{E}\left\{\log_2\left(1+\frac{\gamma_1\gamma_2}{\gamma_2\left(1+\lambda\right)+C\left(1+\chi\right)}\right)\right\}, \quad (37)$$

which can be further expanded as

$$C_E = \frac{\mathrm{E}\left[\ln(1+\gamma_1+\lambda+\frac{C(1+\chi)}{\gamma_2})\right]-\mathrm{E}\left[\ln(1+\lambda+\frac{C(1+\chi)}{\gamma_2})\right]}{2\ln(2)}. \quad (38)$$

In order to give a formal proof of (6), consider the following Taylor series expansion of $\ln(1+z)$ valid for all $z \geq 0$, [16, Eq. (1.512.3)]

$$\ln(1+Z) = \sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{Z}{Z+1}\right)^n, \qquad \forall Z \geq 0. \quad (39)$$

Accordingly, we can write

$$\begin{aligned}\mathrm{E}\left[\ln\left(1+Z\right)\right] &= \mathrm{E}\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{1}{1+\frac{1}{Z}}\right)^n\right] \quad (40)\\ &= \int_0^{\infty}\sum_{n=1}^{\infty}\frac{s^{n-1}}{n!}e^{-s}M_{\frac{1}{Z}}(s)ds\\ &= \int_0^{\infty}\frac{1-e^{-s}}{s}M_{\frac{1}{Z}}(s)ds. \quad (41)\end{aligned}$$

Recalling the MGF-based relation between a random variable Z and its inverse $1/Z$ in [24, Theorme 1] and carrying out the change of variable $\xi^2 = \tan(z)$, we obtain

$$M_Z(s) = 1-2\sqrt{s}\int_0^{\infty}J_1(2\sqrt{s}\xi)M_{Z^{-1}}(\xi^2)d\xi, \quad (42)$$

where $J_1(\cdot)$ is the Bessel function of the first kind, it follows that

$$\begin{aligned}\mathrm{E}\left[\ln\left(1+Z\right)\right] &= \int_0^{\infty}\frac{1-e^{-s}}{s}ds\\ &-2\int_0^{\infty}\underbrace{\int_0^{\infty}\frac{1-e^{-s}}{\sqrt{s}}J_1(2\sqrt{s}\xi)}_{\Sigma}M_Z(\xi^2)dsd\xi, \quad (43)\end{aligned}$$

where, using [16, Eq. (7.811.1)], we get

$$\Sigma = \frac{1}{\xi}\mathrm{G}_{3,2}^{1,2}\left(\frac{1}{\xi^2}\middle|\begin{array}{c}0,1,1\\1,0\end{array}\right) = \xi E_0(\xi^2). \quad (44)$$

Finally, substituting (44) into (43) and exploiting the mutual independency between γ_1, γ_2, λ and χ, we obtain

$$\begin{aligned}\mathrm{E}\left[\ln\left(1+\gamma_1+\lambda+\frac{C(1+\chi)}{\gamma_2}\right)\right] &= \int_0^{\infty}\frac{1-e^{-s}}{s}ds\\ -2\int_0^{\infty}\xi E_0(\xi^2)M_{\gamma_1}(\xi^2)&M_{\lambda}(\xi^2)M_{\frac{1+\chi}{\gamma_2}}(C\xi^2)d\xi. \quad (45)\end{aligned}$$

In a similar fashion, the second expectation form in (38) can be expressed as

$$\begin{aligned}\mathrm{E}\left[\ln\left(1+\lambda+\frac{C(1+\chi)}{\gamma_2}\right)\right] &= \int_0^{\infty}\frac{1-e^{-s}}{s}ds\\ -2\int_0^{\infty}\xi E_0(\xi^2)M_{\lambda}(\xi^2)&M_{\frac{1+\chi}{\gamma_2}}(C\xi^2)d\xi. \quad (46)\end{aligned}$$

Finally, substituting (45) and (46) into (38) and performing the necessary mathematical manipulations, (6) is easily proven.

VIII. Appendix B

The MGF of $U = \frac{1+\chi}{\gamma_2}$ can be expressed as

$$M_U(s) = \int_0^{\infty}\int_0^{\infty}e^{-s\frac{x}{y}}p_{\gamma_2}(y)p_{1+\chi}(x)dxdy \quad (47)$$

where the pdfs of γ_2 and χ can be obtained by applying the inverse Laplace transform to (8) and (9), thereby yielding

$$p_{\gamma_2}(y) = \frac{m_2^{m_2}}{\Gamma(m_2)\bar{\gamma_2}^{m_2}}y^{m_2-1}\exp\left(-\frac{m_2 y}{\bar{\gamma_2}}\right), \quad (48)$$

$$p_{\chi}(x) = \sum_{p=1}^{P}\sum_{k=1}^{m_{Z_p}}\frac{\beta_{p,k}}{(k-1)!}x^{k-1}\exp\left(-\frac{m_{Z_p}}{\bar{Z}_p}x\right). \quad (49)$$

Moreover, by applying the concept of transformation of random variables, we obtain

$$\begin{aligned}p_{1+\chi}(x) &= \sum_{p=1}^{P}\sum_{k=1}^{m_{Z_p}}\sum_{n=0}^{k-1}\exp\left(\frac{m_{Z_p}}{\bar{Z}_p}\right)\frac{(-1)^{n+1}\beta_{p,k}C_n^{k-1}}{(k-1)!}x^n\\ &\exp\left(-\frac{m_{Z_p}}{\bar{Z}_p}x\right). \quad (50)\end{aligned}$$

Then, tacking into account (49) and (50), the MFG of U can be derived as follows

$$\begin{aligned} M_U(s) &= \int_0^\infty p_{\gamma_2}(y) \int_0^\infty e^{-s\frac{x}{y}} p_{1+\chi}(x) dx dy \\ &= \sum_{p=1}^{P} \sum_{k=1}^{m_{Z_p}} \sum_{n=0}^{k-1} \exp\left(\frac{m_{Z_p}}{\bar{Z}_p}\right) \frac{(-1)^{n+1}\beta_{p,k} C_n^{k-1}}{(k-1)!} \\ &\quad \int_0^\infty p_{\gamma_2}(y) \int_0^\infty x^n e^{-x\left(\frac{s}{y}+\frac{m_{Z_p}}{\bar{Z}_p}\right)} dx dy \\ &= \frac{m_2^{m_2}}{\Gamma(m_2)\bar{\gamma}_2^{m_2}} \sum_{p=1}^{P} \sum_{k=1}^{m_{Z_p}} \sum_{n=0}^{k-1} \exp\left(\frac{m_{Z_p}}{\bar{Z}_p}\right) \\ &\quad \frac{(-1)^{n+1}\beta_{p,k}}{\Gamma(k-n)} \int_0^\infty \frac{y^{m_2-1} e^{-\frac{m_2 y}{\bar{\gamma}_2}}}{\left(\frac{s}{y}+\frac{m_{Z_p}}{\bar{Z}_p}\right)^{n+1}} dy. \qquad (51) \end{aligned}$$

Finally, by the help of [16, Eq. (9.211.4)], the MGF of U is obtained as shown in (12).

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Ms. Trigui is currently the recipient of a top-tier Alexander-Graham-Bell Canada graduate scholarship from the National Sciences and Engineering Research Council (2012-2014). She had to decline another Ph.D. scholarship offered over the same period from the "Fonds de recherche du Québec Ű Nature et Technologies" (FRQNT). She was previously the recipient of an Undergraduate Student Fellowship from the Tunisian Ministry of Communications. She also received a best paper award at IEEE VTC'2010-Fall. Ms. Trigui serves regularly as a reviewer for many top international journals and conferences in her field.

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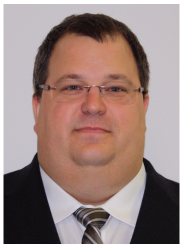
He has been since with INRS-EMT, University of Quebec, Montreal, Canada, as a Research Associate from 1995 till 1997, as an Assistant Professor till 2000, then as an Associate Professor till 2009. Currently he is a Full Professor in the Wireless Communications Group. His research interests are in wireless communications, statistical signal and array processing and adaptive space-time processing. From 1998 to 2002 he has been leading the radio design and signal processing activities of the Bell/Nortel/NSERC Industrial Research Chair in Personal Communications at INRS-EMT, Montreal, Canada. Since 2004, he has been actively involved in major projects in wireless of PROMPT (Partnerships for Research on Microelectronics, Photonics and Telecommunications).

Professor Affes was the co-recipient of the 2002 Prize for Research Excellence of INRS. Since 2003, he holds a Canada Research Chair in Wireless Communications. From 2008 to 2011, he held a Discovery Accelerator Supplement Award from NSERC (Natural Sciences & Engineering Research Council of Canada). In 2006, Professor Affes served as a General Co-Chair of the IEEE VTC'2006-Fall conference, Montreal, Canada. In 2008, he received from the IEEE Vehicular Technology Society the IEEE VTC Chair Recognition Award for exemplary contributions to the success of IEEE VTC. He currently acts as a member of the Editorial Boards of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the IEEE TRANSACTIONS ON SIGNAL PROCESSING, the IEEE TRANSACTIONS ON COMMUNICATIONS, and the *Wiley Journal on Wireless Communications & Mobile Computing*.



Alex Stéphenne was born in Quebec, Canada, on May 8, 1969. He received the B.Eng. degree in electrical engineering from McGill University, Montreal, Quebec, in 1992, and the M.Sc. degree and Ph.D. degrees in telecommunications from INRS-Télécommunications, Université du Québec, Montreal, in 1994 and 2000, respectively. In 1999 he joined SITA Inc., in Montreal, where he worked on the design of remote management strategies for the computer systems of airline companies. In 2000, he became a DSP Design Specialist for Dataradio Inc., Montreal, a company specializing in the design and manufacturing of advanced wireless data products and systems for mission critical applications. In January 2001, he joined Ericsson and worked for over two years in Sweden, where he was responsible for the design of baseband algorithms for WCDMA commercial base station receivers. From June 2003 to December 2008, he was still working for Ericsson, but was based in Montreal, where he was a researcher focusing on issues related to the physical layer of wireless communication systems. Since 2004, he is also an adjunct professor at INRS, where he has been continuously supervising the research activities of multiple students. His current research interests include Coordinated Multi-Point (CoMP) transmission and reception, Inter-Cell Interference Coordination (ICIC) and mitigation techniques in Heterogeneous Networks (HetNets), wireless channel modeling/characterization/estimation, statistical signal processing, array processing and adaptive filtering for wireless telecommunication applications. He joined Huawei Technologies Canada, in Ottawa, in Dec. 2009.

Alex has been a member of the IEEE since 1995 and a Senior member since 2006. He is a member of the “Ordre des Ingénieurs du Québec” (OIQ). He is a member of the organizing committee and a co-chair of the Technical Program Committee (TPC) for the 2012-Fall IEEE Vehicular Technology Conference (VTC’12-Fall), in Quebec City. He has served as a co-chair for the “Multiple antenna systems and space-time processing” track for VTC’08-Fall, in Calgary, and as a co-chair of the TPC for VTC’2006-Fall in Montreal.