On the Capacity of Log-Normal Fading Channels

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***Abstract*—In this letter we provide an analytical expression for the moments of the capacity for the log-normal fading channel. Since the developed expression involves infinite series, we show that the error that results from the truncation of these series is insignificant. We also analyze in more details the ergodic capacity by giving a simpler expression for the remainder of the truncated series. Relying on the fact that the sum of log-normal Random Variables (RV) is well approximated by another log-normal RV, we further utilize the obtained results to approximate the capacity of diversity combining techniques in correlated log-normal fading channels. The results that we provide in this letter are an important tool for measuring the performance of communication links in a log-normal environment.**

***Index Terms*—Information rates, log-normal distributions, diversity methods.**

I. INTRODUCTION

THE capacity of fading channels has attracted an extensive interest in the last decade. This concern is motivated by the need for a valuable tool to assess the achievable performance of communication links over fading channels. Although several studies on the capacity of different kinds of fading channels are available [1]-[6], the results on the capacity of log-normal fading channels are rather scarce. This is to be contrasted to the fact that the log-normal distribution is found to be the best fit to characterize several wireless channels like, to name a few, indoor channels and Ultra Wideband (UWB) channels [7]. In a recent study [8], we have provided formulae to estimate the average capacity of the log-normal channel and we have analyzed the capacity with diversity combining techniques as well as the capacity in interference-limited environments.

Higher order statistics (HOS) play generally an important role in several areas of communications [9]. For example the interested reader is referred to [10] for the use of HOS in antenna subset diversity in fading channels. Due to the random nature of the wireless channel, the capacity is generally viewed as a random variable. As such the average capacity is not enough to characterize the performance of the communication system. One would be, for instance, interested in finding the variance of the capacity, which requires the computation of the second-order moment. Several papers have addressed the moments of the capacity for different types of fading channels [11]-[17]. In this letter we provide a generic expression to compute all the moments of the capacity in log-normal fading channels. Since the obtained formula involves infinite series, we study the error that results from the truncation of these series. For the ergodic capacity, we further give a simpler expression for the remainder of the series. Also, we extend the second approximation given in [8] to allow for the computation of higher order moments of the capacity. Finally, we consider the capacity of diversity combining techniques in a correlated log-normal environment. The results that we provide in this letter are an important tool for measuring the performance of communication links in a log-normal environment, that supplement the performance analysis in terms of outage probability that was conducted in [20], [21] and the references therein. The remainder of the paper is organized as follows, in Section II we derive the moments of the capacity. In Section III, we study the average capacity in more details. Based on the Gaussian approximation, section IV provides a simple estimate of the moments of the capacity. The obtained results are then used in Section V to calculate the capacity of maximum ratio combining and equal gain combining in a correlated environment. Numerical results are provided in Section VI. Conclusions are given in Section VII.

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II. THE MOMENTS OF THE CAPACITY

A. The Moments as an Infinite Series

In this paper, we are interested in deriving the moments of the capacity defined as:

$$E[C^n] = \frac{\xi}{\sigma\sqrt{2\pi}} \int_0^{+\infty} \frac{\ln^n(1+\gamma)}{\gamma} e^{-\frac{(\xi \ln \gamma - \mu)^2}{2\sigma^2}} d\gamma, \qquad (1)$$

where γ is the instantaneous SNR, $\xi = \frac{10}{\ln(10)} = 4.3429$, σ and μ are, respectively, the standard deviation and the mean of $10\log_{10}(\gamma)$ and are expressed in dB. Note that we consider that γ is normalized i.e., $\mu = \Gamma_{\text{dB}} - \frac{\sigma^2}{2\xi}$ with $\Gamma_{\text{dB}} = \xi \ln(\Gamma)$ is the average SNR in dB.

Theorem: The moments of the capacity are given by

$$E[C^n]{=}n!e^{-\frac{\mu^2}{2\sigma^2}} \left(\frac{1}{2} \sum_{k=n}^{+\infty} \frac{S_k^{(n)}}{k!} \operatorname{erfcx}\left(\frac{\sigma k}{\xi\sqrt{2}} + \frac{\mu}{\sqrt{2}\sigma} \right) + \frac{1}{\sqrt{\pi}} \sum_{j=1}^{n} \left(\frac{\sqrt{2}\sigma}{\xi} \right)^{n-j} \sum_{k=j}^{+\infty} \frac{S_k^{(j)}}{k!} H_{-(n-j+1)} \left(\frac{\sigma k}{\sqrt{2}\xi} - \frac{\mu}{\sqrt{2}\sigma} \right) \right) + \frac{n! e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{\pi}} \left(\frac{\sqrt{2}\sigma}{\xi} \right)^n H_{-(n+1)} \left(-\frac{\mu}{\sqrt{2}\sigma} \right), \qquad (2)$$

$$|E[C^n]-E[C^n]_K|<\frac{n!e^{-\frac{\mu^2}{2\sigma^2}}}{(K+1)!}\left(\frac{1}{2}|S_{K+1}^{(n)}|\operatorname{erfcx}\left(\frac{\sigma(K+1)}{\xi\sqrt{2}}+\frac{\mu}{\sqrt{2}\sigma}\right)+\frac{1}{\sqrt{\pi}}\sum_{j=1}^{n}\left(\frac{\sqrt{2}\sigma}{\xi}\right)^{n-j}|S_{K+1}^{(j)}|H_{-(n-j+1)}\left(\frac{\sigma(K+1)}{\sqrt{2}\xi}-\frac{\mu}{\sqrt{2}\sigma}\right)\right) \quad (11)$$

where $S_k^{(n)}$ denotes the Stirling number of the first kind, which is equal to $(-1)^{k-n}$ times the number of permutations of k symbols which have exactly n cycles (Section 9.74 in [19]) and where $H_\nu(x)$ is the Hermite function [26] and $\operatorname{erfcx}(x) = e^{x^2}\operatorname{erfc}(x)$ is called the scaled complementary error function.

Proof: By the change of variable $y = \ln(\gamma)$ in (1), the capacity moments can be rewritten as

$$E[C^n] = \frac{\xi}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty}\ln^n(1+e^y)\exp\left(-\frac{(\xi y-\mu)^2}{2\sigma^2}\right)dy. \quad (3)$$

After some manipulations, the last integral can be rewritten as follows:

$$\begin{aligned} E[C^n] &= \frac{\xi}{\sqrt{2\pi}\sigma}\left(\int_0^{+\infty}(y+\ln(1+e^{-y}))^n e^{-\frac{(\xi y-\mu)^2}{2\sigma^2}}dy \right.\\ &\quad \left. + \int_0^{+\infty}\ln^n(1+e^{-y})e^{-\frac{(\xi y+\mu)^2}{2\sigma^2}}dy\right) \quad (4)\\ &= \frac{\xi}{\sqrt{2\pi}\sigma}\left(\sum_{j=0}^{n}C_n^j\int_0^{+\infty}y^{n-j}\ln^j(1+e^{-y})e^{-\frac{(\xi y-\mu)^2}{2\sigma^2}}dy \right.\\ &\quad \left. + \int_0^{+\infty}\ln^n(1+e^{-y})e^{-\frac{(\xi y+\mu)^2}{2\sigma^2}}dy\right), \quad (5)\end{aligned}$$

where C_n^j is the binomial coefficient. Since, for $y > 0$, we have $e^{-y} < 1$, then we may use the following identity [19, Eq.(9.741.2)]:

$$\ln^j(1+e^{-y}) = j!\sum_{k=j}^{+\infty}\frac{S_k^{(j)}e^{-ky}}{k!}. \quad (6)$$

Plugging this last equality in the expression of $E[C^n]$, we obtain:

$$\begin{aligned} E[C^n] &= \frac{\xi}{\sqrt{2\pi}\sigma}\left(\sum_{j=0}^{n}C_n^j j!\sum_{k=j}^{+\infty}\frac{S_k^{(j)}}{k!}\int_0^{+\infty}y^{n-j}e^{-ky}e^{-\frac{(\xi y-\mu)^2}{2\sigma^2}}dy \right.\\ &\quad \left. + n!\sum_{k=n}^{+\infty}\frac{S_k^{(n)}}{k!}\int_0^{+\infty}e^{-ky}e^{-\frac{(\xi y+\mu)^2}{2\sigma^2}}dy\right). \quad (7)\end{aligned}$$

Using

$$\int_0^{+\infty}e^{-ky}e^{-\frac{(\xi y+\mu)^2}{2\sigma^2}}dy = \sqrt{\frac{\pi}{2}}\frac{\sigma}{\xi}e^{-\frac{\mu^2}{2\sigma^2}}\operatorname{erfcx}\left(\frac{\sigma k}{\xi\sqrt{2}}+\frac{\mu}{\sqrt{2}\sigma}\right), \quad (8)$$

and

$$\begin{aligned}\int_0^{+\infty}y^{n-j}e^{-(k-\frac{\xi\mu}{\sigma^2})y}e^{-\frac{\xi^2y^2}{2\sigma^2}}dy &= \left(\frac{\sqrt{2}\sigma}{\xi}\right)^{n-j+1}(n-j)!\times\\ &\quad H_{-(n-j+1)}\left(\frac{\sigma k}{\sqrt{2}\xi}-\frac{\mu}{\sqrt{2}\sigma}\right), \quad (9)\end{aligned}$$

as well as the fact that $S_k^{(0)} = \delta_k$, we obtain (2).

B. The Effect of the Truncation

In this section we study the error that results from the truncation of the above series. Since this study hinges on the theory of alternating series, it is useful to introduce the following lemma. [19, (0.227)]

Lemma: Let $\sum_k(-1)^k a_k$ be a convergent alternating series, i.e., $a_k > a_{k+1} \geq 0$ and $\lim_{k\to\infty} a_k = 0$. Then, we have the following result:

$$\left|\sum_{k=K+1}^{+\infty}(-1)^k a_k\right| < a_{K+1}. \quad (10)$$

It can be easily shown that the $n+1$ series that intervene in (2) are alternating series. Consequently, if we denote by $E[C^n]_K$ the truncated version of $E[C^n]$ where all the series are truncated at the Kth term, then using the lemma we obtain the inequality given by (11) at the top of this page.

If K is selected large enough, using the fact that for large x we have that

$$\begin{cases}\operatorname{erfcx}(x) \simeq \frac{1}{x\sqrt{\pi}},\\ H_{-l}(x) \simeq \frac{1}{(2x)^l},\end{cases}$$

the last inequality reduces to

$$\begin{aligned}|E[C^n]-E[C^n]_K| &< \frac{n!e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{2\pi}(K+1)!}\left(\frac{(-1)^{K+1-n}S_{K+1}^{(n)}}{\left(\frac{\sigma(K+1)}{\xi}+\frac{\mu}{\sigma}\right)}\right.\\ &\quad \left.+\sum_{j=1}^{n}\left(\frac{\sigma}{\xi}\right)^{n-j}\frac{(-1)^{K+1-j}S_{K+1}^{(j)}}{\left(\frac{\sigma(K+1)}{\xi}-\frac{\mu}{\sigma}\right)^{n-j+1}}\right) \quad (12)\end{aligned}$$

For instance, if $n = 1$ (average capacity), the last inequality reduces to

$$|E[C]-E[C]_K| < \frac{\xi\sqrt{2}e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{\pi}\sigma(K+1+\frac{\xi\mu}{\sigma^2})(K+1-\frac{\xi\mu}{\sigma^2})}. \quad (13)$$

For a relatively large value of K, we will see in the numerical results section that the impact of the truncation of the series will be negligible.

III. The Average Capacity

In this section we are interested in the first moment of the capacity, the so-called ergodic capacity defined as $E[C]$. Setting $n = 1$ in (2), and using the following set of relations:

$$\begin{cases} S_k^{(1)} = (-1)^{k+1}(k-1)! \quad \text{if} \quad k \in \mathcal{N}^+,\\ H_{-1}(x) = \frac{\sqrt{\pi}}{2}\operatorname{erfcx}(x), \quad H_{-2}(x) = \frac{1}{2} - \frac{\sqrt{\pi}x}{2}\operatorname{erfcx}(x),\end{cases}$$

we obtain that the ergodic capacity of the log-normal channel is given by[1]

$$E[C] \;=\; E[C]_K + R_K, \tag{14}$$

where $E[C]_K$ is given by:

$$\begin{aligned} E[C]_K =\; & \frac{e^{-\frac{\mu^2}{2\sigma^2}}}{2}\left[\sum_{k=1}^{K}\frac{(-1)^{k+1}}{k}\operatorname{erfcx}\left(\frac{\sigma k}{\xi\sqrt{2}}+\frac{\mu}{\sqrt{2}\sigma}\right)\right.\\ & \left.+\sum_{k=1}^{K}\frac{(-1)^{k+1}}{k}\operatorname{erfcx}\left(\frac{\sigma k}{\xi\sqrt{2}}-\frac{\mu}{\sqrt{2}\sigma}\right)\right]\\ & +\frac{\mu}{2\xi}\operatorname{erfc}\left(-\frac{\mu}{\sqrt{2}\sigma}\right)+\frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\xi\sqrt{2\pi}}, \end{aligned} \tag{15}$$

and R_K is the remainder of the series given by:

$$\begin{aligned} R_K \;=\; & \frac{e^{-\frac{\mu^2}{2\sigma^2}}}{2}\left[\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k}\operatorname{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}+\frac{\mu}{\sqrt{2}\sigma})\right.\\ & \left.+\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k}\operatorname{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}-\frac{\mu}{\sqrt{2}\sigma})\right]. \end{aligned} \tag{16}$$

For sufficiently large values of K, by using $\operatorname{erfcx}(x) \simeq \frac{1}{x\sqrt{\pi}}$, we obtain that

$$R_K \simeq \frac{e^{-\frac{\mu^2}{2\sigma^2}}\xi}{\sigma\sqrt{2\pi}}\left[\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k(k+\frac{\xi\mu}{\sigma^2})}+\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k(k-\frac{\xi\mu}{\sigma^2})}\right]. \tag{17}$$

Here two cases can be distinguished:

- First case ($\mu \neq 0$):
  It can be easily shown using partial fractional decomposition that

$$\begin{aligned} \sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k(k\pm\frac{\xi\mu}{\sigma^2})} =\; & \pm\frac{\sigma^2}{\xi\mu}\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k}\\ & \mp\frac{\sigma^2}{\xi\mu}(-1)^K\beta(K\pm\frac{\xi\mu}{\sigma^2}+1), \end{aligned} \tag{18}$$

  where $\beta(\cdot)$ is given by [19, Eq. (8.372)] as

$$\beta(x) = \sum_{k=0}^{+\infty}\frac{(-1)^k}{k+x} = \frac{1}{2}\left(\psi\left(\frac{x+1}{2}\right)-\psi\left(\frac{x}{2}\right)\right), \tag{19}$$

  where $\psi(\cdot)$ is the Digamma function defined by [19]

$$\psi(x) = \frac{d}{dx}\ln(\Gamma(x)), \tag{20}$$

  where $\Gamma(\cdot)$ is the Gamma function. Finally, we obtain that

$$R_K \cong \frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\mu\sqrt{2\pi}}(-1)^K\left[\beta(K+1-\frac{\xi\mu}{\sigma^2})-\beta(K+1+\frac{\xi\mu}{\sigma^2})\right]. \tag{21}$$

- Second case ($\mu = 0$):
  If $\mu = 0$, we apply the same procedure and use the fact that ((0.234.1) in [19])

$$\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k^2} = \frac{\pi^2}{12}-\sum_{k=1}^{K}\frac{(-1)^{k+1}}{k^2}, \tag{22}$$

  to obtain that

$$R_K \cong \frac{\xi\sqrt{2}}{\sigma\sqrt{\pi}}\left[\frac{\pi^2}{12}-\sum_{k=1}^{K}\frac{(-1)^{k+1}}{k^2}\right]. \tag{23}$$

IV. A Simple Approximation to the Moments of the Capacity

In [8], by approximating $1+\gamma$ by a log-normal RV, we were able to provide a simple approximation to the average capacity. We extend here this approach to obtain higher order moments of the capacity. Hence we have $C \approx \widehat{C} \sim \mathcal{N}\left(\mu_{\widehat{C}}, \sigma^2_{\widehat{C}}\right)$, where

$$\begin{cases} \mu_{\widehat{C}} = \ln\left(\frac{(1+\Gamma)^2}{\sqrt{1+2\Gamma+e^{\frac{\sigma^2}{\xi^2}}\Gamma^2}}\right), \\ \sigma^2_{\widehat{C}} = \ln\left(\frac{1+2\Gamma+e^{\frac{\sigma^2}{\xi^2}}\Gamma^2}{(1+\Gamma)^2}\right). \end{cases}$$

Using the closed-form expression of the moments of the Gaussian distribution provided in [18], the moments of the capacity can be approximated as follows

$$E[C^n] \approx n!\sigma^n_{\widehat{C}}\sum_{j=0}^{\lfloor n/2\rfloor}\frac{1}{2^j j!(n-2j)!}\left(\frac{\mu_{\widehat{C}}}{\sigma_{\widehat{C}}}\right)^{n-2j}. \tag{24}$$

This approximation has the advantage of being simpler. However, its accuracy heavily depends on the value of σ: for small values of σ this approximation will be more accurate, but for large values of σ the approximation accuracy heavily deteriorates. We will further investigate this point in the numerical results section.

V. Capacity with Maximum Ratio Combining and Equal Gain Combining

At the output of an M-branch maximum ratio combiner and equal gain combiner, the instantaneous received SNR is, respectively, given by:

$$\begin{cases} \gamma_{\text{mrc}} = \sum_{i=1}^{M}\gamma_i, \\ \gamma_{\text{egc}} = \frac{1}{M}(\sum_{i=1}^{M}\sqrt{\gamma_i})^2. \end{cases}$$

Each SNR's branch γ_i is log-normally distributed with a logarithmic standard deviation equal to σ_i and a logarithmic mean equal to $\mu_i = \Gamma_{\text{dB}} - \frac{\sigma_i^2}{2\xi}$. As in [8], we resort to the log-normal approximation [20]-[25] that states that the sum[2] of log-normal random variables can be correctly represented by another log-normal RV. Therefore, γ_{mrc} and γ_{egc} will be viewed as log-normal variates. The analysis in [8] was conducted in independent fading. Since in practical cases correlation exists, we generalize our previous work to account for any possible correlation between the diversity branches. Here, we propose

[1]We should note here that Schwartz and Yeh [22] obtained a similar expression in the context of approximating the distribution of the sum of log-normal random variables.

[2]Since the square, the square root, as well as the multiplication by a constant of a log-normal RV are all log-normal RVs, then γ_{egc} is also a sum of log-normal variates.

to use the extension of Wilkinson's method developed in [20]. According to this method, we have the following expressions:

$$\begin{cases} \mu_{mrc} = \xi \ln(M\Gamma) - \frac{\sigma_{mrc}^2}{2\xi}, \\ \sigma_{mrc}^2 = \xi^2 \ln\left(\frac{1}{M^2}\sum_{i,j=1}^{M} e^{\rho_{ij}\frac{\sigma_i\sigma_j}{\xi^2}}\right), \end{cases}$$

and

$$\begin{cases} \mu_{egc} = \xi \ln\left(\frac{\Gamma}{M}\sum_{i,j=1}^{M} e^{\rho_{ij}\frac{\sigma_i\sigma_j}{4\xi^2} - \frac{(\sigma_i^2+\sigma_j^2)}{8\xi^2}}\right) - \frac{\sigma_{egc}^2}{2\xi}, \\ \sigma_{egc}^2 = 4\xi^2 \ln\left(\frac{1}{M^2}\sum_{i,j=1}^{M} e^{\rho_{ij}\frac{\sigma_i\sigma_j}{4\xi^2}}\right), \end{cases}$$

where ρ_{ij} denotes the correlation coefficient between $10\log_{10}(\gamma_i)$ and $10\log_{10}(\gamma_j)$. Note that for i.i.d. fading, these expressions reduce to those in [8]. Finally, the capacity moments with MRC and EGC can be calculated by substituting these values in the previously obtained results.

VI. Numerical Results

Figs. 1 and 2 show the first and second moments of the capacity. The standard deviation is equal to 3 dB in Fig. 1 and to 6 dB in Fig. 2, and in both figures the sums in the analytical formula are truncated at the 10th term. These figures depict clearly the adequacy between the results obtained by Monte-Carlo simulations and those generated by the analytical formula (2). The approximation given by (24) is only accurate for relatively small standard deviations. This is because (24) is based on the well-known Fenton-Wilkinson approximation which is not very accurate for large standard deviations. When the standard deviation increases (like in Fig. 2), the accuracy of (24) degrades[3]. This is, however, not the case for (2) which retains its accuracy for all the values of σ, even for a small truncation order like $K = 10$.

For the computation of the average capacity, at equal complexity, the first approximation in [8] is the most precise, because the coefficients a_k in [8, (9)] are tailored in such a way to give a very accurate approximation. As seen in Figs. 1 and 2, the accuracy of the second approximation in [8] (which is given in (24) for $n = 1$) heavily depends on the accuracy of the Fenton-Wilkinson method. The formula developed in this letter (2) and [8, (9)] both give approximately the same results and are identical[4] to the results obtained by Monte-Carlo simulations. However, the advantage of (2) is that it provides a generic solution to compute all the moments of the capacity.

The fact that truncating the series does not impair the accuracy is depicted in Fig. 3, where we have plotted the upper bound on the truncation error given by the right-hand side of the inequality (12). It can be seen also that the error decreases as the SNR increases.

Fig. 4 illustrates the capacity versus the SNR over a log-normal fading channel with two different antenna settings; $M = 2$ antennas (dual diversity) and $M = 8$ antennas.

[3]Note that the degradation will be more severe for higher standard deviations.

[4]For the sake of clarity, the curve representing the performance of [8, (9)] is not shown in the figure.

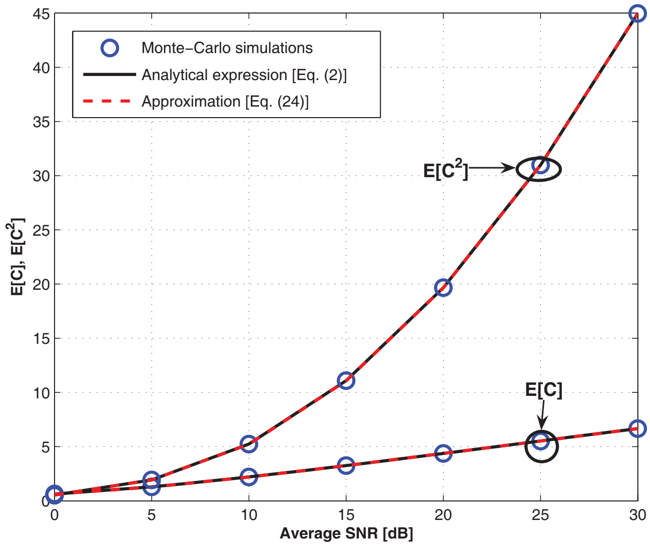

Fig. 1. The first and the second moments of the capacity ($\sigma = 3$; $K = 10$).

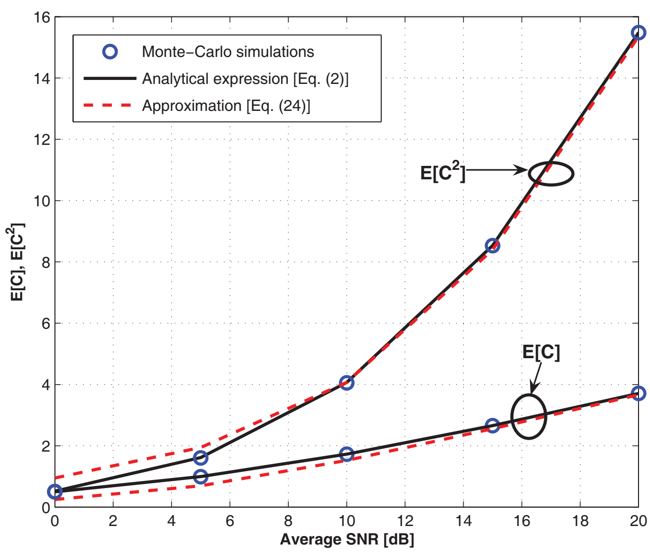

Fig. 2. The first and the second moments of the capacity ($\sigma = 6$; $K = 10$).

The correlation model for these figures is exponential, i.e., $\rho_{ij} = \rho^{|i-j|}$ with ρ taking the value 0.1. As described in the figure, the logarithmic variance of the received power at each antenna was set to either 5 or 6 dB. Here again, it can be seen that the capacity generated by the analytical formula accurately approximates the capacity given by Monte-Carlo simulations.

VII. Conclusion

In this letter, we have provided an analytical expression for the moments of the capacity of log-normal fading channels. Because the developed expression contains infinite series, we showed that the error resulting from truncating these series can be neglected. Since the sum of log-normal RVs is well approximated by another log-normal RV, the developed formula is used as well to evaluate the capacity of uncorrelated/correlated log-normal channels with Maximum Ratio Combining and Equal Gain Combining. The analytical expressions obtained match perfectly the capacity given by simulations.

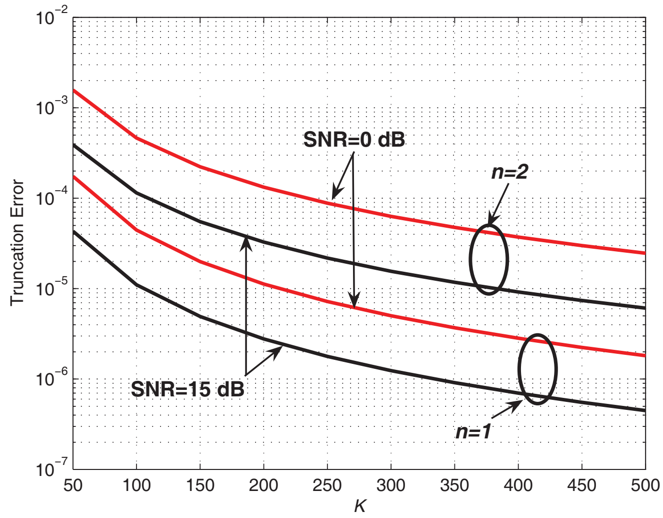

Fig. 3. The truncation error as a function of K ($\sigma = 6$ dB).

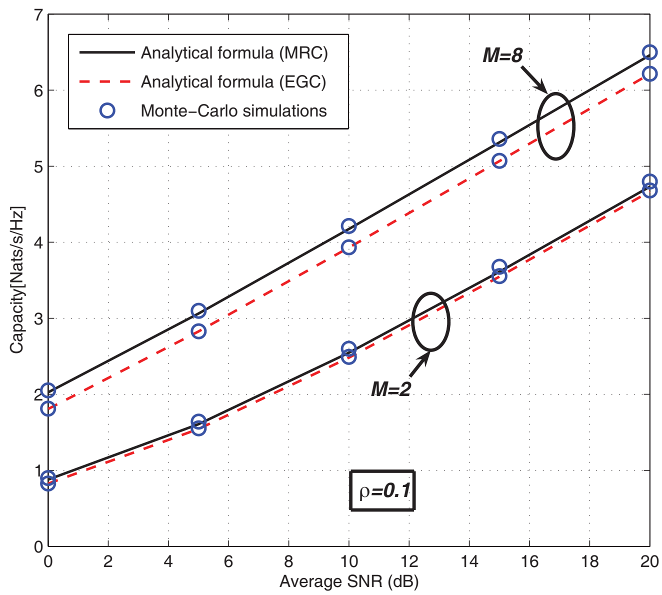

Fig. 4. Capacity of maximum ratio combining and equal gain combining in a correlated log-normal fading environment (ρ=0.1) with $M = 2$ antennas ($\sigma_1 = 5$; $\sigma_2 = 6$) and $M = 8$ antennas ($\sigma_1 = \ldots = \sigma_4 = 5$; $\sigma_5 = \ldots = \sigma_8 = 6$).

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