Interference-Limited Mixed Málaga-$\mathcal{M}$ and Generalized-$\mathcal{K}$ Dual-Hop FSO/RF Systems

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***Abstract*—This paper investigates the impact of radio frequency (RF) cochannel interference (CCI) on the performance of dual-hop free-space optics (FSO)/RF relay network. The considered FSO/RF system operates over mixed Málaga-$\mathcal{M}$/composite fading/shadowing generalized-$\mathcal{K}$ ($\mathcal{GK}$) channels with pointing errors. The H-transform theory, wherein integral transforms involve Fox's H-functions as kernels, is embodied into a unifying performance analysis framework that encompasses closed-form expressions for the outage probability, the average bit error rate (BER), and the ergodic capacity. By virtue of some H-transform asymptotic expansions, the high signal-to-interference-plus-noise ratio (SINR) analysis culminates in easy-to-compute expressions of the outage probability and BER.**

***Index Terms*— Amplify-and-forward (AF), free-space optics (FSO), generalized-$\mathcal{K}$, interference, Málaga-$\mathcal{M}$ distribution, pointing errors.**

I. Introduction

Free-space optics (FSO) communication has recently drawn a significant attention as one promising solution to cope with radio frequency (RF) wireless spectrum scarcity [1]. Though securing high data rates, FSO communications performance significantly degrades due to atmospheric turbulence-induced fading and strong path-loss [2]. Aiming to address these shortcomings, relay-assisted FSO systems have been actually identified as an influential solution to provide more efficient and wider networks. As such, understanding the fundamental system performance limits of mixed FSO/RF architectures has attracted a lot of research endeavor in the past decade (cf. [3]-[4] and references therein).

The performance of dual-hop FSO relaying networks was investigated in [3]-[5] assuming restrictive irradiance probability density function (PDF) models for the FSO link such as lognormal and the Gamma-Gamma fading models, for weak and strong atmospheric turbulence conditions, respectively. The Málaga-$\mathcal{M}$ distribution, recently proposed in [4], proved to be an attractive statistical model offering an excellent fit with the turbulent medium under all range of optical fluctuations. The $\mathcal{M}$-distribution is an extremely versatile distribution encompassing several FSO turbulence models such as Log-normal and Gamma-Gamma distributions. Recently the authors in [6] presented a comprehensive performance analysis of mixed FSO/RF systems while assuming a generalized $\mathcal{M}$-distribution fading model in the FSO link.

On the RF side, previous works typically assume either Nakagami-m [3], [7] or Rayleigh [8], [9] fading, thereby lacking the flexibility to account for disparate signal propagation mechanisms as those characterized in 5G communications which will accommodate a wide range of usage scenarios with diverse link requirements. In fact, in 5G communications design, the combined effect of small-scale and shadowed fading needs to be properly addressed. Shadowing, which is due to obstacles in the local environment or human body (user equipments) movements, can impact link performance by causing fluctuations in the received signal. For instance, the shadowing effect comes to prominence in millimeter wave (mmWave) communications due to the higher carrier frequency. In this respect, the generalized-$\mathcal{K}$ ($\mathcal{GK}$) model was proposed by combining Nakagami-m multi-path fading and Gamma-Gamma distributed shadowing [10],[11].

While FSO transmissions are robust to RF interference, mixed FSO/RF systems are inherently vulnerable to the harmful effect of co-channel interference (CCI) through the RF link (cf. [12] and references therein). Previous contributions pertaining to FSO relay-assisted communications [3]-[9] relied on the absence of CCI. Recently, the recognition of the interference-limited nature of emerging communication systems has motivated [13] to account for CCI in mixed decode and forward RF/FSO systems performance analysis. Besides ignoring the shadowing effect on the RF link, [13] assumes restrictive Gamma-Gamma model on the FSO link.

In this paper, motivated by the aforementioned challenges, we assess the impact of RF CCI on the per-

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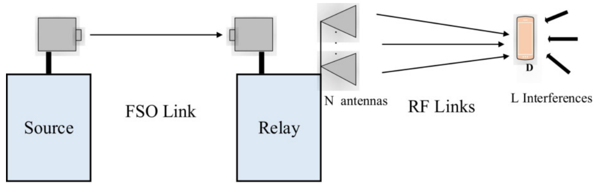

Fig. 1. Dual-hop interference-limited mixed FSO/RF AF relay network.

formance of dual-hop amplify and forward (AF) mixed FSO/RF system operating over Málaga-$\mathcal{M}$ and composite fading shadowing generalized-$\mathcal{K}$ ($\mathcal{GK}$) channels, respectively. Assuming fixed gain relaying and taking into account the effect of pointing errors while considering both heterodyne and intensity modulation/direct (IM/DD) detection techniques, we present a comprehensive performance analysis by exploiting seminal results form the H-transform theory.

II. Channel and System Models

We consider a downlink of a relay-assisted network featuring a mixed FSO/RF communication. We assume that the optical source (S) communicates with the destination (D) in a dual-hop fashion through an intermediate relay (R), able to activate both heterodyne and IM/DD detection techniques at the reception of the optical beam. Using AF relaying, the relay amplify the received optical signal and retransmit it to the destination D using MRT with N antennas. We assume that the destination is subject to inter-cell interference (I) brought by L co-channel RF sources in the network (See Fig.1).

The optical (S-R) channel follows a Málaga-$\mathcal{M}$ distribution for which the CDF of the instantaneous SNR γ_1 in the presence of pointing errors is given by

$$F_{\gamma_1}(x) = \frac{\xi^2 Ar}{\Gamma(\alpha)} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \mathrm{H}_{2,4}^{3,1}\left[\frac{B^r x}{\mu_r}\middle| \begin{matrix}(1,r),(\xi^2+1,r)\\ (\xi^2,r),(\alpha,r),(k,r),(0,r)\end{matrix}\right], \quad (1)$$

where ξ is the ratio between the equivalent beam radius and the pointing error displacement standard deviation (i.e., jitter) at the relay (for negligible pointing errors $\xi \to +\infty$) [2], $A = \alpha^{\frac{\alpha}{2}}\left[g\beta/(g\beta+\Omega)\right]^{\beta+\frac{\alpha}{2}} g^{-1-\frac{\alpha}{2}}$ and $b_k = \binom{\beta-1}{k-1}(g\beta+\Omega)^{1-\frac{k}{2}}\left[(g\beta+\Omega)/\alpha\beta\right]^{\frac{\alpha+k}{2}}(\Omega/g)^{k-1}(\alpha/\beta)^{\frac{k}{2}}$, where α, β, g and Ω are the fading parameters related to the atmospheric turbulence conditions [4]. Moreover in (1), $\mathrm{H}_{p,q}^{m,n}[\cdot]$ and $\Gamma(\cdot)$ stand for the Fox-H function [14, Eq.(1.2)] and the incomplete gamma function [15, Eq.(8.310.1)], respectively, and $B = \alpha\beta h(g+\Omega)/[(g\beta+\Omega)]$ with $h = \xi^2/(\xi^2+1)$. Furthermore, r is the parameter that describes the detection technique at the relay (i.e., $r = 1$ is associated with heterodyne detection and $r = 2$ is associated with IM/DD) and, μ_r refers to the electrical SNR of the FSO hop [4]. In particular, for $r = 1$, $\mu_1 = \mu_{\text{heterodyne}} = \mathbb{E}[\gamma_1] = \bar{\gamma}_1$ and for $r = 2$, $\mu_2 = \mu_{\text{IM/DD}} = \mu_1 \alpha\xi^2(\xi^2+1)^{-2}(\xi^2+2)(g+\Omega)/((\alpha+1)[2g(g+2\Omega)+\Omega^2(1+1/\beta)])$ [4, Eq.(8)].

The RF (R-D) and (I-D) links are assumed to follow generalized-$\mathcal{K}$ fading distribution. Hence the probability density function (PDF) the instantaneous SNR (respectively INR), γ_{XD}, $X \in (R, I)$, follows a generalized-$\mathcal{K}$ probability density function (PDF) given by [10, Eq.(5)]

$$f_{\gamma_{XD}}(x) = \frac{2\left(\frac{m_X\kappa_X}{\bar{\gamma}_{XD}}\right)^{\frac{\kappa_X+\delta_X m_X}{2}} x^{\frac{\kappa_X+\delta_X m_X}{2}-1}}{\Gamma(\delta_X m_X)\Gamma(\kappa_X)} K_{\kappa_X-\delta_X m_X}\left(2\sqrt{\frac{\kappa_X m_X x}{\bar{\gamma}_{XD}}}\right), \quad (2)$$

where $X \in \{R, I\}$ and $K_\nu(\cdot)$ stands for the modified Bessel function of the second kind [15, Eq.(8.407.1)]. Moreover, $m_X \geqslant 0.5$ and $\kappa_X \geqslant 0$ denote the multipath fading and shadowing severity of the X-Dth channel coefficient, respectively. Moreover, $\delta_X = \{N, L\}$ for $X \in \{R, I\}$ follows form the conservation property under the summation of N and L i.i.d $\mathcal{GK}$ random variables, where N is the relay antennas number and L is the number of interfering signals which are assumed to be independent identically distributed (i.i.d) $\mathcal{GK}$ with parameters m_I and κ_I. Using [15, Eq.(9.34.3)], the PDF of the $\mathcal{GK}$ distribution can be represented in terms of the Meijer's-G function as

$$f_{\gamma_{XD}}(x) = \frac{\frac{m_X\kappa_X}{\bar{\gamma}_{XD}}}{\Gamma(\delta_X m_X)\Gamma(\kappa_X)} \mathrm{G}_{0,2}^{2,0}\left[\frac{\kappa_X m_X}{\bar{\gamma}_{XD}} x \middle| \begin{matrix} - \\ \delta_X m_X-1, \kappa_X-1\end{matrix}\right], \quad (3)$$

The CDF of the SIR $\gamma_2 = \gamma_{RD}/\gamma_{ID}$ under $\mathcal{GK}$ fading can be derived from a recent result in [11, Lemma 1] as

$$F_{\gamma_2}(x) = 1 - \frac{1}{\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)} \mathrm{G}_{3,3}^{3,2}\left[\frac{\kappa m x}{\kappa_I m_I \bar{\gamma}_2}\middle| \begin{matrix}1-\kappa_I, 1-Lm_I, 1\\ 0, \kappa, Nm\end{matrix}\right], \quad (4)$$

where $\bar{\gamma}_2 = \bar{\gamma}_{RD}/\bar{\gamma}_{ID}$ is the average signal-to-interference-ratio (SIR) of the RF link where, for consistency, we have dropped the subscript R from the parameters m_R and κ_R.

Hence, for fixed-gain AF relaying protocol, the end-to-end SINR at the destination is [16, Eq.(2)]

$$\gamma = \frac{\gamma_1\gamma_2}{\gamma_2 + \mathrm{C}}, \quad (5)$$

where C stands for the fixed gain at the relay.

III. End-to-End Statistics

A. Cumulative Distribution Function

The CDF of the end-to-end SINR of interference-limited dual-hop FSO/RF systems using fixed-gain relay in Málaga-$\mathcal{M}$/$\mathcal{GK}$ fading under both heterodyne detection and IM/DD is given by

$$F_\gamma(x) = \frac{\xi^2 A\kappa m \mathrm{C}}{\Gamma(\alpha)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \bar{\gamma}_2} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \mathrm{H}^{0,1:0,3:4,3}_{1,0:3,2:4,5}\left[\begin{array}{c|c} \frac{\mu_r}{B^r x} & (0,1,1) \\ & - \\ \frac{\kappa m \mathrm{C}}{\kappa_I m_I \bar{\gamma}_2} & (\delta,\Delta) \\ & (\lambda,\Lambda) \\ & (\chi, X) \\ & (\upsilon,\Upsilon) \end{array}\right], \quad (6)$$

where where $\mathrm{H}^{m_1,n_1:m_2,n_2:m_3,n_3}_{p_1,q_1:p_2,q_2:p_3,q_3}[\cdot]$ denotes the Fox-H function (FHF) of two variables [17, Eq.(1.1)] also known as the bivariate FHF whose Mathematica implementation may be found in [18, Table I], whereby $(\delta,\Delta) = (1-\xi^2, r), (1-\alpha, r), (1-k, r)$; $(\lambda,\Lambda) = (0,1), (-\xi^2, r)$; $(\chi, X) = (-1,1), (-\kappa_I, 1), (-Lm_I, 1), (0,1)$; and $(\upsilon,\Upsilon) = (-1,1), (-1,1), (\kappa-1), (Nm-1), (0,1)$

Proof: See Appendix A ■

B. Probability Distribution Function

The PDF of the end-to-end SINR γ in mixed Málaga-$\mathcal{M}$/$\mathcal{GK}$ is obtained as

$$f_\gamma(x) = \frac{-\xi^2 A\kappa m \mathrm{C} x^{-1}}{\Gamma(\alpha)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \bar{\gamma}_2} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \mathrm{H}^{0,1:0,3:4,3}_{1,0:3,2:4,5}\left[\begin{array}{c|c} \frac{\mu_r}{B^r x} & (0,1,1) \\ & - \\ \frac{\kappa m \mathrm{C}}{\kappa_I m_I \bar{\gamma}_2} & (\delta,\Delta) \\ & (\lambda',\Lambda') \\ & (\chi, X) \\ & (\upsilon,\Upsilon) \end{array}\right], \quad (7)$$

where $(\lambda',\Lambda') = (1,1), (-\xi^2, r)$;

Proof: The result follows from differentiating the Mellin-Barnes integral in (6), applying [14, Eq.(2.57)], over x using $\frac{dx^{-s}}{dx} = -sx^{-s-1}$ with $\Gamma(s+1) = s\Gamma(s)$. ■

IV. Performance Analysis

A. Outage Probability

The quality of service (QoS) of the considered mixed FSO/RF system is ensured by keeping the instantaneous end-to-end SNR, γ, above a threshold γ_{th}.

Proposition 1: The outage probability of the considered mixed FSO/RF system follows from (6) as

$$P_{\text{out}} = F_\gamma(\gamma_{th}). \quad (8)$$

Corollary 1 (Asymptotic Outage Probability): At high normalized average SNR in the FSO link ($\frac{\mu_r}{\gamma_{th}} \to \infty$), the outage probability of the system under consideration is obtained as

$$\begin{aligned} P^{\infty}_{\text{out}} = & \frac{\xi^2 A\kappa m \mathrm{C}}{\Gamma(\alpha)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \bar{\gamma}_2} \\ & \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \left[\frac{\Gamma(\alpha-\xi^2)\Gamma(k-\xi^2)}{r\Gamma(1-\frac{\xi^2}{r})} \quad \Xi\left(\gamma_{th}, \frac{\xi^2}{r}\right) \right. \\ & + \frac{\Gamma(\xi^2-\alpha)\Gamma(k-\alpha)}{r\Gamma(1-\frac{\alpha}{r})\Gamma(1+\xi^2-\alpha)} \quad \Xi\left(\gamma_{th}, \frac{\alpha}{r}\right) \\ & + \frac{\Gamma(\xi^2-k)\Gamma(\alpha-k)}{r\Gamma(1-\frac{k}{r})\Gamma(1+\xi^2-k)} \quad \Xi\left(\gamma_{th}, \frac{k}{r}\right) \\ & \left. + \frac{B^r \gamma_{th}}{\mu_r} \mathrm{H}^{7,3}_{6,8}\left[\frac{\kappa m \mathrm{C} B^r \gamma_{th}}{\kappa_I m_I \bar{\gamma}_2 \mu_r} \middle| \begin{array}{c} (\sigma,\Sigma) \\ (\phi,\Phi) \end{array} \right] \right], \end{aligned} \quad (9)$$

where

$$\Xi(x,y) = \left(\frac{B^r x}{\mu_r}\right)^y \mathrm{G}^{4,4}_{5,5}\left[\frac{\kappa m \mathrm{C}}{\kappa_I m_I \bar{\gamma}_2} \middle| \begin{array}{c} -\kappa_I, -Lm_I, -1, y, 0 \\ \kappa-1, Nm-1, -1, -1, 0 \end{array} \right], \quad (10)$$

$(\sigma,\Sigma) = (-\kappa_I, 1), (-Lm_I, 1), (-1,1), (0,1), (1+\xi^2-r, r), (0,1)$, and $(\phi,\Phi) = (\xi^2-r, r), (\alpha-r, r), (k-r, r), (\kappa-1, 1), (Nm-1, 1), (-1,1), (-1,1), (0,1)$.

Proof: Resorting to the Mellin-Barnes representation of the bivariate FHF [14, Eq.(2.57)] in (6) and applying [19, Theorem 1.7] yield (9) after some additional algebraic manipulations.

Remark: At high SNR values, the outage probability of the mixed FSO/RF relaying system can be expressed as $P_{\text{out}} \simeq (c\text{SNR})^{-d}$, where c and d denote the coding gain and the diversity order of the system, respectively. When $\bar{\gamma}_2 \to \infty$, then by applying [19, Theorem 1.11] to (9) while only keeping the dominant term, it follows that, for a fixed CCI power, the diversity order of mixed FSO/RF systems with pointing errors over Málaga-$\mathcal{M}$/$\mathcal{GK}$ fading conditions is shown to be given by

$$d = \min\left\{Nm, \kappa, \frac{\xi^2}{r}, \frac{\alpha}{r}, \frac{k}{r}\right\}. \quad (11)$$

In particular, for Nakgami-m fading ($\kappa \to \infty$) $d = \min\left\{Nm, \frac{\xi^2}{r}, \frac{\alpha}{r}, \frac{k}{r}\right\}$, while it becomes $d = \min\left\{Nm, \frac{\xi^2}{r}, \frac{\alpha}{r}, \frac{\beta}{r}\right\}$ for mixed FSO/RF under Gamma-Gamma turbulence-induced fading. ■

B. Average Bit-Error Rate

For a variety of modulation schemes, the average BER is expressed in terms of the end-to-end SINR PDF as

$$\overline{P}_e = \frac{\delta}{2\Gamma(p)} \sum_{j=1}^{n} \int_0^\infty \Gamma(p, q_j x) f_\gamma(x) \mathrm{d}x, \quad (12)$$

where $\Gamma(\cdot,\cdot)$ stands for the incomplete Gamma function [15, Eq.(8.350.2)] and the parameters δ, n, p and q_j account for different modulations schemes [10].

Proposition 2: The average BER of mixed Málaga-$\mathcal{M}$/$\mathcal{GK}$ AF relay systems in the presence of CCI and operating under both heterodyne and IM/DD detection techniques with pointing errors at the FSO link is given by

$$\overline{P}_e = \frac{\xi^2 A\delta\kappa m \mathrm{C}}{2\Gamma(\alpha)\Gamma(p)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \bar{\gamma}_2}$$
$$\sum_{j=1}^{n}\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)}\mathrm{H}^{0,1:1,3:4,3}_{1,0:3,3:4,5}\left[\begin{array}{c|c} \frac{\mu_r q_j}{B^r} & (0,1,1) \\ & - \\ & (\delta,\Delta) \\ \frac{\kappa m \mathrm{C}}{\kappa_I m_I \bar{\gamma}_2} & (p,1),(\lambda,\Lambda) \\ & (\chi, X) \\ & (\upsilon,\Upsilon) \end{array}\right]. \quad (13)$$

Proof: The average BER follows after substituting the Mellin-Barnes integral form of (7) using [14, Eq.(2.56)] into (12). More specifically, using the identity $\Gamma(a,bx) = \mathrm{G}^{2,0}_{1,2}[bx|\,{}^{1}_{0,a}]$ and [15, Eq.(7.811.4)], i.e.,

$$\int_0^\infty x^{-s-1}\mathrm{G}^{2,0}_{1,2}\left[q_j x \left| \begin{array}{c} 1 \\ 0,p \end{array}\right.\right]\mathrm{d}x = \frac{\Gamma(-s)\Gamma(p-s)}{\Gamma(1-s)}q_j^s, \quad (14)$$

thereby yielding (13) after some manipulations. ■

Corollary 2 (Asymptotic Average BER): At high SNR regime (i.e. $\mu_r \to \infty$), the asymptotic average BER is obtained as

$$\overline{P}_e^\infty = \frac{\xi^2 A\delta\kappa m \mathrm{C}}{2\Gamma(\alpha)\Gamma(p)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \bar{\gamma}_2}$$
$$\sum_{j=1}^{n}\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)}\left[\frac{\Gamma(\alpha-\xi^2)\Gamma(k-\xi^2)}{r\Gamma(1-\frac{\xi^2}{r})}\Xi\left(\frac{1}{q_j}\frac{\xi^2}{r}\right)\right.$$
$$+\frac{\Gamma(\xi^2-\alpha)\Gamma(k-\alpha)}{r\Gamma(1-\frac{\alpha}{r})\Gamma(1+\xi^2-\alpha)}\quad\Xi\left(\frac{1}{q_j},\frac{\alpha}{r}\right)$$
$$+\frac{\Gamma(\xi^2-k)\Gamma(\alpha-k)}{r\Gamma(1-\frac{k}{r})\Gamma(1+\xi^2-k)}\quad\Xi\left(\frac{1}{q_j},\frac{k}{r}\right)$$
$$\left.+\frac{B^r}{\mu_r q_j}\mathrm{H}^{7,4}_{7,8}\left[\frac{\kappa m \mathrm{C} B^r}{\kappa_I m_I \bar{\gamma}_2 \mu_r q_j}\left| \begin{array}{c}(\sigma',\Sigma') \\ (\phi,\Phi)\end{array}\right.\right]\right], \quad (15)$$

where $(\sigma',\Sigma') = (-\kappa_I,1),(-Lm_I,1),(-1,1),(-p,1),(0,1),(1+\xi^2-r,r),(0,1)$.

Proof: The asymptotic BER follows in the same line of (9). ■

C. Ergodic Capacity

Proposition 3: The ergodic capacity of mixed Málaga-$\mathcal{M}$/interference-limited $\mathcal{GK}$ transmission system under both detection techniques with pointing errors at the FSO link is obtained as

$$\overline{C} = \frac{\xi^2 A\kappa m \mathrm{C}}{2\ln(2)\Gamma(\alpha)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \bar{\gamma}_2}$$
$$\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)}\mathrm{H}^{0,1:1,4:4,3}_{1,0:4,3:4,5}\left[\begin{array}{c|c} \frac{\mu_r}{B^r x} & (0,1,1) \\ & - \\ & (\delta,\Delta),(1,1) \\ \frac{\kappa m \mathrm{C}}{\kappa_I m_I \bar{\gamma}_2} & (0,1)(\lambda',\Lambda') \\ & (\chi, X) \\ & (\upsilon,\Upsilon) \end{array}\right]. \quad (16)$$

Proof: The ergodic capacity $\overline{C} = \frac{1}{2}\mathbb{E}\left[\ln_2(1+\gamma)\right]$ follows from averaging $\ln(1+\gamma) = \mathrm{G}^{1,2}_{2,2}[\gamma|\,{}^{1,1}_{1,0}]$ over the end-to-end SINR PDF obtained in (7) while resorting to [17, Eq.(1.1)] and [15, Eq.(7.811.4)] with some manipulations. ■

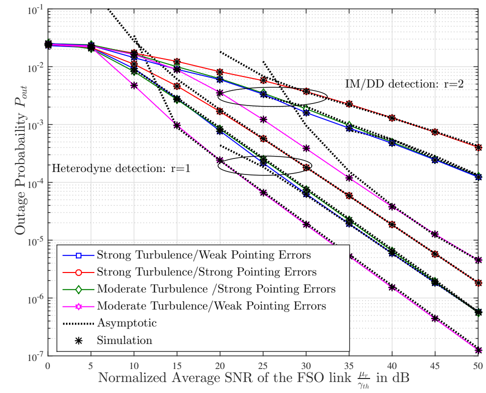

Fig. 2. Outage probability of a fixed gain mixed RF/FSO network with interference under different turbulence and pointing errors severities with $N = L = 2$, $m = 2.5$, and $\kappa = 1.09$.

Corollary 3: Consider mixed Gamma-Gamma/Nakagami-m fading. The Málaga-$\mathcal{M}$ reduces to Gamma-Gamma fading when ($g = 0$, $\Omega = 1$), whence all terms in (1) vanish except for the term when $k = \beta$. Hence, when $\kappa, \kappa_I \to \infty$, (16) reduces, when $r = 1$, to the ergodic capacity of mixed Gamma-Gamma FSO/interference-limited Nakagami-m RF transmission with heterodyne detection given by

$$\overline{C} = \frac{\xi^2}{2\ln(2)\Gamma(Nm)\Gamma(Lm_I)\Gamma(\alpha)\Gamma(\beta)}\mathrm{G}^{1,0:1,4:3,2}_{1,0:4,3:4,3}$$
$$\left[\frac{\mu_1}{\alpha\beta h};\frac{m\mathrm{C}}{m_I\bar{\gamma}_2}\left|\begin{array}{c}1\\-\end{array}\right|\begin{array}{c}1-\xi^2,1-\alpha,1-\beta,1\\1,0,-\xi^2\end{array}\left|\begin{array}{c}1-Lm_I,1,0\\Nm,0,1\end{array}\right.\right], \quad (17)$$

where $\mathrm{G}[\cdot;\cdot]$ is the bivariate Meijer-G function [15].

V. Numerical Results

In this section, numerical examples are shown to substantiate the accuracy of the new unified mathematical framework and to confirm its potential for analyzing mixed FSO/RF communications. Remarkably, all numerical results obtained by the direct evaluation of the analytical expressions developed in this paper, are in very good match with their Monte-Carlo stimulated counterparts showing the accuracy and effectiveness of the performance analysis framework. Unless stated otherwise, all the simulations were carried with the following parameters: $\mathrm{C} = 1.7$, $m_I = 1.5$, $\kappa_I = 3.5$, and $\bar{\gamma}_2 = 20$ dB.

Fig.2 illustrates the outage probability of mixed FSO/RF fixed-gain AF systems versus the FSO link normalized average SNR in strong $\alpha = 2.4$, $\beta = 2$ and weak $\alpha = 5.4$, $\beta = 4$ turbulence conditions, respectively. The figure also investigates the effect of strong ($\xi = 1.1$) and weak ($\xi = 6.8$) pointing errors on the system performance. As expected, the outage probability deteriorates by decreasing the pointing error displacement standard deviation, i.e., for smaller ξ, or

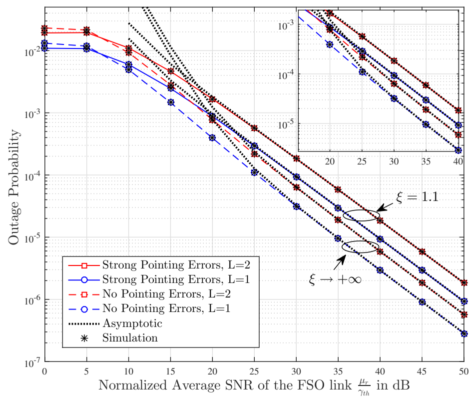

Fig. 3. Outage Probability of an interference-limited fixed-gain mixed RF/FSO network in strong turbulence conditions for different values of L and ξ with $N = L = 2$, $m = 2.5$, and $\kappa = 1.09$.

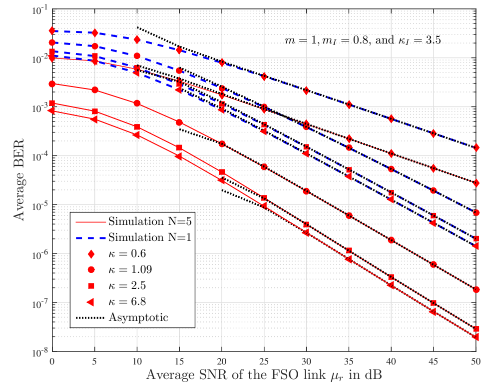

Fig. 5. Average BER of an interference-limited fixed-gain mixed RF/FSO network in strong turbulence conditions for different values of κ and antenna number at the relay N.

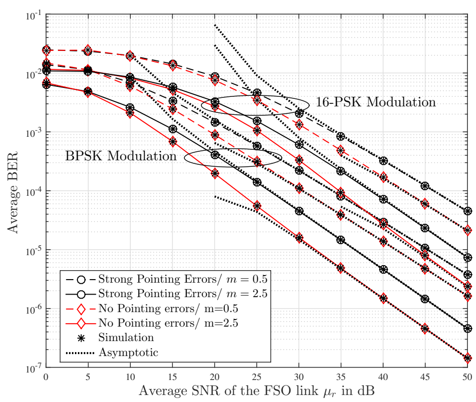

Fig. 4. Average BER of an interference-limited fixed-gain mixed RF/FSO network in strong turbulence conditions for different values of m with $N = L = 2$, and $\kappa = 1.09$.

decreasing the turbulence fading parameter, i.e., smaller α and β. At high SNR, the asymptotic expansion in (9) matches very well its exact counterpart, which confirms the validity of our mathematical analysis for different parameter settings. On the other hand, we observe that the heterodyne detection better performs in turbulent environments than the IM/DD, a behavior also identified in [4].

Fig.3 depicts the outage probability of mixed FSO/intefrence-limited RF systems with $L = \{1, 2\}$ versus the FSO link normalized average SNR. As expected increasing L deteriorates the system performance, by increasing the outage probability, while the diversity gain remains unchanged. Once again we highlight that the exact and asymptotic expansion in (9) agree very well at high SNRs.

Actually, the $\mathcal{GK}$ fading/shadowing parameters m and κ are important parameters that affect the system performance as shown in Fig.4 and Fig.5, respectively. As can be seen, heavy shadowing (small κ) and/or severe fading (small m) are detrimental for the system performance. In Fig.5, we fix $\alpha = 2.4$, $\beta = 2$, $\xi = 6.8$ and $r = 2$. It can be noticed that, expect for $\kappa = 0.6$, all curves have the same slopes thereby inferring that they have the same diversity order. This is due to the fact that the system diversity order is dependent on $d = \min\left\{Nm, \kappa, \frac{\xi^2}{r}, \frac{\alpha}{r}, \frac{k}{r}\right\}$. For the two curves when $\kappa = 0.6$, they have the same slope revealing equal diversity order $d = \kappa$. Fig.4 and Fig.5 also show that the asymptotic expansion in (15) agree very well with the simulation results, hence corroborating its usefulness.

The impact of the relay antennas number N on the system BER is investigated in Fig.5 under several shadowing conditions. As discussed in Corollary 2, spacial diversity resulted from employing higher antennas number N at the relay enhances the overall performance of the system.

Fig.6 shows the impact of the FSO link atmospheric turbulence conditions on system capacity performance. We can see that that decreasing α and β (i.e., stronger turbulence conditions) deteriorates the system capacity, notably when IM/DD is employed. It is clear from this figure that weaker turbulence conditions leads to the situation where the RF link dominates the system performance thereby inhibiting any performance improvement coming from the FSO link.

VI. Conclusion

We have studied the performance of relay-assisted mixed RF/FSO network with RF interference and two different detection techniques. The H-transform theory is involved into a unified performance analysis framework featuring closed-from expressions for the outage probability, the BER and the channel capacity assuming Málaga-$\mathcal{M}$/composite fading/shadowing $\mathcal{GK}$ channel models for the FSO/RF links while taking into account pointing errors. Results show that the system diversity order is related to the the minimum value of the at-

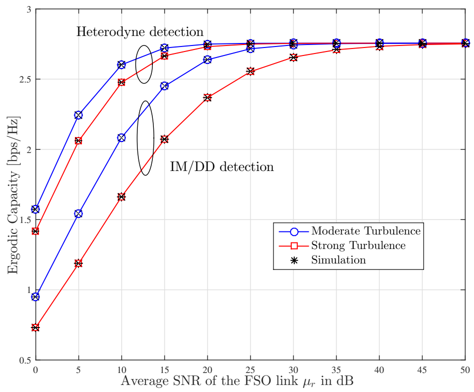

Fig. 6. Ergodic Capacity of an interference-limited fixed-gain mixed RF/FSO network in strong and weak turbulence conditions, with $N = L = 2$, $m = 2.5$, and $\kappa = 1.09$.

mospheric turbulence, small-scale fading, shadowing and pointing error parameters.

Appendix A
CDF of the End-to-End SINR

The CDF of the end-to-end SINR γ with fixed-gain relaying scheme can be derived, using [16, Eq.(8)], as

$$F_\gamma(x) = \int_0^\infty F_{\gamma_1}\left(x\left(\frac{\mathrm{C}}{y}+1\right)\right) f_{\gamma_2}(y)\mathrm{d}y, \quad (18)$$

where F_{γ_1} and f_{γ_2} are the FSO link's CDF and the PDF of RF link, respectively. f_{γ_2} is derived by differentiation of (4) over x as

$$f_{\gamma_2}(x) = \frac{-\kappa m}{\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \bar{\gamma}_2} \mathrm{G}_{4,4}^{3,3}\left[\frac{\kappa m x}{\kappa_I m_I \bar{\gamma}_2}\middle| \begin{matrix} -1, -\kappa_I, -Lm_I, 0 \\ -1, \kappa-1, Nm-1, 0 \end{matrix}\right] \quad (19)$$

Substituting (1) and (19) into (18) while resorting to the integral representation of the Fox-H [14, Eq.(1.2)] and Meijer-G [15, Eq.(9.301)] functions yields

$$\begin{aligned} F_\gamma(x) &= \frac{-\xi^2 A r \kappa m}{\Gamma(\alpha)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \bar{\gamma}_2} \\ &\sum_{k=1}^{\beta}\frac{b_k}{\Gamma(k)}\frac{1}{4\pi^2 i^2}\int_{\mathcal{C}_1}\int_{\mathcal{C}_2}\frac{\Gamma(\xi^2+rs)\Gamma(k+rs)\Gamma(\alpha+rs)}{\Gamma(\xi^2+1+rs)\Gamma(1-rs)} \\ &\times\frac{\Gamma(-rs)\Gamma(-1-t)}{\Gamma(1+t)}\frac{\Gamma(\kappa-1-t)\Gamma(Nm-1-t)}{\Gamma(-t)} \\ &\times\Gamma(2+t)\Gamma(1+\kappa_I+t)\Gamma(1+Lm_I+t)\left(\frac{\kappa m \mathrm{C}}{\kappa_I m_I \bar{\gamma}_2}\right)^t \\ &\left(\frac{B^r x}{\mu_r}\right)^{-s}\int_0^\infty\left(1+\frac{\mathrm{C}}{y}\right)^{-s} y^t \mathrm{d}y \ \mathrm{d}s \ \mathrm{d}t, \end{aligned} \quad (20)$$

where $i^2 = -1$, and $\mathcal{C}_1$ and $\mathcal{C}_2$ denote the s and t-planes, respectively. Finally, simplifying $\int_0^\infty\left(1+\frac{\mathrm{C}}{y}\right)^{-s} y^t\mathrm{d}y$ to $\frac{\mathrm{C}^{1+t}\Gamma(-1-t)\Gamma(1+t+s)}{\Gamma(s)}$ by means of [15, Eqs(8.380.3) and (8.384.1)] while utilizing the relations $\Gamma(1-rs) = -rs\Gamma(-rs)$, and $s\Gamma(s) = \Gamma(1+s)$ then [17, Eq.(1.1)] yield (6).

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