On the Performance Analysis of Mixed Multi-aperture FSO/Multiuser RF Relay Systems with Interference.

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***Abstract*—In this paper, we consider multi-aperture multiuser dual-hop amplify-and-forward (AF) free-space optical/radio frequency (FSO/RF) communication systems where the FSO and RF links, respectively, experience Málaga $\mathcal{M}$-distribution and Nakagami-m fading. Moreover, the effect of co-channel interference is considered at users. Under the assumption of transmit aperture selection at the source and opportunistic scheduling at the destination, we derive the exact expression for the ergodic capacity in terms of bivariate Meijer's-G function. Additionally, resorting to extreme value theory, we derive closed-form expressions for the asymptotic ergodic capacity, all in terms of Meijer's G and elementary functions, whenever the number of apertures at the source and/or users grows large.**

***Index Terms*—Amplify-and-forward (AF), co-channel interference, ergodic capacity, free-space optics (FSO), Málaga $\mathcal{M}$-distribution.**

I. Introduction

Recently, free-space optical (FSO) has drawn a significant attention due to its advantages to higher bandwidth in unregulated spectrum and higher capacity compared to its RF counterpart [1]. Hence, the gathering of both FSO and RF technologies arises as a promising solution for securing connectivity between the RF access network and the fiber-optic-based backbone network. As such, there has been prominent interest in mixed FSO/RF systems where RF transmission is used at one hop and FSO transmission at the other. Most contributions within this research line consider single-aperture single-user communications with many irradiance probability density function (PDF) models for the FSO link. The most commonly utilized models are the lognormal [2] and the Gamma-Gamma [3]. Recently, a new and generalized statistical model, the Málaga $\mathcal{M}$-distribution, was proposed in [4] to model the irradiance fluctuation of an unbounded optical wavefront (plane or spherical waves) propagating through a turbulent medium under all irradiance conditions in homogeneous, isotropic turbulence [4]. The $\mathcal{M}$-distribution is a generalized distribution that unifies most statistical models proposed so far with its ability to better reflect a wider range of turbulence conditions. On the RF side, previous works typically assume Rayleigh fading [5],[6] while Nakagami-m distribution is considered only in [7].

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Aiming to further increase the system capacity and reliability, another line of work dedicated to multiuser relay-assisted networks with multi-aperture FSO communications has been longing for understanding such systems [8]-[9]. In this context, the Málaga $\mathcal{M}$-distribution was only considered in [9] with Rayleigh distributed user links.

In this paper, we consider a mixed multi-aperture multiuser FSO/RF system where the FSO and RF links, respectively, experience Málaga $\mathcal{M}$ and Nakagami-m fading. We further take into account the effect of co-channel interference which might be detrimental in RF links. This paper quantifies accurately the capacity of mixed FSO/RF AF relay-assisted networks with opportunistic scheduling among users and transmit selection at the source's apertures.

The paper is organized as follows. Section II introduces the system model of multi-aperture multiuser mixed FSO/RF interference-limited AF relay communications. Section III derive the exact expression of the ergodic capacity of mixed FSO/RF relay systems with finite-count apertures and finite-count users. In section IV, we derive the scaling-law expressions of the ergodic capacity when either count or both grow very large. Finally, we provide and discuss numerical results in Section V before concluding in Section VI.

II. System Model

We consider a mixed FSO/RF relay system where a multi-aperture source S communicates with multiple users ($D_k, k = 1, \ldots, K$) via a relay R. It is assumed that L RF co-channel interferers impinge on each user D_k (cf Fig. 1). The FSO and RF links are assumed to follow the $\mathcal{M}$ and Nakagami-m distributions, respectively. Assume that the source is equipped with M FSO apertures, then the PDF of the m-th aperture irradiance $I_m, m = 1, \ldots, M$ is given by [4]

$$f_{I_m}(x) = A \sum_{k=1}^{\beta} a_k x^{\frac{\alpha+k}{2}-1} K_{\alpha-k}\left(2\sqrt{\frac{\alpha\beta x}{\mu\beta+\Omega}}\right), \qquad (1)$$

where $A = \frac{2\alpha^{\frac{\alpha}{2}}}{\mu^{1+\frac{\alpha}{2}}\Gamma(\alpha)}\left(\frac{\mu\beta}{\mu\beta+\Omega}\right)^{\beta+\frac{\alpha}{2}}$ and $a_k = \binom{\beta-1}{k-1}\frac{(\mu\beta+\Omega)^{1-\frac{k}{2}}}{(k-1)!}\left(\frac{\Omega}{\mu}\right)^{k-1}\left(\frac{\alpha}{\beta}\right)^{\frac{k}{2}}$ with α, β, μ, and Ω are the fading parameters related to the atmospheric

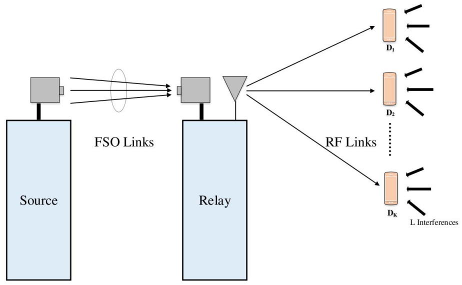


Fig. 1. Two-hop multiuser mixed FSO/RF AF relay network. Each RF user is affected by co-channel interference.

turbulence conditions [10] where $\mu = 2b_0(1-\rho)$ with $2b_0$ is the average power of the LOS term and ρ represents the amount of scattering power coupled to the LOS component $(0 \leqslant \rho \leqslant 1)$. Moreover, in (1) $K_\nu(\cdot)$ denotes the νth-order modified Bessel function of the second kind [11, Eq.8.407.1]. It is worth highlighting that the $\mathcal{M}$ distribution unifies most of the proposed statistical models characterizing the optical irradiance in homogeneous and isotropic turbulence (cf. [4, ch. 8. Table 1]). Hence both Gamma-Gamma and $\mathcal{K}$ models are special cases of the Málaga-$\mathcal{M}$ distribution, as they mathematically derive from (1) by setting ($\rho = 1$, $\Omega = 1$) and ($\rho = 0$, $\Omega = 0$ or $\beta = 1$), respectively.

The instantaneous SNR of the m-th FSO link is $\bar{\gamma}_1 I_m$, where $\bar{\gamma}_1$ is the average electrical SNR of the FSO link. With transmit selection at the source, the aperture with the highest received channel gain is selected among the M available apertures at the optical transmitter. As a result, the SNR of the selected aperture is given by

$$\gamma^{FSO} = \bar{\gamma}_1 \max_{m=1,\ldots,M}(I_m). \tag{2}$$

The average SNR of the RF link is $\bar{\gamma}_{RD}$. Moreover, the instantaneous SNR is given by $\gamma_{RD_k} = \bar{\gamma}_{RD}|h_k|^2, k = 1,\ldots,K$, where h_k is the channel fading gain between the relay R and the k-th user. Under the assumption of Nakagami-m distribution, we have $|h_k|^2 \sim \mathfrak{g}(m_1, 1/m_1)$, where $\mathfrak{g}(m, \Omega/m)$ is the Gamma distribution with shape and scale parameters m and Ω, respectively. The PDF of the SNR is given by

$$f_{\gamma_{RD_k}}(x) = \frac{m_1^{m_1}}{\bar{\gamma}_{RD}^{m_1}\Gamma(m_1)} x^{m_1-1} e^{-\frac{m_1}{\bar{\gamma}_{RD}}x}. \tag{3}$$

On the other hand, the SNR of the RF link between an interferer $\mathcal{I}_l, l = 1,\ldots,L$ and the destination user D_k is given by $\gamma_{\mathcal{I}_l,D_k} \overset{d}{\sim} \mathfrak{g}(m_2, \bar{\gamma}_{ID}/m_2)$, where $\bar{\gamma}_{ID}$ is the average interference-noise-ratio (INR), and m_2 is the Nakagami parameter of the $\mathcal{I}_l - D_k$ link. It is known that the sum of L i.i.d Gamma random variables (RVs) with shape parameter ϱ and scale parameter δ, is also a Gamma RV with parameters $L\varrho$ and δ. By defining $\gamma_{\mathcal{I},D} \triangleq \sum_{l=1}^{L} \gamma_{\mathcal{I}_l,D}$ as the overall INR, the PDF of $\gamma_{\mathcal{I},D_k}$ can be written as

$$f_{\gamma_{\mathcal{I},D_k}}(x) = \frac{m_2^{Lm_2}}{\bar{\gamma}_{ID}^{Lm_2}\Gamma(Lm_2)} x^{Lm_2-1} e^{-\frac{m_2}{\bar{\gamma}_{ID}}x}. \tag{4}$$

With opportunistic scheduling at the relay, the user with the largest signal-to-interference-ratio (SIR) is selected. The SIR of the RF link under consideration can be written as

$$\gamma^{RF} = \bar{\gamma}_2 \max_{k=1,\ldots,K}\left(\frac{\gamma_{RD_k}}{\gamma_{\mathcal{I},D_k}}\right), \tag{5}$$

where $\bar{\gamma}_2 = \frac{\bar{\gamma}_{RD}}{\bar{\gamma}_{ID}}$ is the average SIR of the RF link. Under the assumption of variable-gain relaying, the SINR of the mixed FSO/RF link can be written as [7, Eq.(28)]

$$\gamma^{FSO,RF} = \frac{\gamma^{FSO}\gamma^{RF}}{\gamma^{FSO} + \gamma^{RF} + 1}. \tag{6}$$

Hereafter, we provide capacity formulas for the considered system by using the complementary moment generation function CMGF-based approach

$$\begin{aligned} C &\triangleq \frac{\mathrm{E}[\ln(1+\gamma^{FSO,RF})]}{2\ln(2)} \\ &\overset{(a)}{=} \frac{1}{2\ln(2)} \int_0^\infty s e^{-s} M^{(c)}_{\gamma^{FSO}}(s) M^{(c)}_{\gamma^{RF}}(s) ds, \end{aligned} \tag{7}$$

where $M_X^{(c)}(s) = \int_0^\infty e^{-sx} F_X^{(c)}(x)dx$ stands for the CMGF with $F_X^{(c)}(x)$ denoting the complementary cumulative distribution function (CCDF) of X.

Capitalizing on (7), a new mathematical framework investigating the average capacity of the considered multiuser multi-aperture mixed FSO/RF relay network with opportunistic user scheduling is presented in the next section.

III. Exact Analysis of the Capacity with Finite-Count Apertures and Users

According to (7), the exact capacity analysis amounts to studying the CMGFs of the first and second hop SNR and SIR, respectively. As a result, the analysis will be simplified as shown subsequently.

Lemma 1: Let $\gamma_m^{FSO} = \bar{\gamma}_1 I_m, \quad m = 1,\ldots,M$ be the instantaneous SNR of m-th FSO link following the $\mathcal{M}$-distribution. Then the CMGF of $\gamma^{FSO} = \max_{m=1,\ldots,M}(\gamma_m^{FSO})$ is obtained as

$$M^{(c)}_{\gamma^{FSO}}(s) = \sum_{t=1}^{M} \sum_{\Upsilon} \frac{\Theta_t}{s^{\frac{\delta_t}{2}+1}} G^{1,2}_{2,1}\left(\frac{\bar{\gamma}_1(\mu\beta+\Omega)s}{t^2\alpha\beta} \middle| \begin{matrix} 1, \frac{1}{2} \\ \frac{\delta_t}{2}+1 \end{matrix}\right), \tag{8}$$

where $\sum_\Upsilon = \sum_{\Upsilon_{t,\beta}} \sum_{\Upsilon_{t_p,\alpha-t_p+\frac{1}{2}}} \sum_{\Upsilon_{t_{pq}\alpha+t_p-t_{pq}-\frac{1}{2}}}$, with $\Upsilon_{z,l} = \{(z_1,\ldots,z_l) : z_i \geqslant 0, \sum_{i=1}^{l} z_i = z\}$; Θ_t is given in (9); $\delta_t = \sum_{l=0}^{\alpha+t_p-t_{pq}-\frac{3}{2}} l t_{p_{q_{l+1}}}$, and $G^{m,n}_{p,q}(\cdot,\cdot)$ is the Meijer-G function [11, Eq.9.301].

Proof: From (1) and using [11, Eq 8.468], the PDF of the m-th FSO link I_m can be written as

$$f_{I_m}(x) = A\sqrt{\pi} \sum_{k=1}^{\beta} \sum_{j=0}^{\alpha-k-\frac{1}{2}} \frac{a_k(\alpha-k-\frac{1}{2}+j)!}{\left(4\sqrt{\frac{\alpha\beta}{(\mu\beta+\Omega)}}\right)^{j+\frac{1}{2}} (\alpha-k-\frac{1}{2}-j)!j!} x^{\frac{\alpha+k}{2}-\frac{5}{4}-\frac{j}{2}} \exp\left(-2\sqrt{\frac{\alpha\beta x}{(\mu\beta+\Omega)}}\right). \tag{10}$$

$$\Theta_t = \frac{\prod_{p=1}^{\beta}\prod_{q=1}^{\alpha-t_p+\frac{1}{2}}\prod_{r=1}^{\alpha+t_p-t_{p_q}-\frac{1}{2}}\left(\frac{2^{\frac{1}{2}-\alpha-t_p-t_{p_q}-t_{p_{q_r}}}Aa_{t_p}\sqrt{\pi}(\alpha-t_p-\frac{3}{2}+t_{p_q})!(\alpha+t_p-\frac{1}{2}-t_{p_q})!(\mu\beta+\Omega)^{\frac{\alpha+t_p-t_{p_{q_r}}+1}{2}}}{(\alpha\beta)^{\frac{\alpha+t_p-t_{p_{q_r}}+1}{2}}(\alpha-t_p+\frac{1}{2}-t_{p_q})!(t_{p_q}-1)!(t_{p_{q_r}}-1)!}\right)^{t_{p_{q_r}}}\binom{M}{t}(-1)^{t+1}t!}{\prod_{k=1}^{\alpha+t_p-t_{p_q}-\frac{1}{2}}t_{p_{q_k}}!\sqrt{\pi}\bar{\gamma}_1^{\frac{\delta_t}{2}}}. \tag{9}$$

Therefore, its CCDF is obtained by resorting to the identity [11, Eq 3.351.2] as

$$F_{I_m}^{(c)}(x) = A\sqrt{\pi}\sum_{k=1}^{\beta}\sum_{j=0}^{\alpha-k-\frac{1}{2}}\frac{a_k(\alpha-k-\frac{1}{2}+j)!2^{\frac{1}{2}-\alpha-k-j}}{\left(\sqrt{\frac{\alpha\beta}{(\mu\beta+\Omega)}}\right)^{\alpha+k}(\alpha-k-\frac{1}{2}-j)!j!}$$
$$\Gamma\left(\alpha+k-j-\frac{1}{2},2\sqrt{\frac{\alpha\beta x}{(\mu\beta+\Omega)}}\right), \tag{11}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function [11, Eq.(8.350.2)]. By substituting the incomplete Gamma function in (11) by its series expansion in [11, Eq 8.352.2] and applying the multinomial expansion, the CCDF of $\gamma^{FSO} = \bar{\gamma}_1 \max_{m=1,\dots,M}(I_m)$ is obtained as

$$F_{\gamma^{FSO}}^{(c)}(x) = \sqrt{\pi}\sum_{t=1}^{M}\sum_{\Upsilon}\Theta_t x^{\frac{\delta_t}{2}}\exp\left(-2t\sqrt{\frac{\alpha\beta}{\bar{\gamma}_1(\mu\beta+\Omega)}x}\right). \tag{12}$$

The Laplace transform of the FSO link CCDF yields its CMGF obtained as in (8) after applying [11, Eq.3.462.1] with some manipulations.

Lemma 2: Let the instantaneous SIR of the k-th RF link be $\gamma_k^{RF} = \bar{\gamma}_2\left(\frac{\gamma_{RD_k}}{\gamma_{\mathcal{I},D_k}}\right), k = 1,\dots,K$, as the ratio of two Gamma-distributed RVs. The CMGF of $\gamma^{RF} = \max_{k=1,\dots,K}(\gamma_k^{RF})$ is given by

$$M_{\gamma^{RF}}^{(c)}(s)=\sum_{q=0}^{\Delta-1}\phi_q\Psi\left(q+1,q+2-\Delta,\frac{\bar{\gamma}_2 m_2 s}{m_1}\right)-\sum_{r=1}^{K}\sum_{\Upsilon_{r,Lm_2-1}}\varphi_r$$
$$\Psi\left(\delta_r+2-r(m_1-1),\delta_r+\Delta+r(1-m_1)+1,\frac{\bar{\gamma}_2 m_2 s}{m_1}\right), \tag{13}$$

where $\Delta = K(Lm_2+m_1-1)$; $\phi_q = \binom{\Delta}{q}\frac{m_1}{m_2}\Gamma(q+1)\bar{\gamma}_2$; $\Psi(a,b,x)$ is the Triconomi confluent Hypergeometric function [11, Eq.9.211.1]; and $\varphi_r = \frac{\binom{K}{r}m_1 r!\prod_{p=0}^{Lm_2-2}\left(\frac{(1-Lm_2)_p(-1)^p}{(1+m_1)_p}\right)^{r_{p+1}}\bar{\gamma}_2\Gamma(\delta_r+\Delta+1)}{m_2(m_1B(Lm_2,m_1))^r\prod_{m=1}^{Lm_2-1}r_m!}$.

Proof: Let $r_k = \frac{\gamma_{RD_k}}{\gamma_{\mathcal{I},D_k}}, k = 1,\dots,K$, then from (3) and (4) and resorting to [11, Eq.1.194.1], the CCDF of r_k follows as

$$F_{r_k}(x) = \frac{\left(\frac{m_1x}{m_2}\right)^{m_1}{}_2F_1\left(Lm_2+m_1,m_1;m_1+1;-\frac{m_1x}{m_2}\right)}{m_1B(Lm_2,m_1)}$$
$$\overset{(a)}{=}\frac{\left(1+\frac{m_1x}{m_2}\right)^{1-Lm_2-m_1}}{m_1B(Lm_2,m_1)}\sum_{p=0}^{Lm_2-1}\frac{(1-Lm_2)_p(-1)^p}{(1+m_1)_p}\left(\frac{\frac{m_1x}{m_2}}{\frac{m_1x}{m_2}+1}\right)^{p+m_1}, \tag{14}$$

where $B(a,b)$ and ${}_2F_1(a,b;c;x)$ denote the incomplete Beta function and the Gauss Hypergeometric function [11, Eq. 9.100.1], respectively, and (a) follows from substituting the Gauss Hypergeometric function by its finite series expansion [11]. The CCDF of $\gamma^{RF} = \bar{\gamma}_2 \max_{k=1,\dots,K}(r_k)$ is then obtained as

$$F_{\gamma^{RF}}^{(c)}(x) \overset{(b)}{=} \sum_{q=0}^{\Delta-1}\binom{\Delta}{q}\left(\frac{m_1x}{m_2\bar{\gamma}_2}\right)^q\left(1+\frac{m_1x}{m_2\bar{\gamma}_2}\right)^{-\Delta}$$
$$-\sum_{r=1}^{K}\sum_{\Upsilon_{r,Lm_2-1}}\frac{\varphi_r m_1\left(\frac{m_1x}{m_2\bar{\gamma}_2}\right)^{\delta_r+\Delta+r(1-Lm_2)}}{m_2\bar{\gamma}_2\Gamma(\delta_r+\Delta+1)\left(1+\frac{m_1x}{m_2\bar{\gamma}_2}\right)^{\Delta}}, \tag{15}$$

where (b) follows from applying the multinomial expansion [11, Eq.(1.111.1)]. The CMGF of γ^{RF} is obtained after resorting to [11, Eq. 9.211.1], thereby leading to (13) after some manipulations.

Proposition 1 (Closed-form expression of C): The ergodic capacity for arbitrary M and K in mixed FSO/RF AF systems with interference is obtained as in (16), where $G_{A,[C,E],B,[D,F]}^{p,q,r,s,t}(\cdot,\cdot)$ stands for the bivariate generalized Meijer-G function [14].

Proof: Plugging (8) and (13) into (7) reveals that the computation of C requires the resolution of integrals of the form

$$I = \int_0^{\infty}x^p e^{-x}G_{m,n}^{p,q}\left(yx\,\middle|\,\begin{matrix}(a)\\(b)\end{matrix}\right)\Psi(a_1,b_1,zx)dx. \tag{17}$$

where utilizing the identity $\Psi(a,b,x) = \frac{1}{\Gamma(a)\Gamma(a-b+1)}G_{1,2}^{2,1}\left(x\,\middle|\,\begin{matrix}1-a\\0,1-b\end{matrix}\right)$, (7) can be evaluated by means of the bivariate generalized Meijer-G function, as can be seen from a more general integral formula due to [14, Eq.(2.1)], thereby leading to (16) after some manipulations.

IV. Large-Scale Analysis of the Capacity

In this section, the ergodic capacity of the mixed FSO/RF relay network with massive aperture selection and/or user scheduling is considered. Therefore, by using extreme value theory, the asymptotic CMGF expressions are derived, for the case in which the numbers of apertures and users grow without bound. The new computed CMGF expressions of the two hops are given by the following lemmas.

Lemma 3: As the number of apertures M at the FSO link grows large, the CMGF expression of γ^{FSO} can be written as

$$M_{\gamma^{FSO}}^{(c)}(s) = \frac{1-\exp\left(-\bar{\gamma}_1\left(b_M-c_M\right)s\right)}{s}, \tag{18}$$

where b_M and c_M are constants such that

$$F_{I_m}^{(c)}(b_M) = \frac{1}{M}, \quad \text{and} \quad c_M = \frac{F_{I_m}^{(c)}(b_M)}{f_{I_m}(b_M)}, \tag{19}$$

$$C = \frac{1}{2\ln(2)} \sum_{t=1}^{M} \sum_{\Upsilon} \Theta_t \left(\sum_{q=0}^{\Delta-1} \frac{\phi_q}{\Gamma(q+1)\Gamma(\Delta)} G^{1,1,2,2,1}_{1,[1,2],0,[2,1]} \left(\frac{\bar{\gamma}_1(\mu\beta+\Omega)}{t^2\alpha\beta}, \frac{\bar{\gamma}_2 m_2}{m_1} \middle| \begin{matrix} 1-\frac{\delta_t}{2}; 1, \frac{1}{2}; -q \\ \frac{\delta_t}{2}+1; 0, \Delta-q-1 \end{matrix} \right) \right.$$

$$\left. - \sum_{r=1}^{K} \sum_{\Upsilon_{r,Lm_2-1}} \frac{\varphi_r}{\Gamma(\delta_r+2-r(m_1-1))\Gamma(2-\Delta)} G^{1,1,2,2,1}_{1,[1,2],0,[2,1]} \left(\frac{\bar{\gamma}_1(\mu\beta+\Omega)}{t^2\alpha\beta}, \frac{\bar{\gamma}_2 m_2}{m_1} \middle| \begin{matrix} 1-\frac{\delta_t}{2}; 1, \frac{1}{2}; r(1-m_1)-\delta_r-1 \\ \frac{\delta_t}{2}+1; 0, -\delta_r-\Delta-r(1-m_1) \end{matrix} \right) \right). \quad (16)$$

with $F^{(c)}_{I_m}$ and f_{I_m} given in (11) and (1), respectively.

Proof: We denote by the growth function $g_{I_m}(x) = F^{(c)}_{I_m}(x)/f_{I_m}(x)$. Then, by resorting to the Hospital's rule, it follows that

$$\lim_{x\to\infty} g_{I_m}(x) = \lim_{x\to\infty} \left(-1 - \frac{f'_{I_m}(x)(F^{(c)}_{I_m}(x))}{f^2_{I_m}(x)} \right),$$

$$\overset{(a)}{=} -1 - \lim_{x\to\infty} \frac{d(F^{(c)}_{I_m})}{d\left(\frac{f^2_{I_m}(x)}{f'_{I_m}(x)}\right)} = 0, \quad (20)$$

where in (a), f'_{I_m} is the derivative of f_{I_m}. and (b) follows from several mathematical manipulations. Accordingly, I_m belongs to the maximum domain of attraction ($\mathcal{MDA}$) of Gumbel distribution. As a result, for a massive apertures selection at the source, the CDF of the FSO link can be written as

$$\lim_{M\to\infty} F_{\gamma^{FSO}}(x) = \exp\left[-\exp\left(-\frac{x-\bar{\gamma}_1 b_M}{\bar{\gamma}_1 c_M} \right) \right]. \quad (21)$$

The Laplace transform of the CCDF of γ^{FSO} yields its CMGF given by

$$M^{(c)}_{\gamma^{FSO}}(s) = \int_0^\infty e^{-sx} \left(1 - e^{-e^{-\frac{x-\bar{\gamma}_1 b_M}{\bar{\gamma}_1 c_M}}} \right) dx,$$

$$\overset{(a)}{=} c_M \bar{\gamma}_1 \left[\int_0^{\frac{4}{\zeta}} t^{s c_M \bar{\gamma}_1 - 1} \left(1 - e^{-\zeta t} \right) dt + \int_{\frac{4}{\zeta}}^1 t^{s c_M \bar{\gamma}_1 - 1} dt \right], (22)$$

where (a) follows from letting $t = e^{-\frac{x}{c_M \bar{\gamma}_1}}$ and defining $\zeta = e^{\frac{b_M}{c_M}}$. We can easily see that the limit of the first term on the R.H.S of (22) becomes vanishingly small as $\lim_{M\to\infty} \frac{4}{\zeta} \approx 0$. Hence, the second integral on the R.H.S of (22) is obtained easily, thereby leading to (18) after some manipulations.

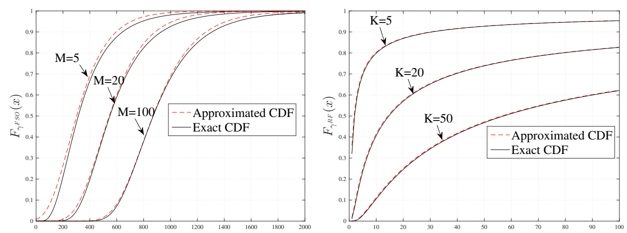
Lemma 4: As the number of users K at the RF link grows large, the CMGF of $\gamma^{RF} = \bar{\gamma}_2 \max_{k=1,\ldots,K}(r_k)$ is obtained as

$$M^{(c)}_{\gamma^{RF}}(s) = \frac{1 - \exp\left(-\bar{\gamma}_2 \left(\frac{m_2}{m_1} \left(\frac{K}{B(Lm_2, m_1) Lm_2} \right)^{\frac{1}{Lm_2}} - 1 \right) s \right)}{s}. \quad (23)$$

Proof: Applying ${}_2F_1(a, b; b+1; x) = b x^{-b} B_x(b, 1-a)$ to (14) and resorting to the Hospitals' rule yield

$$\lim_{x\to\infty} \frac{1 - F_{r_n}(x)}{1 - F_{r_n}(tx)} = \frac{t^{-m_1}(1+tx)^{Lm_2+m_1}}{(1+x)^{Lm_2+m_1}} = t^{Lm_2}, \quad (24)$$

which implies that $F_{\gamma^{RF}}(x)$ lies in the $\mathcal{MDA}$ of the Fréchet distribution [15, Th 11.5.2]. As a result, the CDF of the RF link can be written as



(a) (b)

Fig. 2. The exact and approximated CDF of (a): γ^{FSO} for different values of M, and (b): γ^{RF} for different values of K.

$$\lim_{K\to\infty} F_{\gamma^{RF}}(x) = \exp\left(-\left(\frac{x}{\bar{\gamma}_2 a_K} \right)^{-Lm_2} \right), \quad x \geqslant 0, \quad (25)$$

where $a_K = F^{(c)^{-1}}_{r_k}\left(\frac{1}{K}\right) = \frac{m_2}{m_1}\left(\frac{K}{B(Lm_2,m_1)Lm_2}\right)^{\frac{1}{Lm_2}} - 1$, obtained after applying ${}_2F_1(a,b,c,x) = (1-x)^{c-a-b}{}_2F_1(c-a, c-b, c, x)$ to (14) and resorting to the Gauss hypergeometric asymptotic expansion. Finally the CMGF of the second hop SIR under massive user scheduling is obtained as shown in (23).

Proposition 2 (Capacity for large M and K): For massive FSO apertures selection and user scheduling, the ergodic capacity of mixed FSO/RF AF system with interference is given in (26).

Proof: The result follows after plugging (18) and (23) into (7) and resorting to [11, Eq 3.421.4].

Proposition 3 (Capacity for fixed M and large K): The ergodic capacity for finite-count apertures and a massive user scheduling in mixed FSO/RF AF relay system with interference is obtained as in (27).

Proof: The result follows after plugging (8) and (23) into (7) and resorting to [11, Eq.7.813.1].

V. Numerical Results

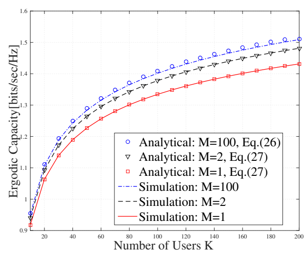
Here, we provide some numerical examples to illustrate the tightness of the closed-form expression of the ergodic capacity for the mixed FSO/RF AF relay system. The simulation setup is summarized in the caption of each figure.

Fig. 2 shows the exact and asymptotic CDFs of the two hops, γ^{FSO} and γ^{RF} for different values of M and K, respectively. We observe that the asymptotic distributions in (21) and (25) are a good approximations even for small values of M and K, respectively.

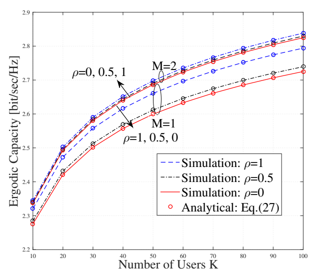
Fig. 3(a) depicts the ergodic capacity for large K with fixed and large M using (27) and (26), respectively. We

$$C = \frac{1}{2\ln(2)}\Bigg[\ln\left(1+\bar{\gamma}_1(b_M-c_M)\right)+\ln\Bigg(1+\bar{\gamma}_2\Bigg(\frac{m_2}{m_1}\left(\frac{K}{B(Lm_2,m_1)Lm_2}\right)^{\frac{1}{Lm_2}}-1\Bigg)\Bigg)$$
$$-\ln\Bigg(1+\bar{\gamma}_1(b_M-c_M)+\bar{\gamma}_2\Bigg(\frac{m_2}{m_1}\left(\frac{K}{B(Lm_2,m_1)Lm_2}\right)^{\frac{1}{Lm_2}}-1\Bigg)\Bigg)\Bigg]. \quad (26)$$

$$C = \frac{1}{2\ln(2)}\sum_{t=1}^{M}\sum_{\Upsilon}\Theta_t\left(G_{3,1}^{1,3}\left(\frac{\bar{\gamma}_1(\mu\beta+\Omega)}{t^2\alpha\beta}\,\middle|\,\begin{matrix}\frac{\delta_t}{2}+1,1,\frac{1}{2}\\ \frac{\delta_t}{2}+1\end{matrix}\right)-(1+a_K\bar{\gamma}_2)^{\frac{\delta_t}{2}}G_{3,1}^{1,3}\left(\frac{\bar{\gamma}_1(\mu\beta+\Omega)}{t^2\alpha\beta\left(1+a_K\bar{\gamma}_2\right)}\,\middle|\,\begin{matrix}\frac{\delta_t}{2}+1,1,\frac{1}{2}\\ \frac{\delta_t}{2}+1\end{matrix}\right)\right). \quad (27)$$



(a) Setup: $\alpha = 3.5, \beta = 2, \rho = 0.7, \bar{\gamma}_1 = 20$ dB, $m_1 = 3, m_2 = 2, L = 5, \bar{\gamma}_2 = 10$ dB.



(b) Setup: $\alpha = 3.5, \beta = 2, \bar{\gamma}_1 = 30$ dB, $m_1 = 1, m_2 = 2, L = 2, \bar{\gamma}_2 = 10$ dB.

Fig. 3. Ergodic capacity of mixed FSO/RF systems for different values of (a): M, and (b): ρ.

observe that the analytical curves approach the simulated curves for small to moderate values of K, thereby providing an attractive alternative to the cumbersome expression of the average capacity shown for any K in (16).

When M and K grow large, the obtained curves are in very good match with their stimulated counterparts showing the accuracy and effectiveness of the new approximation proposed in (26).

Fig. 3(b) investigates the effect of the atmospheric turbulence severity ρ on the system performance. As ρ increases, the atmospheric-turbulence over the FSO link is reduced thereby leading to better performance.

VI. Conclusion

In this paper, we derived closed-form expressions of the ergodic capacity for interference-limited mixed FSO/RF AF relay systems under the assumption of transmit aperture selection at the source and opportunistic scheduling at the destination. The system operates over mixed Málaga/Nakagami-*m* distributions. The large scale analysis reveals simpler and very accurate results thereby providing an attractive alternative to the cumbersome expression obtained in the case of finite aperture and user counts. The accuracy of the derived expressions were unambiguously illustrated numerically.

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