Opportunistic Scheduling in Downlink Multiple-Antenna AF Interfered Networks

Imène Trigui, Imen Mechmeche, Sofiène Affes, and Alex Stéphenne

***Abstract*—In this paper, multiuser multiple-antenna (MU-MIMO) relay networks employing opportunistic scheduling and operating in the presence of rayleigh fading and co-channel interference are analyzed. Notwithstanding the system complexity, due to the newly found complementary moment generating function transform (CMGF) operator, an exact expression for the capacity under general conditions is obtained. Moreover, driven by the fact that communication devices have grown much faster than the infrastructure relay support, a specific wireless setup, which consists of a large number of users K and a relatively small antenna number is investigated. Simulation results indicate a rather fast convergence to the asymptotic limits with the system's size, thereby demonstrating the practical importance of the scaling results.**

***Index Terms*—Amplify-and-forward, MIMO systems, multiuser diversity, opportunistic scheduling, co-channel interference.**

I. Introduction

Driven by the surge of shared data volume and connected devices, multiuser multiple-antenna (MU-MIMO) relaying networks have drawn recently a significant attention, as a promising solution to cope with the necessities of more efficient and larger networks. Aiming to enhance multiuser capacity, multiple antenna communications have been actually identified as a key enabling technique to secure the unprecedented data deluge these large networks are deemed to convey. As such, there has been prominent activity in the past decade toward understanding the fundamental system capacity limits of such architectures, notably when interference-limited, the ultimate nature of future cellular networks.

Many contributions spearheaded this line of research by considering the combination of cooperative and multiuser diversities in the context of single-antenna communications [1]-[3]. It has been shown in multiuser dual-hop amplify-and-forward (AF) relaying networks, that the end-to-end signal-to-noise ratio (SNR) is an inadequate criterion for reduced-feedback approaches with threshold-based SNR. Alternatively, the second-hop SNR which requires less attendant in complexity at the relay, turns out to be more promising in terms of achieved capacity.

Aiming to further increase the system capacity and reliability, another line of work dedicated to multiuser relay-assisted networks with multi-antenna communications is longing for understanding such systems. For instance, in [4], the capacity in the range of bits per second per hertz (b/s/Hz) is achieved by allowing the communication, through an AF relay, of multiple antenna devices and a source, using receive antenna diversity. A full knowledge of local channels (backward channels from all source antennas and forward channels to all destinations) was assumed at the relay, making thereby distributed beamforming possible. It has been shown, only through empirical trials, that spatial diversity at the destinations deteriorates the system capacity and, further, burdens the feedback cost.

Although these works have made great strides toward understanding MU-MIMO relay-assisted communications, they all rely on the absence of the harmful effect of co-channel interference (CCI). The recognition of the interference-limited nature of emerging communication systems, such as heterogeneous cellular networks, has motivated several works to investigate the impact of CCI on the performance of relay networks for different fading models and communication setups [5]-[8]. In [6], a novel analytical capacity expression for two-hop multiple antenna AF relaying systems have been proposed. The more general case of multihop interference-limited communications has also been treated in [7]. However the works in [5] and [8] only consider a single-user scenario. So far, CCI assessment in the context of multiuser relaying networks has only recently been considered in [9] by harnessing on opportunistic scheduling. This work, however, provides only bounds on the system capacity without characterizing its scaling laws.

II. System model

Consider a half-duplex multiple-antenna AF relay network with a source and a relay equipped with M and N antennas, respectively, and K single-antenna receivers (or users). The system setup operates in an interference-limited environment over frequency-flat Rayleigh fading. Let $\mathbf{Y}_i$ be the $1 \times N$ received signal vector at the relay from the i-th source's antenna element given by

$$\mathbf{Y}_i = \sqrt{P_S}\mathbf{h}_i^S x + \underbrace{\sum_{l=1}^{L} \sqrt{\mu_l}\mathbf{v}_l b_l}_{\mathbf{i}_{T_1}}, \qquad i = 1, \ldots, M \qquad (1)$$

where $\mathbf{h}_i^S$ is a $1 \times N$ complex channel vector from the i-th source's antenna element to the relay and $\mathbf{v}_l, l = 1, \ldots, L$ is the l-th interference vector. The entries of $\mathbf{h}_i^S$ and $\mathbf{v}_l$ are assumed, in what follows, to be independent and identically distributed (i.i.d.) zero mean complex Gaussian random variables. Let us denote by ρ the received power from the i-th transmit antenna and, hence, from (1), $\rho = P_s E\left\{\left|\mathbf{h}_i^S x\right|^2\right\}$. We also denote by $\mathbf{i}_{T_1}$ the aggregate interference vector due to L interferers at the relay. Upon reception of the source's signal, the relay employs the receive-MRC reconstitution to obtain $y_i = \mathbf{w}_i^\dagger \mathbf{Y}_i$ where $\mathbf{w}_i = \mathbf{h}_i^S / \|\mathbf{h}_i^S\|$. Afterward, y_i is amplified with a gain $G = \sqrt{\left(P_S|\mathbf{h}_i^S|^2 + \sum_{l=1}^{L}\mu_l|\mathbf{v}_l|^2\right)^{-1}}$ before transmission to the j-th user which is surrounded by

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Q_j interferences. The received signal at the j-th user is then given by

$$z_j = \mathbf{F}_j^{\dagger}\sqrt{P_r}G\mathbf{h}_j^R y_i + \underbrace{\sum_{q=1}^{Q_j}\sqrt{\nu_{j,q}}u_{j,q}c_{j,q}}_{i_{T_{2j}}}, \qquad j=1,\ldots,K \tag{2}$$

where $\mathbf{h}_j^R$ is a $1\times N$ complex channel vector from the relay to the j-th user and $u_{j,l}$ is the channel between the latter and the q-th interference. It follows from (2) that the received power at the j-th user is $\lambda_j = P_r E\left\{\left|\mathbf{h}_j^R\right|^2\right\}, j=1,\ldots,K$. In (2), $i_{T_{2j}}$ the aggregate interference due to Q_j interferers at the j-th user. In the sequel, we assume that the entries of $\mathbf{h}_i^R$ and $u_{j,l}$ $j=1,\ldots,K; l=1\ldots,Q_j$ are i.i.d. zero mean complex Gaussian random variables. In order to recover the original signal at the j-th user, the relay recurs to the transmit beamforming vector $\mathbf{F}_j = \mathbf{h}_j^R/\|\mathbf{h}_j^R\|$. The channel gains $|\mathbf{h}_i^S|^2, i=1\ldots,M$ and $|\mathbf{h}_j^R|^2, j=1,\ldots,K$ are assumed here to be quasi-static, i.e., they remain unchanged during a complete two-hop transmission session but vary independently in different sessions. We also assume that the relay and the j-th user are aware of their backward channels $\mathbf{h}_i^S$ and $\mathbf{h}_j^R$, respectively, i.e., the CSI at the receiver (CSIR) assumption is adopted. It is noteworthy that the assumption on the channel knowledge entails acceptable signaling overhead deemed reasonable even for decentralized implementation. In this paper, $\{x, b_i, c_{j,l}\}, i=1\ldots L, j=1,\ldots,K, l=1\ldots,Q_j$ are letters from codewords of a Gaussian capacity-achieving codebook.

Now, let us describe the scheduling scheme employed at each hop. As far as the first hop (i.e., source to relay communication) is concerned, the relay is aware of $\mathbf{h}_i^S, i=1,\ldots,M$ and, hence, able to schedule the transmission of the strongest source antenna, say $i_M = \underset{i=1\ldots M}{\operatorname{argmax}}|\mathbf{h}_i^S|^2$, by feeding back the index i_M at the beginning of the data bloc. The overhead incurred at this phase of the protocol does not exceed a single integer. Commonly known as transmit antenna selection (TAS), this technique requires an instantaneous and error-free feedback from the relay. Using TAS, the relay receives a superposition of the i_M-th transmitted signal and CCI. Its received signal-to-interference ratio (SIR) is then given by

$$SIR^{H_1} = \frac{P_s|\mathbf{h}_{i_M}^S|^2}{\sum_{l=1}^{L}\mu_l|\mathbf{v}_l|^2}. \tag{3}$$

where the superscript H_1 refers to the first hop. At the second hop, due the CSIR assumption, the relay is oblivious to the interference pertaining to the K users. The latter are, therefore, responsible for making scheduling decisions. According to (2), the SIR at receiver j is given by

$$SIR_j^{H_2} = \frac{P_r|\mathbf{h}_j^R|^2}{\sum_{l=1}^{Q_j}\nu_l|u_{jl}|^2}, \quad j=1,\ldots,K, \tag{4}$$

where the superscript H_2 refers the second hop. At this hop, please note that each user feeds back its SIR value to the relay which selects the scheduled user (i.e., with the highest SIR value).

The achieved capacity of the two-hop AF relaying system with TAS/MRC on the first hop and interference-aware user scheduling on the second hop is given by

$$C = \frac{1}{2}\mathbb{E}\left\{\ln_2\left(1+\frac{\frac{P_s X_M}{\sum_{l=1}^{L}\mu_l|\mathbf{v}_l|^2}Z_K}{1+\frac{P_s X_M}{\sum_{l=1}^{L}\mu_l|\mathbf{v}_l|^2}+Z_K}\right)\right\}, \tag{5}$$

where $X_M = \max_{i=1\ldots,M}|\mathbf{h}_i^S|^2$ and $Z_K = \max_{k=1,\ldots,K}\mathrm{SIR}_j^{H_2}$. Drawing a comparison, in this paper, between interference-aware (i.e., SIR based) and interference-oblivious (i.e., CSI based) user scheduling will be certainly useful for quantifying the potential gain that originates from interference-awareness and, further, understanding whether this gain justifies the required high-complexity signal processing (demodulation and parameters estimation).

In this paper, without loss of generality, a homogeneous network in which all users are clustered together, whereby $\lambda_j = \lambda, j=1\ldots K$ are considered. This policy guarantees a uniform user experience, saves valuable energy at terminals, and avoids near-far blockage where the receiver's limited dynamic range makes weak signals drown in stronger ones. Moreover, we assume that all users have the same statistical behavior with equal interference number $Q_j = Q, j=1\ldots K$ and similar interference distribution.

III. Exact Analysis of The Capacity

The objective herein is to compute the end-to-end capacity and to study the asymptotic regime of the dual-hop multiuser network described in Section II. Unfortunately, it turns out that it is impossible to express this capacity in closed-form, owing to the complexity of such a system. In order to circumvent this issue, we recur to a two-step methodology using which the end-to-end capacity C may be formulated as a linear combination of integrals and, hence, given by

$$\begin{aligned} C &\overset{(a)}{=} \frac{1}{2\ln(2)}\int_0^{\infty} s e^{-s}\left(1-s\mathbb{E}_{\mathbf{h}^S,i_{T_1}}\{e^{-sSIR^{H_1}}\}\right)\times \\ &\qquad \left(1-s\mathbb{E}_{\mathbf{h}^R,i_{T_2}}\{e^{-sSIR^{H_2}}\}\right)ds \\ &\overset{(b)}{=} \frac{1}{2\ln(2)}\int_0^{\infty} s e^{-s} M_{SIR^{H_1}}^{(c)}(s) M_{SIR^{H_2}}^{(c)}(s), \end{aligned} \tag{6}$$

where $M_X^{(c)}(s) = 1 - sM_X(s)$ is the complementary MGF of X. Please note that the two-step methodology applied in (6) is a powerful tool allowing to gain insights on the overall system performance while separately treating its components (hops and Rvs pertaining to them). The equalities in (6.a) and (6.b) correspond to the first and second step of this methodology, respectively. As far as (6.a) is concerned, its computation may be avoided by resorting to

$$M_X^{(c)}(s) \triangleq \int_0^{\infty} e^{-sx}F_X^{(c)}(x)dx, \tag{7}$$

where $F_X^{(c)}(z) = 1 - F_X(z)$ is the complementary cumulative distribution function (CCDF) of X.

Proof: Interested readers are referred to the proof in [8, Theorem 1] omitted here for conciseness.

It is noteworthy that (7) is very convenient since the SIR's CCDF of single-hop communication systems over fading channels has been widely studied in the literature ([7] and references therein)[1]. Exploiting the methodology in (6), the rest of this section is devoted then to the calculation of the capacity achieved by the system model in Section II. The obtained result is stated in the following lemma.

Lemma 1: Let the system model in section II, then for any cellular consideration (pathloss, antenna number, CCI), the achievable capacity is given by (8) at the top of the next page, where

$$\Xi_m=\int_0^\infty se^{-s}\Psi\left(m+1,m+2-K(Q+N-1),\frac{\lambda s}{\nu}\right)\Psi\left(k+1,2-j,\frac{\rho s}{\mu_{<i>}(n+1)}\right)ds, \quad (9)$$

and

$$\Pi_n=\int_0^\infty se^{-s}\Psi\left(\delta_n+K(Q+N-1)+n(1-Q)+1,\delta_n+2-n(Q-1),\frac{\lambda s}{\nu}\right)\Psi\left(k+1,2-j,\frac{\rho s}{\mu_{<i>}(n+1)}\right)ds. \quad (10)$$

Also in (8) we denote $\Theta_n = \frac{(-1)^n n!\prod_{p=0}^{N-1}\left(\frac{1}{p!}\right)^{n_p+1}}{\prod_{k=1}^{N} n_k!}$, and $\Omega(n,N) = \{(n_1,...,n_N) : n_k \geq 0; \sum_{k=1}^{N} n_k = n\}$; and $\delta_n = \sum_{l=0}^{N-1} l\ n_{l+1}$. Moreover $\mathbf{D} = \mathrm{diag}(\mu_1,\mu_2,\ldots,\mu_L)$, $\rho(\mathbf{D})$ is the number of distinct diagonal elements of $\mathbf{D}$, $\mu_{<1>} > \mu_{<2>} > \ldots > \mu_{<L>}$ are the distinct diagonal elements in decreasing order, $\tau_{\mathbf{i}}(\mathbf{D})$ is the multiplicity of $\mu_{<i>}$ and $\zeta_{i,j}(\mathbf{D})$ is the (i,j)-th characteristic coefficient of $\mathbf{D}$. For instance, when non-equal-power interferers are considered, we have $\tau_i(\mathbf{D}) = 1$ and $\zeta_{i,1}(\mathbf{D}) = \prod_{k=1,k\neq i}^{\rho(\mathbf{D})} 1/\left(1-\frac{\mu_{<k>}}{\mu_{<i>}}\right)$.

Proof: The direct application of the two-step purposed method in (6) using the results in [12] pertaining to the per hop SIR CMGF calculation completes the proof.

It is worthwhile to mention that integrals like in (9)-(10) can be evaluated by means of the generalized Meijer's G-function of two variables, as can be seen from a more general integral formula due to [13, Eq. (3.2)]. We begin by expressing the Triconomi functions $\Psi(a,b,c)$ in the integrands of Ξ_m and Π_n in terms of the Meijers's G function as

$$\Psi\ (a,b,z) = \frac{1}{\Gamma(a)\Gamma(a-b+1)} G_{1,2}^{2,1}\left(z \,\middle|\, \begin{matrix} 1-a \\ 0, 1-b \end{matrix}\right). \quad (11)$$

Then performing Laplace transform over the product of two Meijer's functions (see [13, Eq. (3.2)]), (8) can be expressed according to (12) at the top of the next page, where $G[\cdot,\cdot]$ denotes the generalized Meijer's G function of two variables defined in [13, Eq. (1.2)].

The result accounts for all relevant cellular considerations including the antenna and user numbers, path loss power gain and interference power. To gain more insight into these parameters we propose in the next section more convenient capacity expressions obtained when large user number K.

[1]Although for brevity we limit our discussion to Rayleigh fading channels, many other channels, such as Nakagami-m channels, may be used here.

IV. Asymptotic Analysis of The Capacity: Large K

In this section, we study the asymptotic capacity for large number of users K but for a small and finite number of antennas at the source M and at the relay N. We will state a theorem that gives the closed-form expression for the achievable capacity for large K. After proving the theorem, we summarize the insights in some key results. To this end, the asymptotic behavior of the distribution of the maximum of K i.i.d. random variables is studied.

Lemma 2: When M, N and ASIRs[2] are fixed, the capacity of AF MU-MIMO with interference aware scheduling for large K is asymptotically given by

$$C=\frac{1}{2\ln(2)}\sum_{n=0}^{M-1}\sum_{\Omega(n,N)}\sum_{i=1}^{\rho(\mathbf{D})}\sum_{j=1}^{\tau_i(\mathbf{D})}\sum_{k=0}^{N+\delta_n-1}\frac{\Theta_p\zeta_i^j(\mathbf{D})\rho}{(n+1)^{N+\delta_n+1}\mu_{<i>}}\frac{M(N+\delta_n-1)!}{\Gamma(N)(k+j)}\left({}_2\mathrm{F}_1\left(k+1,1,k+j+1,1-\frac{\frac{\rho}{\mu_{<i>}}}{(n+1)}\right)-\left(\frac{NK\lambda}{4\nu}+1\right)^{-1}{}_2\mathrm{F}_1\left(k+1,1,k+j+1,1-\frac{\frac{\rho}{\mu_{<i>}}}{\left(\frac{NK\lambda}{4\nu}+1\right)(n+1)}\right)\right), \quad (13)$$

when $Q=1, \forall\ N$, while it is given by

$$C = \frac{1}{2\ln(2)}\sum_{n=0}^{M-1}\sum_{i=1}^{\rho(\mathbf{D})}\sum_{j=1}^{\tau_i(\mathbf{D})}\frac{M\Theta_p\zeta_i^j(\mathbf{D})\rho}{j(n+1)\mu_{<i>}}\left({}_2\mathrm{F}_1\left(1,1,1+j,1-\frac{\frac{\rho}{\mu_{<i>}}}{(n+1)}\right)-\left(\frac{\lambda}{\nu}\left(\frac{K^{\frac{1}{Q}}-1}{4^{\frac{1}{Q}}}\right)+1\right)^{-1}{}_2\mathrm{F}_1\left(1,1,1+j,1-\frac{\frac{\rho}{\mu_{<i>}}}{\left(\frac{\lambda}{\nu}\left(\frac{K^{\frac{1}{Q}}-1}{4^{\frac{1}{Q}}}\right)+1\right)(n+1)}\right)\right), \quad (14)$$

when $N=1, \forall\ Q$, where ${}_2\mathrm{F}_1(a,b,c,x)$ denote the Gauss hypergeometric function [11, Eq.(9.100)].

Proof: In order to evaluate the capacity we have to obtain the asymptotic distribution of the maximum K i.i.d SIRs denoted by $Z_K = \frac{\lambda}{\nu}\max_{j=1,\ldots,K}\frac{|\mathbf{h}_j^R|^2}{\sum_{l=1}^{Q}|u_l|^2}$. Before embarking on the proof, it is worth examining the SIRs $Z_j = \frac{|\mathbf{h}_j^R|^2}{\sum_{l=1}^{Q}|u_l|^2}$. We have $Z_j \overset{d}{=} \frac{\chi(2N)}{\chi(2Q)}, j = 1\ldots,K$, where "$\chi(2p)$" denotes a chi-square random variable with $2p$ degrees of freedom, are K i.i.d random variable with CDF

$$F_Z(x) = \frac{x^N\ {}_2\mathrm{F}_1\left(N+Q,N,1+N,-x\right)}{NB(N,Q)}, \quad (15)$$

implying that $F_{Z_K}(x) = [F_Z(\frac{\nu}{\lambda}x)]^K$. Next, we use results form [10] to find the asymptotic distribution of Z_K. We show that

$$\lim_{x\to\infty}\frac{1-F_Z(x)}{1-F_Z(tx)} = \lim_{x\to\infty}\frac{t^{-N}(1+tx)^{N+Q}}{(1+x)^{N+Q}} \quad (16)$$

$$= t^Q, \quad (17)$$

where after substituting the hypergeometric function by its equivalent ${}_2\mathrm{F}_1(a,b;b+1;z) = bz^{-b}B_z(b,1-a)$ with

[2]ASIR refers to the ratios $\frac{\rho}{\mu}$ and $\frac{\lambda}{\nu}$.

$$C = \frac{1}{2\ln(2)} \sum_{n=0}^{M-1} \sum_{\Omega(n,N)} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \sum_{k=0}^{N+\delta_n-1} \frac{\Theta_p \zeta_i^j(\mathbf{D})\Gamma(k+j)\rho}{(j-1)!\mu_{<1i>}} \frac{\lambda M(N+\delta_n-1)!}{\nu(n+1)^{N+\delta_n+1}\Gamma(N)} \left(\sum_{m=0}^{K(Q+N-1)-1} \binom{K(Q+N-1)}{m} \Gamma(m+1)\Xi_m \right.$$

$$\left. + \sum_{n=1}^{K} \sum_{\Omega(n,Q-1)} \binom{K}{n} \alpha_n \Gamma(\delta_n + K(L+N-1) + n(1-L) + 1)\Pi_n \right), \quad (8)$$

$$C = \frac{1}{2\ln(2)} \sum_{n=0}^{M-1} \sum_{\Omega(n,N)} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \sum_{k=0}^{N+\delta_n-1} \frac{\Theta_p \zeta_i^j(\mathbf{D})\Gamma(k+j)\rho}{(j-1)!\mu_{<i>}} \frac{\lambda M(N+\delta_n-1)}{\nu(n+1)^{N+\delta_n+1}\Gamma(N)} \sum_{m=0}^{K(Q+N-1)-1} \frac{\binom{K(Q+N-1)}{m}}{\Gamma(K(Q+N-1))}$$

$$G^{1,1,1,1,2}_{1,[1,1],0,[1,2]} \left(\frac{\frac{\rho}{\mu_{<i>}}}{n+1}, \frac{\lambda}{\nu} \middle| \begin{matrix} \ldots; 1+k; 1+m \\ \ldots; 0, j-1; 0, K(Q+N-1)-m-1 \end{matrix} \right) + \sum_{n=1}^{K} \sum_{\Omega(n,Q-1)} \frac{\binom{K}{n}\alpha_n}{\Gamma(K(Q+N-1))}$$

$$G^{1,1,1,1,2}_{1,[1,1],0,[1,2]} \left(\frac{\frac{\rho}{\mu_{<i>}}}{n+1}, \frac{\lambda}{\nu} \middle| \ldots; 1+k; 1+\delta_n+K(Q+N-1)+n(1-Q); 0, j-1; 0, -\delta_n-1+n(1-Q) \right), \quad (12)$$

$B_z(c,d)$ being the beta function [11, Eq.(8.380.1)], we use the l'Hospital rule to get form (16) to (17). (17) implies that the limiting distribution of Z_K lies in the domain of maximal attraction of Frechet distribution [10, Theroem 10.5.2], i.e.

$$[F_Z(a_K x)]^K = e^{-x^{-Q}}, \quad x \geq 0, \quad (18)$$

where a_K is a normalizing parameter defined via the equation $F_Z(a_K) = 1 - \frac{1}{K}$. In order to find a_K while keeping the analytical complexity tractable, we concentrate the following analysis on a system of high practical interest i.e. i) $Q = 1, \forall\ N$, and ii) $N = 1, \forall\ Q$. The results can be extended to other values of N, Q, however the analysis is very challenging. The arbitrary N, Q case can be handled using bounding techniques but thwarting the paper goal of exact capacity analysis. For case i), by exploiting the ${}_2\mathrm{F}_1$ reduction formulas ${}_2\mathrm{F}_1(b,a;a;z) = (1-z)^{-b}$, we show that a_K satisfies

$$\begin{aligned} \frac{a_K^N}{(1+a_K)^N} &= 1 - \frac{1}{K} \\ \Longrightarrow a_K &= \frac{(1-1/K)^{1/N}}{1-(1-1/K)^{1/N}} \\ &\underset{K\to\infty}{\approx} NK. \end{aligned} \quad (19)$$

For case ii), we have $N = 1$ thereby enabling the simplification of $F_Z(x)$ using the fact that ${}_2\mathrm{F}_1(1,b;2;z) = \frac{(1-z)^{-b}-1}{(b-1)z}$. The parameter a_k is therefore obtained as follows

$$\begin{aligned} (1+a_K)^{-Q} &= \frac{1}{K} \\ \Longrightarrow a_K &\underset{K\to\infty}{\approx} K^{1/Q} - 1. \end{aligned} \quad (20)$$

Substituting a_K into (18), we obtain as $K \to \infty$

$$F_{Z_K}(x) = \begin{cases} e^{-\frac{NK\lambda}{\nu x}}, & Q = 1, \forall\ N \\ e^{-\left(\frac{\left(K^{1/Q}-1\right)\lambda}{\nu x}\right)^Q}, & N = 1, \forall\ Q \end{cases} \quad (21)$$

Fig. 1 shows the exact and asymptotic CDFs of Z_K for different values of N and Q. We observe that the asymptotic distribution in (21) is a good approximation even for small values of N and Q and the approximation becomes more accurate by increasing the value of K.

As for the CMGF of Z_K, replacing (21) into (7) and performing some algebraic manipulations, we get

$$M^{(c)}_{Z_K}(s) \overset{(a)}{\approx} \begin{cases} \frac{1}{s}\left(1 - e^{-\frac{NK\lambda}{4\mu}s}\right), & Q = 1, \forall\ N \\ \frac{1}{s}\left(1 - e^{-\frac{\lambda}{\mu}\left(\frac{K^{\frac{1}{Q}}-1}{4^{\frac{1}{Q}}}\right)s}\right), & N = 1, \forall\ Q \end{cases} \quad (22)$$

where (a) follows from the fact that $1 - e^{-x} \underset{x\geq 4}{\approx} 1$. At this step, recalling the first hop CMGF derived in section III , the end-to-end capacity of tow-hop AF relaying with interference aware scheduling as K goes large is obtained from applying (6). For instance, case i) yields

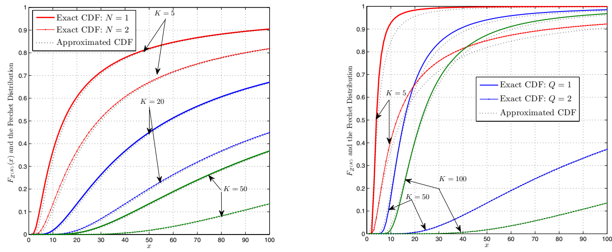
$$\begin{aligned} C = {} & \frac{1}{2\ln(2)} \sum_{n=0}^{M-1} \sum_{\Omega(n,N)} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \sum_{k=0}^{N+\delta_n-1} \frac{\Theta_p \zeta_i^j(\mathbf{D})\Gamma(k+j)\rho}{(j-1)!\mu_{<1i>}} \\ & \frac{M(N+\delta_n-1)!}{(n+1)^{N+\delta_n+1}\Gamma(N)} \int_0^\infty e^{-s}\left(1 - e^{-\frac{NK\lambda}{4\mu}s}\right) \\ & \Psi\left(k+1, 2-j, s\frac{\rho}{\mu_{<i>}(n+1)}\right) ds, \end{aligned} \quad (23)$$

where to reach the closed-form expression of C shown in Theorem 1, [11, Eq.(8.380.1)] was employed. The capacity expression for for large K when case ii) i.e., $N = 1$ and arbitrary Q can be derived similarly.

Remark 1: For fixed M, N and ASIRS, the capacity for large K reach its maximum value C_{max} given by

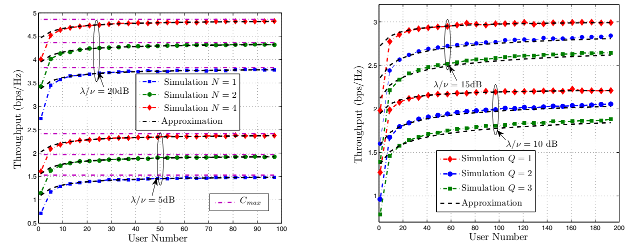
$$\begin{aligned} C_{max} = {} & \frac{1}{2\ln(2)} \sum_{n=0}^{M-1} \sum_{\Omega(n,N)} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \sum_{k=0}^{N+\delta_n-1} \frac{\Theta_p \zeta_i^j(\mathbf{D})\rho}{(n+1)^{N+\delta_n+1}\mu_{<i>}} \\ & \frac{M(N+\delta_n-1)!}{\Gamma(N)(k+j)} {}_2\mathrm{F}_1\left(k+1, 1, k+j+1, 1 - \frac{\frac{\rho}{\mu_{<i>}}}{(n+1)}\right). \end{aligned} \quad (24)$$

Proof: (24) follows by resorting to the series expansion of the ${}_2\mathrm{F}_1$ function near one and performing the limit operation as K goes to infinity.



(a) (b)

Fig. 1. The Exact and asymptotic distribution of F_{z_K} for different values of N:(a) and different values of Q:(b)



(a) (b)

Fig. 2. System average capacity as a function of the number of users K for different values of N:(a) and different values of Q:(b)

Remark 2: From (14) and since $K^{1/Q} - 1 \underset{Q \gg 1}{\approx} \frac{\ln(K)}{Q}$, then unless the multiuser diversity gain, which scales as $\ln(K)$, is high enough to compensate for the nominal spectral efficiency decrease, the ergodic capacity will decrease with Q.

V. SIMULATION RESULTS

Here, we provide some numerical examples to illustrate: 1) the tightness of the proposed approximations for large scale MU-MIMO relay networks; and 2) the impact of interference on spatial and multiuser diversity. The simulations set-up consists of an (M-N-K) MU-MIMO AF relaying system where the relay and each destination is subject to L and Q i.i.d. interferers, respectively. We also assume, without loss of generality, equal per-hop ASIR $\frac{\rho}{\mu} = \frac{\lambda}{\nu}$.

We show in Fig. 2 the average capacity for two-hop AF network where a source with $M = 3$ antennas is communicating with K users through a relay with $N = 1, 2, 4$ antennas under Rayleigh fading. The simulated curves were obtained by averaging over 10000 channel realizations. The "approximation" curves refer to the average capacity using (13) in case i) $Q = 1, \forall\ N$, and (14) in case ii) $N = 1, \forall\ Q$. These two curves match each other very well even for small K, which establishes the fact that considering the large-K assumption leads to tight capacity approximations thereby alleviating the need for evaluating the cumbersome Meijer's G-function in (12). In Fig. 2 (a), we highlight the system's behavior under equitable interference conditions on the two hops, i.e., $L = Q = 1$ and for different relay antenna numbers N. One can see that, as K goes large, the capacity of opportunistic scheduling continuously approaches the asymptotic value C_{max} in (24) which constitutes the bottleneck of the system capacity. In Fig. 2 (b), the system capacity is depicted for different values of Q with $L = 2$. It can be seen that, for fixed K, as Q increases the capacity gain from multiuser diversity cannot compensate for the harmful interference; thus, the system capacity begins to decline monotonically. In fact, the capacity is governed by the order of multiuser diversity defined as the ratio of the desired signal strength (i.e., the multiuser diversity gain), which we have shown in Section V to scale as $\ln(K)$ for Rayleigh fading, to the aggregate interference power which is proportional to the interference number Q.

VI. CONCLUSION

In this work, we have considered opportunistic scheduling capacity analysis for two hop MIMO amplify and forward relay networks in interference-limited environments. The scheme entails a two-hop communication protocol, in which an M-antenna source can communicate with K destinations only through a half-duplex multi-antenna relay. We derived exact and asymptotic (large K) capacity expressions over general cellular considerations. The analysis found that the ergodic capacity scales with the order of multiuser diversity defined as the ratio of multiuser diversity gain to interference number.

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