Capacity of Cognitive AF Relay Networks with Multiple Primary Receivers

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***Abstract*—This paper evaluates the ergodic capacity of secondary dual-hop cognitive amplify-and-forward (AF) relay networks (CRNs) subject to arbitrary Nakagami-m fading. Considering a spectrum-sharing environment, we assume that the transmit power of secondary users is solely governed by the interference power constraint at N primary users. A closed-form expression for the ergodic capacity of the secondary AF CRN is derived for the first time and shown to be affected by the distance ratio of the interference link (from the secondary transmitter to the closest primary receiver) to the relaying link (between the secondary transmitter and the secondary receiver). This new ergodic capacity expression is validated and assessed by insightful simulations against different key CRN parameters.**

***Index Terms*—Amplify-and-forward (AF), dual-hop cognitive relay network (CRN), Nakagami-m fading, spectrum sharing.**

I. Introduction

Spectrum-sharing cognitive radio (CR) communication has arisen as a promising technique for overcoming the spectrum shortage in current wireless networks [1]. Roughly speaking, in this communication paradigm, unlicensed users (secondary users or SUs) are permitted to use the licensed spectrum band, as long as the generated interference aggregated at the licensed users (primary users or PUs) is below acceptable levels. Recently, due to the enormous performance gains obtained with its use, the concept of relaying [2] has been introduced in cognitive networks as a potential way to remarkably improve the secondary user throughput. The advantage of relaying lies in enabling high capacity where traditional architectures are unsatisfactory due to location constraints (e.g., cell-edge, shadowing, indoor), leading to a more homogenous user experience. Motivated by these promising performance gains, several works have investigated the performance of combining these two promising technologies known as cognitive relay networks (CRNs) [3].

In [4], the outage probability (OP) of CRNs with a suitable relay selection was examined. There it was demonstrated that a CRN outperforms a conventional relay network. In [5], assuming the presence of a direct link in the secondary network, a tight lower-bound expression for the OP of CRNs was derived in Rayleigh fading. Recently the authors of [6] performed an outage analysis of CRNs over Nakagami-m fading. This work was recently extended in [7] assuming the presence of the direct link. In [8], the outage performance of CRNs with multiple SU relays and destinations was analyzed assuming that the overall transmit power is solely governed by the interference at the PU receiver.

While all of the aforementioned contributions substantially provide a good understanding of CRNs, most of them provide OP analysis. In this paper, in contrast to previous works, we investigate the ergodic capacity of SUs dual-hop AF-relaying link in spectrum-sharing CR networks while adhering to the interference constraint at N PUs. Considering a Nakagami-m fading scenario, a closed-form expression for the ergodic capacity of the secondary AF CRN is derived for the first time and shown to be affected by the distance ratio of the interference link (from the secondary transmitter to the closest primary receiver) to the relaying link (between the secondary transmitter and the secondary receiver).

In Section II, we present the system and channel models of the proposed cooperative CR system and the assumed interference constraint. In Section III, we obtain a closed-from expression for the secondary network capacity in Nakagami-m fading. It turns out that the ergodic capacity belongs to a special class of generalized hypergeometric series for which we propose a new implementation method. Then in Section IV, making use of these formulas, the overall secondary network ergodic capacity is investigated and numerical results and comparisons are provided. Finally, concluding remarks are drawn in Section V.

II. Network and Channel Models

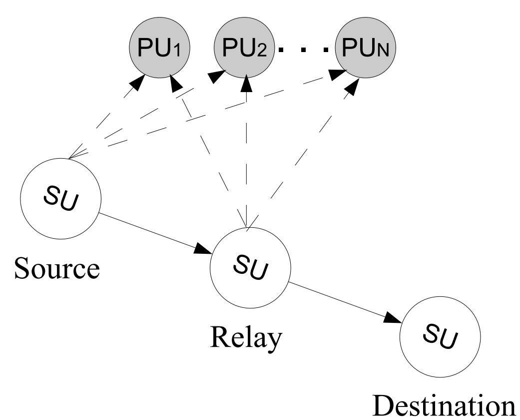

Fig. 1. System model of a secondary two-hop CRN in the presence of multiple PU receivers.

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The CRN's model of interest is shown in Fig. 1. There, a pair of SU source and destination nodes S and D, respectively, are sharing the same spectrum band with N PU receivers $\{PU_1, \ldots, PU_N\}$ and one SU AF relay R. The SU source S has no direct link with the SU destination D due to the unsatisfactory quality of the channel, and transmission is performed only through the relay R. As the primary and secondary users share the same frequency band, S and R are allowed to operate in the licensee's spectrum, as long as the interference impinged on the PU receivers remains below the interference temperature constraint Q. Let P_s and P_r be the maximum transmit powers at the SU source and the SU relay, respectively. Thus, based on the underlay approach [4] that dictates compliance only to the interference constraint on the primary users, the transmit powers at S and R can be written as $P_s = Q/\max_{i=1,\ldots,N}\{|g_{1,i}|^2\}$ and $P_r = Q/\max_{i=1,\ldots,N}\{|g_{2,i}|^2\}$, respectively, where $g_{1,i}$ and $g_{2,i}$ are the channel coefficients of the $S - PU_i$ and $R - PU_i$ interference links, respectively. The instantaneous channel of the link between the SU source and the SU relay is represented by h_1, and the one between the SU relay and the SU destination is represented by h_2. Finally, we consider that the interference generated by the primary user operating in the secondary transmission area is modeled as an additive zero-mean Gaussian noise at R and D with mean power N_0. The relay mode is non-regenerative with a variable gain in which the amplification factor is determined by the instantaneous channel statistics of the source-relay link, so that the end-to-end instantaneous signal-to-noise ratio (SNR) can be written as

$$\gamma_T = \left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)^{-1}, \tag{1}$$

where $\gamma_i = \frac{\bar{\gamma_Q}|h_i|^2}{\max_{j=1,\ldots,N}\{|g_{i,j}|^2\}}$, and $\bar{\gamma_Q} = Q/N_0, i = 1, 2$. The channels of the relay links are assumed to be independent and non-identically Nakagami-m fading distributed (i.n.i.d.) with arbitrary fading parameters and arbitrary average powers. Nonetheless, for the sake of simplicity, we assume that the interference links are subject to identically-distributed Nakagami-m fading. As a result, $|h_1|^2, |h_2|^2, |g_{1,i}|^2$, and $|g_{2,i}|^2$ are Gamma distributed with fading severity parameters m_1 , m_2 , m_{I_1} , and m_{I_2} and channel powers Ω_1 , Ω_2 , Ω_{I_1}, Ω_{I_2} , respectively. Thus, the probability density function and cumulative density function of X, $X \in \{|h_1|^2, |h_2|^2, |g_{1,i}|^2, |g_{2,i}|^2\}$ can be formulated in compact forms as

$$\begin{aligned} f_X(x) &= \frac{m^m}{\Gamma(m)\Omega^m} x^{m-1} \exp\left(-\frac{mx}{\Omega}\right), \\ F_X(x) &= 1 - \frac{\Gamma(m, \alpha x)}{\Gamma(m)}, \end{aligned} \tag{2}$$

where $\alpha = m/\Omega$, $\Gamma(\cdot)$, and $\Gamma(\cdot,\cdot)$ denote the Gamma function [11, Eq.(8.310.1)] and the upper incomplete Gamma function [11, Eq.(8.350.2)], respectively. Furthermore, it can be shown that the pdf of $Y_k = \max_{i=1,\ldots,N}\{|g_{k,i}|^2\}, k = 1, 2$ is given by

$$f_{Y_k}(y) = \frac{N\alpha_{I,k}^{m_{I_k}}}{\Gamma(m_{I_k})} y^{m_{I_k}-1} e^{-\alpha_{I_k} y}\left(1 - \frac{\Gamma(m_{I_k}, \alpha_{I_k} x)}{\Gamma(m_{I_k})}\right)^{N-1}. \tag{3}$$

Making use of the binomial theorem [11, Eq.(1.111.1)], the incomplete Gamma function involved in (3) can be rewritten in terms of finite sums by means of [11, Eq.(8.352.2)]. Then recalling the multinomial theory [11, Eq.(1.111.1)], a more tractable mathematical form of (3) is attained as

$$\begin{aligned} f_{Y_k}(y) &= \frac{N\alpha_{I_k}^{m_{I_k}}}{\Gamma(m_{I_k})} \sum_{n=0}^{N-1} \binom{N-1}{n} (-1)^n \sum_{\Omega(n, m_{I_k})} \frac{n!}{\prod_{i=1}^{m_{I_k}} n_i!} \\ &\left(\prod_{p=0}^{m_{I_k}-1} \left(\frac{\alpha_{I_k}^p}{p!}\right)^{n_{p+1}}\right) y^{m_{I_k}+\delta_n-1} e^{-\alpha_{I_k}(n+1)y}, \end{aligned} \tag{4}$$

where $\Omega(n, m_{I,k})$ is the set of m_{I_k}-tuples such that $\Omega(n, m_{I,k}) = \{(n_1, ..., n_{m_{I_k}}) : n_k \geq 0, \sum_{k=1}^{m_{I_k}} n_k = n\}$ and $\delta_n = \sum_{j=0}^{m_{I(k-1)}} j n_{j+1}$.

III. ERGODIC CAPACITY ANALYSIS

Since the secondary source transmits data with the help of the relay R in a dual-hop fashion, known to be the most efficient multi-hop transmission with respect to system capacity [9], that of the SU based on a unit bandwidth is given by

$$C_E = \frac{1}{2}\mathbb{E}\left[\log_2(1 + \gamma_T)\right], \tag{5}$$

where $\mathbb{E}[\cdot]$ stands for the mathematical expectation and $1/2$ arises from the dual-hop transmission in two time slots.

A. MGF-Based General Capacity Expression

An alternative expression for C_E can be established in terms of the moment generating function (MGF) of γ_T as [10]

$$C_E = \frac{1}{2\ln(2)} \int_0^\infty \frac{1 - e^{-s}}{s} M_{\gamma_T^{-1}}(s) ds, \tag{6}$$

where $M_{\gamma_T^{-1}}(s) = M_{\gamma_1}^{-1}(s) M_{\gamma_2}^{-1}(s)$. The terms $M_{\gamma_k^{-1}}(s), k = 1, 2$ can be evaluated using (2) and (4), and by exploiting [11, Eqs. (3.351.3) and (9.211.4)], as

$$\begin{aligned} M_{\gamma_k^{-1}}(s) &= \frac{N\alpha_{I_k}^{m_{I_k}}}{\Gamma(m_{I_k})\Gamma(m_k)} \sum_{n=0}^{N-1} \binom{N-1}{n} (-1)^n \sum_{\Omega(n, m_{I,k})} \tau_\Omega^n \\ &\Psi\left(\delta_n + m_{I_k}, 1 - m_k, \frac{\alpha_k s}{\alpha_{I_k}(n+1)\bar{\gamma_Q}}\right), \end{aligned} \tag{7}$$

where

$$\begin{aligned} \tau_\Omega^n = &\frac{n!}{\prod_{i=1}^{m_{I_k}} n_i!} \left(\prod_{p=0}^{m_{I_k}-1} \left(\frac{\alpha_{I_k}^p}{p!}\right)^{n_{p+1}}\right) \\ &\times \frac{\Gamma(\delta_n + m_{I_k})\Gamma(\delta_n + m_{I_k} + m_k)}{(\alpha_{I_k}(n+1))^{\delta_n + m_{I_k}}}, \end{aligned} \tag{8}$$

In practice, the channel state information (CSI) of the links between the secondary and primary nodes can be obtained through a direct feedback from the PU or through an indirect feedback by a band manager which mediates the exchange of information between the primary and secondary networks.

with $\Psi(a;b;z)$ being the Triconomi confluent hypergeometric function [11, Eq. (9.211.1)]. The ergodic capacity is then obtained by replacing (7) into (6) as

$$C_E = \frac{N^2}{2\ln(2)} \frac{\alpha_{I_1}^{m_{I_1}} \alpha_{I_2}^{m_{I_2}}}{\Gamma(m_{I_1})\Gamma(m_{I_2})\Gamma(m_1)\Gamma(m_2)} \sum_{n,p=0}^{N-1} \binom{N-1}{n} \binom{N-1}{p} (-1)^{n+p} \sum_{\Omega(n,m_{I_1})} \sum_{\Omega(p,m_{I_2})} \tau_\Omega^n \tau_\Omega^p I_{n,p}, \quad (9)$$

where

$$I_{n,p} = \int_0^\infty \frac{1-e^{-s}}{s} \Psi\left(\delta_n + m_{I_1}, 1-m_1, \frac{\alpha_1 s}{\alpha_{I_1}(n+1)\bar{\gamma_Q}}\right) \Psi\left(\delta_p + m_{I_2}, 1-m_2, \frac{\alpha_2 s}{\alpha_{I_2}(p+1)\bar{\gamma_Q}}\right) ds. \quad (10)$$

Yet, to the best of the author's knowledge, no closed-form solution for this integral has been established so far.

B. Closed-Form Solution

To circumvent this challenging obstacle, we start by representing the Triconomi function in terms of the confluent Hypergeometric function ${}_1\mathrm{F}_1(a;b;z)$ [11, Eq. (9.210.1)], as

$$\Psi\left(\delta_i + m_{I_k}, 1-m_k, \frac{\alpha_k s}{\alpha_{I_k}(i+1)\bar{\gamma}_{Q_k}}\right) = \frac{\Gamma(m_k)}{\Gamma(m_k+\delta_i+m_{I_k})} {}_1\mathrm{F}_1\left(\delta_i+m_{I_k};1-m_k;\frac{\alpha_k s}{\alpha_{I_k}(i+1)\bar{\gamma_Q}}\right) + \frac{\Gamma(-m_k)\left(\frac{\alpha_k s}{\alpha_{I_k}(i+1)\bar{\gamma_Q}}\right)^{m_k}}{\Gamma(\delta_i+m_{I_k})} {}_1\mathrm{F}_1\left(m_k+\delta_i+m_{I_k};1+m_k;\frac{\alpha_k s}{\alpha_{I_k}(i+1)\bar{\gamma_Q}}\right). \quad (11)$$

Likewise, the function $(1-e^{-s})/s$ can be expressed in terms of ${}_1\mathrm{F}_1(a;b;z)$ as $(1-e^{-s})/s = e^{-s}{}_1\mathrm{F}_1(1;2;s)$. Thus, by performing the necessary substitutions and simplifying, we obtain

$$I_{n,p} = \frac{\Gamma(m_1)\Gamma(m_2)}{\Gamma(m_1+\delta_n+m_{I_1})\Gamma(m_2+\delta_p+m_{I_2})} \int_0^\infty e^{-s}{}_1\mathrm{F}_1(1;2;s) \left[{}_1\mathrm{F}_1\left(\delta_n+m_{I_1};1-m_1;\frac{\alpha_1 s}{\alpha_{I_1}(n+1)\bar{\gamma_Q}}\right) + \rho_n {}_1\mathrm{F}_1\left(m_1+\delta_n+m_{I_1};1+m_1;\frac{\alpha_1 s}{\alpha_{I_1}(n+1)\bar{\gamma_Q}}\right)\right] \times \left[{}_1\mathrm{F}_1\left(\delta_p+m_{I_2};1-m_2;\frac{\alpha_2 s}{\alpha_{I_2}(p+1)\bar{\gamma_Q}}\right) + \rho_p {}_1\mathrm{F}_1\left(m_2+\delta_p+m_{I_2};1+m_2;\frac{\alpha_2 s}{\alpha_{I_2}(p+1)\bar{\gamma_Q}}\right)\right] ds, \quad (12)$$

where $\rho_i = \frac{\Gamma(-m_k)\left(\frac{\alpha_k s}{\alpha_{I_k}(i+1)\bar{\gamma_Q}}\right)^{m_k}}{\mathrm{B}(\delta_i+m_{I_k},m_k)}$ with $(i,k) = \{(n,1),(p,2)\}$ and $\mathrm{B}(a,b)$ denotes the Beta function. A careful inspection of (12) reveals that the modified version of the first Lauricella hypergeometric function, which is given by [12, Eq. (2.4.2)]

$$\mathrm{F}_\mathrm{A}^{(\mathrm{r})}\left(a;b_1,\ldots,b_r;c_1,\ldots,c_r;\frac{x_1}{\nu},\ldots,\frac{x_r}{\nu}\right) = \frac{\nu^a}{\Gamma(a)} \int_0^\infty e^{-\nu t} t^{a-1} \left(\prod_{k=1}^r {}_1F_1(b_k;c_k,x_k t)\right) dt; \quad (\mathfrak{Re}(a)>0), \quad (13)$$

can be applied to solve the integrals involved in (12). Subsequently, by setting $X_n = \alpha_1/(\alpha_{I_1}(n+1)\bar{\gamma_Q})$ and $X_p = \alpha_2/(\alpha_{I_2}(p+1)\bar{\gamma_Q})$, we show that a closed-form expression for $I_{p,n}$ is obtained, using (13), as shown in (14) at the top of the next page. Now, by replacing (14) into (9) and after some algebraic manipulations, a closed-form expression for the ergodic capacity of a dual-hop CRN with multiple primary users can be obtained for the very first time as shown in (15) at the top of the next page.

To the best of the authors' knowledge, this ergodic capacity expression is totally new. More than that, it is worthwhile to mention that even in the Rayleigh case, such closed-form expression was not established previously in the technical literature.

It is worth noting that since the average fading powers of the relaying and interfering links are proportional to $d^{-\eta}$ with d denoting the distance between the respective transceivers and $\eta \in \{2,4\}$ signifying the path loss exponent, we can see, form (15), that the ergodic capacity depends not on the distances of the interference links, but on the distance ratio of the interference link to the relaying link. This means that the relative distance of the interference link based on the relaying link distance has more of an impact on the ergodic capacity than the absolute distance of the interference link.

C. One-Term Continuation Relation for $\mathrm{F}_\mathrm{A}^{(\mathrm{r})}$

The multivariable Lauricella function $\mathrm{F}_\mathrm{A}^{(\mathrm{r})}$ is usually defined via its series representation given by [12, Eq. (2.1.1)], and its convergence is guaranted whenever $\sum_{i=1}^r |x_i| < 1$. Under its Laplace-type integral representation (13), $\mathrm{F}_\mathrm{A}^{(\mathrm{r})}$ is convergent whenever $\mathfrak{Re}\left(\sum_{i=1}^r x_i\right) < 1$ [13, Eq.(2)]. Nevertheless, the convergence of $\mathrm{F}_\mathrm{A}^{(\mathrm{r})}$ may often be improved by the use of analytic continuation formulas. In this section, a one-term continuation relation is obtained for the Lauricella $\mathrm{F}_\mathrm{A}^{(\mathrm{r})}$ by making use of its Barnes integral representation. In what follows, let us assume that only one argument of the Lauricella function is greater that one (say x_r) and the remaining are less than one with $\mathfrak{Re}\left(\sum_{i=1}^{r-1} x_i\right) < 1$. We shall obtain one-term continuation relation for the function F_A by using the Barnes integral representation given by [12, Eq. (2.5.3)]

$$\frac{\Gamma(a)\Gamma(b_r)}{\Gamma(c_r)} \mathrm{F}_\mathrm{A}^{(\mathrm{r})}(a;b_1,\ldots,b_r;c_1,\ldots,c_r;x_1,\ldots,x_r) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \mathrm{F}_\mathrm{A}^{(\mathrm{r-1})}(a+t;b_1,\ldots,b_{r-1};c_1,\ldots,c_{r-1};x_1,\ldots,x_{r-1}) \times \frac{\Gamma(a+t)\Gamma(b_r+t)}{\Gamma(c_r+t)} \Gamma(-t)(-x_r)^t dt. \quad (16)$$

Note that (11) is valid only for real-valued non-integer values of $m_k, k = 1,2$. However, practically, very similar results are obtained for integer values of m_k and $m_k+\epsilon$ for sufficiently small ϵ values.

$$
\begin{aligned}
I_{n,p} \;&=\; \frac{\Gamma(m_1)\Gamma(m_2)}{\Gamma(m_1+\delta_n+m_{I_1})\Gamma(m_2+\delta_p+m_{I_2})}\Bigg(\mathrm{F}_{\mathrm{A}}^{(3)}\left(1;\delta_n+m_{I_1},\delta_p+m_{I_2};1-m_1,1-m_2;1,X_n,X_p\right)+\rho_n\Gamma(1+m_1)\\
&\mathrm{F}_{\mathrm{A}}^{(3)}\left(1+m_1;\delta_n+m_{I_1}+m_1,\delta_p+m_{I_2};1+m_1,1-m_2;1,X_n,X_p\right)+\;\rho_p\Gamma(1+m_2)\mathrm{F}_{\mathrm{A}}^{(3)}\Bigg(1+m_2;\\
&\delta_p+m_{I_2}+m_2,\delta_n+m_{I_1};1+m_2,1-m_1;1,X_n,X_p\Bigg)+\rho_n\rho_p\Gamma(1+m_1+m_2)\mathrm{F}_{\mathrm{A}}^{(3)}\Bigg(1+m_1+m_2;\\
&\delta_n+m_{I_1}+m_1,\delta_n+m_{I_2}+m_2;1+m_1,1+m_2;1,X_n,X_p\Bigg)\Bigg). \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
C_E \;&=\; \frac{N^2}{2\ln(2)}\sum_{n,p=0}^{N-1}\binom{N-1}{n}\binom{N-1}{p}(-1)^{n+p}\sum_{\Omega(n,m_{I,1}),\Omega(p,m_{I,2})}\frac{\frac{n!p!}{\prod_{i=1}^{m_{I_1}}n_i!\prod_{i=1}^{m_{I_2}}p_i!}\Gamma(\delta_n)\Gamma(\delta_p)}{\mathrm{B}(\delta_n,m_{I_1})\mathrm{B}(\delta_p,m_{I_2})(n+1)^{\delta_n+m_{I_1}}(p+1)^{\delta_p+m_{I_2}}}\\
&\left(\prod_{t=0}^{m_{I,1}-1}\left(\frac{1}{t!}\right)^{n_{t+1}}\right)\left(\prod_{l=0}^{m_{I,2}-1}\left(\frac{1}{l!}\right)^{n_{l+1}}\right)\Bigg(\mathrm{F}_{\mathrm{A}}^{(3)}\left(1;\delta_n+m_{I_1},\delta_p+m_{I_2};1-m_1,1-m_2;1,X_n,X_p\right)\\
&+\rho_n\Gamma(1+m_1)\mathrm{F}_{\mathrm{A}}^{(3)}\left(1+m_1;\delta_n+m_{I_1}+m_1,\delta_p+m_{I_2};1+m_1,1-m_2;1,X_n,X_p\right)+\;\rho_p\Gamma(1+m_2)\\
&\mathrm{F}_{\mathrm{A}}^{(3)}\Bigg(1+m_2;\delta_p+m_{I_2}+m_2,\delta_n+m_{I_1};1+m_2,1-m_1;1,X_n,X_p\Bigg)+\rho_n\rho_p\Gamma(1+m_1+m_2)\\
&\mathrm{F}_{\mathrm{A}}^{(3)}\Bigg(1+m_1+m_2;\delta_n+m_{I_1}+m_1,\delta_n+m_{I_2}+m_2;1+m_1,1+m_2;1,X_n,X_p\Bigg)\Bigg). \qquad (15)
\end{aligned}
$$

Apart from the numerical integration of (13), the integrand $\mathrm{F}_{\mathrm{A}}^{(\mathrm{r}-1)}$ in (16), which converges uniformly, can readily be computed via Gauss-Laguerre quadrature (GLQ), as suggested in [15, Eq. (44)], according to

$$
\begin{aligned}
&\mathrm{F}_{\mathrm{A}}^{(\mathrm{r}-1)}\left(a+t;b_1,\ldots,b_{r-1};c_1,\ldots,c_{r-1};x_1,\ldots,x_{r-1}\right)\approx\\
&\qquad\sum_{k=0}^{N_p}w_k\xi_k^{a+t-1}\prod_{i=1}^{r-1}{}_1F_1(b_i;c_i;x_i\xi_k), \qquad (17)
\end{aligned}
$$

where t_k and w_k are, respectively, the $k-$th zero and weight of the Laguerre polynomial of order N_p. Then, after plugging (17) into (16) and using [12, Eq. (1.21.7)], we obtain

$$
\begin{aligned}
&\mathrm{F}_{\mathrm{A}}^{(\mathrm{r})}\left(a;b_1,\ldots,b_r;c_1,\ldots,c_r;x_1,\ldots,x_r\right)\approx\\
&\sum_{k=0}^{N_p}w_k\xi_k^{a-1}\left(\prod_{i=1}^{r-1}{}_1F_1(b_i;c_i;x_i\xi_k)\right){}_2F_1(a,b_r;c_r,x_r\xi_k),\\
&\qquad\qquad\left(x_r\geq 1,\mathfrak{Re}\left(\sum_{i=1}^{r-1}x_i\right)<1\right), \qquad (18)
\end{aligned}
$$

which is the the continuation of $\mathrm{F}_{\mathrm{A}}^{(\mathrm{r})}$ to a different region of its arguments x_r. Note that one can always modify the arguments x_i in (16) in order for the convergence of the integrand $\mathrm{F}_{\mathrm{A}}^{(\mathrm{r}-1)}$ to be satisfied, by making use of the following Euler integral transformation [12, Eq. (4.2.2)]

$$
\begin{aligned}
&\mathrm{F}_{\mathrm{A}}^{(\mathrm{r}-1)}(a;b_1,\ldots,b_{r-1},c_1,\ldots,c_{r-1};x_1,\ldots,x_{r-1})=\\
&(1-x_j)^{-a}\mathrm{F}_{\mathrm{A}}^{(\mathrm{r}-1)}\Bigg(a;b_1,\ldots,c_j-b_j,b_{r-1};c_1,\ldots,c_{r-1};\\
&\qquad\frac{x_1}{1-x_j},\ldots,\frac{x_j}{x_j-1},\frac{x_{r-1}}{x_j-1}\Bigg). \qquad (19)
\end{aligned}
$$

Note that an r-fold repetition of the preceding operations can lead to the continuation of the Lauricella function outside its region of convergence. Nevertheless, the result is not necessarily the most convenient in form. Note that the new continuation formula obtained in (18) was derived here for the first time with the prime purpose of establishing the ergodic capacity expression obtained in (15). Nevertheless, it is worth mentioning that it can find meaningful and practical use in many other significant contexts.

IV. Illustrative Numerical Results

In this section, some key numerical examples are presented in order to evaluate the ergodic capacity of the considered dual-hop AF spectrum-sharing CRN.

For the plots, without loss of generality, the statistical average mean power of each link in the network is assumed to be proportional to $d^{-\eta}$, with d being the distance between the considered transceivers and η denoting the path loss exponent.

Note that one could also use the semi-infinite GLQ method presented in [14] for higher accuracy.

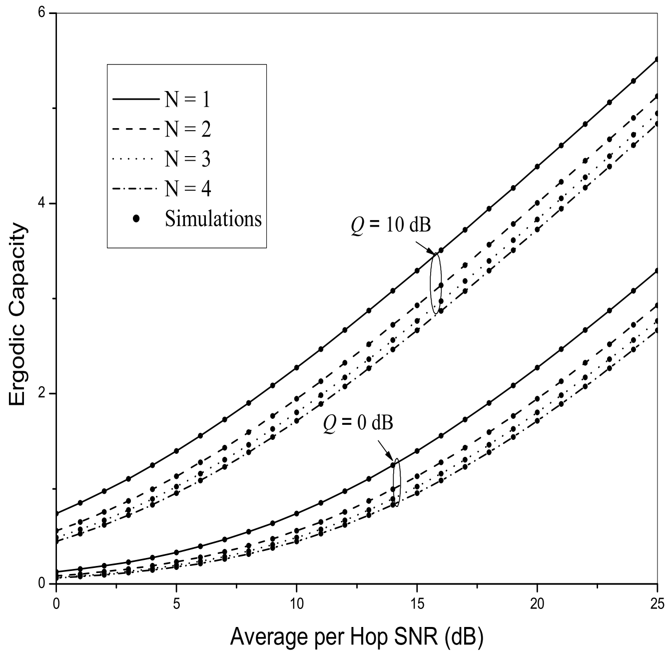

Fig. 2. Impact of interference constraints on the ergodic capacity of two-hop spectrum-sharing CRN with AF relaying for different numbers of PU receivers PU_{Rx} [S, R, D, and PU_{Rx} are located, respectively, at $(0,0)$, $(0.5,0)$, $(1,0)$, and $(0,1)$].

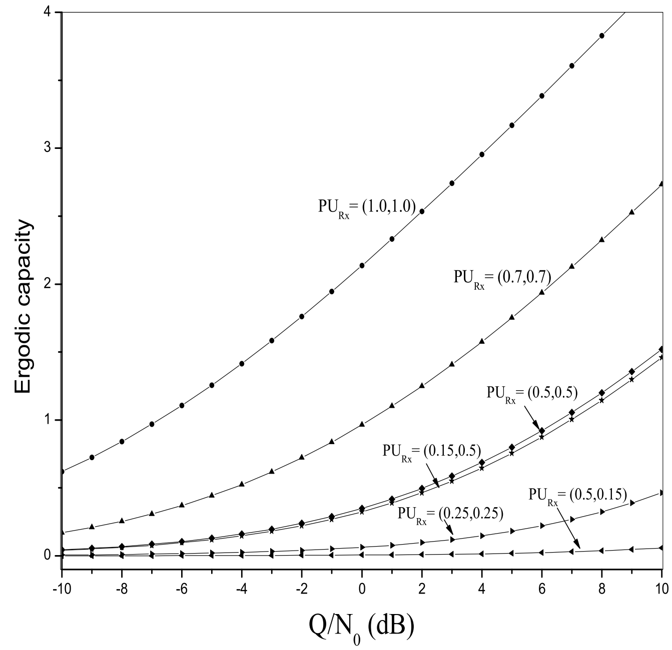

Fig. 3. Ergodic capacity of two-hop spectrum-sharing systems with AF relaying and $N = 3$ primary receivers vs. the interference constraint Q/N_0 with $m_1 = m_2 = 1.5$, $m_{I_1} = m_{I_2} = 2$ and $\eta = 4$ [S, R and D are located, respectively, at $(0,0)$, $(0.5,0)$, and $(1,0)$].

The network is generated in a two dimensional topology, where each node is defined by its coordinates (x, y). Moreover, all the PU receivers are assumed to be clustered together so that they can be identified using a single coordinate.

Fig. 2 depicts the ergodic capacity under different interference temperature constraints Q with $m_1 = m_2 = 1.5$, $m_{I_1} = m_{I_2} = 2$, $\Omega_{I_1} = 1$ and $\Omega_{I_2} = 0.5$. The achievable ergodic capacity is examined in two cases: (i) varying the number N of the PU receivers PU_{Rx} while the interference temperature constraint Q is kept constant, (ii) varying the interference constraint while keeping N constant. As can be easily verified in both cases, the analytical and simulation results are in excellent agreement thereby validating our new ergodic capacity expressions. Moreover, as expected, an increase of the number of PU_{Rx} in Fig. 2 results into the system capacity degradation. In fact, as the number N of PU_{Rx} increases, it becomes even more difficult for the SU transmitters to satisfy the interference temperature constraint Q for all the PU receivers. Indeed, the SU transmitters must limit their power to satisfy the worst PU constraint, which leads to degradation in capacity performance. Moreover, as Q gets larger, the ergodic capacity improves, approaching the no interference case.

In Fig. 3, the ergodic capacity is investigated against the interference constraint Q/N_0 for different PU_{Rx} positions. It can be seen that the ergodic capacity of cognitive relay networks improves when the primary receiver is located farther away from the secondary users. In addition, as observed from this figure, the SU relay is more vulnerable to a close PU receiver than the source. This implies that the farthest relay from the primary receiver is the most suitable for dual-hop relaying in CRNs with relay selection.

V. Conclusion

The overall contribution of this paper is the derivation of a new analytical expression for the ergodic capacity of two-hop spectrum-sharing systems with AF relaying and multiple PU receivers under interference power constraints satisfying the transmission protection requirements at the PUs' side. In contrast to conventional relay networks, the ergodic capacity in CRNs is affected by the distance ratio of the interference link to the relaying link, and not the absolute distances. Our theoretical analysis has been sustained by simulation results highlighting several important insights on how the primary network affects the ergodic capacity of the spectrum-sharing secondary relay network.

Our analysis is currently being extended for scenarios with multiple available relays with different relay selection strategies and multiple PU transmitters inflicting interference on the SU network.

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