Efficient Range-Free Localization Algorithm for Randomly Distributed Wireless Sensor Networks

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***Abstract*—In this paper, we propose a novel range-free localization algorithm able to reduce errors due to mapping the hops into distance units. Using the proposed algorithm, the mean hop size $\bar{h}_s$ is locally derived at each regular or position-unaware node, thereby avoiding its broadcast by anchors (i.e., a few nodes aware of their exact position) as usually required in current state-of-the-art solutions and, hence, resulting in less battery power depletion. The analytical expression of $\bar{h}_s$ is derived for different node distributions. Furthermore, it is shown that it is possible to locally compute $\bar{h}_s$ at each regular node with or even without prior knowledge of the node distribution. Simulations results show that the proposed scheme outperforms the most representative range-free localization schemes in terms of accuracy.**

***Index Terms*—wireless sensor networks, localization accuracy, range-free, nonparametric approach.**

I. Introduction

Due to their reliability, low cost, and ease of deployment, wireless sensor networks (WSNs) are emerging as a key tool for many applications such as environment monitoring, disaster relief, and target tracking [1]. A WSN is a set of small battery-powered sensors able to collect data from the surrounding environment and transmit it to a base station or an access point [2]. However, the sensing data are very often useless if the location from where they have been measured is unknown, making the localization a fundamental and essential issue in WSNs. So far, several localization algorithms have been proposed in the literature. These algorithms can be roughly classified into two categories: range-based and range-free.

To properly localize the regular or position-unaware node positions, range-based algorithms exploit the measurements of the received signals' characteristics such as the time of arrival (TOA) [3], the angle of arrival (AOA) [4], or the received signal strength (RSS) [5]. These signals are, in fact, transmitted by nodes with prior knowledge of their positions called anchors (or landmarks). Although the range-based algorithms stand to be very accurate, they are unsuitable for WSNs. Indeed, these algorithms require high power to ensure communication between anchors and regular nodes which are small battery-powered units. Furthermore, additional hardware is usually required at both anchors and regular nodes [6], thereby increasing the overall cost of the network. Moreover, the performance of these algorithms can be severely affected by noise, interference, and/or fading. Unlike range-based algorithms, range-free algorithms, which rely on the network connectivity to estimate the regular node positions, are more power-efficient and do not require any additional hardware and, hence, are suitable for WSNs. Due to these practical merits, range-free localization algorithms have garnered the attention of the research community.

So far, many range-free schemes have been proposed in the literature [7]- [11]. Most solutions are based on variations of the distance vector-hop algorithm (DV-Hop) [7] which is often considered as a benchmark. Unfortunately, like other range-free algorithms, DV-Hop does not provide sufficient accuracy due to errors occurring when mapping the hops into distance units. Furthermore, with DV-Hop each anchor has to compute an estimate of the network hop size and broadcast it to the other nodes, resulting in unnecessary high power consumption.

In this paper, we propose a new efficient and low-complexity localization algorithm which is able to reduce the errors due to mapping hops into distance, thereby increasing the localization accuracy. Using the proposed algorithm, each regular node locally computes an exact mean hop size $\bar{h}_s$, thereby avoiding its broadcast by anchors and, hence, the depletion of battery power. The analytical expression of $\bar{h}_s$ is derived for different node distributions. Furthermore, it is shown that it is possible to locally compute $\bar{h}_s$ at each regular node even without prior knowledge of the node distribution. It is also proven that the proposed algorithm outperforms the best representative range-free localization algorithms currently available in the literature in terms of localization accuracy.

The remainder of this paper is organized as follows: In Section II the system model is described. In Section III a novel range-free algorithm is proposed. Section IV derives the expression of the average hop size $\bar{h}_s$ for different node distributions. Section IV shows how $\bar{h}_s$ can be computed without prior knowledge of the node distribution. Simulation results are discussed in Section VI and concluding remarks are made in section VII.

II. Network model

Fig. 1 illustrates the system model of N WSN nodes randomly deployed in a 2-D square area with side length A. We assume that all nodes have the same transmission radius denoted by R. Hence, each node can only communicate with any other node located within its coverage area πR^2.

We assume that only a few nodes commonly known as anchors are aware of their positions. The other nodes, called hereafter position-unaware, or for the sake of simplicity regular nodes are oblivious of this information. As shown in Fig. 1, the anchor nodes are marked with red triangle and the regular

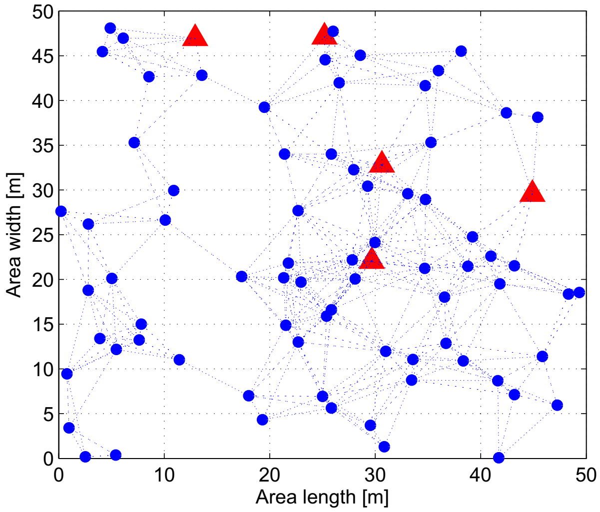

Fig. 1. Network model.

nodes are marked with blue circle. When the nodes are located within the communication range of each other, they are linked with a dashed line that represents the hop. Let N_a and N_u denote the number of anchors and regular nodes, respectively. Let (x_i, y_i) be the coordinates of the i-th regular node and (a_k, b_k) those of the k-th anchor.

In the following, we propose an efficient range-free localization algorithm aiming to accurately estimate the regular nodes' coordinates $(x_i, y_i), i = 1, 2, \ldots, N_u$.

Notation: Uppercase and lowercase bold letters denote matrices and vectors, respectively. $[\cdot]_{il}$ and $[\cdot]_i$ are the (i, l)-th entry of a matrix and i-th entry of a vector, respectively. $\mathbf{I}$ is the identity matrix. $(\cdot)^T$ denote the transpose.

III. The Proposed Algorithm

In this section, we propose a two-step localization algorithm. In the first step, the k-th anchor broadcasts through the network a message containing (a_k, b_k, n) where n is the hop-count value initialized to one. When a node receives this message, it stores the k-th anchor position as well as the received hop-count $n_k = n$ in its database, adds one to the hop-count value and broadcasts the resulting message. Once this message is received by an another node, its database information is checked. If the k-th anchor information exists and the received hop-count value n is smaller than the stored n_k, the node updates n_k by n, increments by 1 then broadcasts the resulting message. If n_k is smaller than n, the node discards the received message. However, when the node is oblivious to the k-th anchor position, it adds this information to its database and forwards the received message after incrementing n by 1. This mechanism will continue until all nodes become aware of all anchors' positions and their corresponding minimum hop count.

The i-th regular node computes then an estimate of its distance to the k-th anchor as $\hat{d}_{ik} = n_{ik}^{\min}\bar{h}_{\rm s}$ where $n_{ik}^{\min}$ is the minimum hop count value corresponding to the k-th anchor and $\bar{h}_{\rm s}$ is the mean hop size value depending on the node distribution. Unlike the well-known DV-Hop which derives at the anchors several estimates of $\bar{h}_{\rm s}$ (called corrections) then broadcasts them to the rest of the WSN, each regular node is able, owing to the new proposed algorithm, to locally compute the exact value of $\bar{h}_{\rm s}$, thereby avoiding its broadcast and reducing battery-power depletion. In the next sections, the expression of $\bar{h}_{\rm s}$ is derived for different node distributions. Using $\hat{d}_{ik}$, $k = 1 \ldots N_a$, the i-th regular node is now able to compute an initial guess $(\hat{x}_i, \hat{y}_i)$ of its 2-D coordinates by performing trilateration [12], provided that $N_a \geq 3$.

Unfortunately, errors are expected to occur when estimating the distance between each regular node-anchor pair, thereby hindering localization accuracy. In the second step, we propose to minimize the aforementioned errors. Let ϵ_{ik} denotes the estimation error of the distance between the i-th regular node and the k-th anchor node as

$$\epsilon_{ik} = \hat{d}_{ik} - d_{ik}, \tag{1}$$

where d_{ik} is the true distance between the two nodes. As discussed above, this error hinders localization accuracy. As such, we have

$$\begin{cases} x_i = \hat{x}_i + \delta_{x_i} \\ y_i = \hat{y}_i + \delta_{y_i} \end{cases}, \tag{2}$$

where δ_{x_i} and δ_{y_i} are the location coordinates' errors to be determined. Exploiting the Taylor series expansion and retaining the first two terms, the following approximation holds:

$$d_{ik} \approx d_{ik}^{\dagger} + \alpha_{k1}\delta_{x_i} + \alpha_{k2}\delta_{y_i}, \tag{3}$$

where

$$d_{ik}^{\dagger} = \sqrt{(\hat{x}_i - a_k)^2 - (\hat{y}_i - b_k)^2} \tag{4}$$

and

$$\alpha_{k1} = \left.\frac{\partial d_{ik}^{\dagger}}{\partial x}\right|_{\hat{x}_i, \hat{y}_i} = \frac{\hat{x}_i - a_k}{\sqrt{(\hat{x}_i - a_k)^2 - (\hat{y}_i - b_k)^2}} = \frac{\hat{x}_i - a_k}{d_{ik}^{\dagger}}, \tag{5}$$

$$\alpha_{k2} = \left.\frac{\partial d_{ik}^{\dagger}}{\partial y}\right|_{\hat{x}_i, \hat{y}_i} = \frac{\hat{y}_i - b_k}{\sqrt{(\hat{x}_i - a_k)^2 - (\hat{y}_i - b_k)^2}} = \frac{\hat{y}_i - b_k}{d_{ik}^{\dagger}}, \tag{6}$$

for $k = 1, 2, \ldots, N_a$. Note that $d_{ik}^{\dagger}$ is different from $\hat{d}_{ik}$ due to the error incurred by trilateration [12]. Therefore, rewriting (3) in a matrix form yields

$$\boldsymbol{\Gamma}_i \boldsymbol{\delta}_i = \boldsymbol{\zeta}_i - \boldsymbol{\epsilon}_i, \tag{7}$$

where

$$\boldsymbol{\Gamma}_{\mathbf{i}} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \vdots & \vdots \\ \alpha_{N_a 1} & \alpha_{N_a 2} \end{bmatrix} = \begin{bmatrix} \frac{\hat{x}_i - a_1}{d_{i1}^{\dagger}} & & \frac{\hat{y}_i - b_1}{d_{i1}^{\dagger}} \\ \frac{\hat{x}_i - a_2}{d_{i2}^{\dagger}} & & \frac{\hat{y}_i - b_2}{d_{i2}^{\dagger}} \\ \vdots & \vdots & \vdots \\ \frac{\hat{x}_i - a_m}{d_{iN_a}^{\dagger}} & & \frac{\hat{y}_i - b_m}{d_{iN_a}^{\dagger}} \end{bmatrix}, \tag{8}$$

$$\boldsymbol{\zeta}_i = \begin{bmatrix} \hat{d}_{i1} - d^{\dagger}_{i1} \\ \hat{d}_{i2} - d^{\dagger}_{i2} \\ \vdots \\ \hat{d}_{iN_a} - d^{\dagger}_{iN_a} \end{bmatrix}, \tag{9}$$

$\boldsymbol{\epsilon}_i = [\epsilon_{i1}, \epsilon_{i2}, \ldots, \epsilon_{iN_a}]^T$, and $\boldsymbol{\delta}_i = [\delta_{x_i}, \delta_{y_i}]^T$.

Many methods such as the weighted least squares (WLS) might be used to properly derive $\boldsymbol{\delta}_i$. Using WLS, the solution of (7) is given by :

$$\boldsymbol{\delta}_i = \left(\boldsymbol{\Gamma}_i^T \mathbf{P}_i^{-1} \boldsymbol{\Gamma}_i\right)^{-1} \boldsymbol{\Gamma}_i^T \mathbf{P}_i^{-1} \boldsymbol{\zeta}_i, \tag{10}$$

where $\mathbf{P}_i$ is the covariance matrix of $\boldsymbol{\epsilon}_i$. Since ϵ_{ik} $k = 1, \ldots, N_a$ are independent random variables, $\mathbf{P}_i$ boils down to $\text{diag}\left\{\sigma^2_{i1}, \ldots, \sigma^2_{iN_a}\right\}$ where σ^2_{ik} is the variance of ϵ_{ik}. However, assuming a high node density in the network, d_{ik} could be approximated as follows

$$d_{ik} \approx \sum_{j=1}^{n_{ik}^{\min}} h_j, \tag{11}$$

where h_j is the real size of the j-th hop which is a random variable itself. Substituting (11) in (1), we obtain that $\epsilon_{ik} \approx n_{ik}^{\min}\bar{h}_s - \sum_{j=1}^{n_{ik}^{\min}} h_j$ and, hence, $\sigma^2_{ik} = n_{ik}^{\min}\sigma^2_h$ where σ^2_h is the variance of h_j. Consequently,

$$\boldsymbol{\delta}_i = (\boldsymbol{\Gamma}_i^T \boldsymbol{\Lambda}_i \boldsymbol{\Gamma}_i)^{-1} \boldsymbol{\Gamma}_i^T \boldsymbol{\Lambda}_i \boldsymbol{\zeta}_i, \tag{12}$$

where $\boldsymbol{\Lambda}_i = \text{diag}\{1/n_{i1}^{\min}, \ldots 1/n_{ik}^{\min}\}$. A straightforward inspection of (10) reveals that $\boldsymbol{\delta}_i$ solely depends on the information locally available at the i-th regular node and, therefore, is locally computable at this node and does not require any additional information exchange between nodes. Moreover, since $\boldsymbol{\Gamma}_i^T \boldsymbol{\Lambda}_i \boldsymbol{\Gamma}_i$ is a 2-by-2 matrix, the entries of its inverse can be analytically and easily derived. Thus, the computation of $\boldsymbol{\delta}_i$ does not burden neither the implementation complexity of the proposed algorithm nor the overall cost of the network. Once we get $\boldsymbol{\delta}_i$, the value of $(\hat{x}_i, \hat{y}_i)$ is updated as $\hat{x}_i$=$\hat{x}_i + \delta_{x_i}$ and $\hat{y}_i = \hat{y}_i + \delta_{y_i}$. The computations are repeated until δ_{x_i} and δ_{y_i} approach zero. In such a case, we have from (2) that $x_i \approx \hat{x}_i$ and $y_i \approx \hat{y}_i$ and, hence, more accurate localization is performed. Note that, from (12), $\boldsymbol{\delta}_i$ is independent of ϵ_{ik} $k = 1, \ldots, N_a$. Consequently, the proposed algorithm is able to reduce the error due to mapping hops into distance units without requiring any distance error estimation.

IV. Parametric evaluation of the average hop-size

As discussed above, to work properly, the proposed algorithm requires the average hop size $\bar{h}_s$ to be available at each regular node. It is easy to show that

$$\bar{h}_s = \frac{\int_0^R z f_Z(z)\, dz}{\int_0^R f_Z(z)\, dz}, \tag{13}$$

where Z denotes the distance between any two nodes in the network and $f_Z(z)$ is its probability density function (pdf). As expected, $\bar{h}_s$ depends on $f_Z(z)$ which in turn depends on the node distribution. In the following, the average hop size $\bar{h}_s$ is derived considering the most used node distributions in WSN: Uniform and Gaussian.

Without loss of generality, let us denote by (x_1, y_1) and (x_2, y_2) the coordinates of two nodes in the area of concern, where x_1, y_1, x_2, y_2 are assumed to be identically and independently distributed random variables. Z can be then expressed as $Z = \sqrt{X^2 + Y^2}$ where $X = x_1 - x_2$ and $Y = y_1 - y_2$.

A. Uniform distribution

Assuming that the nodes are uniformly distributed, one can prove that the pdf of X^2 and Y^2 are $f_{X^2}(x) = \frac{1}{A^2}\left(\frac{A}{\sqrt{x^2}} - 1\right)$ and $f_{Y^2}(y) = \frac{1}{A^2}\left(\frac{A}{\sqrt{y^2}} - 1\right)$, respectively. Thus, Z has a cumulative density function (CDF) given by

$$F_Z(z) = \frac{z^2(6\pi A^2 - 16zA + 3z^2)}{6A^4}, \tag{14}$$

and, therefore, $f_Z(z)$ is given by

$$f_Z(z) = \frac{2z(\pi A^2 - 4zA + z^2)}{A^4}. \tag{15}$$

Using (15), $\bar{h}_s$ is hence given by

$$\bar{h}_s = \frac{4R(5\pi A^2 - 15AR + 3R^2)}{5(6\pi A^2 - 16AR + 3R^2)}. \tag{16}$$

It is straightforward to show from (16) that when A is large enough, $\bar{h}_s$ is reduced to $2R/3$. As expected, $\bar{h}_s$ increases proportionately with the transmission range R.

B. Gaussian distribution

Consider now that x_i, y_i, x_j, y_j are normally distributed random variables with the same standard deviation σ. In such a case, X and Y are also normally distributed random variables with variance equal to 2σ. Consequently, Z follows a Chi-distribution with 2 degrees of freedom. and, hence,

$$f_Z(z) = \frac{z}{2\sigma^2} e^{-\frac{z^2}{2\sigma^2}}. \tag{17}$$

$\bar{h}_s$ is then given by

$$\bar{h}_s = \sqrt{\pi}\sigma\left[1 - 2Q\left(\frac{R}{\sqrt{2}\sigma}\right)\right] - Re^{-\frac{R^2}{4\sigma^2}}. \tag{18}$$

From (18), $\bar{h}_s$ increases with R as observed in the case of the Uniform node distribution.

It follows from (16) and (18) that $\bar{h}_s$ can be readily computed at each regular node given the *a priori* knowledge of the node distribution before WSN deployment.

V. Nonparametric average hop-size evaluation

In the previous section, $\bar{h}_s$ was derived using prior knowledge of the node distribution. However, in practice, this distribution is often unknown before the deployment of the WSN. In such a case, we propose to exploit the distances between anchors, which are available at each regular nodes, as observations and use them to properly estimate the pdf $f_Z(z)$.

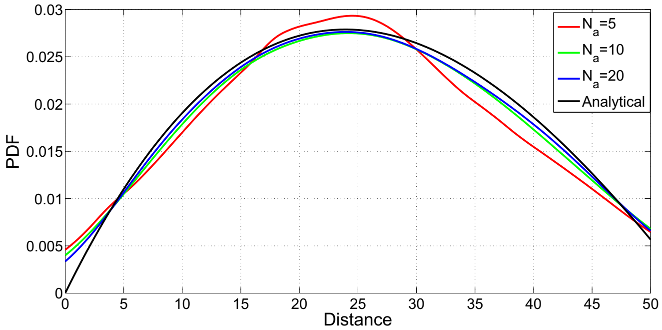

Fig. 2. Distance's pdf estimation under a Uniform distribution.

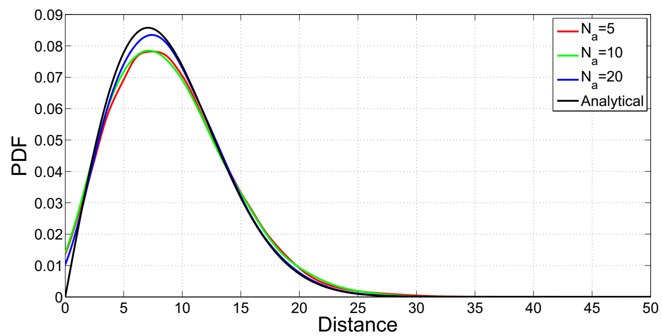

Fig. 3. Distance's pdf estimation under a Gaussian distribution.

Known as nonparametric pdf estimation, this technique plays a key role in enabling many applications such as image signal processing, speech recognition, etc. [13].

Recently, many nonparametric techniques have been proposed in the literature, for instance the histogram [14] and the well-known kernel density estimation (KDE) techniques [15]. In this paper, we are only concerned by the latter.

Assuming that N_a anchors exist in the network, the total number of distances (i.e, observations) available at each regular node is $p = \frac{N_a \times (N_a - 1)}{2}$. Let us denote by $z_1, z_2, \ldots, z_p$ such observations. Hence, the distance's pdf can be approximated by

$$\hat{f}_Z(z) = \frac{1}{pS} \sum_{i=1}^{p} K\left(\frac{z - z_i}{S}\right), \qquad (19)$$

where S is a smoothing parameter determined using the method in [14] and $K(z)$ is the Gaussian kernel given by

$$K(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right). \qquad (20)$$

The estimated pdf is computed by averaging the Gaussian density over all observations. Substituting (20) in (19) and using the resulting pdf to compute $\bar{h}_s$ yields

$$\bar{h}_s = \frac{\sum_{i=1}^{p} \mathrm{X}_i}{\sum_{i=p}^{p} \mathrm{A}_i}, \qquad (21)$$

where

$$A_i = s\sqrt{2\pi}\left[1 - Q\left(\frac{z_i}{S}\right) - Q\left(\frac{R - z_i}{S}\right)\right], \qquad (22)$$

and

$$\mathrm{X}_i = \left(S^2 + z_i^2\right) A_i - S^2\left[\left(R + z_i\right) e^{-\frac{(R - z_i)^2}{2S^2}} - z_i e^{-\frac{z_i^2}{2S^2}}\right], \qquad (23)$$

where $Q(x)$ is the Q-function. As can be observed from Figs. 2 and 3, it is possible to accurately estimate the distance's pdf for both the Uniform and Gaussian node distributions using a few anchors (i.e., observations). Moreover, from these figures, when N_a increases, the estimated pdf approaches the analytical one. This gives a sanity check for the proposed nonparametric method. Note that this method can be locally performed at each node using the information already available locally without any additional information exchange between nodes.

VI. Simulations and Results

In this section, we evaluate the performance of the proposed algorithm in terms of localization accuracy by simulations using Matlab. These simulations are conducted to compare, under the same network settings, the proposed algorithm with some of the best representative range-free methods currently available in the literature, i.e., DV-Hop [7] and DV-RSD [8]. All simulation results are obtained by averaging over 100 trials. We randomly deploy $N = 60$ nodes in a 2-D square area with $A = 50$ m. In order to obtain a connected network with high probability, one should select a suitable value of R. To this end, we exploit the results in [16] to fix $R = 12$ in our WSN setting.

As an evaluation criterion, we propose to use the normalized root mean square error (NRMSE) defined as follows

$$e = \frac{\sum_{i=1}^{N_u} \sqrt{\left(x_i - \hat{x}_i\right)^2 + \left(y_i - \hat{y}_i\right)^2}}{N_u R}, \qquad (24)$$

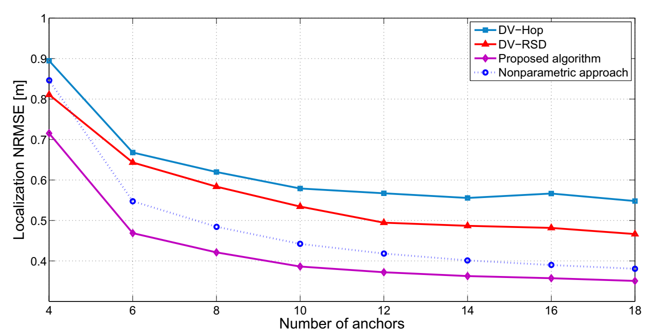

Fig. 4. Localization NRMSE vs. the number of anchors with a Uniform distribution.

Figs. 4 and 5 plot the localization NRMSE for different numbers of anchors N_a. From these figures, e decreases when N_a increases. This is expected since the trilateration becomes more efficient for large N_a. Moreover, as it can be seen from

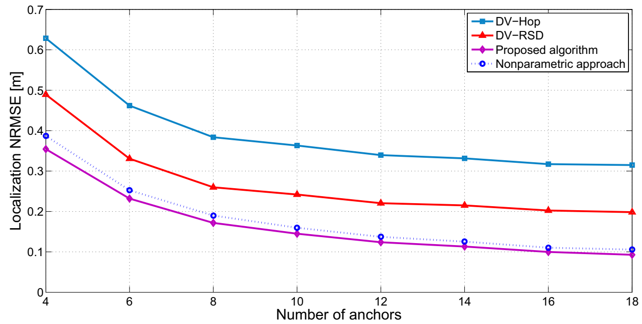

Fig. 5. Localization NRMSE vs. the number of anchors with a Gaussian distribution.

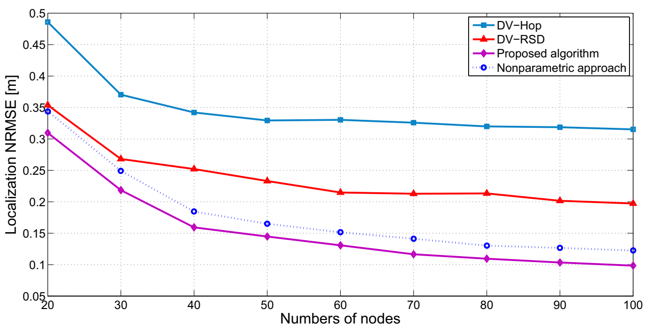

Fig. 6. Localization NRMSE vs. the number of nodes with a Uniform distribution.

Figs. 4, and 5, the proposed algorithm outperforms both the DV-Hop and DV-RSD algorithms for both the Uniform and Gaussian node distributions. Indeed, our algorithm turns out to be until about four and six times more accurate than DV-RSD and DV-Hop, respectively. Furthermore, the NRMSE curves for the parametric and nonparametric approaches of our algorithm are in quasi-perfect match even for small numbers of anchors. This is hardly surprising since the small error incurred when estimating $\bar{h}_s$ using the nonparametric approach is, in fact, added to ϵ_{ik}, $k = 1, \ldots, N_a$ which are reduced at the second step of our proposed algorithm. . This further verifies the effectiveness of our nonparametric approach.

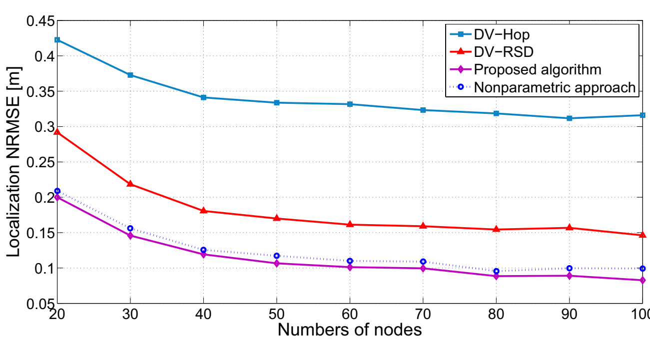

Fig. 7. Localization NRMSE vs. the number of nodes with a Gaussian distribution.

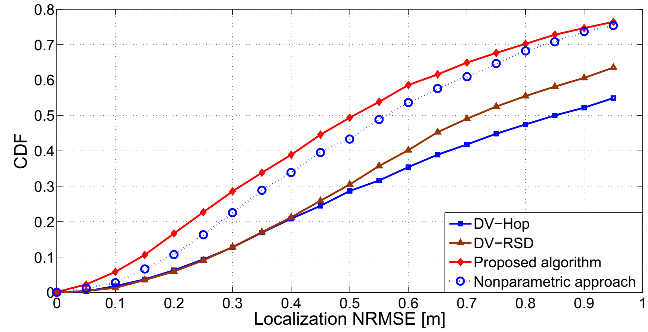

Fig. 8. Localization NRMSE's CDF with a Uniform distribution.

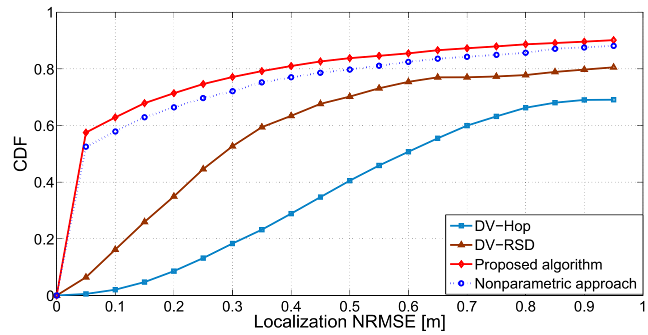

Fig. 9. Localization NRMSE's CDF with a Gaussian distribution.

Figs. 6 and 7 plot the localization NRMSE achieved by DV-Hop, DV-RSD and the proposed algorithm for different values of N when the number of anchors $N_a = 20\% \times N$ and $R = 28$. Note that $R = 28$ is selected to be adequate to the lowest density (i.e. $N = 20$). These figures show that the NRMSE decreases when the nodes' density increases. As can be shown from Figs. 6 and 7, the proposed algorithm, whether parametric or not, achieves almost identically the lowest localization NRMSE when compared to the two other benchmarks.

Figs. 8 and 9 illustrate the localization NRMSE's CDF. As it can be seen from Fig. 8 (Fig. 9, respectively), using the proposed algorithm, 50% (85%) of the regular nodes could estimate their position within half of the transmission range. While using the DV-RSD, 30% (72%) of the nodes achieve the same accuracy, and only 29% (40%) with DV-Hop.

VII. Conclusion

In this paper, we proposed a novel range-free localization algorithm able to reduce the error due to mapping the hops into distance units. Using the proposed algorithm, the mean hop size $\bar{h}_s$ is locally derived analytically at each regular (i.e., position-unaware) node, thereby avoiding its broadcast by anchors and, hence, resulting in reduced battery power depletion. The analytical expression of $\bar{h}_s$ is actually derived for both the Uniform and Gaussian node distributions. Furthermore, it was shown that it is possible to compute it locally at each regular node with or without prior knowledge of the node distribution.

It was also proved that the proposed scheme outperforms the well-known DV-Hop and DV-RSD in terms of localization accuracy.

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