# Enhancing the Performance of Spectrum-Sharing Systems via Collaborative Distributed Beamforming and AF Relaying*

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***Abstract*—In this paper, we use a distributed beamforming method in cognitive radio relay networks in an effort to enhance the spectrum efficiency and improve the performance of the cognitive (secondary) system. In particular, we consider a spectrum sharing system where a set of potential relays are employed to help a pair of secondary users in the presence of a licensed (primary) user. A selection relaying scenario in an amplify and forward (AF) scheme is investigated. In this context, we obtain the exact expressions for the cumulative distribution function (CDF) and the moment generating function (MGF) of the equivalent end-to-end SNR at the secondary destination. Then, to analyze the performance, we derive closed-form expressions for the outage probability and bit error rate (BER) over independent and identically distributed (i.i.d.) Rayleigh fading channels. Numerical results demonstrate the efficacy of beamforming in improving the secondary system performance in addition to limiting the interference to the primary users.**

## I. Introduction

Cognitive Radio (CR) is a promising solution to enhance the wireless spectrum utilization efficiency. In this regard, spectrum sharing is proposed to allow unlicensed users (secondary users) to share the spectrum with the licensed users (primary users) without causing harmful interference to the latter [1]. It is obliged to impose a constraint on the interference inflicted from the secondary users (SUs) onto the primary users (PUs). In this respect, [2] investigated the achievable capacity and outage probability of a spectrum-sharing system with amplify and forward (AF) relaying considering the average received-interference at the PU.

Beamforming is an effective technology to mitigate the inflicted interference in cognitive radio networks (CRNs). However, beamforming needs multi-antennas to be deployed in the unit to be realized which is prohibited in the first CR based IEEE standard (IEEE 802.22) [3], hence creating a virtual antenna array via cooperative relaying becomes a necessity. Our work mainly tackle these obstacles by using a collaborative distributed beamforming method in an AF selection relaying scheme in a spectrum sharing environment. In [4], Laneman et al., first introduced the selection relaying protocols where the relay decides either to amplify and forward or decode and forward (DF) selectively according to the received signal-to-noise ratio (SNR) in order to reduce the probability of error propagation. Recently, there have been a few articles applying beamforming in cooperative CRNs [5]– [7]. In [5], an iterative alternating optimization-based algorithm has been developed to obtain the optimal beamforming weights in order to maximize the worst signal to interference noise ratio. In [6], convex optimization tools are used to find the sub-optimal beamformers in relay assisted CRNs. However, these algorithms and tools suffer from high computational complexity and time consuming. Zero forcing beamforming (ZFB) is a simple sub-optimal approach that can be practically implemented. In [7], a zero forcing beamforming approach is applied to improve the primary system performance in an overly CR scenario. However, in [7], the ZFB is used in a single relay with collocated multi-antenna system.

In this paper, a distributed ZFB approach is applied to null the inflicted interference to the PU in the relaying phase beside improving the performance of secondary system. We also limit the interference from the secondary source by imposing a peak constraint on the interference received at the PU in the broadcasting phase. To analyze the performance, we derive the cumulative distribution functions (CDF) and the moment generating function (MGF) of the end-to-end equivalent SNR. Making use of these statistics, we derive closed form expressions for the outage probability and the bit error rate (BER). As a result, the ZFB approach has a potential for improving the secondary performance and limiting the interference in a simple practical manner compared to other complex approaches.

The rest of this paper is organized as follows. Section II describes the system model. Section III presents the statistics analysis. The system performance is analyzed in Section IV. Numerical results are given in Section V. Section VI concludes the paper. Finally, Appendices are introduced in Section VII. Throughout this paper, the Frobenius norm of the vectors are denoted by $||.||$. The Transpose and the Conjugate Transpose operations are denoted by $(.)^T$ and $(.)^\dagger$, respectively. $|x|$ means the magnitude of a complex number $x$. $\mathcal{CN} \sim (0,1)$ refers to

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a complex Gaussian normal random variable with zero-mean and unit variance. Diag($\mathbf{x}$) denotes a diagonal matrix whose diagonal elements are $\mathbf{x}$'s elements.

## II. System and Channel Models

Consider a relay-assisted CRN shown in Fig.1 where each SU and PU is equipped with a single antenna. Specifically, our system model consists of a secondary source (SS), a secondary destination (SD) and a set of $M$ Relays $R_i$, $i = 1, .., M$. There is no direct link between the source and destination, and they only communicate via potential relays $L_s \leq M$ that decide to forward the source's message. A primary system coexists in the same area with the secondary system. The SUs are allowed to share the same frequency spectrum with the PU as long as the interference to the PU is limited to a predefined threshold. They are transmitting simultaneously in underlay manner. The transmission protocol consists of two orthogonal time slots and is divided into two phases as shown in Fig. 1.

In the first phase, based on the interference channel state information (CSI) between the SS and $PU_1$, SS adjusts its transmit power under a predefined threshold $Q$ and broadcasts its message to the set of relays. So a peak power constraint is imposed on the interference received at $PU_1$ .

In the second phase, the potential relays, which are selected during the first-hop transmission, become members of the potential relays set $\mathcal{C}$ where ZFB is applied to null the interference from $\mathcal{C}$ to $PU_2$. By applying the ZFB approach, the synchronized set of potential relays are able to always transmit without interfering with $PU_2$. It is assumed that SS and $\mathcal{C}$ have perfect knowledge of their interference channel power gains which can be acquired through a spectrum-band manager that mediates between the primary and secondary users [2]. It is also assumed that perfect channel information is available at nodes SS, $\mathcal{C}$ and SD. The interference from the primary transmitter is neglected and can be represented in terms of noise when its message is generated by random Gaussian codebooks [2].

### A. CR Channel Model

All channel coefficients are assumed to be independent Rayleigh flat fading. Let $h_{a,b}$ denote the channel coefficient between nodes $a$ and $b$, which is modeled as a zero mean, circularly symmetric complex Gaussian (CSCG) random variable with variance $\lambda_{a,b}$. $n_a$ denotes additive white Gaussian noise which is also modeled as a zero mean, CSCG random variable with variance $\sigma^2$. Let $h_{s,r_i}$ denote the channel coefficient between the source's transmit antenna and the receive antenna of the $i^{th}$ relay and its channel power gain is $|h_{s,r_i}|^2$ which is exponentially distributed with parameter $\lambda_{s,r_i}$. Denote $h_{s,p}$ as the interference channel coefficient between SS and $PU_1$ and its channel power gain $|h_{s,p}|^2$, which is also exponentially distributed with parameter $\lambda_{s,p}$. Let $h_{r_i,p}$ and $h_{r_i,d}$ represent the interference channel coefficients between the $i^{th}$ relay and $PU_2$ and between the $i^{th}$ relay and SD, respectively.

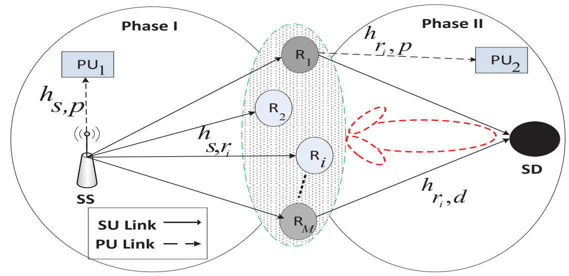


Fig. 1: System Model.

### B. Mathematical Model (Size of set $\mathcal{C}$)

In the underlay approach of this model, the SU can utilize the PU's spectrum as long as the interference it generates at the PUs remains below the interference threshold $Q$, which is the maximum tolerable interference level at which the PU can still maintain reliable communication [1]. Hence, the SS's power $P_s$ is constrained as $P_s \leq \frac{Q}{|h_{s,p}|^2}$ where $P_s$ is the maximum transmission power of SS.

The received SNR $\gamma_{s,r_i}$ at the $i^{th}$ relay is given as

$$\gamma_{s,r_i} = \frac{Q|h_{s,r_i}|^2}{\sigma^2|h_{s,p}|^2} \tag{1}$$

where $\sigma^2$ is the noise variance at each relay. As both of the channel power gains $|h_{s,r_i}|^2$ and $|h_{s,p}|^2$ are independent exponential distribution random variables, the PDF of $\gamma_{s,r_i}$ is represented as $\frac{\lambda_{s,r_i}\lambda_{s,p}\gamma_q}{(\lambda_{s,r_i}\gamma+\lambda_{s,p}\gamma_q)^2}$ [2]. Then, we find the CDF of $\gamma_{s,r_i}$ as

$$F_{\gamma_{s,r_i}}(\gamma) = \frac{\lambda_{s,r_i}\gamma}{\lambda_{s,r_i}\gamma + \lambda_{s,p}\gamma_q} \tag{2}$$

where $\gamma_q = \frac{Q}{\sigma^2}$.

We define $\mathcal{C}$ to be the set of relays which have their received instantaneous SNRs exceed a certain threshold in the first time slot. This translates to the fact that the mutual information between SS and each relay is above a specified target value. In this case, the potential $i^{th}$ relay is only required to meet the following constraint given as [4]

$$\Pr\left[R_i \in \mathcal{C}\right] = \Pr\left[\frac{1}{2}\log_2(1+\gamma_{s,r_i}) \geq R_{min}\right], \quad i = 1, ..., M \tag{3}$$

where $(1/2)$ is from the dual-hop transmission in two time slots and $R_{min}$ denotes the minimum target rate below which outage occurs. According to (2), we can get

$$\begin{aligned} \Pr\left[R_i \in \mathcal{C}\right] &= 1 - F_{\gamma_{s,r_i}}(\gamma_{min}) \\ &= 1 - \frac{\lambda_{s,r_i}\gamma_{min}}{\lambda_{s,r_i}\gamma_{min} + \lambda_{s,p}\gamma_q} \end{aligned} \tag{4}$$

where $\gamma_{min} = 2^{2R_{min}} - 1$ is the SNR threshold.

Without loss of generality, for all sub-channels are symmetrical, i.e., $\lambda_{s,r_i} = \lambda_{s,r}\,\forall i$, then $\Pr\left[R_i \in \mathcal{C}\right]$ is exactly the same for all $i$. Let $\Pr[R_i \in \mathcal{C}] = q$, and denote the cardinality of the set $\mathcal{C}$ as $|\mathcal{C}|$, then according to the Bionomial Law, $\Pr[|\mathcal{C}| = L_s]$ becomes

$$\Pr[|\mathcal{C}| = L_s] = \binom{M}{L_s} q^{L_s}(1-q)^{M-L_s}. \quad (5)$$

*C. ZFB Weights Design*

Our aim is to maximize the received power at the destination in order to maximize the mutual information of the secondary system. ZFB scheme is used as an alternative for the optimal scheme because of its simplicity and low complexity. To be able to apply ZFB, we consider the general assumption that the number of relays must be greater than or equal to the number of primary receivers plus secondary destination, hence $L_s \geq 2$. Let the ZFB vector be $\mathbf{w_{zf}}^T = [w_1, w_2, ...., w_{L_s}]$. Also let $\mathbf{h_{rd}}^T = [h_{r_1,d}, ..., h_{r_{L_s},d}]$, and $\mathbf{h_{rp}}^T = [h_{r_1,p}, ..., h_{r_{L_s},p}]$ be the channel vectors between the relays and both SD and $\text{PU}_2$, respectively. According to the ZFB principles, the transmit weight vector $\mathbf{w_{zf}}$ is chosen to lie in the orthogonal space of $\mathbf{h_{rp}}^\dagger$ such that $|\mathbf{h_{rp}}^\dagger \mathbf{w_{zf}}| = 0$ and $|\mathbf{h_{rd}}^\dagger \mathbf{w_{zf}}|$ is maximized. So the problem formulation for finding the optimal weight vector is as follows.

$$\begin{aligned} \max_{\mathbf{w_{zf}}} \quad & |\mathbf{h_{rd}}^\dagger \mathbf{w_{zf}}| \\ \text{s.t.:} \quad & |\mathbf{h_{rp}}^\dagger \mathbf{w_{zf}}| = 0 \\ & \|\mathbf{w_{zf}}\| = 1. \end{aligned} \quad (6)$$

To find the optimal vector, we consider the following Lemma from projection matrix theory [8].
*Lemma 1*: Let $\mathbf{T}$ be an $n \times k$ matrix with full column rank $k$, $k < n$, then the nonzero matrix $\mathbf{T}(\mathbf{T^H T})^{-1}\mathbf{T^H}$ is an idempotent symmetric matrix and its orthogonal projection matrix is $\mathbf{I} - \mathbf{T}(\mathbf{T^H T})^{-1}\mathbf{T^H}$ with rank $(n-k)$ [8, Theorems 4.21, 4.22].
By using Lemma 1 and applying a standard Lagrangian multiplier method, the weight vector that satisfies the above optimization method is given as

$$\mathbf{w_{zf}} = \frac{\mathbf{T}^\perp \mathbf{h_{rd}}}{\|\mathbf{T}^\perp \mathbf{h_{rd}}\|}, \quad (7)$$

where $\mathbf{T}^\perp = \left(\mathbf{I} - \mathbf{h_{rp}}(\mathbf{h_{rp}}^\dagger \mathbf{h_{rp}})^{-1}\mathbf{h_{rp}}^\dagger\right)$ is the projection idempotent matrix with rank $(L_s - 1)$.

## III. Statistics of the End-to-End SNR

In the first phase, the SS broadcasts its signal to all $M$ relays, then the received signal at the $i^{th}$ relay is given as

$$y_r = \sqrt{P_s} h_{s,r_i} x_s + n_r, \quad (8)$$

where $P_s$ is the source transmit power, $x_s$ is the information symbol with $\mathrm{E}[|x_s|^2] = 1$ and $n_r$ denotes the zero-mean complex Gaussian noise at the $i^{th}$ relay with variance $\sigma^2$. When the potential relays $L_s \leq M$ decide to participate in the second phase, the received $L_s \times 1$ vector at the relays $\mathbf{y_r}$ can be written in a vector form as

$$\mathbf{y_r} = \sqrt{P_s}\mathbf{h_{sr}} x_s + \mathbf{n_r} \quad (9)$$

where $\mathbf{h_{sr}}$ is the $L_s \times 1$ source- relays channel vector and $\mathbf{n_r}$ is $L_s \times 1$ CSCG noise vector with its elements are $\sigma^2$. In the second phase, to allow concurrent transmission of the secondary relays and $\text{PU}_2$, we first apply the $L_s \times 1$ ZFB vector denoted by $\mathbf{w_{zf}}$, and then the weighted signals are forwarded to SD. The received signal at SD is given as

$$y_d = \sqrt{P_s} A_r \mathbf{h_{rd}}^\dagger \text{Diag}(\mathbf{w_{zf}})\mathbf{h_{sr}} x_s + A_r \mathbf{h_{rd}}^\dagger \text{Diag}(\mathbf{w_{zf}})\mathbf{n_r} + n_d,$$

where $n_d$ denotes the zero-mean CSCG noise at SD with variance $\sigma^2$ and $A_r$ is the normalization constant designed to ensure that the long-term total transmit power at the relays is constrained and it is given by [11]

$$A_r = \sqrt{\frac{P_r}{\mathbf{w_{zf}}^\dagger (P_s \mathbf{h_{sr}} \mathbf{h_{sr}}^\dagger + \sigma^2 \mathbf{I})\mathbf{w_{zf}}}}. \quad (10)$$

Then the total received signal to noise ratio at SD is given as

$$\gamma_{eq} = \frac{P_s A_r^2 |\mathbf{h_{rd}}^\dagger \text{Diag}(\mathbf{w_{zf}})\mathbf{h_{sr}}|^2}{A_r^2 |\mathbf{h_{rd}}^\dagger \mathbf{w_{zf}}|^2 \sigma^2 + \sigma^2}. \quad (11)$$

Substituting (7) and (10) into (11), and after simple manipulations, the equivalent SNR at SD can be written in the general form of $\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}$ as:

$$\gamma_{eq} = \frac{\frac{P_s}{\sigma^2}\|\mathbf{h_{sr}}\|^2 \gamma_r \left\|\mathbf{T}^\perp \mathbf{h_{rd}}\right\|^2}{\frac{P_s}{\sigma^2}\|\mathbf{h_{sr}}\|^2 + \gamma_r \|\mathbf{T}^\perp \mathbf{h_{rd}}\|^2 + 1}. \quad (12)$$

Now considering the peak constraint on the received power at the $\text{PU}_1$, we substitute in SS's power $P_s$, then $\gamma_{eq}$ becomes

$$\gamma_{eq} = \frac{\gamma_q \frac{\|\mathbf{h_{sr}}\|^2}{|h_{s,p}|^2}\gamma_r \left\|\mathbf{T}^\perp \mathbf{h_{rd}}\right\|^2}{\gamma_q \frac{\|\mathbf{h_{sr}}\|^2}{|h_{s,p}|^2} + \gamma_r \|\mathbf{T}^\perp \mathbf{h_{rd}}\|^2 + 1}. \quad (13)$$

*A. First Order Statistics of $\gamma_{eq}$:*
We first present the statistics of the new random variables. Then, we derive the CDF and MGF of $\gamma_{eq}$ which will be used for the the derivation of the performance metrics. To continue, let $\gamma_1 = \gamma_q \frac{\|\mathbf{h_{sr}}\|^2}{|h_{s,p}|^2}$ and $\gamma_2 = \gamma_r \left\|\mathbf{T}^\perp \mathbf{h_{rd}}\right\|^2$.
*Lemma 2: (PDF and CDF of $\gamma_1$)*: Let each entry of $\mathbf{h_{sr}}$ be i.i.d. $\mathcal{CN} \sim (0,1)$, then $\|\mathbf{h_{sr}}\|^2$ is a chi squared random variable with $2L_s$ degrees of freedom, and given that $|h_{s,p}|^2$ is an exponential random variable, then the PDF and CDF of $\gamma_1$ are given respectively by:

$$f_{\gamma_1}(\gamma) = \frac{\lambda_{s,p} L_s}{(\gamma_q)^{L_s}} \frac{(\gamma)^{L_s - 1}}{(\frac{\gamma}{\gamma_q} + \lambda_{s,p})^{L_s+1}}, \quad (14)$$

$$F_{\gamma_1}(\gamma) = (\frac{\gamma}{\gamma_q \lambda_{s,p}})^{L_s} \; {}_2F_1(L_s+1, L_s; L_s+1; -\frac{\gamma}{\gamma_q \lambda_{s,p}}), \quad (15)$$

where ${}_2F_1(\,,\,;\,;\,)$ is the Gauss hypergeometric function defined in [13].
*Proof*: See appendix A.
*Lemma 3:(CDF of $\gamma_2$)*: Let each entry of $\mathbf{h_{rd}}$ be i.i.d. $\mathcal{CN} \sim (0,1)$, then $\left\|\mathbf{T}^\perp \mathbf{h_{rd}}\right\|^2$ is a chi squared random variable with $2(L_s - 1)$ degrees of freedom [9, theorem 2

Ch.1] then the CDF of $\gamma_2$

$$F_{\gamma_2}(\gamma) = 1 - \frac{1}{(L_s-2)!}\Gamma\left(L_s-1, \frac{\gamma}{\gamma_r}\right), \ \gamma \geq 0. \quad (16)$$

To proceed, we compute the statistics of the random variable $\gamma_{eq}$ defined by $\gamma_{eq} = \frac{\gamma_1\gamma_2}{\gamma_2+\gamma_2}$ [2], which can be considered as a tractable tight upper bound to the actual equivalent SNR.

*Theorem 1: (CDF of $\gamma_{eq}$)*: The CDF of the tight upper bounded $\gamma_{eq}$ is given by

$$\begin{aligned} F_{\gamma_{eq}}(\gamma) &= 1 - d\, e^{-c\gamma} \sum_{n=0}^{Ls-1}\sum_{k=0}^{Ls-2}\sum_{v=0}^{k} \frac{(c)^{k+\frac{n-v}{2}}}{k!}\binom{L_s-1}{n} \\ &\times \binom{k}{v}\Gamma(L_s-n+v)(\gamma)^{L_s-1+k}\ (\gamma+b)^{\frac{n-v}{2}-L_s} \\ &\times e^{\left(\frac{c\gamma^2}{2(\gamma+b)}\right)} W_{\frac{n-v}{2}-L_s, \frac{-n+v-1}{2}}\left(\frac{c\gamma^2}{\gamma+b}\right), \quad (17) \end{aligned}$$

where $b = \lambda_{s,p}\gamma_q$, $c = \frac{1}{\gamma_r}$, $d = \frac{\lambda_{s,p}}{\gamma_q}$ and $W_{.,.}(.)$ is the Whittaker function as defined in [13]. It is worth noting that the Whittaker function is implemented in many mathematical softwares such as Matlab and Mathematica.

*Proof:* See appendix B.

*B. Moment Generating Function (MGF) of $\gamma_{eq}$:*

In order to obtain the average BER for the secondary system, the MGF based approach [10] will be used in this article. Let $\gamma_{eq}^{-1} = \gamma_1^{-1} + \gamma_2^{-1} = X_1 + X_2$ where $X_1 = \gamma_1^{-1}$ and $X_2 = \gamma_2^{-1}$. As $\gamma_{eq}^{-1}$ is the sum of two independent random variables, the MGF of the $\gamma_{eq}^{-1}$ results simply from the product of the two MGFs of $X_1$ and $X_2$.

The MGF of a random variable $X$ is defined as

$$\phi_X(s) = E_X\{\exp(-sX)\} = \int_0^\infty e^{-sz} f_X(z)dz, \quad (18)$$

where $f_X(z)$ is the PDF of the random variable $X$. Firstly, we need to find the PDFs of $X_1$ and $X_2$. For the PDF of $X_1$, we follow the same mathematical approach as applied in (33) Which after some mathematical manipulations is obtained as

$$f_{X_1}(z) = \frac{\lambda_{s,p}L_s}{(\gamma_q)^{L_s}}\frac{1}{(\lambda_{s,p}z+\frac{1}{\gamma_q})^{L_s+1}}. \quad (19)$$

The PDF of $X_2$ is the PDF of the inverse chi-square random variable which leads to the following expression

$$f_{X_2}(z) = \frac{e^{\frac{-1}{\gamma_r z}}}{(\gamma_r)^{L_s-1}(L_s-2)!z^{L_s}}. \quad (20)$$

Substituting (19) into (18), and using [13, 3.382.4], the MGF for $X_1$ is

$$\phi_{X_1}(s) = \frac{L_s}{(\lambda_{s,p})^{L_s}(\gamma_q)^{L_s}} s^{L_s} e^{\frac{s}{\gamma_q\lambda_{s,p}}}\Gamma(-L_s, \frac{s}{\gamma_q\lambda_{s,p}}). \quad (21)$$

Similarly, substituting (20) into (18), and using [13, 3.471.9], the MGF for $X_2$ is

$$\phi_{X_2}(s) = \frac{2}{(\gamma_r)^{L_s-1}(L_s-2)!}\left(\frac{s}{\gamma_r}\right)^{\frac{L_s-1}{2}} K_{L_s-1}\left(2\sqrt{\frac{s}{\gamma_r}}\right) \quad (22)$$

where $K_v(.)$ is the modified Bessel function [13].

Now, we can easily compute the MGF of $\gamma_{eq}^{-1}$ as the product of $\phi_{X_1}(s)$ and $\phi_{X_2}(s)$ which is given as

$$\phi_{\gamma_{eq}^{-1}}(s) = \delta s^{\frac{3s-1}{2}} e^{\frac{s}{\gamma_q\lambda_{s,p}}}\Gamma(-L_s, \frac{s}{\gamma_q\lambda_{s,p}}) K_{L_s-1}\left(2\sqrt{\frac{s}{\gamma_r}}\right) \quad (23)$$

where $\delta = \frac{2L_s}{(\lambda_{s,p}\gamma_q)^{L_s}(\gamma_r)^{\frac{L_s-1}{2}}(L_s-2)!}$. We can make use of the following formula to find the MGF of the $\gamma_{eq}^{AF}$ utilizing the MGF of $\gamma_{eq}^{-1}$ [14, Eq. 18]

$$\phi_{\gamma_{eq}}(s) = 1 - 2\sqrt{s}\int_0^\infty J_1(2\beta\sqrt{s})\phi_{\gamma_{eq}^{-1}}(\beta^2)d\beta, \quad (24)$$

where $J_1(.)$ is the Bessel function of the first kind [13]. Although this formula seems to be difficult, we can still use it to study the performance of the BER based on the relationship that exists between the MGF and symbol error rate [10].

## IV. Performance Analysis

*A. Outage Probability Analysis*:

The mutual information at SD, $I_{AF}$, can be written as [4]

$$I_{AF} = \frac{1}{2}\log_2(1+\gamma_{eq}), \quad (25)$$

where $\gamma_{eq}$ represents the end to end received SNR at SD. An outage event occurs when $I_{AF}$ falls below a certain target rate. For a given rate $R_{min}$, the outage probability, $P_{out}$, can be rewritten using the total probability theorem as

$$P_{out} = \sum_{L_s=0}^{M} \Pr(|\mathcal{C}| = L_s)\Pr(I_{AF} < R_{min}|\,|\mathcal{C}| = L_s). \quad (26)$$

There exist two exclusive outage events for the secondary system using the distributed ZFB. Event A: failing to apply ZFB when $L_s < 2$ [1], and Event B: failing to achieve the target rate when $L_s \geq 2$. The probability of event A is given as $\sum_{L_s=0}^{1}\binom{M}{L_s}q^{L_s}(1-q)^{M-L_s}$ and the probability of event B is

$$\begin{aligned} \Pr(B) &= \Pr(I_{AF} < R_{min}|\,|\mathcal{C}| = L_s) \\ &= \Pr\left[\frac{1}{2}\log_2(1+\gamma_{eq}^{AF}) < R_{min}\right] \\ &= F_{eq}(\gamma_{min}). \quad (27) \end{aligned}$$

where $\gamma_{min} = 2^{2R_{min}} - 1$. The corresponding total outage probability can be computed by substituting (5), Pr(A) and Pr(B) into (26)

$$\begin{aligned} P_{out} &= \sum_{L_s=0}^{1}\binom{M}{L_s}q^{L_s}(1-q)^{M-L_s} \\ &+ \sum_{L_s=2}^{M}\binom{M}{L_s}q^{L_s}(1-q)^{M-L_s} \times F_{eq}^{AF}(\gamma_{min}) \quad (28) \end{aligned}$$

*B. Bit Error Rate Analysis*:

Exploiting the MGF-based form, the average BER of coherent

[1] In this case, system can limit the interference following the same approach as applied in the first phase

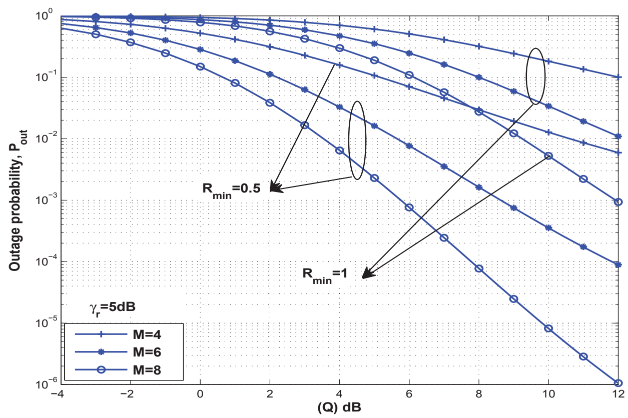


Fig. 2: Outage Probability vs. Q(dB) for different numbers of AF relays $M$=4, 6, 8 for $R_{min}$= 0.5, 1 bits/s/Hz and fixed $\gamma_r = 5$dB.

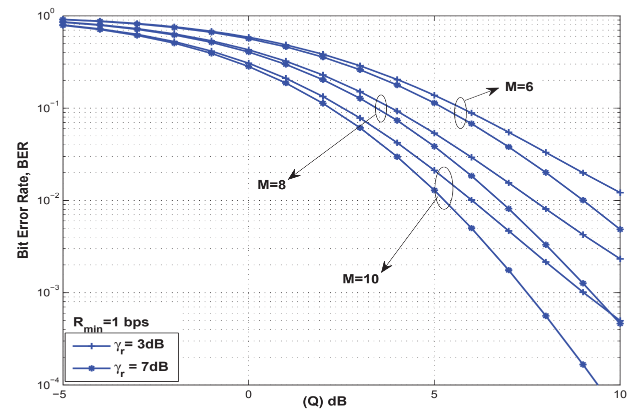


Fig. 3: Average BER vs. Q(dB) for different numbers of AF relays $M$=6, 8, 10 for two values of $\gamma_r = 3, 7$dBs for $R_{min} = 1$ bit/s/Hz

binary signaling is given by [10]

$$P_e = \frac{1}{\pi}\int_0^{\pi/2} \phi_{\gamma_{eq}}\left(\frac{A}{\sin^2\varphi}\right) d\varphi \tag{29}$$

where $A = 1$ for BPSK scheme. Now, Substituting (24) into (29) and after some manipulations, the formula of the BER becomes

$$P_e = \frac{1}{2} - \frac{2}{\pi}\int_0^{\infty} \phi_{\gamma_{eq}^{-1}}(\beta^2)\int_0^{\pi/2}\sqrt{\frac{A}{\sin^2\varphi}}J_1\left(\sqrt{\frac{4\beta^2 A}{\sin^2\varphi}}\right)d\varphi d\beta \tag{30}$$

The inner integral of (30) can be solved by using variable change and equation [15, eq. 2.12.4.15] which leads to the value $\frac{\sin(2\beta\sqrt{A})}{2\beta}$. So the BER can be evaluated according to the following formula

$$P_e = \frac{1}{2} - \frac{2}{\pi}\int_0^{\infty} \phi_{\gamma_{eq}^{-1}}(\beta^2)\frac{\sin(2\beta\sqrt{A})}{2\beta}d\beta, \tag{31}$$

where $\phi_{\gamma_{eq}^{-1}}$ is the MGF of the inverse SNR given in (23). Following the same approach as in the outage analysis, the total average BER is given as

$$P_e^{AF} = \sum_{L_s=0}^{M} \Pr(|\mathcal{C}| = L_s)\Pr(P_e||\mathcal{C}| = L_s), \tag{32}$$

where $\Pr(|\mathcal{C}| = L_s)$ is the same as (5) and $\Pr(P_e||\mathcal{C}| = L_s)$ is calculated as in (31).

## V. Numerical Results

In this section, we study the performance of some of the derived results through numerical evaluation. We assume that $\lambda_{s,p} = \lambda_{s,r} = 1$. Fig. 2 shows the outage performance of the AF selection relaying system versus the predefined threshold $Q$ for different number of available relays, $M = 4, 6, 8$ at the minimum rates $R_{min} = 0.5, 1$ bits/s/Hz. It can be readily seen that as the values of $Q$ becomes less restrict, the outage performance improves substantially. Moreover, by increasing the number of potential relays with ZFB approach applied, we observe significant improvement on the outage performance. It is translated to the effect of the combined cooperative diversity and beamforming on enhancing the total received SNR and hence the mutual information which is not the case of non-beamforming systems that use time division multiple access (TDMA) to have orthogonal channels. Using ZFB, it needs only two time slots to transmit meanwhile TDMA non-beamforming systems need number of time slots as the number of relaying channels . Fig. 3 illustrates the average BER performance versus the tolerable interference threshold $Q$ for for different numbers of relays $M$= 6, 8, 10 and two values of $\gamma_r = 3, 7$ dBs at $R_{min} = 1$ . It is obvious that the BER improves substantially as the number of potential relays increases and $Q$ becomes looser. For less transmit power of relays, $\gamma_r = 3$, this means less interference to PU and at the same time less received SNR at SD and hence less performance than when $\gamma_r = 7$ is used.

## VI. Conclusion

We investigated an AF selection relaying system model in CRNs that limits the interference to the primary system by imposing a peak interference power constraint in the first phase and applying a distributed ZFB approach in the second phase. The beamforming weights are optimized to maximize the received SNR at secondary destination and to null the interference inflicted on the primary user. We analyzed the performance of the secondary system by deriving the outage and BER probabilities. Our numerical results showed that the distributed ZFB method improves the outage and BER performances by increasing number of participating relays in addition to limiting interference to PU.

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## VII. APPENDIX

### A. Proof of Lemma 2

Using [12, equ. 6.60], $f_{\gamma_1}(\gamma)$ is found as follows:

$$f_{\gamma_1}(\gamma) = \int_{y=0}^{\infty} y f_{\|\mathbf{h_{sr}}\|^2}(y\gamma) f_{|h_{s,p}|^2}(y) dy$$

$$= \int_{y=0}^{\infty} y \frac{(y\gamma)^{L_s-1} e^{(-y\gamma/\gamma_q)}}{(L_s-1)!(\gamma_q)^{L_s}} \lambda_{s,p} e^{-\lambda_{s,p} y} dy \qquad (33)$$

$$= \frac{\lambda_{s,p}(\gamma)^{L_s-1}}{(\gamma_q)^{L_s}(L_s-1)!} \int_{y=0}^{\infty} y^{L_s} e^{(-y\gamma/\gamma_q)} e^{-\lambda_{s,p} y} dy$$

Using [13, eq. 3.326.1], we get (14).
To find the CDF, we just integrate the PDF as follows:

$$F_{\gamma_1}(\gamma) = \frac{\lambda_{s,p} L_s}{(\gamma_q)^{L_s}} \int_0^{\gamma} \frac{(x)^{L_s-1}}{(\frac{x}{\gamma_q} + \lambda_{s,p})^{L_s+1}} dx \qquad (34)$$

Using [13, eq. 3.194.1] to solve the integral, resulting in (15).

### B. Proof of Theorem 1

Using the definition of CDF $F_{\gamma_{eq}}^{AF}(\gamma) = F\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \le \gamma\right)$,

$$F_{\gamma_{eq}}^{AF}(\gamma) = \int_0^{\infty} \Pr\left[\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \le \gamma | \gamma_1\right] f_{\gamma_1}(\gamma_1) d\gamma_1 \qquad (35)$$
$$= F_{\gamma_1}(\gamma) + \underbrace{\int_{\gamma}^{\infty} F_{\gamma_2}\left(\frac{\gamma\gamma_1}{\gamma_1-\gamma}\right) f_{\gamma_1}(\gamma_1) d\gamma_1}_{I_1}$$

Then substitute into (16) and (14) , $I_1(\gamma)$ becomes as

$$I_1(\gamma) = \int_{\gamma}^{\infty} \left(1 - \frac{\Gamma\left(L_s-1, \frac{\gamma}{\gamma_1-\gamma}\frac{\gamma_1}{\gamma_r}\right)}{(L_s-2)!}\right) f_{\gamma_1}(\gamma_1) d\gamma_1$$
$$= 1 - \int_0^{\gamma} f_{\gamma_1}(\gamma_1) d\gamma_1 - \frac{1}{(L_s-2)!}$$
$$\times \int_{\gamma}^{\infty} \Gamma(L_s-1, \frac{\gamma}{\gamma_1-\gamma}\frac{\gamma_1}{\gamma_r}) f_{\gamma_1}(\gamma_1) d\gamma_1 \qquad (36)$$

then substitute (36) into (35),

$$F_{\gamma_{eq}}^{AF}(\gamma) = 1 - I_2 \qquad (37)$$

where

$$I_2 = \int_{\gamma}^{\infty} \frac{\Gamma\left(L_s-1, \frac{\gamma}{\gamma_1-\gamma}\frac{\gamma_1}{\gamma_r}\right)}{(L_s-2)!} f_{\gamma_1}(\gamma_1) d\gamma_1 \qquad (38)$$

By using the variable change $u = \gamma_1 - \gamma$, the integral in (38) can be written as

$$I_2 = a_1 \int_0^{\infty} \frac{\Gamma(L_s-1, c\gamma + \frac{c\gamma^2}{u})(u+\gamma)^{L_s-1}}{(u+\gamma+\lambda_{s,p}\gamma_q)^{L_s+1}} du \qquad (39)$$

where $a_1 = \frac{\lambda_{s,p} L_s}{(L_s-2)!(\gamma_q)^{2L_s+1}}$.
By using [13, eq. 8.352.2] and [13, eq. 1.111], the incomplete gamma function of the integral in (39) can be expressed as

$$\Gamma(L_s-1, c\gamma + \frac{c\gamma^2}{u}) = (Ls-1)!\mathrm{e}^{\left(-c\gamma - \frac{c\gamma^2}{u}\right)} \sum_{k=0}^{Ls-1} \sum_{v=0}^{k} \frac{1}{k!}$$
$$\times \binom{k}{v} (c\gamma)^{k-v} (c\gamma^2)^v \frac{1}{(u)^v} \qquad (40)$$

By using (40)and using [13, eq. 1.111] again for the term $(u+\gamma)^{L_s-1}$, $I_2$ can be expressed as

$$I_2 = a_1 (Ls-2)!\mathrm{e}^{(-c\gamma)} \sum_{n=0}^{Ls-1} \sum_{k=0}^{Ls-2} \sum_{v=0}^{k} \frac{(c)^k}{k!} \binom{k}{v} \binom{L_s-1}{n}$$
$$\times (\gamma)^{L_s-1-n+k+v} \int_0^{\infty} \frac{\mathrm{e}^{\left(-\frac{c\gamma^2}{u}\right)} (u)^{n-v}}{(u+\gamma+\lambda_{s,p}\gamma_q)^{L_s+1}} du \qquad (41)$$

The inner integral of $I_2$ can be solved by exploiting [13, eq. 3.471.7], resulting in

$$I_2 = a_1 (Ls-2)!\mathrm{e}^{-c\gamma} \sum_{n=0}^{Ls-1} \sum_{k=0}^{Ls-2} \sum_{v=0}^{k} \frac{(c)^{k+\frac{n-v}{2}}}{k!} \binom{L_s-1}{n}$$
$$\times \binom{k}{v} \Gamma(L_s-n+v)(\gamma)^{L_s-1+k} (\gamma+b)^{\frac{n-v}{2}-L_s}$$
$$\times \mathrm{e}^{\left(\frac{c\gamma^2}{2(\gamma+b)}\right)} W_{\frac{n-v}{2}-L_s, \frac{-n+v-1}{2}}\left(\frac{c\gamma^2}{\gamma+b}\right) \qquad (42)$$

With the help of (37) and (42), we get the CDF expression of $F_{\gamma_{eq}}^{AF}(\gamma)$ as given in (17).