Distributed Beamforming for Wireless Sensor Networks in Local Scattering Environments

Slim Zaidi and Sofiène Affes
INRS-EMT, 800, de la Gauchetière West, Suite 6900, Montréal, H5A 1K6, Qc, Canada,
Email: zaidi, affes@emt.inrs.ca

***Abstract*—In this paper, transmit and receive collaborative beamforming (CB) techniques are considered to achieve a dual-hop communication from a source to a receiver, through a wireless sensor network (WSN). Whereas the previous works assumed a model of plane wavefronts, here, a local scattering in the source or receiver vicinity is considered, thereby broadening the range of applications in real-world environments. Taking into account the local scattering, these beamformers aim to maintain the beamforming response in the desired direction equal to unity. It is shown that the so-obtained beamformers are not suitable for a distributed implementation in WSNs. We hence propose a novel distributed collaborative beamforming (DCB) technique that can be implemented in a distributed fashion and, further, well-approximates both transmit and receive CB techniques. The performance of the proposed DCB is analyzed and its advantages against the conventional DCB, which is designed without taking into account the presence of local scattering in the source or receiver vicinity, are analytically proved and are further verified by simulations.**

***Index Terms*—wireless sensor networks, distributed collaborative beamforming, local scattering.**

I. Introduction

In WSNs, the implementation of the collaborative beamforming (CB) technique can enable reliable communication over long distances [1]- [6]. Using this technique, a set of K small battery-powered WSN nodes multiply their received signals with the complex conjugates of properly selected beamforming weights, and forward the resulting signals to the destination. When the beamforming response in the desired direction is fixed, it has been shown that the forwarded noise power from the WSN nodes decreases inversely proportional to K and, hence, the achieved signal-to-noise ratio (SNR) increases with K [1], [2], [11], [13].

Due to these significant practical merits, the CB technique has garnered the attention of the research community. Assuming that the WSN nodes are uniformly distributed, the conventional distributed CB (DCB) technique has been presented in [1] and the characteristics of its beampattern have been analyzed. Beampattern characteristics of the conventional DCB have been also evaluated in [2], in the case that WSN nodes are Gaussian distributed. To improve the beampattern proprieties, nodes selection algorithms aiming to narrow down the mainbeam and minimize the effect of sidelobes have been, respectively, presented in [3] and [4]. CB techniques that improve the energy-efficiency and the WSN nodes' lifetime have been, respectively, proposed in [5] and [6]. In spite of their significant contributions, all the above papers neglect the effect of the scattering and reflection and assume a simple model of plane wavefronts. Unfortunately, in practice, the propagation environments are often more complicated than this model. In fact, when the source or the receiver is scattered by a large number of scatterers within its vicinity, as in rural and suburban environments, several replicas of the transmit or receive signal are generated [7]-[10]. Thus, this signal can be modeled as a superposition of independent, and identically distributed (i.i.d.) rays [7]. In [10], the effect of the local scattering in the CB technique for WSN has been investigated. It has been shown that this phenomenon causes a substantial decrease in terms of average power in the desired direction. However, the author did not offer any specific treatment to improve the robustness of the CB technique against it.

In this paper, we consider both transmit and receive CB that aim to fix the beamforming response in the desired direction to unity. Depending on the scheme, the source S or the receiver is assumed to be scattered by a large number of scatterers within its vicinity that generate L i.i.d rays from the transmit or the receive signal. Taking into account this phenomenon, the beamforming vectors corresponding to the both beamformers are derived. Unfortunately, it is shown that the so-obtained beamformers are not suitable for a distributed implementation in WSNs. Using the fact that the number of nodes is typically large in practice [11], [12], we propose a new DCB that not only can be implemented in a distributed fashion, but also well-approximates its transmit and receive CB counterparts. The performance of the proposed DCB technique is analyzed and its advantages against the conventional DCB technique, which is designed without taking into account the presence of local scattering in the source or receiver vicinity, is proved. It is shown that the proposed DCB is more robust than its vis-a-vis against the presence of local scattering and that it is able to achieve until 3 dB of SNR gain.

The rest of this paper is organized as follows. The system model is described in Section II. The CB technique in the presence of local scattering is described in Section III. A novel DCB technique is proposed in Section IV. Section V analyzes the performance of the proposed technique while Section VI provides computer simulations to support the theoretical results. Concluding remarks are given in Section VII.

Notation: Uppercase and lowercase bold letters denote matrices and vectors, respectively. $[\cdot]_{il}$ and $[\cdot]_i$ are the (i,l)-

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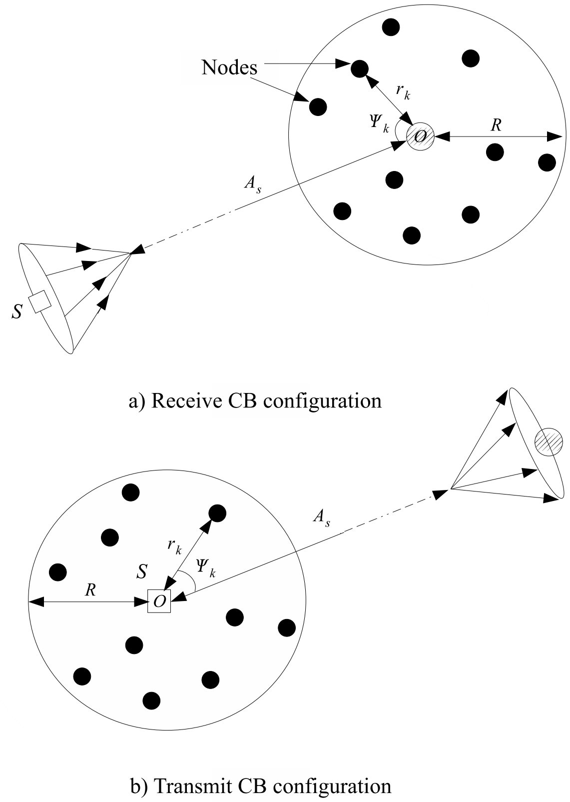

Fig. 1. Receive and transmit system configurations.

th entry of a matrix and i-th entry of a vector, respectively. $\mathbf{I}$ is the identity matrix and $\mathbf{e}_l$ is a vector with one in the l-th position and zeros elsewhere. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the Hermitian transpose, respectively. $\|\cdot\|$ is the 2-norm of a vector and $|\cdot|$ is the absolute value. $\mathrm{E}\{\cdot\}$ stands for the statistical expectation and ($\xrightarrow{ep1}$) $\xrightarrow{p1}$ denotes (element-wise) convergence with probability one. $J_1(\cdot)$ is the first order Bessel function of the first kind and $\odot$ is the element-wise product.

II. System model

As can be observed form Fig. 1, in this work, both receive and transmit CB configurations are of concern. As illustrated in Fig. 1.a, the system of interest in the receive configuration consists of a WSN comprised of K uniformly and independently distributed WSN nodes on $D(O,R)$, the disc with center at O and radius R, a receiver at O, and a source S located in the same plane containing $D(O,R)$ [1]-[6]. We assume that there is no direct link from the source to the receiver due to pathloss attenuation. Moreover, let (r_k,ψ_k) denote the polar coordinates of the k-th node and (A_s,ϕ_s) denote those of the source. Without loss of the generality, the latter is assumed to be at $\phi_s=0$ and to be located in the far-field region, hence, $A_s \gg R$. Description of the transmit configuration in Fig. 1.b is straightforward from the previous, where only the source and receiver switch positions. The following assumptions are further considered with respect to the receive CB configuration in Fig. 1.a (transmit CB configuration in Fig. 1.b):

A1) The far-field source (receiver) is scattered by a large number of scatterers within its vicinity that generate L equal power rays [7]- [10]. The l-th ray is characterized by its direction θ_l and its complex amplitude $\alpha_l=\rho_l e^{j\xi_l}$ where the amplitudes $\rho_l,\ l=1,\ldots,L$ and the phases $\xi_l,\ l=1,\ldots,L$ are i.i.d. random variables, and each phase is uniformly distributed over $[-\pi,\pi]$. The $\theta_l,\ l=1,\ldots,L$ are also i.i.d. random variables distributed with variance σ_θ^2 and probability density functions (pdf) $p(\theta_l)$. All θ_ls, ξ_ls, and ρ_ls are mutually independent [7]-[10].

A2) The channel gain $[\mathbf{f}]_k$ between the k-th node and the receiver (source) is a zero-mean unit-variance circular Gaussian random variable [11].

A3) The source signal s is a zero-mean random variable with power p_s while noises at nodes and the receiver are zero-mean Gaussian random variables with variances ${\sigma_v}^2$ and ${\sigma_n}^2$, respectively. The source signal, noises, and the nodes forward (backward) channel gains are mutually independent.

A4) The k-th node is aware of its own coordinates (r_k,ψ_k), its forward (backward) channel $[\mathbf{f}]_k$, the directions of the source (receiver) ϕ_s, K, and σ_θ while being oblivious to the locations and the forward (backward) channels of *all* other nodes in the network.

Using A1 and the fact that $A_s \gg R$, the channel gain between the k-th node and the source (receiver) can be represented as

$$[\mathbf{g}]_k=\sum_{l=1}^{L}\alpha_l e^{-j\frac{2\pi}{\lambda}r_k\cos(\theta_l-\psi_k)} \tag{1}$$

where λ is the wavelength.

III. CB techniques in the presence of local scattering

A. Receive CB Configuration

In this scheme, a dual-hop communication is established from the source S to the receiver. In the first time slot, the source sends its signal s to the WSN. Let $\mathbf{y}$ denotes the received signal vector at the nodes given by

$$\mathbf{y}=\mathbf{g}s+\mathbf{v}, \tag{2}$$

where $\mathbf{v}$ is the nodes' noise vector. In the second time slot, the k-th node multiplies its received signal with the complex conjugate of the beamforming weight w_k and forwards the resulting signal to the receiver. It follows from (2) that the received signal at O is

$$\begin{aligned} r &= \mathbf{f}^T(\mathbf{w}^*\odot\mathbf{y})+n=\mathbf{w}^H(\mathbf{f}\odot\mathbf{y})+n \\ &= \mathbf{w}^H(\mathbf{f}\odot\mathbf{g}s+\mathbf{f}\odot\mathbf{v})+n \\ &= s\mathbf{w}^H\mathbf{h}+\mathbf{w}^H(\mathbf{f}\odot\mathbf{v})+n, \end{aligned} \tag{3}$$

where $\mathbf{w}\triangleq[w_1\ldots w_K]$ is the beamforming vector, $\mathbf{h}\triangleq\mathbf{f}\odot\mathbf{g}$, $\mathbf{f}\triangleq[[\mathbf{f}]_1\ldots[\mathbf{f}]_K]^T$ and n is the receiver noise.

Note that several approaches can be adopted to properly select the beamforming weights such as minimizing the total transmit power subject to the received quality of service constraint, maximizing the received SNR subject to two different types of power constraints, namely the total transmit power constraint [11] and individual node power constraint [13], or simply matching the channel from the source to the receiver [1]-[4]. In this paper, we are only concerned with the latter approach that aims to maintain the beamforming response in the source direction equal to unity. In what follows, we design a collaborative receive beamformer and discuss its implementability in WSNs. Mathematically, we have to solve the following problem:

$$\mathbf{w}_{\mathrm{r}}^H \mathbf{h} = 1, \tag{4}$$

where $\mathbf{w}_{\mathrm{r}}$ is the beamforming vector associated with the receive CB technique. Since the noises at nodes are zero-mean Gaussian random variables, $\mathbf{w}_{\mathrm{r}} = \mathbf{w}_{\mathrm{o}}$ the optimum beamforming vector which satisfies

$$\mathbf{w}_{\mathrm{o}} = \arg\min P_{\mathbf{w},n}^{\mathrm{r}} \qquad \text{s.t.} \qquad \mathbf{w}^H \mathbf{h} = 1, \tag{5}$$

where $P_{\mathbf{w},n}^{\mathrm{r}}$ is the aggregate noise power due to the thermal noise at the receiver and the forwarded noises from the nodes given by

$$P_{\mathbf{w},n}^{\mathrm{r}} = \mathbf{w}^H \mathbf{\Lambda} \mathbf{w} + \sigma_n^2, \tag{6}$$

where $\mathbf{\Lambda} \triangleq \sigma_v^2 \mathrm{diag}\{|[\mathbf{f}]_1|^2 \ldots |[\mathbf{f}]_K|^2\}$. Using (6) in (5) we obtain the following optimization problem

$$\mathbf{w}_{\mathrm{o}} = \arg\min \mathbf{w}^H \mathbf{\Lambda} \mathbf{w} \qquad \text{s.t.} \qquad \mathbf{w}^H \mathbf{h} = 1. \tag{7}$$

Obliviously, since the channel is the sum of random variables, there is no practical purpose from claiming that the optimal solution of (7) is simply $\mathbf{h}$ due to prohibitive overhead required for its instantaneous estimation. Using the fact that $\mathbf{w}^H \mathbf{h} = 1$, one can rewrite (7) as

$$\mathbf{w}_{\mathrm{o}} = \arg\max \frac{\mathbf{w}^H \mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\} \mathbf{w}}{\mathbf{w}^H \mathbf{\Lambda} \mathbf{w}} \ \text{ s.t. } \ \mathbf{w}^H \mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\} \mathbf{w} = 1, \tag{8}$$

where the expectation is taken with respect to the rays' directions θ_ls and their complex amplitudes α_ls. It can be readily shown that $\mathbf{w}_{\mathrm{o}}$ is the principal eigenvector of the matrix $\mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}$ scaled to satisfy the constraint [11]-[13]. However, it can be observed from (8) that $\mathbf{w}_{\mathrm{o}}$ cannot be directly derived using the actual form of the latter matrix. Therefore, for the sake of analytical tractability, a useful approximation may be developed. This requires a more in-depth analytical study of the matrix $\mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}$. Using the assumption A1, one can deduce the following property:

$$\mathrm{E}\left\{\alpha_l^* \alpha_m\right\} = \left\{ \begin{array}{ll} 0 & l \neq m \\ \frac{1}{L} & l = m \end{array} \right. . \tag{9}$$

Consequently, $\mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}$ is reduced to

$$\mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\} = \int p(\theta) \mathbf{a}(\theta) \mathbf{a}^H(\theta) d\theta, \tag{10}$$

where $\mathbf{a}(\theta) \triangleq \left[[\mathbf{a}(\theta)]_1 \ldots [\mathbf{a}(\theta)]_K\right]^T$ with $[\mathbf{a}(\theta)]_k = [\mathbf{f}]_k e^{-j\frac{2\pi}{\lambda} r_k \cos(\theta - \psi_k)}$.

Nevertheless, if σ_θ is relatively small as in typical suburban environments, the relationship between $\mathbf{a}(\theta)$ and θ can be accurately described by the first three non-zero terms of the Taylor series of $\mathbf{a}(\theta)$ at 0 and, hence,

$$\mathbf{a}(\theta) = \mathbf{a} + \mathbf{a}'\theta + \mathbf{a}''\theta^2, \tag{11}$$

where $\mathbf{a} = \mathbf{a}(0)$, and $\mathbf{a}'$ and $\mathbf{a}''$ are, respectively, the first and the second derivatives of $\mathbf{a}(\theta)$ at 0. Therefore,

$$\begin{aligned} \mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\} &\approx \mathbf{a}\mathbf{a}^H + \frac{\sigma_\theta^2}{2}\left(\mathbf{a}\mathbf{a}''^H + \mathbf{a}''\mathbf{a}^H + 2\mathbf{a}'\mathbf{a}'^H\right) \\ &\approx \frac{1}{2}\left(\mathbf{a}(\sigma_\theta)\mathbf{a}(\sigma_\theta)^H + \mathbf{a}(-\sigma_\theta)\mathbf{a}(-\sigma_\theta)^H\right). \end{aligned} \tag{12}$$

It is noteworthy that the approximation in (12), previously exploited differently in angular spread and direction of arrival estimation of scattered sources [8], [9], is independent of the pdf $p(\theta)$. Using (12), the optimization problem in (8) can be written as

$$\mathbf{w}_{\mathrm{o}} \approx \arg\max \frac{\mathbf{w}^H \mathbf{\Xi} \mathbf{w}}{\mathbf{w}^H \mathbf{\Lambda} \mathbf{w}} \qquad \text{s.t} \quad \mathbf{w}^H \mathbf{\Xi} \mathbf{w} = 2, \tag{13}$$

where $\mathbf{\Xi} = \left(\mathbf{a}(\sigma_\theta)\mathbf{a}(\sigma_\theta)^H + \mathbf{a}(-\sigma_\theta)\mathbf{a}(-\sigma_\theta)^H\right)$. It can be shown that $\mathbf{w}_{\mathrm{o}} = \mu \mathbf{\Lambda}^{-1} \rho_{\max}(\mathbf{\Xi})$ where $\rho_{\max}(\mathbf{\Xi})$ is the principal eigenvector of the matrix $\mathbf{\Xi}$ and μ is the factor chosen such that the constraint in (13) is satisfied [13]. In the sequel, the expression of $\rho_{\max}(\mathbf{\Xi})$ is derived.

It is easy to note that the rank of $\mathbf{\Xi}$ is inferior or equal to two, which means that this matrix has at most two eigenvectors. In addition, it can be seen that

$$\begin{aligned} \mathbf{\Xi}\left(\mathbf{a}(\sigma_\theta) + \mathbf{a}(-\sigma_\theta)\right) = \ & \mathbf{a}(\sigma_\theta)\left(\|\mathbf{a}(\sigma_\theta)\|^2 + \mathbf{a}(\sigma_\theta)^H \mathbf{a}(-\sigma_\theta)\right) + \\ & \mathbf{a}(-\sigma_\theta)\left(\|\mathbf{a}(-\sigma_\theta)\|^2 + \mathbf{a}(-\sigma_\theta)^H \mathbf{a}(\sigma_\theta)\right), \end{aligned} \tag{14}$$

and

$$\begin{aligned} \mathbf{\Xi}\left(\mathbf{a}(\sigma_\theta) - \mathbf{a}(-\sigma_\theta)\right) = \ & \mathbf{a}(\sigma_\theta)\left(\|\mathbf{a}(\sigma_\theta)\|^2 - \mathbf{a}(\sigma_\theta)^H \mathbf{a}(-\sigma_\theta)\right) - \\ & \mathbf{a}(-\sigma_\theta)\left(\|\mathbf{a}(-\sigma_\theta)\|^2 - \mathbf{a}(-\sigma_\theta)^H \mathbf{a}(\sigma_\theta)\right). \end{aligned} \tag{15}$$

It is direct to show from the definition of $\mathbf{a}(\theta)$ that $\|\mathbf{a}(\sigma_\theta)\| = \|\mathbf{a}(-\sigma_\theta)\|$ and, further, $\mathbf{a}(-\sigma_\theta)^H \mathbf{a}(\sigma_\theta) \approx \mathbf{a}(\sigma_\theta)^H \mathbf{a}(-\sigma_\theta)$ for small σ_θ. Therefore, from (14) and (15), $\mathbf{a}(\sigma_\theta) + \mathbf{a}(-\sigma_\theta)$ and $\mathbf{a}(\sigma_\theta) - \mathbf{a}(-\sigma_\theta)$ are both eigenvectors of $\mathbf{\Xi}$ and, additionally, $\rho_{\max}(\mathbf{\Xi}) \approx \mathbf{a}(\sigma_\theta) + \mathbf{a}(-\sigma_\theta)$, when σ_θ is relatively small. Consequently, $\mathbf{w}_{\mathrm{o}}$ is given by

$$\mathbf{w}_{\mathrm{o}} = \frac{\mu}{K} \mathbf{\Lambda}^{-1}\left(\mathbf{a}(\sigma_\theta) + \mathbf{a}(-\sigma_\theta)\right), \tag{16}$$

where

$$\mu \ = \ \left(\frac{1}{\sigma_v^2} + \frac{\mathbf{a}(\sigma_\theta)^H \mathbf{\Lambda}^{-1} \mathbf{a}(-\sigma_\theta)}{K}\right)^{-1}. \tag{17}$$

Note that $\mathbf{w}_{\mathrm{o}}$ is valid for any given pdf $p(\theta)$.

Nevertheless, the receive CB technique is implementable in WSNs only if the k-th node can compute its corresponding beamforming weight $[\mathbf{w}_{\mathrm{o}}]_k$ that depends on μ and the k-th entry of $\mathbf{\Lambda}^{-1}\left(\mathbf{a}(\sigma_\theta) + \mathbf{a}(-\sigma_\theta)\right)/K$. According to A4, the latter depends only on the information available at the k-th node while μ is function of all nodes' locations and forward channels and, hence, cannot be computed at each node. Therefore, $\mathbf{w}_{\rm o}$ is not suitable for a distributed implementation in WSNs.

B. Transmit CB configuration

In this scheme, a dual-hop communication is also considered from the source S to the receiver. In the first time slot, the source sends its signal s to the nodes while, in the second time slot, the k-th node multiplies its received signal with the complex conjugate of the beamforming weight w_k and forwards the resulting signal to the far-field receiver. In order to select w_k for $k = 1 \ldots K$, the same criterion as above is used and, hence, we have to solve

$$\mathbf{w}_{\rm t} = \arg\min P^{\rm t}_{\mathbf{w},n} \qquad \text{s.t.} \qquad \mathbf{w}^H\mathbf{h} = 1 \tag{18}$$

where $\mathbf{w}_{\rm t}$ is the beamforming vector associated with the transmit CB technique and $P^{\rm t}_{\mathbf{w},n}$ is the aggregate noise power given by

$$P^{\rm t}_{\mathbf{w},n} = \mathbf{w}^H \mathrm{E}\left\{(\mathbf{h}\odot\mathbf{v})\left(\mathbf{h}^H\odot\mathbf{v}^H\right)\right\}\mathbf{w} + \sigma_n^2. \tag{19}$$

The expectation in (19) is taken with respect to the rays' directions θ_ls, their complex amplitudes α_ls and the forwarded noises from the nodes $\mathbf{v}$. Using the property in (9), it is straightforward to prove that $P^{\rm t}_{\mathbf{w},n} = P^{\rm r}_{\mathbf{w},n}$ and, therefore, $\mathbf{w}_{\rm t} = \mathbf{w}_{\rm o} = \mathbf{w}_{\rm r}$. Thus, both transmit and receive CB techniques are not suitable for a distributed implementation in WSNs.

IV. Proposed DCB technique

In order to get around the problem underlined above, one can substitute μ with a quantity that can be computed at each individual node and, in addition, well-approximates its original counterpart. Using the fact that the number of nodes K could be typically large in many practical cases [11], [12], μ can be substituted with $\mu_p = \lim_{K\to\infty}\mu$ in (16). Although μ_p is a good approximation of μ, it must also solely depend on the information commonly available at all the nodes. This will be proved in the following.

It is direct to show, from (17), that

$$\mu_p = \left(\frac{1}{\sigma_v^2} + \lim_{K\to\infty}\frac{\mathbf{a}(\sigma_\theta)^H\,\mathbf{\Lambda}^{-1}\mathbf{a}(-\sigma_\theta)}{K}\right)^{-1}. \tag{20}$$

It follows from the definition of $\mathbf{a}(\theta)$ that

$$\frac{\mathbf{a}(-\sigma_\theta)^H\mathbf{\Lambda}^{-1}\mathbf{a}(\sigma_\theta)}{K} = \frac{\sum_{k=1}^{K} e^{j\frac{2\pi}{\lambda}r_k(\cos(\psi_k+\sigma_\theta)-\cos(\psi_k-\sigma_\theta))}}{K\sigma_v^2}. \tag{21}$$

Using the strong law of large numbers and the fact that r_k, ψ_k and $[\mathbf{f}]_k$ are all mutually statistically independent, we obtain

$$\lim_{K\to\infty}\frac{\mathbf{a}(-\sigma_\theta)^H\mathbf{\Lambda}^{-1}\mathbf{a}(\sigma_\theta)}{K} \xrightarrow{p1} \frac{1}{\sigma_v^2}\mathrm{E}\left\{e^{j\frac{2\pi}{\lambda}r_k(\cos(\psi_k+\sigma_\theta)-\cos(\psi_k-\sigma_\theta))}\right\}. \tag{22}$$

Since the nodes are uniformly distributed on $D(O,R)$, it can be shown that [1]

$$\mathrm{E}\left\{e^{j\frac{2\pi}{\lambda}r_k(\cos(\psi_k+\sigma_\theta)-\cos(\psi_k-\sigma_\theta))}\right\} = 2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}, \tag{23}$$

where $\alpha(\phi) \triangleq (4\pi R/\lambda)\sin(\phi/2)$. By substituting (23) in (22), we have

$$\lim_{K\to\infty}\frac{\mathbf{a}(-\sigma_\theta)^H\mathbf{\Lambda}^{-1}\mathbf{a}(\sigma_\theta)}{K} \xrightarrow{p1} \frac{2}{\sigma_v^2}\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}. \tag{24}$$

Therefore, it follows from (20)-(24) that

$$\mu_{\rm p} \xrightarrow{p1} \sigma_v^2\left(1+2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}\right)^{-1}, \tag{25}$$

when the number of nodes K is large enough. As it can be observed from (25), $\mu_{\rm p}$ does not depend on the locations and the forward channels of any nodes and, therefore, it is locally computable at each individual node. Substituting μ by $\mu_{\rm p}$ in (16), we introduce a new DCB technique whose beamforming vector

$$\mathbf{w}_{\rm p} = \frac{\mu_{\rm p}}{K}\mathbf{\Lambda}^{-1}\left(\mathbf{a}(\sigma_\theta) + \mathbf{a}(-\sigma_\theta)\right) \tag{26}$$

not only can be implemented in a distributed fashion in WSNs, but also well-approximates its counterpart $\mathbf{w}_{\rm o}$, when K is large enough[1]. Moreover, it is valid for any given pdf $p(\theta)$. Note that in the conventional scenario, when there is no local scattering phenomenon (i.e. plane-wave propagation), $\sigma_\theta \longrightarrow 0$ and hence (26) is reduced to

$$\mathbf{w}_{\rm c} = \frac{1}{K}\mathbf{a}, \tag{27}$$

the beamforming vector associated with the well-known conventional DCB technique [1], [2].

V. Performance analysis of the proposed DCB technique

As discussed above, the proposed DCB well-approximates its transmit and receive CB counterparts and achieves the same performances in both considered configurations. Thus, for the sake of simplicity, in what follows, we only focus on the receive CB scheme. One way to prove the efficiency of the proposed DCB technique is undoubtedly comparing its achieved SNR with the SNR performed when the conventional DCB technique, which is designed without taking into account the presence of local scattering in the source vicinity, is used. To this end, we introduce the following performance measure:

$$\Upsilon(\sigma_\theta) = \frac{\xi_{\mathbf{w}_{\rm c}}}{\xi_{\mathbf{w}_{\rm p}}}, \tag{28}$$

where

$$\xi_{\mathbf{w}} = \frac{P_{\mathbf{w}}(\phi_s)}{P^{\rm r}_{\mathbf{w},n}}, \tag{29}$$

is the achieved SNR when the beamforming vector $\mathbf{w}$ is used. In (29), commonly known as the beampattern, $P_{\mathbf{w}}(\phi_\star) =$

[1]We will see in Section VI that K in the range of 20 is already sufficient to perfectly fit the asymptotic solution while K in the range of 10 readily offers an acceptable approximation within a dB fraction.

$p_\star \left|\mathbf{w}^H \sum_{l=1}^{L} \alpha_l \mathbf{a}(\phi_\star + \theta_l)\right|^2$ is the received power from a transmitter at direction $\phi_\star$ with power $p_\star$. It is noteworthy that Υ is an excessively complex function of the random variables r_k, ψ_k, $[\mathbf{f}]_k$ for $k = 1, \ldots, K$ and α_l, θ_l for $l = 1, \ldots, L$ and, hence, a random quantity of its own. Therefore, it is practically more appealing to investigate the behavior and the properties of $\tilde{\Upsilon}$ given by [7]

$$\tilde{\Upsilon}(\sigma_\theta) = \frac{\tilde{\xi}_{\mathbf{w}_\mathrm{c}}}{\tilde{\xi}_{\mathbf{w}_\mathrm{p}}}, \tag{30}$$

where $\tilde{\xi}_{\mathbf{w}} = \tilde{P}_{\mathbf{w}}(\phi_s)/\tilde{P}^{\mathrm{r}}_{\mathbf{w},n}$ is the achieved average-signal-to-average-noise ratio (ASANR) when $\mathbf{w}$ is implemented with $\tilde{P}_{\mathbf{w}}(\phi_\star) = \mathrm{E}\left\{P_{\mathbf{w}}(\phi_\star)\right\}$, called the average beampattern, and $\tilde{P}^{\mathrm{r}}_{\mathbf{w},n} = \mathrm{E}\left\{P^{\mathrm{r}}_{\mathbf{w},n}\right\}$ is the average noise power. Thus, using the proposed DCB technique, it can be shown that

$$\tilde{P}^{\mathrm{r}}_{\mathbf{w}_\mathrm{p},n} = \frac{2\sigma_v^2}{K}\left(1 + 2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}\right)^{-1} + {\sigma_n}^2, \tag{31}$$

and

$$\tilde{P}_{\mathbf{w}_\mathrm{p}}(\phi_\star) = \frac{2p_\star}{K\left(1 + 2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}\right)}\left(1 + \frac{2(K-1)\Omega(\phi_\star)}{\left(1 + 2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}\right)}\right), \tag{32}$$

with

$$\Omega(\phi) = \int p(\theta)\left(\frac{J_1(\alpha(\phi+\theta+\sigma_\theta))}{\alpha(\phi+\theta+\sigma_\theta)} + \frac{J_1(\alpha(\phi+\theta-\sigma_\theta))}{\alpha(\phi+\theta-\sigma_\theta)}\right)^2 d\theta. \tag{33}$$

When the conventional DCB technique is implemented in the WSN, it can be proved that

$$\tilde{P}^{\mathrm{r}}_{\mathbf{w}_\mathrm{c},n} = \frac{\sigma_v^2}{K} + {\sigma_n}^2, \tag{34}$$

and

$$\tilde{P}_{\mathbf{w}_\mathrm{c}}(\phi_\star) = \frac{p_\star}{K}\left(1 + (K-1)\Gamma(\phi_\star)\right), \tag{35}$$

with

$$\Gamma(\phi) = \int p(\theta)\left(2\frac{J_1(\alpha(\phi+\theta))}{\alpha(\phi+\theta)}\right)^2 d\theta. \tag{36}$$

Note that, from (31)-(36), $\tilde{\Upsilon}(\sigma_\theta)$ is independent of the pdf $p(\theta)$. It is also noteworthy that the integrals in (33) and (36) can be computed numerically with any desired accuracy by using the most popular mathematical software packages such as Matlab and Mathematica, after properly choosing the pdf $p(\theta)$. In fact, several statistical distributions for θ_l have been proposed so far such as the Laplace, Gaussian or Uniform distribution [7]-[10]. Moreover, from (31)-(36), when K is large enough, it holds for small σ_θ that

$$\tilde{\Upsilon}(\sigma_\theta) \approx \frac{1}{4}\left(1 +_0 F_1\left(;2;-4\pi^2\left(\frac{R}{\lambda}\sigma_\theta\right)^2\right)\right)^2. \tag{37}$$

Since the hypergeometric function ${}_0F_1\left(;2;-4\pi^2x^2\right)$ has a maximum peak value of 1 at $x = 0$, the above expression indicates that regardless of the value of R/λ, $\tilde{\xi}_{\mathbf{w}_\mathrm{p}} \approx \tilde{\xi}_{\mathbf{w}_\mathrm{c}}$, when there is no local scattering in the source vicinity. This is expected since $\mathbf{w}_\mathrm{p}$ boils down to $\mathbf{w}_\mathrm{c}$ in such a case. In addition, for small x, ${}_0F_1\left(;2;-4\pi^2x^2\right)$ decreases inversely proportional to x while for large x, it has an oscillatory tail but converges to 0 as x increases. Thus, it can be observed from (37) that, as σ_θ increases, the ASANR gain achieved using $\mathbf{w}_\mathrm{p}$ instead of $\mathbf{w}_\mathrm{c}$ increases and can reach as much as 3 dB. Therefore, the proposed DCB technique is much more efficient in terms of achieved ASANR compared to the conventional DCB technique, which is designed without taking into account the presence of local scattering, and this holds for any given pdf $p(\theta)$.

Further, it can be shown that

$$\lim_{K\to\infty} \tilde{\Upsilon}(\sigma_\theta) = \lim_{K\to\infty} \Upsilon^{\mathrm{av}}(\sigma_\theta), \tag{38}$$

where $\Upsilon^{\mathrm{av}}(\sigma_\theta)$ is the average SNR (ASNR). Therefore, $\tilde{\Upsilon}(\sigma_\theta)$ is a meaningful performance measure that asymptotically converges to $\Upsilon^{\mathrm{av}}(\sigma_\theta)$ when K is large enough. Consequently, the proposed DCB technique is also much more efficient in enhancing the achieved ASNR, and this holds for any distribution of θ_ls.

VI. Simulation Results

Computer simulations are provided to support the theoretical results. All the simulation results are obtained by averaging over 10^6 random realizations of r_k, ψ_k, $[\mathbf{f}]_k$ for $k = 1, \ldots, K$ and α_l, θ_l for $l = 1, \ldots, L$. In all examples, we assume that the noises' powers σ_n^2 and σ_v^2 are 10 dB below the source transmit power p_s. It is also assumed that the number of rays is $L = 6$ and that their phases are uniformly distributed. Fig. 2 displays $\tilde{P}_{\mathbf{w}_\mathrm{p}}(\phi_\star)$, $\tilde{P}_{\mathbf{w}_\mathrm{c}}(\phi_\star)$ and $\tilde{P}_{\mathbf{w}_\mathrm{o}}(\phi_\star)$ for $R = 1$, $K = 20$, and $\sigma_\theta = 17$ (deg). As can be observed from this figure, the proposed DCB is much more efficient than the conventional DCB in keeping the beamforming response in the source direction equal to unity. Fig. 3 plots $\tilde{\xi}_{\mathbf{w}_\mathrm{p}}$ and $\tilde{\xi}_{\mathbf{w}_\mathrm{o}}$ versus σ_θ for $R = 1$ and different values of K. From this figure, the performance of the proposed DCB technique fits perfectly with that of the receive CB technique when K is in the range of 20 while it looses only a fraction of a dB when K is in the range of 10. Fig. 4 displays $\tilde{\xi}_{\mathbf{w}_\mathrm{p}}$, $\tilde{\xi}_{\mathbf{w}_\mathrm{c}}$ and $\tilde{\xi}_{\mathbf{w}_\mathrm{o}}$ versus σ_θ for $R = 1$ and $K = 20$. It can be verified from Fig. 4 that the proposed DCB technique is able to perform the maximum achievable SNR even in suburban environments where σ_θ is in the range of 20 degrees, while the SNR performed by its conventional vis-a-vis decreases by 0.5 dB for $\sigma_\theta = 7$ degrees and becomes unsatisfactory in suburban environments. In such environments, the proposed technique achieves until 3 dB of SNR gain.

VII. Conclusion

We consider both transmit and receive CB techniques to achieve a dual-hop communication from a source to a receiver, through a WSN. Here, local scattering in the source or receiver vicinity is assumed while in the previous works the effect of scattering and reflection was neglected. Taking into account this phenomenon, the beamforming vectors corresponding to

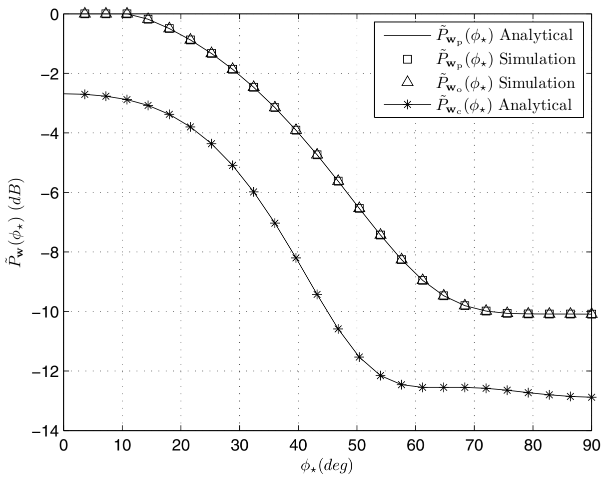

Fig. 2. $\tilde{P}_{\mathbf{w}_{\rm p}}(\phi_\star)$, $\tilde{P}_{\mathbf{w}_{\rm c}}(\phi_\star)$ and $\tilde{P}_{\mathbf{w}_{\rm o}}(\phi_\star)$ for $\sigma_\theta = 17$ (deg) and $R = 1$.

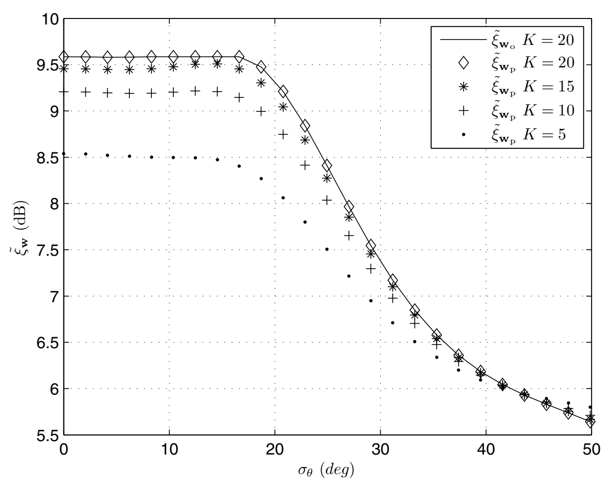

Fig. 3. $\tilde{\xi}_{\mathbf{w}_{\rm p}}$ and $\tilde{\xi}_{\mathbf{w}_{\rm o}}$ versus σ_θ for $R = 1$ and $K = 5, 10, 15, 20$.

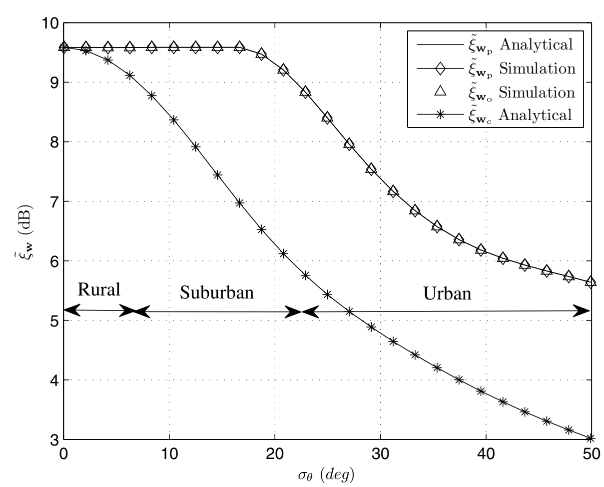

Fig. 4. $\tilde{\xi}_{\mathbf{w}_{\rm p}}$, $\tilde{\xi}_{\mathbf{w}_{\rm c}}$ and $\tilde{\xi}_{\mathbf{w}_{\rm o}}$ versus σ_θ for $R = 1$.

the transmit and receive configurations are derived. Unfortunately, it is shown that the so-obtained beamformers are not suitable for a distributed implementation in WSNs. Using the fact that the number of nodes is typically large in practice, we propose a novel DCB that can be implemented in a distributed fashion and, further, well-approximates its transmit and receive CB counterparts. The performance of the proposed DCB technique is analyzed and its advantages against the conventional DCB technique, which is designed without taking into account the presence of local scattering in the source or receiver vicinity, was proved. It is shown that the proposed DCB is not only more robust than its vis-a-vis against the presence of local scattering in the source or receiver vicinity but also, is able to achieve until 3 dB of SNR gain.

REFERENCES

[1] H. Ochiai, P. Mitran, H. V. Poor, and V. Tarokh, "Collaborative beamforming for distributed wireless ad hoc sensor networks," *IEEE Trans. Signal Process.,* vol. 53, pp. 4110-4124, Nov. 2005.

[2] M. F. A. Ahmed and S. A. Vorobyov, "Collaborative beamforming for wireless sensor networks with Gaussian distributed sensor nodes," *IEEE Trans. Wireless Commun.,* vol. 8, pp. 638-643, Feb. 2009.

[3] K. Zarifi, S. Affes, and A. Ghrayeb, "Distributed beamforming for wireless sensor networks with improved graph connectivity and energy efficiency," *IEEE Trans. Signal Process.,* vol. 58, pp. 1904-1921, Mar. 2010.

[4] M. F. A. Ahmed and S. A. Vorobyov, "Sidelobe control in collaborative beamforming via node selection," *IEEE Trans. Signal Process.,* vol. 58, pp. 6168-6180, Dec. 2010.

[5] M. Pun, D. R. Brown III, and H. V. Poor, "Opportunistic collaborative beamforming with one-bit feedback," *IEEE Trans. Wireless Commun.,* vol. 8, pp. 2629-2641, May 2009.

[6] Z. Han and H. V. Poor, "Lifetime improvement in wireless sensor networks via collaborative beamforming and cooperative transmission," *IET Microw. Antennas Propag.,* vol. 1, pp. 1103-1110, Dec 2007.

[7] D. Astly and B. Ottersten, "The effects of local scattering on direction of arrival estimation with MUSIC," *IEEE Trans. Signal Process.,* vol. 47, pp. 32203234, Dec. 1999.

[8] M. Souden, S. Affes, and J. Benesty, "A two-stage approach to estimate the angles of arrival and the angular spreads of locally scattered sources," *IEEE Trans. Signal Process.,* vol. 56, pp. 19681983, May 2008.

[9] M. Bengtsson and B. Ottersten, "Low-complexity estimators for distributed sources," *IEEE Trans. Signal Process.,* vol. 48, pp. 21852194, Aug. 2000.

[10] A. Amar, "The effect of local scattering on the gain and beamwidth of a collaborative beampattern for wireless sensor networks," *IEEE Trans. Wireless Commun.,* vol. 9, pp. 27302736, Sep. 2010.

[11] K. Zarifi, S. Zaidi, S. Affes, and A. Ghrayeb, "A distributed amplify-and-forward beamforming technique in wireless sensor networks," *IEEE Trans. Signal Process.,* vol. 59, pp. 36573674, Aug. 2011.

[12] K. Zarifi, S. Affes, and A. Ghrayeb, "Collaborative null-steering beamforming for uniformly distrubuted wireless sensor networks," *IEEE Trans. Signal Process.,* vol. 58, pp. 1889-1903, Mar. 2010.

[13] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z.-Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Trans. Signal Process.,* vol. 56, pp. 43064316, Sep. 2008.