Reactive Relay Selection in Cooperative Spectrum-Sharing Systems

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***Abstract*—We consider a dual-hop cooperative spectrum-sharing system with multiple relays that are available to opportunistically help the secondary communication system. In this primary/secondary cooperative system, we propose using a *reactive* relay selection (RRS) technique at the secondary system in which the best relay is chosen based on both the first and second hops transmission conditions. In fact, the best relay is selected as the relay node that not only satisfies a minimum required rate on the first-hop transmission, but can also achieve the highest signal-to-noise ratio (SNR) at the destination node. In this context, we first derive an expression for the cumulative distribution function (CDF) of the received SNR at the secondary destination node while assuming that the secondary transmission is limited by the appropriate interference constraints. Then, we use this CDF expression to obtain a closed-form expression for the end-to-end outage probability of the proposed cooperative system. Finally, illustrative numerical examples are shown and the benefits of using the proposed RRS technique in different channel propagation conditions are discussed.**

I. INTRODUCTION

Relaying communication has shown great potential to overcome the insufficiency of the spectrum resources in cognitive radio (CR) networks [1]. Earlier, the cooperative transmission technique has been used in wireless communication systems in order to improve the system performance and coverage area [2], [3]. In CR systems, when the available spectrum resources are not sufficient to guarantee reliable transmission for the cognitive users, the resource allocation policy may not be able to satisfy the cognitive users' requirements. In such cases, cooperative communication can be employed by the CR (secondary) system to more effectively and efficiently utilize the available transmission resources, such as power, while adhering to the interference constraints at the primary receivers. This has recently been shown to be of great interest in CR systems [4]–[6].

In this regard, in [5], an optimization technique has been developed for a multipoint-to-multipoint spectrum-sharing system in order to jointly optimize the secondary transmit power and also the relays' beamforming weights. The latter was investigated subject to the constraints on the interference inflicted on the existing PRs. The work in [4], on the other hand, considered that the secondary communication is assisted by some intermediate relays that implement the decode-and-forward (DF) technique onto the SU's relayed signal. That paper also investigated the end-to-end performance of the dual-hop cooperative spectrum-sharing system under resource constraints defined so as to ensure that the primary's quality of service (QoS) is unaffected.

Regarding the scenario that a cluster of relays is available between the source and destination nodes, how to choose a proper relay is an important issue. Several centralized relay selection schemes were developed in [7], where the selection of an appropriate relay node was performed based on different assumptions about the channel knowledge. In this context, an antenna/relay selection strategy for coded cooperative networks has been introduced in [8] to improve the detection reliability at the relay nodes. In [9] and [10], a threshold-based relaying approach has been presented to mitigate the error propagation inherent in cooperative communication networks. Moreover, in [11] and [12], a simple relaying protocol, called *reactive* relay selection (RRS), was presented in which the relay selection is performed after the first-hop transmission. In the RRS strategy, only the relays whose received SNRs are higher than a certain threshold will be taken into account in the selection. Then, the single relay with the maximum received SNR at the destination node is chosen to forward the signal to the destination. It is worth noting that in the last-mentioned works, the selection is basically performed based on the SNR values calculated using the forwarding channel information, i.e., source-relays-destination. In cooperative spectrum-sharing CR networks, however, only limited interference with primary systems is allowed. Hence, the relay selection method must be different from the conventional ones. In cooperative CR systems, the secondary source chooses the relay that maximizes the SNR at the secondary destination node, while taking the interference generated at the primary system into consideration to ensure that no harmful interference is caused to the primary system.

In this paper, we propose using the RRS strategy in cooperative relaying spectrum-sharing systems while taking into account the necessary limitations on the generated interference on the primary system. In fact, the RRS is chosen to be implemented in the secondary communication system, since it selects the best relay based on both the first and second hops transmission conditions. Specifically, in Section II, we

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consider a dual-hop cooperative spectrum-sharing system with multiple intermediate relays available to help the secondary communication system between the source and destination nodes. In Section III, a useful expression for the cumulative distribution function (CDF) of the received SNR at the secondary destination is obtained while taking into account the average interference constraints at the PRs. Then, in Section IV, making use of the obtained CDF, we derive a closed-form expression for the end-to-end outage probability of the proposed cooperative spectrum-sharing system. Moreover, we provide insightful discussion about the benefits of using the RRS scheme in Section V. Numerical results show an improvement on the overall system performance when the second transmission link is in strong propagation conditions or when the number of potential relays increases. Finally, concluding remarks are drawn in Section VI.

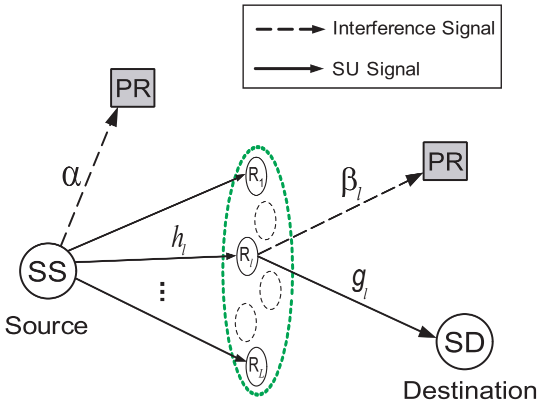

Fig. 1: The proposed cooperative relaying spectrum-sharing system with *reactive* relay selection.

II. System and Channel Models

We consider a dual-hop cooperative spectrum-sharing CR system where the DF relays are employed to help in the SU's communication process, as shown in Fig. 1. It is assumed that a cluster of relays are available for the secondary source (SS) communication from where one relay is chosen based on the RRS selection technique [11]. In this technique, the selection is performed after the first-hop transmission. Specifically, we assume that during the first-hop transmission, all relays listen to the SS's signal and only those with the received SNR higher than a certain threshold, denoted by $\gamma_{\min}$, will decode the received signal, where $\gamma_{\min}$ can be calculated in terms of the minimum service-rate constraint ($R_{\min}$) on the first-hop transmission as given by $\gamma_{\min} = 2^{R_{\min}} - 1$. Then, during the second-hop transmission, the single relay with the maximum received SNR at the secondary destination (SD) is chosen to forward the secondary signal to the destination. It is assumed that the relays within a cluster are located close together so that the SS-relay links experience the same average SNR, but distinct instantaneous SNR values.

We consider a discrete-time flat-fading channel with perfect channel state information (CSI) at the secondary transmitters and receivers. As illustrated in Fig. 1, we assume that the channel power gains between the SS and relays is given by h_l, $\forall\ l = 1, ..., L$, with mean τ^{f} and the one between the l-th relay and the SD by g_l with mean τ^{s}. In this paper, it is assumed, as in underlay spectrum sharing technique [4], that the SUs have knowledge of the interference they cause to the primary users (PUs). Hence, we assume that the secondary and primary users are allowed to simultaneously operate within the same spectrum band as long as the interference generated by the SUs at the PRs is below certain acceptable levels. The level of interference inflicted on the PRs can be estimated by the fact that the secondary transmitters (STs) can hear the uplink signal of the PRs. In this regard, we assume the availability of the interference channel power gains between the SS and the PR, α, and the one between the l-th relay and the SD, β_l, both at their respective STs. α and β_l are mutually independent with unit mean distribution functions. The information about the interference channels may be carried out directly by the licensee or indirectly through a band manager which mediates between the two parties. Furthermore, we consider that the interference generated by the primary transmitter is modeled as an additive zero-mean Gaussian noise at the relays and SD, with variance σ_1^2 and σ_2^2, respectively.

Regarding the secondary system communication, at the first-hop transmission, the SS listens to the interference channel and adjusts its transmit power under a predefined interference constraint to make sure that the primary system operation is unaffected. Similarly, at the second-hop transmission, the selected relay node uses the same spectrum band originally assigned to the primary system in order to communicate with the SD. Particularly, during the second-hop transmission, the selected relay listens to the interference channel in order to adhere to the interference constraint with the primary system and forwards the decoded received signal to the SD node. It is assumed that the first and second hops' transmissions are independent, e.g., through a time-division channel allocation scheme, and the secondary transmissions in both hops are limited by the average received-interference power at the PRs. We define the average received-interference power constraints for the first- and second-hop transmissions as:

$$\mathrm{E}_{h_l,\alpha}\left[S_{\mathrm{sr}}\left(h_l,\alpha\right)\alpha\right] \le W_1, \tag{1}$$

$$\mathrm{E}_{g_l,\beta_l}\left[S_{\mathrm{rd}}\left(g_l,\beta_l\right)\beta_l\right] \le W_2, \tag{2}$$

where $S_{\mathrm{sr}}\left(h_l,\alpha\right)$ and $S_{\mathrm{rd}}\left(g_l,\beta_l\right)$ represent the instantaneous transmit powers at the SS and l-th relay, respectively, and $\mathrm{E}_X[\cdot]$ denotes the statistical average with respect to X. Furthermore, W_1 and W_2 are the average received-interference power limits pertaining to the first- and second-hop transmissions, respectively.

III. Signal-to-Noise Ratio Statistics

In this section, we investigate the first-order statistic pertaining to the instantaneous SNR at the SD node under average interference constraints at the PRs. Specifically, we obtain the CDF of the received SNR at the SD node which has key importance in the performance analysis of the cooperative spectrum-sharing system under consideration. In this context, considering the proposed system model, it can be shown that the optimal power allocation policy that maximizes the ergodic capacity of the first link is obtained through the Lagrangian optimization technique such that the aforementioned interference constraint is met. Therefore, the first-hop power transmission policy can be obtained as

$$S_{\mathrm{sr}}\left(h_l,\alpha\right)=\left(\frac{\lambda^{\mathrm{f}}}{\alpha}-\frac{\sigma_1^2}{h_l}\right)^+,\quad \forall\ \frac{\sigma_1^2}{\lambda^{\mathrm{f}}}\leq\frac{h_l}{\alpha}, \tag{3}$$

where $(X)^+\triangleq\max(X,0)$ and λ^{f} is the first-hop optimization parameter and must be determined such that the average interference constraint in (1) is satisfied with equality. Substituting the optimal power allocation policy in (3) into the average interference constraint given by (1) with equality, we have

$$\int\limits_{\frac{\sigma_1^2}{\lambda^{\mathrm{f}}}\leq z}\left(\lambda^{\mathrm{f}}-\frac{\sigma_1^2}{z}\right)f_Z\left(z\right)dz=W_1, \tag{4}$$

where the random variable Z is defined as $Z\triangleq h_l/\alpha$. Then, considering that the fading follows the Rayleigh distribution, i.e., h_l and α are exponentially distributed random variables, it can be shown that the probability distribution function (PDF) of Z is given by $f_Z\left(z\right)=\tau^{\mathrm{f}}/\left(\tau^{\mathrm{f}}+z\right)^2$ [13]. Thus, applying the latter in (4), the interference constraint may be calculated with equality as

$$\lambda^{\mathrm{f}}-\frac{\sigma_1^2}{\tau^{\mathrm{f}}}\ln\left(1+\frac{\tau^{\mathrm{f}}\lambda^{\mathrm{f}}}{\sigma_1^2}\right)=W_1. \tag{5}$$

Now, using the power allocation policy in (3), we can express the instantaneous received SNR at the l-th relay as

$$\gamma_{\mathrm{sr},l}=\frac{S_{\mathrm{sr}}\left(h_l,\alpha\right)h_l}{\sigma_1^2}=\left(\frac{\lambda^{\mathrm{f}}}{\sigma_1^2}Z-1\right)^+, \tag{6}$$

where $Z\triangleq h_l/\alpha$ with PDF $f_Z\left(z\right)$. Then, it is easy to show that the PDF of $\gamma_{\mathrm{sr},l}$ can be obtained using the *fundamental* theorem as [14]

$$f_{\gamma_{\mathrm{sr},l}}\left(\gamma\right)=\left.\frac{\sigma_1^2}{\lambda^{\mathrm{f}}}f_Z\left(z\right)\right|_{z\,=\,\frac{\sigma_1^2}{\lambda^{\mathrm{f}}}\left(\gamma+1\right)}. \tag{7}$$

After substituting $f_Z\left(z\right)$ and some mathematical manipulations, the PDF of the received SNR pertaining to the link between the SS and the l-th relay is obtained as

$$f_{\gamma_{\mathrm{sr},l}}\left(\gamma\right)=\frac{\sigma_1^2\lambda^{\mathrm{f}}\tau^{\mathrm{f}}}{\left(\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\gamma\right)^2},\quad \gamma_{\mathrm{sr},l}\geq 0. \tag{8}$$

Following the same approach for the second-hop transmission, the PDF of the instantaneous received SNR at the SD node from the l-th relay, $f_{\gamma_{\mathrm{rd},l}}\left(\gamma\right)$, can be expressed as

$$f_{\gamma_{\mathrm{rd},l}}\left(\gamma\right)=\frac{\sigma_2^2\lambda^{\mathrm{s}}\tau^{\mathrm{s}}}{\left(\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma\right)^2},\quad \gamma_{\mathrm{rd},l}\geq 0 \tag{9}$$

where λ^{s} is the second-hop optimization parameter and must satisfy the interference constraint in (2) at equality.

To obtain the CDF functions of $\gamma_{\mathrm{sr},l}$ and $\gamma_{\mathrm{rd},l}$, using $F_\gamma\left(\gamma\right)=\int_0^\gamma f_\gamma\left(\gamma\right)d\gamma$, we have

$$\begin{cases} F_{\gamma_{\mathrm{sr},l}}\left(\gamma\right)=\dfrac{\sigma_1^2\gamma}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\gamma}, & \mathrm{First-hop}\\ F_{\gamma_{\mathrm{rd},l}}\left(\gamma\right)=\dfrac{\sigma_2^2\gamma}{\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma}. & \mathrm{Second-hop}\end{cases} \tag{10}$$

Now, as mentioned earlier about the RRS technique, this selection strategy first chooses the relays whose link quality satisfies the threshold $\gamma_{\min}$. Then, during the second-hop transmission, only the best relay with the maximum $\gamma_{\mathrm{rd},l}$ is selected among the relay candidates to transmit the secondary signal to the destination node. Making use of the Binomial theorem, when there are k relay candidates that satisfy the threshold condition $\gamma_{\mathrm{sr},l}\geq\gamma_{\min}$, the probability that k relays pass the minimum threshold can be calculated according to

$$P_{\gamma_{\min},k}=\binom{L}{k}\left(1-F_{\gamma_{\mathrm{sr},l}}\left(\gamma_{\min}\right)\right)^k\left(F_{\gamma_{\mathrm{sr},l}}\left(\gamma_{\min}\right)\right)^{L-k}. \tag{11}$$

After substituting the CDF expression in (10), the probability $P_{\gamma_{\min},k}$ can be expressed as

$$P_{\gamma_{\min},k}=\binom{L}{k}\left(\frac{\sigma_1^2\gamma_{\min}}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\gamma_{\min}}\right)^L\left(\frac{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}}{\sigma_1^2\gamma_{\min}}\right)^k. \tag{12}$$

Let us assume that $\gamma_{\mathrm{sr},l_k^*}$ is the best relay selected among k relay candidates that passed the threshold test, i.e., $\gamma_{\mathrm{sr},l_k}\geq\gamma_{\min}$. In this case, $\Pr\left(\gamma_{\mathrm{sr},l_k^*}<\gamma\right)=\Pr\left(\gamma_{\mathrm{sr},l}<\gamma\left|\gamma_{\mathrm{sr},l}\geq\gamma_{\min}\right.\right)=F_{\gamma_{\mathrm{sr},l}}\left(\gamma-\gamma_{\min}\right)$, and hence, the CDF of the received SNR for the first-hop transmission can be written as

$$F_{\gamma_{\mathrm{sr},l_k^*}}\left(\gamma\right)=\frac{\sigma_1^2\left(\gamma-\gamma_{\min}\right)}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\left(\gamma-\gamma_{\min}\right)}\,\mathcal{U}\left(\gamma-\gamma_{\min}\right), \tag{13}$$

where $\mathcal{U}\left(\cdot\right)$ denotes the unit step function.

Regarding the second-hop transmission, since $\gamma_{\mathrm{rd},l_k^*}=\max\left\{\gamma_{\mathrm{rd},l_k}\right\}$ and making use of the order statistics theorem [14], the CDF of $\gamma_{\mathrm{rd},l_k^*}$ pertaining to the second-hop transmission can be obtained as follows:

$$F_{\gamma_{\mathrm{rd},l_k^*}}\left(\gamma\right)=\left(\frac{\sigma_2^2\gamma}{\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma}\right)^k. \tag{14}$$

Considering the fact that the end-to-end instantaneous SNR at the SD for the RRS technique is defined as the minimum of the channel strength among the SS-$R_{l_k^*}$ and $R_{l_k^*}$-SD links, i.e., $\gamma_{l_k^*}^{\mathrm{tot}}=\min\left\{\gamma_{\mathrm{sr},l_k^*},\gamma_{\mathrm{rd},l_k^*}\right\}$, the CDF of the end-to-end received SNR at the SD node can be expressed in terms of the CDF expressions $F_{\gamma_{\mathrm{sr},l_k^*}}\left(\gamma\right)$ and $F_{\gamma_{\mathrm{rd},l_k^*}}\left(\gamma\right)$, as follows [4]

$$F_{\gamma,l_k^*}^{\mathrm{tot}}\left(\gamma\right)=F_{\gamma_{\mathrm{sr},l_k^*}}\left(\gamma\right)+F_{\gamma_{\mathrm{rd},l_k^*}}\left(\gamma\right)-F_{\gamma_{\mathrm{sr},l_k^*}}\left(\gamma\right)\cdot F_{\gamma_{\mathrm{rd},l_k^*}}\left(\gamma\right). \tag{15}$$

By substituting the CDF functions in (13) and (14) into (15),

$$P_{\mathrm{out}}=\left(\frac{\sigma_1^2\gamma_{\min}}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\gamma_{\min}}\right)^L\underbrace{\sum_{k=1}^{L}\binom{L}{k}\left(\frac{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}\sigma_2^2\gamma_{\mathrm{th}}}{\sigma_1^2\gamma_{\min}\left(\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma_{\mathrm{th}}\right)}\right)^k}_{\Sigma_1}+\left(\frac{\sigma_1^2\gamma_{\min}}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\gamma_{\min}}\right)^L\left(\frac{\sigma_1^2\left(\gamma_{\mathrm{th}}-\gamma_{\min}\right)}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\left(\gamma_{\mathrm{th}}-\gamma_{\min}\right)}\right)$$

$$\times\left(\underbrace{\sum_{k=1}^{L}\binom{L}{k}\left(\frac{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}}{\sigma_1^2\gamma_{\min}}\right)^k}_{\Sigma_2}-\underbrace{\sum_{k=1}^{L}\binom{L}{k}\left(\frac{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}\sigma_2^2\gamma_{\mathrm{th}}}{\sigma_1^2\gamma_{\min}\left(\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma_{\mathrm{th}}\right)}\right)^k}_{\Sigma_1}\right)\mathcal{U}\left(\gamma_{\mathrm{th}}-\gamma_{\min}\right). \tag{20}$$

$$P_{\mathrm{out}}=\left(\frac{\sigma_1^2\gamma_{\min}}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\gamma_{\min}}\right)^L\left(\left(\frac{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}\sigma_2^2\gamma_{\mathrm{th}}}{\sigma_1^2\gamma_{\min}\left(\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma_{\mathrm{th}}\right)}+1\right)^L-1\right)$$

$$+\left(\frac{\sigma_1^2\left(\gamma_{\mathrm{th}}-\gamma_{\min}\right)}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\left(\gamma_{\mathrm{th}}-\gamma_{\min}\right)}\right)\left(1-\left(\frac{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}\sigma_2^2\gamma_{\mathrm{th}}+\sigma_1^2\gamma_{\min}\left(\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma_{\mathrm{th}}\right)}{\left(\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\gamma_{\min}\right)\left(\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma_{\mathrm{th}}\right)}\right)^L\right)\mathcal{U}\left(\gamma_{\mathrm{th}}-\gamma_{\min}\right). \tag{23}$$

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and after some mathematical manipulations, we have

$$F_{\gamma,l_k^*}^{\mathrm{tot}}\left(\gamma\right)=\left(\frac{\sigma_2^2\gamma}{\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma}\right)^k+\left(\frac{\sigma_1^2\left(\gamma-\gamma_{\min}\right)}{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\left(\gamma-\gamma_{\min}\right)}\right)\times\left(1-\left(\frac{\sigma_2^2\gamma}{\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma}\right)^k\right)\mathcal{U}\left(\gamma-\gamma_{\min}\right). \tag{16}$$

In the following section, making use of the derived statistics for the RRS scheme, we investigate the end-to-end performance of the secondary system and obtain a closed-form expression for the outage probability of the proposed dual-hop cooperative spectrum-sharing system.

IV. End-to-End Outage Probability Performance

In spectrum-sharing systems, the outage probability can be interpreted as the fraction of time that the secondary communication link (SS-Relay-SD) experiences an outage due to fading. In the cooperative system under consideration, the statistical variation of both the received signal at the SD and the received interference power at the PRs must be considered in the outage probability analysis. Mathematically speaking, the end-to-end outage probability of the proposed system is defined as the probability that the received SNR at the SD node, $\gamma_{l_k^*}^{\mathrm{tot}}$, falls below a predetermined threshold, γ_{th}, i.e.,

$$P_{\mathrm{out}}=\sum_{k=1}^{L}P_{\gamma_{\min},k}\cdot F_{\gamma,l_k^*}^{\mathrm{tot}}\left(\gamma_{\mathrm{th}}\right), \tag{17}$$

where the probability $P_{\gamma_{\min},k}$ is given in (12) and $F_{\gamma,l_k^*}^{\mathrm{tot}}\left(\gamma_{\mathrm{th}}\right)$ can be obtained from (16). Now, by substituting the latter and (12) into (17) and after some mathematical manipulations, the outage probability P_{out} is given in (20) at the top of the next page.

The summations in (20) denoted by Σ_1 and Σ_2, can be solved by considering the sum of binomial coefficients rule [15, Eq. 0.155.3], namely, using $\sum_{k=1}^{L}\binom{L}{k}A^k=\left(A+1\right)^L-1$, thus yielding

$$\Sigma_1=\left(\frac{\lambda^{\mathrm{f}}\tau^{\mathrm{f}}\sigma_2^2\gamma_{\mathrm{th}}}{\sigma_1^2\gamma_{\min}\left(\lambda^{\mathrm{s}}\tau^{\mathrm{s}}+\sigma_2^2\gamma_{\mathrm{th}}\right)}+1\right)^L-1, \tag{21}$$

and

$$\Sigma_2=\frac{\left(\lambda^{\mathrm{f}}\tau^{\mathrm{f}}+\sigma_1^2\gamma_{\min}\right)^L-\left(\sigma_1^2\gamma_{\min}\right)^L}{\left(\sigma_1^2\gamma_{\min}\right)^L}. \tag{22}$$

Finally, substituting (21) and (22) into (20) and after some manipulations, the closed-form expression for the end-to-end outage probability of the cooperative relaying spectrum-sharing system can be obtained as given in (23).

V. Numerical Results

In this section, we numerically illustrate the outage probability performance of the proposed cooperative spectrum-sharing system with the RRS selection strategy utilized by the secondary system in order to establish a dual-hop communication between the SS and SD nodes. We consider that the channel gains $\sqrt{h_l}$ and $\sqrt{g_l}$ are modeled according to Rayleigh PDFs with expected values of τ^{f} and τ^{s}, respectively. It is also assumed that the interference channels α and β_l experience Rayleigh fading with unit variances. Furthermore, in our numerical results, we assume that $\sigma_1^2=\sigma_2^2=1$.

In Fig. 2, we illustrate the outage probability of the proposed cooperative system for different values of the average interference limit, $W_1=W_2=W$. In this figure, we set the number of relays participating in the selection $L=3$ and the outage threshold $\gamma_{\mathrm{th}}=-5$ dB, and investigate the variation of τ^{s} with $\tau^{\mathrm{f}}=0$ dB, and for different values of the minimum required rate at the first hop transmission $R_{\min}=0.5$, 0.8, 1 bit/s/Hz. As shown in the figure, for a given $R_{\min}$ value, we observe a significant improvement on the overall outage probability performance of the proposed cooperative system as the second transmission link is in strong propagation conditions, i.e., $\tau^{\mathrm{s}}>\tau^{\mathrm{f}}$, or by decreasing the minimum required rate $R_{\min}$. This can be interpreted by the fact that the lower required

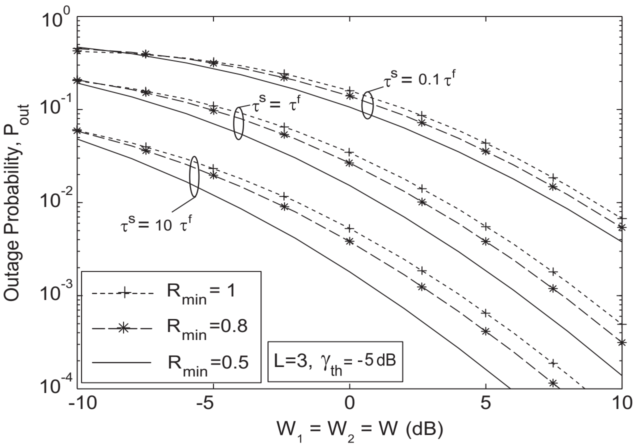

Fig. 2: Outage probability of the cooperative spectrum-sharing system for equal interference limits ($W_{1,2} = W$) with $R_{\min} = 0.5$, 0.8, 1 bit/s/Hz, for different τ^{s} and $\tau^{\mathrm{f}} = 0$ dB ($L = 3$, $\gamma_{\mathrm{th}} = -5$ dB).

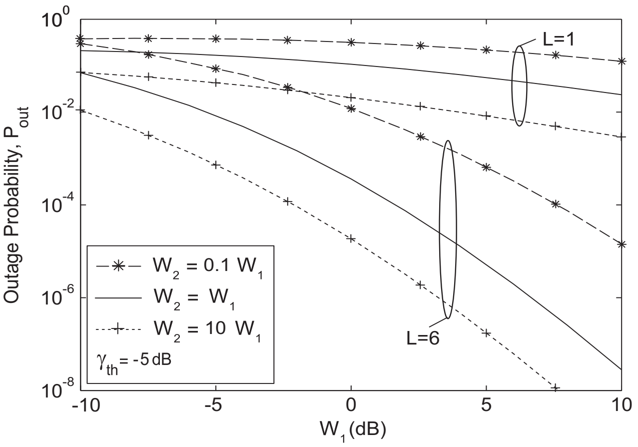

Fig. 3: Outage probability of the cooperative spectrum-sharing system versus W_1, with $L = 1$, 6, and imbalanced resource limits ($R_{\min} = 0.5$, $\gamma_{\mathrm{th}} = -5$ dB).

rate at the first hop and the stronger channel condition at the second hop provide a higher chance for intermediate relays to participate in the selection, and consequently, this improves the overall performance in the proposed cooperative system. On the other hand, it is observed that, for given values of τ^{s} and $R_{\min}$, the outage probability decreases as the average interference limit, W, increases.

In Fig. 3, we fix the outage threshold at $\gamma_{\mathrm{th}} = -5$ dB and set the minimum required rate to $R_{\min} = 0.5$. We investigate the effect of imbalanced resource limits, W_1 and W_2, on the end-to-end outage probability performance of the cooperative spectrum-sharing system when τ^{f} and τ^{s} are set to 0 dB. As shown in the figure, we observe the significant effect of imbalanced resource limits on the performance of the cooperative system utilizing the RRS selection strategy. For instance, for a fixed value of W_1, we observe a substantial improvement on the outage probability of the secondary system as the average interference limit W_2 increases. Furthermore, the number of relays used in the selection is $L = 1$, 6. Analysis of the number of relays shows considerable improvements in outage probability performance as L increases.

VI. Concluding Remarks

We considered a cooperative relaying spectrum-sharing system where L relays are available to opportunistically assist the communication between the secondary source and destination nodes while ensuring that no harmful interference is caused to the primary users of the spectrum band. Hence, the proposed spectrum-sharing system was assumed to operate under appropriate constraints on average received-interference power at the PRs. In this context, assuming the RRS technique in which the selection is performed during the second-hop transmission, we investigated the performance of the proposed cooperative system under a Rayleigh fading environment. In this regard, we first obtained the CDF of the instantaneous received SNR at the secondary destination node. Then, using the obtained CDF expression, we investigated the end-to-end outage probability of the cooperative relaying spectrum-sharing system. Finally, numerical results and comparisons were provided to illustrate the benefits of using the RRS technique in different channel propagation conditions.

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