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DOA Estimation for ULA Systems from Short Data Snapshots: an Annihilating Filter Approach

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Abstract— **In this paper we derive a new method of DOA estimation for ULA configurations using the annihilating filter technique. The new method is non-data-aided (NDA) and does not therefore impinge on the whole throughput of the system. The noise components are assumed spatially and temporally white. The transmitted signals are also assumed to be temporally and spatially white (across the transmitting sources). The new method is compared in performance to the root-MUSIC algorithm, a powerful DOA estimation technique for ULA configurations. Simulations will show that the new method performs well over a wide SNR range. The main advantage of the new method is that it succeeds in accurately estimating the DOAs for fast moving sources or for short data snapshots, and even from a single snapshot where the root-MUSIC method fails completely.**

I. Introduction

Direction of arrival (DOA) estimation of multiple plane waves impinging on an arbitrary array of antennas is an important task in array signal processing [1, 2]. In modern communication systems for example, based only on the data received at the antenna array, estimating the DOAs of the desired users and those of the interference signals allows their extraction and cancellation, respectively, by beamforming technologies [3, 4] in order to improve the systems' performance. Roughly speaking, depending on the *a priori* knowledge of the transmitted signals, DOA estimators can be categorized as data-aided (DA) or non-data-aided (NDA) estimators. In fact, contrarily to DA DOA estimation approaches which assume the transmitted symbols to be perfectly known at the array receiver, NDA DOA estimators base the estimation process only on the received samples. NDA estimators themselves are called deterministic or stochastic if the unknown transmitted signal is assumed deterministic or random, respectively. So far, many DOA estimators have been extensively studied in the literature [5, 6, 7]. Among all the existing DOA estimation methods, root-MUSIC [8] is the most accurate technique for ULA configurations. It is a high-resolution technique that is based on finding the roots of a polynomial objective function. Our new method is also based on finding the roots of an annihilating filter (AF); which are directly related to the unknown DOAs. It should be noted that the *annihilating filter* technique has been well known for a very long time in the mature field of spectral estimation. About a decade ago, it was successfully used to develop the so-called finite-rate-of-innovation (FRI) sampling method [9]. In this contribution, we apply for the first time the AF approach to DOA estimation for ULA configurations. Therefore, we will henceforth refer to our new method as the AF-based method. The coefficients of the corresponding AF are calculated by the singular value decomposition (SVD) of a matrix whose elements are built from second-order cross moments (across the receiving antenna elements) of the received samples.

It will be shown by Monte-Carlo computer simulations that the new AF-based method is a high resolution technique. Its main advantage is its capability to accurately resolve the DOAs from short data snapshots and even from a single-shot measurement where root-MUSIC fails completely even for sufficiently high SNR values. The new method is therefore very useful for DOA estimation of fastly moving sources where the DOAs may change from one snapshot to another.

We organize the rest of this paper as follows. In section II, we introduce the system model that will be used throughout this article. Then in section III, we will develop our new AF-based DOA estimation technique. In section IV, we assess the performance of the new method and compare it to the Root-MUSIC technique. Finally, some concluding remarks will be drawn out in section V.

II. System Model

We consider a uniform linear array (ULA) with N_a antenna elements receiving multiple planar waves impinging from K sources. The K transmitted signals are assumed to be temporally white and uncorrelated between transmitting sources. Assuming perfect frequency synchronization, the received signal on the $\{i^{th}\}_{i=1}^{N_a}$ antenna element, at the output of the matched filter, can be modeled as a complex signal as follows[1]:

$$y_i(n) = \sum_{k=1}^{K} h_k e^{j(k-1)\pi \sin(\theta_k)} a_k(n) + w_i(n), \qquad (1)$$

where at time index n, $a_k(n)$ is the signal transmitted by the k^{th} source and $w_i(n)$ is the noise component on the i^{th} antenna branch that is modelled by a zero-mean complex Gaussian random variable with independent real and imaginary parts, each of variance σ^2. $\{h_k\}_{k=1}^{K}$ is the unknown complex channel

[1]Note here that receiving antenna elements are supposed to be spaced by half the wavelength, i.e., $d = \lambda/2$ where d is the distance between two consecutive antenna branches and λ is the wavelength.

coefficient (i.e., $h_k = |h_k|e^{j\phi_k}$ with ϕ_k standing for any possible channel distortion phase) of the $\{k^{th}\}_{k=1}^{K}$ source. Moreover $\{\theta_k\}_{k=1}^{K}$ are the unknown DOAs of the waves impinging from the K sources. We assume hereafter that, at each instant n, the transmitted signals, $\boldsymbol{a}(n) = [a_1(n),\ a_2(n), \cdots,\ a_K(n)]^T$ and the noise components $\boldsymbol{w}(n) = [w_1(n),\ w_2(n), \cdots,\ w_{N_a}(n)]^T$ are spatially uncorrelated, i.e., $E\{\boldsymbol{w}(n)^H\boldsymbol{w}(n)\} = \sigma^2\boldsymbol{I}_{N_a}$ and $E\{\boldsymbol{a}(n)^H\boldsymbol{a}(n)\} = \boldsymbol{I}_K$ (where in the last equality we assume that the energy of the transmitted signals are normalized to one, i.e., $E\{|a_k(n)|^2\} = 1$). Finally, we assume for each source k that the transmitted symbols $\{a_k(n)\}_{n=1}^{N}$ are independent and equally likely. Then we define the SNR of the source k as follows:

$$\rho_k = \frac{\mathrm{E}\{|h_k|^2|a_k(n)|^2\}}{2\sigma^2} = \frac{|h_k|^2}{2\sigma^2}. \qquad (2)$$

III. Formulation of the New AF-Based DOA Estimator

The new estimator is primarily based on the second-order cross-moments of received signals. In fact, gathering all the unknown DOAs in the vector $\boldsymbol{\theta} = [\theta_1,\ \theta_2, \cdots,\ \theta_K]^T$, the cross-moment between the received signals on the i^{th} and the l^{th} receiving antenna elements, is given by:

$$M_{\boldsymbol{\theta}}(i,l) = E\{y_i(n)y_l^*(n)\},\ i,l = 1,2,\cdots,N_a. \qquad (3)$$

Then using the fact that the transmitted signals and the noise components are spatially and temporally white, $M_{\boldsymbol{\theta}}(i,l)$ reduces simply to:

$$M_{\boldsymbol{\theta}}(i,l) = \begin{cases} \sum_{k=1}^{K}|h_k|^2 + 2\sigma^2, & \text{for } \ i = l \\ \sum_{k=1}^{K}|h_k|^2 e^{j\pi(i-l)\sin(\theta_k)}, & \text{for } \ i \neq l \end{cases} \qquad (4)$$

Actually, $\{M_{\boldsymbol{\theta}}(i,l)\}_{i,l=1}^{N_a}$ are the entries, $[\bar{\boldsymbol{M}}(\boldsymbol{\theta})]_{i,l}$, of the autocovariance matrix, $\bar{\boldsymbol{M}}(\boldsymbol{\theta})$, of the received signal $\boldsymbol{y}(n) = [y_1(n),\ y_2(n), \cdots,\ y_{N_a}(n)]^T$ which is defined as $\bar{\boldsymbol{M}}(\boldsymbol{\theta}) = E\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\}$.
Next, since $\bar{\boldsymbol{M}}(\boldsymbol{\theta})$ is a Hermetian matrix, we will only consider the elements contained in the lower triangular matrix obtained from $\bar{\boldsymbol{M}}(\boldsymbol{\theta})$. Therefore, from now on, the counters i and l always verify $i \geq l$. Then, using the notation $u_k = e^{j\pi\sin(\theta_k)}$, we define the sequence $\{R_{\boldsymbol{\theta}}^{(l)}[m]\}_m$ for each column l with $l = 1,2,\cdots,N_a-1$ as follows:

$$R_{\boldsymbol{\theta}}^{(l)}[m] \;=\; M_{\boldsymbol{\theta}}(l+m,l),\ m = 1,2,\cdots,N_a-l, \qquad (5)$$

which is simply given by:

$$R_{\boldsymbol{\theta}}^{(l)}[m] = \sum_{k=1}^{K}|h_k|^2u_k^m,\ m = 1,2,\cdots,N_a-l. \qquad (6)$$

Then we define the (N_a-l)-dimensional vector $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}$ — that will be used subsequently — as follows[2]:

$$\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)} = \left[R_{\boldsymbol{\theta}}^{(l)}[1], R_{\boldsymbol{\theta}}^{(l)}[2], \cdots, R_{\boldsymbol{\theta}}^{(l)}[N_a-l]\right]^T. \qquad (7)$$

At this stage we introduce the definition of the *annihilating filer*. A filter $A[m]$ is called an annihilating filter [10] of a signal $S[m]$ when:

$$(A * S)[m] = 0, \forall m \ \in \mathbb{N}, \qquad (8)$$

where $\{*\}$ stands for the discrete convolution. Next, for each column l, the cross-moments sequence $\{R_{\boldsymbol{\theta}}^{(l)}[m]\}_m$ will play the role (interpreted as) of the signal sequence $S[m]$ in (8). Note from (6) that the $\{R_{\boldsymbol{\theta}}^{(l)}[m]\}_m$ has the special form of linear combinations of exponentials. This property is actually the main idea behind this work since, as shown subsequently, for such sequences the roots of the corresponding annihilating filters are exactly the involved elementary exponentials. In fact, consider the following filter[3]:

$$\begin{aligned} A(z) &\triangleq \prod_{k=1}^{K}(1-u_kz^{-1}), \\ &= \sum_{n=0}^{K}A[n]z^{-n}. \end{aligned} \qquad (9)$$

Then, we have:

$$\begin{aligned} \left(A * R_{\boldsymbol{\theta}}^{(l)}\right)[m] &= \sum_{n=0}^{K}A[n]R_{\boldsymbol{\theta}}^{(l)}[m-n], \\ &= \sum_{n=0}^{K}\sum_{m=1}^{K}|h_k|^2A[n]u_k^{m-n} \\ &= \sum_{m=1}^{K}|h_k|^2\underbrace{\left(\sum_{n=0}^{K}A[n]u_k^{-n}\right)}_{A(u_k)=0}u_k^m, \\ &= 0. \end{aligned} \qquad (10)$$

Therefore, $A[n]$ — as constructed in (9) — is indeed an annihilating filter for the sequence $\{R_{\boldsymbol{\theta}}^{(l)}[m]\}_1^{N_a-1}$. Then, after finding the coefficients $\{A[n]\}_{n=0}^{K}$, the roots of the corresponding polynomial $\boldsymbol{A}$ in (9) can be easily computed and the DOAs can be easily estimated from the arguments of these roots. Next, we describe an SVD procedure that enables finding the desired coefficients. First notice that the filter coefficients $\{A[n]\}_{n=0}^{K}$ in $A(z) = \sum_{n=0}^{K}A[n]z^{-n}$ must be such that (8) is satisfied for all $m \in$ and in particular for $m > n$:

$$\sum_{n=0}^{K}A[n]R_{\boldsymbol{\theta}}^{(l)}[m-n] = 0 \ \ \forall\ m > n. \qquad (11)$$

Then, using the elements, $R_{\boldsymbol{\theta}}^{(l)}[m-n]$, of the l^{th} column of the covariance matrix, we estimate from (11) the $K+1$ unknown filter coefficients that will be henceforth gathered in the vector $\boldsymbol{A} = \left[A[0], A[2], \cdots, A[K]\right]^T$. Hence, we need $K+1$ equations in order to obtain at least one estimate, $\hat{\boldsymbol{A}}^{(l)}$, of the desired vector $\boldsymbol{A}$ considering the l^{th} column. Thus, the

[2]Note that the vector $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}$ contains all the N_a-l elements of the l^{th} column that are lying under the main diagonal of the covariance matrix.

[3]Note here that the $(K+1)$ unknown filter coefficients can be reduced to K unknown normalized filter coefficients (by setting the first to 1) and the algorithm can be optimized accordingly. In this paper however we decided to treat the natural problem by considering all the unknown coeifficients.

l^{th} vector $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}$ should contain at least $2K+1$ elements. Recall from (7) that the size of $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}$ is $N_l = N_a - l$ which results in $N_a - l \geq 2K+1$. Therefore, l must verify:

$$1 \leq l \leq N_a - 2K - 1, \quad (12)$$

meaning that only the first $N_a - 2K - 1$ columns of the covariance matrix contain a sufficient number of cross-moments that allow having at least one estimate, $\hat{\boldsymbol{A}}^{(l)}$ of $\boldsymbol{A}$, per-column $\big($or per-vector $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}\big)$. We will see later that we can actually obtain (in an iterative way) P_l+1 estimates $\big\{\hat{\boldsymbol{A}}_p^{(l)}\big\}_{p=0}^{P_l}$ for each candidate column where $P_l = N_l-(2K+1) = N_a-l-2K-1$ is the number of samples exceeding the necessary first $2K+1$ cross-moments in the l^{th} column. Observe also form (12) that it is necessary to have at least $N_a \geq 2K+2$ receiving antenna elements. Thus our estimator needs a number of antennas that is almost twice the number of sources.

Now we elaborate further on the iterative scheme that allow us to obtain $P_l+1 = N_l-(2K+1)+1 = N_a-2K-l$ estimates of the vector $\boldsymbol{A}$ for each column among the first N_a-2K-1 columns $\big($or equivalently the N_a-2K-1 vectors $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}\big)$. In fact, fix l and we have the $\{l^{th}\}_{l\leq N_a-2K-1}$ vector $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}$ contains $N_l = 2K+1+P_l$ cross-moments. Then, for each $p = 0,1,2,\cdots,P_l$, consider $2K+1$ consecutive samples of these moments, $\big\{R_{\boldsymbol{\theta}}^{(l)}[m_p+r]\big\}_{r=-K}^{K}$, that are centred around $m_p = K+1+p$. Now, replacing m by m_p+r, the system (11) yields:

$$\sum_{n=0}^{K} A[n]R_{\boldsymbol{\theta}}^{(l)}[m_p+r-n] = 0, \;\; r=-K,-K+1,\cdots,K-1,K. \quad (13)$$

Therefore, for each p, the system (13) can be more conveniently written in the matrix/vector form given by (14) in the top of the next page. In practice the system (14) can be solved via a singular value decomposition (SVD) where the matrix $\boldsymbol{S}_p^{(l)}(\boldsymbol{\theta})$ is decomposed into $\boldsymbol{S}_p^{(l)}(\boldsymbol{\theta}) = \boldsymbol{U}_p^{(l)}(\boldsymbol{\theta})\boldsymbol{\Sigma}_p^{(l)}(\boldsymbol{\theta})\boldsymbol{V}_p^{(l)}(\boldsymbol{\theta})^H$. Then for each $l = 1,2,\cdots,N_a-2K-1$ and $p = 0,1,\cdots,(N_a-2K-1)-l$ we obtain an estimate $\hat{\boldsymbol{A}}_p^{(l)}$ for $\boldsymbol{A}$ as follows:

$$\hat{\boldsymbol{A}}_p^{(l)} = \boldsymbol{V}_p^{(l)}(\boldsymbol{\theta})\boldsymbol{e}_{K+1}, \quad (15)$$

where $\boldsymbol{e}_{K+1}$ is a vector with 1 on position $K+1$ and 0 elsewhere. Afterward, solving for the K roots of $\hat{\boldsymbol{A}}_p^{(l)}$ we obtain a set of estimates for $\{u_k = e^{j\pi\sin(\theta_k)}\}_{k=1}^{K}$. We denote these estimates as $\{\hat{u}_k^{(l,p)}\}_{k=1}^{K}$ from which a set of estimates for the unknown DOAs are obtained for each l and p as follows:

$$\hat{\theta}_k^{(l,p)} = \frac{1}{\pi}\arcsin\big(\angle\hat{u}_k^{(l,p)}\big). \quad (16)$$

Finally recall that for the $\{l^{th}\}_{l=1}^{N_a-2k-1}$ column we have $P_l+1 = N_a-2K-l$ estimates for the same DOA θ_k; which means that by considering all the elegible columns, we have $\sum_{l=1}^{N_a-2K-1}(N_a-2K-l) = \frac{(N_a-2K-1)(N_a-2K)}{2}$ estimates for each DOA. Therefore, one can average over all these estimates (obtained column wise) to obtain more refined estimates for the unknown DOAs as follows:

$$\hat{\theta}_k^{\text{column}} = \frac{2}{(N_a-2K-1)(N_a-2K)}\sum_{l=1}^{N_a-2K-1}\sum_{p=0}^{N_l-2K-1}\hat{\theta}_k^{(l,p)},$$

Now that we have used only the first N_a-2K-1 columns, it can be seen that the elements of some remaining columns can also be used, to further refine the estimates. This can be achieved by considering the rows that contain more than $2K+1$ elements below the main diagonal of the covariance matrix $\bar{\boldsymbol{M}}(\boldsymbol{\theta})$. Indeed, it is possible to take advantage of these elements by applying the previous procedure to the last N_a-2K-1 rows where the m^{th} elements of the corresponding vectors[4], $\boldsymbol{R}_{\boldsymbol{\theta}}'^{(l)}$ are defined for the l^{th} row, $l = N_a, N_a-1,\cdots,2K+2$, as follows:

$$R_{\boldsymbol{\theta}}'^{(l)}[m] \;=\; M_{\boldsymbol{\theta}}(l,l-m), \;\; m=1,2,\cdots,l-1. \quad (17)$$

This is because all the crossmoments that belong to any given secondary diagonal of the covariance matrix have the same expression. Therefore, for each $l = N_a, N_a-1,\cdots,2K+2$ the sequence $\{R_{\boldsymbol{\theta}}'^{(l)}[m]\}_{m=1}^{l-1}$, inherits the important structure of linear combinations of wighted exponentials. Then applying the same procedure using the vectors $\boldsymbol{R}_{\boldsymbol{\theta}}'^{(l)}$ instead of $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}$, we obtain an additional set of estimates for the DOA which we denote $\hat{\theta}_k^{\text{row}}$. Lastly, the final estimates of the DOAs are obtained as:

$$\hat{\theta}_k = \frac{\hat{\theta}_k^{\text{column}} + \hat{\theta}_k^{\text{row}}}{2}, \quad k = 1,2,\cdots,K. \quad (18)$$

IV. Simulation Results

In this section we assess the performance of the new estimator using the mean square error (MSE) as a performance measure. The MSE is computed for each estimator $\hat{\theta}_k$ of the k^{th} DOA, θ_k, as follows:

$$\text{MSE}(\hat{\theta}_k) = \frac{1}{M_c}\sum_{q=1}^{M_c}(\hat{\theta}_k^{(q)} - \theta_k)^2, \quad (19)$$

where M_c is the number of Monte-Carlo simulations which is set to $M_c = 1000$ for all simulations and $\hat{\theta}_k^{(q)}$ is the estimate of θ_k during the q^{th} Monte-Carlo run. We also consider the root-MUSIC (RM) estimator as a benchmark for the assessment of our newly developed method. In fact, the RM estimator remains so far among the best DOA estimators (in terms of accuracy) for ULA configurations. We also consider the case of equipowered sources and we mention that all the simulations are presented for the first source (first DOA) for the sake of briefness only. Yet, we emphasize that the same performance behaviour was observed for the other DOAs.

In Fig. 1, we plot for the two estimators the MSE of the DOA estimates for the first source from $N = 1000$ received samples versus the SNR of the same source.

[4]We mention here that $\boldsymbol{R}_{\boldsymbol{\theta}}'^{(l)}$ plays the role of $\boldsymbol{R}_{\boldsymbol{\theta}}^{(l)}$ that was previously used to perform the estimation process column-wise.

$$\underbrace{\begin{pmatrix} \vdots & \vdots & \cdots & \vdots \\ R_{\boldsymbol{\theta}}^{(l)}[K+p+1] & R_{\boldsymbol{\theta}}^{(l)}[K+p] & \cdots & R_{\boldsymbol{\theta}}^{(l)}[p+1] \\ R_{\boldsymbol{\theta}}^{(l)}[K+p+2] & R_{\boldsymbol{\theta}}^{(l)}[K+p+1] & \cdots & R_{\boldsymbol{\theta}}^{(l)}[p+2] \\ \vdots & \vdots & \ddots & \vdots \\ R_{\boldsymbol{\theta}}^{(l)}[2K+p+1] & R_{\boldsymbol{\theta}}^{(l)}[2K+p] & \cdots & R_{\boldsymbol{\theta}}^{(l)}[K+1+p] \\ \vdots & \vdots & \cdots & \vdots \end{pmatrix}}_{\boldsymbol{S}_p^{(l)}(\boldsymbol{\theta})} \begin{pmatrix} A[0] \\ A[1] \\ \vdots \\ A[K] \end{pmatrix} = \mathbf{0}. \quad (14)$$

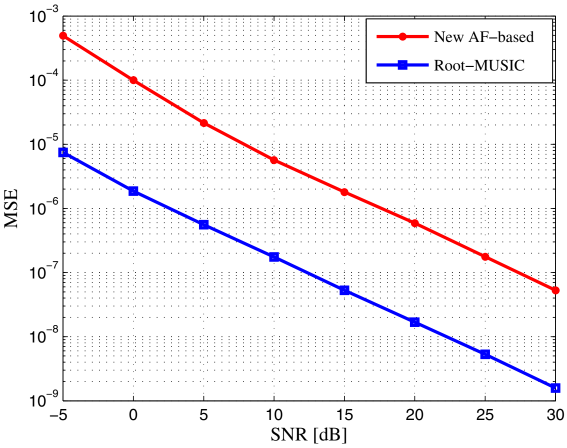

Fig. 1. MSE for the first DOA versus the SNR, 16-QAM, $K = 2$ sources: $\theta_1 = 0.1\pi$, $\theta_2 = 0.2\pi$, $N = 1000$, $N_a = 8$.

We see that the new estimator exhibits almost the same performance behavior as the high-resolution RM technique. But we can see in the next figure the major advantage of the newly developed AF-based technique over the classical RM method in the case of single-shot estimation. Indeed, we plot in Fig. 2, the MSE for the two estimators versus the SNR when only one sample is available at the receiver side, i.e., $N = 1$.

We see clearly from this figure that the RM technique fails completely to estimate the DOAs contrarily to our new AF-based estimator which still provides very accurate estimates over the entire SNR range. Therefore, our new method can be successfully applied in many practical situations of DOA estimation for fastly moving sources where the DOAs may vary from one snapshot to another. This is usually encountered in many real-life scenarios, typically in spatial navigation and satellite communications, among others. To illustrate this capacity of our new estimator, we plot in Figs. 3 and 4 the estimated DOAs of two moving sources for two different SNR values, i.e., $SNR = 15$ and 20 dB. The DOAs were generated assuming that both sources move linearly from 20 to 30 and from 70 to 60 degrees, respectively, in a step (or radial speed $\dot{\theta}$) of 0.1 degree per sample, over 100 data snapshots. The new AF estimator was applied using $N = 1$ (i.e., single snapshot). The AF estimates follow accurately the trajectories of the the two time-varying DOAs more so for relatively higher SNR values (Fig. 4).

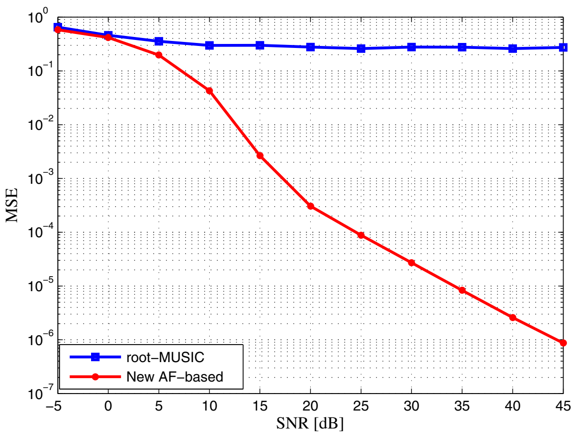

Fig. 2. MSE for the first DOA versus the SNR, $K = 2$ sources, 16-QAM: $\theta_1 = 0.1\pi$, $\theta_2 = 0.2\pi$, $N = 1$, $N_a = 8$.

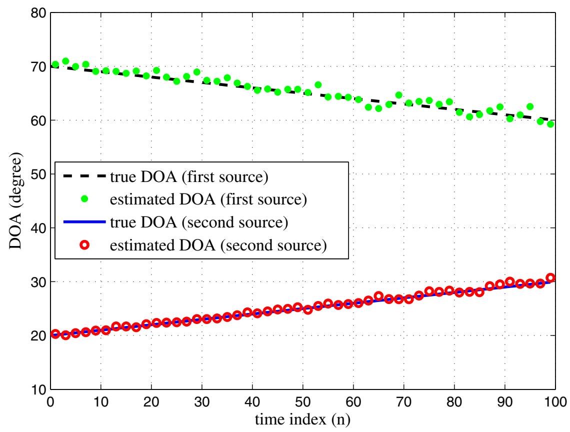

Fig. 3. True DOAs and their AF-based estimates, QPSK, $K = 2$ sources: $N = 1$, $\dot{\theta}_1 = 0.1$, $\dot{\theta}_2 = -0.1$, $N_a = 16$, $SNR = 15$ dB.,

Furthermore, since the new estimator performs well with short data snapshots, then its performance will be very robust to DOA time variations. This is illustrated in Fig. 5 in which we consider two sources moving linearly from -60 and -30 degrees to 40 and 70 degrees, respectively, at a speed ($\dot{\theta}$) as high as 1 degree per sample. Again, we see the very good agreement between the estimated DOAs and their actual trajectories.

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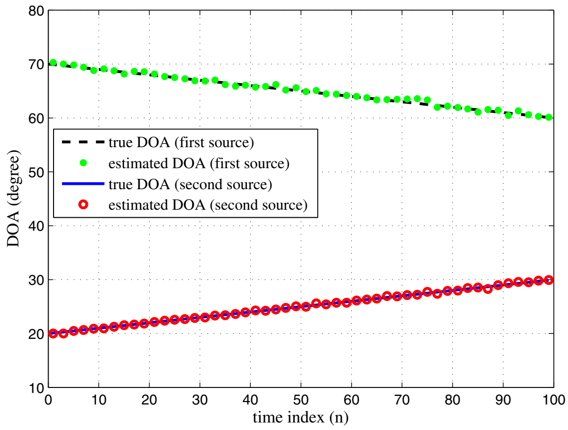

Fig. 4. True DOAs and their AF-based estimates, QPSK, $K = 2$ sources: $N = 1$, $\dot{\theta}_1 = 0.1$, $\dot{\theta}_2 = -0.1$, $N_a = 16$, $SNR = 20$ dB.

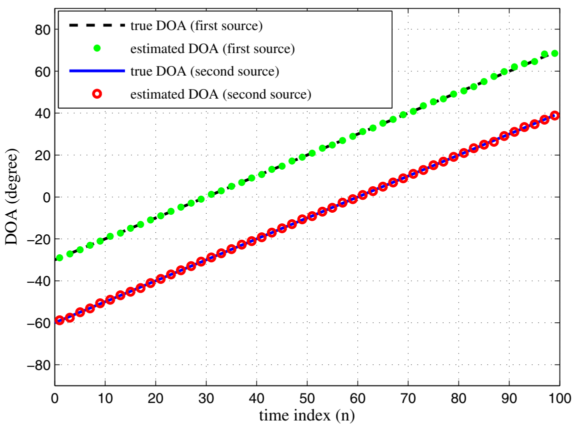

Fig. 5. True DOAs and their AF-based estimates, $K = 2$ sources: $N = 1$, $\dot{\theta}_1 = \dot{\theta}_2 = 1$, degrees, $N_a = 16$, $SNR = 20$ dB.

Now recall from Fig. 1 that, for constant DOAs, the root-MUSIC estimator outperforms our new AF-based estimator for large window sizes.This is no longer true for time-varying DOAs. Actually, when the DOAs are not constant over a time period, the root-MUSIC algorithm can always be applied locally — over a time window of short size — in order to estimate the average DOAs over the considered window. Clearly, in this case, the performance of the root-MUSIC algorithm is affected by the size of the local window and the DOAs speed. In fact, as the speed increases the DOAs vary appreciably over the local window. Hence, the performance of the root-MUSIC estimator degrades since the approximation of locally constant DOAs is no longer valid. Our new estimator does not suffer from this drawback since it succeds to estimate the DOAs even from short data snapshots and since it is also very robust to DOA time vraiations (i.e., it accurately estimates the DOAs even for high variation speeds as seen from Fig. 4 and 5). This behvior is illustrated in Fig. 6 where we show the operational regions, in terms of window sizes (N) and DOA speeds ($\dot{\theta}$), for each estimator. A region is attributed to a given estimator when this estimator shows lower MSE for all the couples (N, $\dot{\theta}$) in this region.

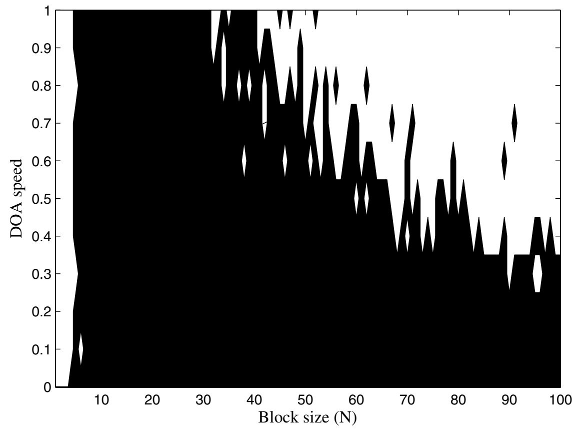

Fig. 6. Operational regions for the AF-based (white surface) and the root-MUSIC (black surface) estimators, $K = 2$, $N_a = 8$, $SNR = 25$ dB.

V. Conclusion

In this paper, we derived a new DOA estimation method for multiple planar waves impinging on a ULA antenna array. The transmitted sources and the noise components are assumed to be spatially and temporally white. The new method is based on the *annihilating filter* technique. It was seen that the new method exhibits good statistical performances. Its major advantage is its capability of accurately resolving closely spaced DOAs from short data snapshots and even from a single snapshot. This makes this new estimator well geared toward applications that require DOA estimation of fast moving sources.

References

[1] D. H. Johnson, and D. E. Dudgeon, *Array Signal Processing Concepts and Techniques*, Prentice Hall, Englewood Cliffs, NJ, 1993.
[2] H. L. Van Trees, *Optimum array processing*, John Wiley, 1st edition, 2003.
[3] T. S. Rappaport, "Smart antennas: adaptive arrays, algorithms, and wireless position location," *IEEE Press*, 1998.
[4] S. D. Blostein and H. Leib, "Multiple antenna systems: role and impact in future wireless access," *IEEE Commun. Mag.*, vol. 41, no. 7, pp. 94-101, Jul. 2003.
[5] R. Roy, A. Paulraj, and T. Kailath, "ESPRIT - A subspace rotation approach to estimation of parameters of cisoids in noise," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-34, pp. 1340-1342, Oct. 1986.
[6] P. Stoica and K. C. Sharman, "Maximum likelihood methods for direction-of-arrival estimation," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 38, pp. 1132-1143, July 1990.
[7] M. Agrawal and S. Prasad, "A modified likelihood function approach to DOA estimation in the presence of unknown spatially correlated Gaussian noise using a uniform linear array," *IEEE Trans. Sign. Process.*, vol. 48, no. 10, pp. 2743-2749, Oct. 2000.
[8] H. Krim and M. Viberg, "Two decades of array signal processing research," *IEEE Signal Process. Mag.*, pp. 67-93, 1996.
[9] M. Veterelli, P. Marziliano, and T. Blu "Sampling signals with finite rate of inoovation," *IEEE Trans. Sign. Process.*, vol. 50, no. 6, pp. 1417-1428, June 2002.
[10] P. Stoica and R. Moses, *Introduction to Spectral Analysis.* Englewood Cliffs, NJ: Prentice-Hall, 2000.