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Closed-Form Error Analysis of Dual-Hop Relaying Systems over Nakagami-m Fading Channels

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***Abstract*—In this paper we investigate the end-to-end performance of dual-hop relaying systems over non identical-Nakagami-m fading channels. Our analysis considers channel state information (CSI-) assisted relays that just amplify and retransmit the information signal also known as ”non-regenerative” relays. New closed-form expressions for the average bit error probability (ABEP) are derived. The proposed expressions apply to general operating scenarios with distinct Nakagami-m fading parameters and average signal to noise ratios (SNRs) between the hops. When the fading parameter is an odd multiple of one half, the ABEP is expressed in terms of hypergeometric functions. When m takes any real non integer value, the obtained results involve the fourth Appell's hypergeometric function. For an arbitrary fading parameter, an analysis of such a scheme is performed using the well known moment-based approach.**

I. Introduction

In recent years, dual-hop transmissions have gained new interest in cooperative wireless networks, due to their numerous benefits over single hop relaying such as increasing the the system connectivity and capacity. Relaying systems has emerged as a promising wireless access solution in cellular, ad-hoc networks and military communications [1]. Amplify-and-forward (AF) is one of the two main schemes for relaying [2] where relays without performing any decoding, retransmit a scaled replica of the received signal. This kind of relaying is useful when the information is time sensitive such as voice and live video. AF relays can be classified into two categories, namely, channel state information CSI-assisted relays and blind relays. In the first case, the relay use the channel information of the preceding hop to control the relay gain. In contrast, systems with blind relays use amplifiers with fixed gains resulting in a signal with variable power at the relay output. Systems with CSI-assisted relays are expected to perform better than systems equipped with blind relays, even thought the latter are more attractive from a practical standpoint due to their ease of deployment.

The performance analysis of multi-hop wireless networks operating under different fading conditions has been an important field of research in the past few years. Indeed, in [2], Hasna and Alouini have studied the average bit error probability (ABEP) of dual-hop systems with CSI-assisted AF relays over Rayleigh and identically distributed Nakagami-m fading channels. Recently in [3], closed-from error analysis of the non identical Nakagami relay fading channel was provided for a dual hop fixed gain relay model. In [4], Karagiannidis et al. have studied the performance bounds of CSI-assisted AF multi-hop transmissions over non-identically distributed Nakagami-m fading channels. Nevertheless, to the best of the authors' knowledge, no exact closed-form ABEP expressions for the non-identical Nakagami-m CSI-assisted AF relaying were previously reported.

In this paper, we present new closed-form expressions for the ABEP of an AF CSI-assisted dual-hop relay link in non-identical Nakagami-m fading channels. It turns out that, for arbitrary non intger values of the Nakagami-m fading parameters, the ABEP belongs to a special class of Appell hypergeometric functions. While for arbitrary fading parameters an approximate performance analysis of such scheme is conducted using the well-known moments-based approach .

This paper is organized as follows. In section II we describe the system and channel model for the dual-hop AF CSI-assisted relay system. In section III, closed-form expressions for the ABEP of different modulation schemes are presented. Some numerical results are provided in section IV. Finally, we conclude the paper while summarizing the main results in section V.

II. System and Channel Models

Consider a dual-hop wireless relaying system where a source S communicates with the destination node D through the help of a relay R [2]. We assume that there is no direct link between the source and the destination, which may result from high shadowing between the nodes S and D. We also assume that the source and the destination have perfect channel state information (CSI) of the source-to-relay and the relay-to-destination links, respectively. Let the modulated signal transmitted by S during the first time slot be denoted as x. Then the signal received by the relay R denoted by y_R is given by [2]

$$y_R = \sqrt{E_1} v_1 x + n_1, \tag{1}$$

where v_1 denotes the fading gain between the source and the relay and n_1 is an additive white Gaussian noise (AWGN) component with single sided power spectral density N_0. In the second time slot, the relay amplifies its received signal by a gain α and then retransmits it to D. Hence, at the destination, the received signal is given by [2]

$$y = \sqrt{E_2} v_2 \alpha (\sqrt{E_1} v_1 x + n_1) + n_2, \tag{2}$$

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where v_2 is the fading amplitude of the R-D link and n_2 is the AWGN component with power N_0 at the input of D. E_1 and E_2 represent the source and the relay transmit power, respectively. The instantaneous end-to-end SNR can be written as

$$\gamma = \frac{\frac{E_1 v_1^2}{N_0} \frac{E_2 v_2^2}{N_0}}{\frac{E_2 v_2^2}{N_0} + \frac{1}{\alpha^2 N_0}}. \tag{3}$$

In the case of available CSI at the relay, a gain of

$$\alpha^2 = \frac{1}{E_1 v_1^2}, \tag{4}$$

was proposed in [5]. Plugging the gain expressions into (3), it follows that

$$\gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}, \tag{5}$$

where $\gamma_1 = v_1^2 E_1/N_0$ and $\gamma_2 = v_2^2 E_2/N_0$ denote the instantaneous SNRs of the S-R and R-D hops, respectively. Since the hops are subject to non-identical Nakagami fading, we model v_1 and v_2 according to the Nakagami-m distribution. Hence, the instantaneous SNR γ_i is gamma distributed with pdf given by

$$P_{\gamma_i}(y_i) = \frac{m_i^{m_i} y_i^{m_i - 1}}{\Gamma(m_i)\bar{\gamma_i}^{m_i}} \exp\left(-\frac{m_i y_i}{\bar{\gamma_i}}\right), \tag{6}$$

where $\bar{\gamma_i} = E\langle \gamma_i \rangle$ and $\Gamma(\cdot)$ is the gamma function [6] defined as $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt, \forall z \geq 1$. m_i is real-valued and denotes the Nakagami-m factor of the i-th hop. As the value of m_i increases, the severity of the fading increases.

III. Error Analysis

The average bit error probability (ABEP) constitutes probably the most important performance measure of a digital communication system and is traditionally computed by determining the pdf of γ and then averaging the conditional BEP in AWGN, $P_b(e|\gamma)$, over this pdf. Mathematically, $P_b(e)$ is given by

$$P_b(e) = \int_0^\infty P_b(e|y) P_\gamma(y) dy. \tag{7}$$

Practical systems often employe Gray bit-mapped constellations with a generic expression of the $P_b(e|\gamma)$ given by $Q(\sqrt{\beta\gamma})$ with $Q(x)$ being the Gaussian Q-function defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t^2} dt$ and β is a constant. Different modulation schemes can be evaluated using the same $P_b(e|\gamma)$ expression by just changing the value of the specified constant. We have $\beta = 2$ for BPSK, $\beta = 1$ for BFSK and in the case of square/rectangular M-QAM, $P_b(e|\gamma)$ can be written as a finite weighted sum of $Q(\sqrt{\beta\gamma})$ terms [7].

By exploiting the Graig's alternative expression for the Gaussian Q function [8, Eq: 9], it is possible to express the integral in (7) in terms of the moment generating function MGF of γ as follows

$$P_b(e) = \frac{1}{\pi} \int_0^{\pi/2} M_\gamma\left(\frac{\beta}{\sin^2\phi}\right) d\phi. \tag{8}$$

Since we are concerned with independent fading between the hops, then the MGF of $\gamma^{-1} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$ can be expressed as the product of the individual MGFs pertaining to the different hops and is given by [9]

$$\begin{aligned} M_{\gamma^{-1}}(s) = 4\frac{(\frac{m_1}{\bar{\gamma_1}})^{m_1/2}(\frac{m_2}{\bar{\gamma_2}})^{m_2/2}}{\Gamma(m_1)\Gamma(m_2)} s^{\frac{m_1+m_2}{2}} \\ K_{m_1}\left(2\sqrt{\frac{s m_1}{\bar{\gamma_1}}}\right) K_{m_2}\left(2\sqrt{\frac{s m_2}{\bar{\gamma_2}}}\right), \end{aligned} \tag{9}$$

where $K_\alpha(\cdot)$ is the modified Bessel function of the second kind and order α [7]. To compute $M_\gamma(s)$, the authors in [9] have proposed a general integral formula to compute the MGF of a RV from the MGF of its inverse [9, Theorem 1]

$$M_\gamma(s) = 1 - 2\sqrt{s} \int_0^\infty J_1(2\sqrt{s}\xi) M_{\gamma^{-1}}(\xi^2) d\xi. \tag{10}$$

Using (9) and (10), the MGF of γ is shown to be given by

$$\begin{aligned} M_\gamma(s) = 1 - 8\frac{(\frac{m_1}{\bar{\gamma_1}})^{m_1/2}(\frac{m_2}{\bar{\gamma_2}})^{m_2/2}}{\Gamma(m_1)\Gamma(m_2)} \sqrt{s} \int_0^\infty \xi^{m_1+m_2} J_1(2\sqrt{s}\xi) \\ K_{m_1}\left(2\xi\sqrt{\frac{m_1}{\bar{\gamma_1}}}\right) K_{m_2}\left(2\xi\sqrt{\frac{m_2}{\bar{\gamma_2}}}\right) d\xi. \end{aligned} \tag{11}$$

Substituting appropriately (11) in (8) and considering the change of variable $t = \frac{1}{\sin(\phi)}$ yield

$$\begin{aligned} P_b(e) = \frac{1}{2} - \frac{4\sqrt{\beta}}{\pi} \frac{(\frac{m_1}{\bar{\gamma_1}})^{m_1/2}(\frac{m_2}{\bar{\gamma_2}})^{m_2/2}}{\Gamma(m_1)\Gamma(m_2)} \int_0^\infty \xi^{m_\Sigma} K_{m_1}\left(2\xi\sqrt{\frac{m_1}{\bar{\gamma_1}}}\right) \\ K_{m_2}\left(2\xi\sqrt{\frac{m_2}{\bar{\gamma_2}}}\right) \underbrace{\int_1^\infty \frac{J_1(2\sqrt{\beta}\xi t)}{\sqrt{t^2-1}} dt}_{K} d\xi, \end{aligned} \tag{12}$$

where $m_\Sigma = m_1 + m_2$ is defined for the sake of notational convenience. By the help of [10, Eq: 6.552.6] and [10, Eq: 8.464.1, Eq: 8469.1], K can be expressed in closed-form as

$$K = -\frac{\pi}{2} J_{\frac{1}{2}}(\sqrt{\beta}\xi) Y_{\frac{1}{2}}(\sqrt{\beta}\xi) = \frac{\sin(2\sqrt{\beta}\xi)}{2\sqrt{\beta}\xi}, \tag{13}$$

where $J_\alpha(\cdot)$ and $Y_\nu(\cdot)$ are the Bessel functions of the first and second kind. Now, recognizing the fact that

$$\begin{aligned} K_\nu(z) = 2^{\nu-1}\Gamma(\nu) z^{-\nu} {}_0F_1(;1-\nu, \tfrac{z^2}{4}) + \\ 2^{-\nu-1}\Gamma(-\nu) z^{\nu} {}_0F_1(;1+\nu, \tfrac{z^2}{4}), \end{aligned} \tag{14}$$

and

$$\sin(z) = z {}_0F_1\left(;\frac{3}{2}, -\frac{z^2}{4}\right), \tag{15}$$

where ${}_0F_1(;b,z)$ denotes the confluent hypergeometric function [6], an alternative expression for the ABEP is shown to be given by

$$P_b(e) = \frac{1}{2} - \frac{4\sqrt{\beta}}{\pi}\left\{\frac{(\frac{m_1}{\bar{\gamma_1}})^{m_1/2}}{\Gamma(m_1)}\Psi + \frac{(\frac{m_1}{\bar{\gamma_1}})^{m_1/2}(\frac{m_2}{\bar{\gamma_2}})^{m_2}\Gamma(-m_2)}{\Gamma(m_1)\Gamma(m_2)}\Phi\right\}, \tag{16}$$

where

$$\Psi = \int_0^\infty \xi^{m_1} K_{m_1}\left(2\xi\sqrt{\frac{m_1}{\bar{\gamma_1}}}\right) {}_0F_1(;\frac{3}{2}, -\xi^2\beta) {}_0F_1(;1-m_2, \xi^2\frac{m_2}{\bar{\gamma_2}}) d\xi, \tag{17}$$

and

$$\Phi = \int_0^\infty \xi^{2m_2+m_1} K_{m_1}\left(2\xi\sqrt{\frac{m_1}{\bar{\gamma_1}}}\right) {}_0F_1(;\frac{3}{2}, -\xi^2\beta) {}_0F_1(;1+m_2, \xi^2\frac{m_2}{\bar{\gamma_2}}) d\xi. \tag{18}$$

Notice that the transformation in (14) is only valid for non-integer ν. Nevertheless, practically, very similar performances can be obtained at ν and $\nu + \epsilon$ for sufficiently small ϵ values.

The integrals Ψ and Φ can be solved by expressing the integrand $K_{m_1}(\cdot)$ in terms of Meijer's G-functions , namely, using $K_{m_1}(z) = \frac{1}{2}G_{0,2}^{2,0}\left[\frac{z^2}{4}\Big|\begin{matrix} - \\ \frac{m_1}{2}, -\frac{m_1}{2}\end{matrix}\right]$ and carrying out the change of variable $z = \xi^2\frac{m_1}{\bar{\gamma}_1}$. In the obtained expressions, we recognize a special instance of the Appell's function [11] given by

$$F_4(a,b;c_1,c_2;x_1,x_2) = \frac{1}{\Gamma(a)\Gamma(b)}\int_0^\infty G_{0,2}^{2,0}\left(t\,|a,b\right) \\ {}_0F_1(;c_1,x_1t){}_0F_1(;c_2,x_2t)\frac{1}{t}dt, \tag{19}$$

where $G_{p,q}^{m,n}[\cdot], m,n,p,q \in N$ is the Meijer's G-function [6]. A closed-form expression of the ABEP is therefore obtained, after some manipulations, as

$$P_b(e) = \frac{1}{2} - \frac{\sqrt{\beta\bar{\gamma}_1}}{\sqrt{m_1}B(\frac{1}{2},m_2)}\left[\left(\frac{m_2\bar{\gamma}_1}{m_1\bar{\gamma}_2}\right)^{m_2}\frac{B(m_2+\frac{1}{2},-m_2)}{B(m_2+\frac{1}{2},m_1)} \\ F_4(m_1+m_2+\tfrac{1}{2},m_2+\tfrac{1}{2},\tfrac{3}{2},1+m_2,-\beta\tfrac{\bar{\gamma}_1}{m_1},\tfrac{m_2\bar{\gamma}_1}{m_1\bar{\gamma}_2})+ \\ F_4(m_1+\tfrac{1}{2},\tfrac{1}{2},\tfrac{3}{2},1-m_2,-\beta\tfrac{\bar{\gamma}_1}{m_1},\tfrac{m_2\bar{\gamma}_1}{m_1\bar{\gamma}_2})\right], \tag{20}$$

where F_4 is the fourth Appell function defined as the multiple hypergeometric series

$$F_4[\alpha,\beta;\gamma,\gamma',x,y] = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{(\alpha)_{j+k}(\beta)_{j+k}}{(\gamma)_j(\gamma')_k}\frac{x^j}{j!}\frac{y^k}{k!}, \\ |x|^{1/2}+|y|^{1/2}<1. \tag{21}$$

The Appell functions are well known, and numerical routines for their exact computation are available in packages such as Mathematica. Notice that, the individual series in (20) converge when $\sqrt{\beta}+\sqrt{\frac{m_2}{\bar{\gamma}_2}} < \sqrt{\frac{m_1}{\bar{\gamma}_1}}$. Nevertheless, using the analytical continuation formulas of the Appell F_4, a similar result to (20) could be found in the region $\sqrt{\beta}+\sqrt{\frac{m_2}{\bar{\gamma}_2}} > \sqrt{\frac{m_1}{\bar{\gamma}_1}}$. The aforementioned continuation formula is given in [11] as follows

$$F_4(a,b;c_1,c-2;z_1,z_2) = \frac{\Gamma(c_2)\Gamma(b-a)}{\Gamma(b)\Gamma(c_2-a)}(-z_2)^{-a} \\ F_4(a,1+a-c_2;c_1,1-b+a;x_1,x_2) \\ +\frac{\Gamma(c_2)\Gamma(a-b)}{\Gamma(a)\Gamma(c_2-b)}(-z_2)^{-b} \\ F_4(b,1+b-c_2;c_1,1-a+b;x_1,x_2). \tag{22}$$

where $x_2 = z_1/z_2$ and $x_2 = 1/z_2$. It is worthwhile to note that (20) constitutes the first closed-from expression for the ABEP of a dual-hop CSI-assisted AF-based relayed transmission over non identically distributed Nakagami-m fading. When the hops are identically distributed, the ABEP specializes when $m_1 = m_2 = m$ and $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}$ to

$$P_b(e) = \frac{1}{2} - \frac{\sqrt{\beta\bar{\gamma}}}{\pi\sqrt{m}B(\frac{1}{2},m)}\left[\frac{B(2m+\frac{1}{2},-m)}{B(\frac{1}{2},m)}F_4(2m+\tfrac{1}{2},m+\tfrac{1}{2},\tfrac{3}{2}, \\ 1+m,-\beta\tfrac{\bar{\gamma}}{m},1)+F_4(m+\tfrac{1}{2},\tfrac{1}{2},\tfrac{3}{2},1-m,-\beta\tfrac{\bar{\gamma}}{m},1)\right]. \tag{23}$$

The Meijer's G-functions are implemented in most popular computing software,such as Matlab and Mathematica.

A. ABEP for an odd multiple of a half fading parameter

When $m_l, l = 1,2$ is equal to an integer plus one-half, i.e., $m_l = n_l + 1/2$, where n_l is an integer, we have

$$K_{m_l}\left(2\xi\sqrt{\frac{m_l}{\bar{\gamma}_l}}\right) = \sqrt{\pi}e^{-2\xi\sqrt{\frac{m_l}{\bar{\gamma}_l}}}\sum_{p=0}^{n_l}\frac{\Gamma(n_l+1+p)}{\Gamma(n_l+1-p)\Gamma(p+1)} \\ \left(4\sqrt{\frac{m_l}{\bar{\gamma}_l}}\right)^{-p-1/2}\xi^{-p-1/2}. \tag{24}$$

The MGF of γ is therefore given by

$$M_\gamma(s) = 1-2\pi\sqrt{s}\frac{(\frac{m_1}{\bar{\gamma}_1})^{m_1/2}(\frac{m_2}{\bar{\gamma}_2})^{m_2/2}}{\Gamma(m_1)\Gamma(m_2)}\int_0^\infty \xi^{n_\Sigma}e^{-2(\sqrt{\frac{m_1}{\bar{\gamma}_1}}+\sqrt{\frac{m_2}{\bar{\gamma}_2}})\xi} \\ J_1(2\sqrt{s}\xi)\left(\sum_{p=0}^{n_1}b_{1,p}\xi^{-p}\right)\left(\sum_{p=0}^{n_2}b_{2,p}\xi^{-p}\right)d\xi, \tag{25}$$

where $n_\Sigma = n_1 + n_2$ and $b_{l,p} = \frac{\Gamma(n_l+1+p)}{\Gamma(n_l+1-p)\Gamma(p+1)}(4\sqrt{\frac{m_l}{\bar{\gamma}_l}})^{-p}$. The MGF expression above encompasses the product of two polynomials of $x = \xi^{-1}$. It is well known that the product of two polynomials is another polynomial whose degree is the sum of the degrees of the two polynomials and the coefficient of x^p in the resulting polynomial is the sum of terms of the form $b_{1,p_1} \times b_{2,p_2}$ such that $p_1 + p_2 = p$. Consequently, we obtain

$$\left(\sum_{p=0}^{n_1}b_{1,p}\xi^{-p}\right)\left(\sum_{p=0}^{n_2}b_{2,p}\xi^{-p}\right) = \sum_{p=0}^{n_\Sigma}\left(\sum_{w_p}b_{1,p_1}b_{2,p_2}\right)\xi^{-p}, \tag{26}$$

where w_p is the set of pairs such that $w_p = \{(p_1,p_2) : p_1 \in \{0,1,...,n_1\}, p_2 \in \{0,1,...,n_2\}, p_1+p_2 = p\}$. Using (26) and after some manipulations, (25) reduces to

$$M_\gamma(s) = 1-2\pi\sqrt{s}\frac{(\frac{m_1}{\bar{\gamma}_1})^{m_1-1/2}(\frac{m_2}{\bar{\gamma}_2})^{m_2-1/2}}{\Gamma(m_1)\Gamma(m_2)}\sum_{p=0}^{n_\Sigma}\left(\sum_{w_p}b_{1,p_1}b_{2,p_2}\right) \\ \int_0^\infty \xi^{n_\Sigma-p}e^{-2(\sqrt{\frac{m_1}{\bar{\gamma}_1}}+\sqrt{\frac{m_2}{\bar{\gamma}_2}})\xi}J_1(2\sqrt{s}\xi)d\xi. \tag{27}$$

By the help of [10, Eq. 6.621], the integral in (27) can be expressed in closed-form and the MGF of γ is found to be given by

$$M_\gamma(s) = 1-\frac{2s\pi(\frac{m_1}{\bar{\gamma}_1})^{m_1-1/2}(\frac{m_2}{\bar{\gamma}_2})^{m_2-1/2}}{\Gamma(m_1)\Gamma(m_2)}\sum_{p=0}^{n_\Sigma}\left(\sum_{w_p}b_{1,p_1}b_{2,p_2}\right) \\ \frac{\Gamma(2+n_\Sigma-p)}{2^{n_\Sigma-p+2}(\sqrt{\frac{m_l}{\bar{\gamma}_l}}+\sqrt{\frac{m_2}{\bar{\gamma}_2}})^{n_\Sigma-p+2}}F(1+\tfrac{n_\Sigma-p}{2},1+\tfrac{n_\Sigma-p+1}{2},2, \\ -\frac{s}{(\sqrt{\frac{m_1}{\bar{\gamma}_l}}+\sqrt{\frac{m_2}{\bar{\gamma}_2}})^2}), \tag{28}$$

where $F(a,b,c,z)$ is the Gauss hypergeometric function [6]. The distribution of γ can be derived as

$$P_\gamma(y) = L^{-1}[M_\gamma(s),y], \tag{29}$$

where $L^{-1}(\cdot;\cdot)$ is the inverse Laplace transform [10]. By representing the hypergeometric function in (28) as a Meijer's G-function and using [10, Eq. 7.813], the resulting end-to-end SNR distribution can be written as in (30), where $\delta(\cdot)$ is the Dirac-Delta function [6]. Using the obtained closed-form expression for the end-to-end SNR, important performance metrics such as the outage probability, the average SNR, the amount of fading and the channel capacity can be easily computed. Nevertheless, in this manuscript, for the sake of

$$P_\gamma(y) = \delta(y) - \frac{2\sqrt{\pi}(\frac{m_1}{\bar{\gamma_1}})^{m_1-1/2}(\frac{m_2}{\bar{\gamma_2}})^{m_2-1/2}}{\Gamma(m_1)\Gamma(m_2)} \sum_{p=1}^{n_\Sigma} \frac{\left(\sum_{w_p} b_{1,p_1} b_{2,p_2}\right)\Gamma(2+n_\Sigma-p)}{\Gamma(1+\frac{n_\Sigma-p}{2})\Gamma(1+\frac{n_\Sigma-p+1}{2})2^{n_\Sigma-p+2}(\sqrt{\frac{m_1}{\bar{\gamma_l}}}+\sqrt{\frac{m_2}{\bar{\gamma_2}}})^{n_\Sigma-p+2}} \frac{1}{y^2} G_{2,3}^{2,1}\left((\sqrt{\frac{m_1}{\bar{\gamma_1}}} + \sqrt{\frac{m_2}{\bar{\gamma_2}}})^2 y \middle| \begin{array}{c} 1,2 \\ 1+\frac{n_\Sigma-p}{2}, 1+\frac{n_\Sigma-p+1}{2}, 2 \end{array} \right), \tag{30}$$

conciseness, we mainly focus on error analysis. Thus we only report on the following obtained results related to the ABEP. By inserting (30) into (7) and recognizing the fact that

$$Q(\sqrt{\beta y}) = \frac{1}{\sqrt{2\pi}} G_{1,2}^{2,0}\left(\frac{\beta}{2} z \middle| \begin{array}{c} 1 \\ 0, \frac{1}{2} \end{array} \right), \tag{31}$$

we show after some manipulations, using the integration formulas of two Meijer G functions [12, Eq. 21], that $P_b(e)$ specializes, in the case of Nakagami fading where the fading parameter is an odd multiple of one half, to (32). When the average symbol energy is the same at every hop and the channel gains of the S-R and R-D links are identically distributed, then (32) simplifies to (33), where $A_{p_l} = \frac{\Gamma(n+1+p_l)}{\Gamma(n+1-p_l)\Gamma(p_l+1)}$ and $m = n + \frac{1}{2}$.

B. Approximate MGF-based error analysis for arbitrary m

In order to derive the ABEP for arbitrary m, we represent the MGF of γ defined as $M_{\gamma(s)} = E\left\langle e^{s\gamma}\right\rangle$ as a formal power series (e.g. Taylor)

$$M_\gamma(s) = \sum_{n=0}^{\infty} \frac{E\left\langle \gamma^n \right\rangle}{n!} s^n, \tag{34}$$

where $E\left\langle \gamma^n \right\rangle$ is the generalized n-th moment of the end-to-end SNR denoted by μ_n and obtained as

$$\mu_n = E\langle\gamma^n\rangle = \int_0^\infty \int_0^\infty \left(\frac{y_1 y_2}{y_1+y_2}\right)^n P_{\gamma_1}(y_1)P_{\gamma_2}(y_2)dy_1 dy_2. \tag{35}$$

By substituting appropriately (6) in (35), μ_n can be written as

$$\mu_n = \frac{m_1^{m_1} m_2^{m_2}}{\Gamma(m_1)\Gamma(m_2)\bar{\gamma_1}\bar{\gamma_2}} \int_0^\infty y_2^{n+m_2-1} e^{-\frac{m_2}{\bar{\gamma_2}} y_2} \times \int_0^\infty \frac{y_1^{n+m_1-1}}{(y_1+y_2)^n} e^{-\frac{m_1}{\bar{\gamma_1}} y_1} dy_1 dy_2. \tag{36}$$

The first integral in (36), i.e., with respect to y_1 and denoted I_1 can be solved by expressing its integrands in terms of Meijer's G-functions [12, Eqs. 10 and 11], namely using $(1+\frac{y_2}{y_1})^{-n} = (1/\Gamma(n)) G_{1,1}^{1,1}\left[y_1/y_2 \middle| \begin{array}{c} 1 \\ n \end{array}\right]$ and $\exp(-m_1 y_1/\bar{\gamma_1}) = G_{0,1}^{1,0}[m_1 y_1/\bar{\gamma_1}|0]$, thus yielding

$$I_1 = \int_0^\infty \frac{y_1^{m_1-1}}{\Gamma(n)} G_{1,1}^{1,1}\left[\frac{y_1}{y_2} \middle| \begin{array}{c} 1 \\ n \end{array}\right] G_{0,1}^{1,0}\left[\frac{m_1 y_1}{\bar{\gamma_1}} \middle| \begin{array}{c} - \\ 0 \end{array}\right] dy_1. \tag{37}$$

Now, recognizing the fact that the integral of a power term and two Meijer's G-functions is also a Meijer's G-function [12, Eq. 21], (37) can be expressed as

$$I_1 = \frac{y_2^{m_1}}{\Gamma(n)} G_{1,2}^{2,1}\left(\frac{m_1}{\bar{\gamma_1}} y_2 \middle| \begin{array}{c} 1-n-m_1 \\ -m_1, 0 \end{array}\right). \tag{38}$$

After substituting I_1 in (36), the second integral, i.e., with respect to y_2 and denoted by I_2 is given by

$$I_2 = \int_0^\infty y_2^{m_1+m_2+n-1} G_{1,2}^{2,1}\left[\frac{m_1 y_2}{\bar{\gamma_1}} \middle| \begin{array}{c} 1-n-m_1 \\ -m_1, 0 \end{array}\right] G_{0,1}^{1,0}\left[\frac{m_2 y_2}{\bar{\gamma_2}} \middle| \begin{array}{c} - \\ 0 \end{array}\right] dy_2. \tag{39}$$

Thus, by the help of [12, Eq. 21], we obtain the generalized moment of the end-to end SNR as given by

$$\mu_n = \frac{\left(\frac{m_1 \bar{\gamma_2}}{m_2 \bar{\gamma_1}}\right)^{m_1} \frac{m_2}{\bar{\gamma_2}}^{-n}}{\Gamma(m_1)\Gamma(m_2)\Gamma(n)} G_{2,2}^{2,2}\left(\frac{m_2 \bar{\gamma_1}}{m_1 \bar{\gamma_2}} \middle| \begin{array}{c} 1+m_1, 1 \\ n+m_1, m_1+m_2+n \end{array}\right). \tag{40}$$

Although the moments of all orders are finite and can be evaluated in closed-form, the convergence of the series in (34) cannot be always assured. Hence, we confine ourselves to obtaining an approximation of $M_\gamma(s)$ by utilizing the Padé approximation method. Such method is widely used in several scientific fields, serving as an alternative means of approximating series similar to that in (34) where practically only few coefficients are known and the series converges too slowly or diverges [12]. A Padé approximant is a rational function approximation to $M_\gamma(s)$, namely $\Delta[A/B](s)$, whose nominator and denominator are of order A and B, respectively, and whose power expansion is identical with the $A+B$ order expansion of $M_\gamma(s)$, i.e.,

$$\Delta[A/B](s) = \sum_{n=0}^{A+B} \frac{\mu_n s^n}{n!} + O(s^{A+B+1}) = \frac{\sum_{j=1}^{A} a_i s_i}{1+\sum_{j=1}^{B} b_i s^i}, \tag{41}$$

where $O(s^{A+B+1})$ is the remainder after the truncation of the infinite-term series to that of order $A+B$. Consequently, only the first $A+B$ moments of the end-to-end SNR are needed in order to construct $\Delta[A/B](s)$ and therefore to obtain an approximation for $M_\gamma(s)$. We note that Padé method is available in the majority of the well-known mathematical software packages, such as Mathematica and Maple. In the sequel, from (41) and using the well-known MGF-based approach [14], the ABEP can be easily evaluated. For Binary Frequency Shift Keying modulation (FSK), the ABEP can be approximated as follows using [15]

$$P_b(e) \cong \frac{1}{12} M_\gamma(\frac{1}{2}) + \frac{1}{4} M_\gamma(\frac{4}{6}), \tag{42}$$

which in turn can be directly computed form the MGF without an additional numerical integration. For non-coherent BFSK (NCBFSK) and differential BPSK (DBPSK), the bit error rate is given by [13]

$$P_b(e) = 0.5 M_\gamma(-b), \tag{43}$$

with $b = 1$ for DBPSK and $b = 0.5$ for NCBFSK. For for other modulation schemes, including M-QAM and M-PSK types, single integrals with finite limits and integrands composed of elementary functions can be readily evaluated by numerical integration.

IV. Numerical Results

In this section, we present illustrative numerical examples for the ABEP metric obtained previously. Fig. 1 depicts the

$$P_b(e) = \frac{1}{\sqrt{2}} - \frac{\beta\sqrt{\pi}}{\sqrt{2}} \frac{(\frac{m_1}{\bar{\gamma}_1})^{m_1-1/2}(\frac{m_2}{\bar{\gamma}_2})^{m_2-1/2}}{\Gamma(m_1)\Gamma(m_2)} \sum_{p=1}^{n_\Sigma} \frac{\left(\sum_{w_p} b_{1,p_1} b_{2,p_2}\right)\Gamma(2+n_\Sigma-p)}{\Gamma(1+\frac{n_\Sigma-p}{2})\Gamma(1+\frac{n_\Sigma-p+1}{2})2^{n_\Sigma-p+2}(\sqrt{\frac{m_l}{\bar{\gamma}_l}}+\sqrt{\frac{m_2}{\bar{\gamma}_2}})^{n_\Sigma-p+2}} G^{2,3}_{4,4}\left(\frac{(\sqrt{\frac{m_1}{\bar{\gamma}_1}}+\sqrt{\frac{m_2}{\bar{\gamma}_2}})^2}{\beta} \middle| \begin{array}{c} 1,2,\frac{3}{2},2 \\ 1+\frac{n_\Sigma-p}{2}, 1+\frac{n_\Sigma-p+1}{2}, 1, 2 \end{array} \right). \quad (32)$$

$$P_b(e) = \frac{1}{\sqrt{2}} - \frac{\beta\sqrt{\pi}}{\sqrt{2}} \frac{(\frac{m}{4\bar{\gamma}})^{n+1}}{\Gamma(n+\frac{1}{2})^2} \sum_{p=1}^{2n} \frac{\left(\sum_{w_p} A_{p_1} A_{p_2}\right)\Gamma(2+2n-p)}{\Gamma(1+n-\frac{p}{2})\Gamma(1+n-\frac{p+1}{2})} G^{2,3}_{4,4}\left(\frac{4m}{\bar{\gamma}\beta} \middle| \begin{array}{c} 1,2,\frac{3}{2},2 \\ 1+n-\frac{p}{2}, 1+n-\frac{p+1}{2}, 1, 2 \end{array} \right). \quad (33)$$

ABEP obtained form (32) for an arbitrary non integer fading parameter m and from (33) for an odd multiple of a half fading parameter m, against the average end-to-end SNR. The influence of the Nakagami-m fading parameter on the performance of the dual hop relaying system is analyzed considering, without loss of generality, that the channels are independent and identically distributed, i.e., $m_1 = m_2 = m$ and $\bar{\gamma}_i = \bar{\gamma} = \bar{\gamma}$. As expected, the error rate diminishes as the fading parameter increases. Fig. 2 plots the ABEP for binary NCFSK and binary DPSK against the first hop average SNR $\bar{\gamma}_1$. The hops have fading parameters given by $m_1 = 0.7$ and $m_2 = 1.5$. Here the power imbalance between the hops is investigated. The higher average SNR resulting from one of the relays may be due to a Line Of Sight (LOS) signal component between the source terminal and the relay, or equivalently between the relay and the destination. Such a situation may occur in a cell-site scheme. Note that such imbalance may be either beneficial or harmful for the overall system performance. Indeed, for $\bar{\gamma}_2 > \bar{\gamma}_1$, it is advantageous compared to the balanced case, otherwise, it is detrimental.

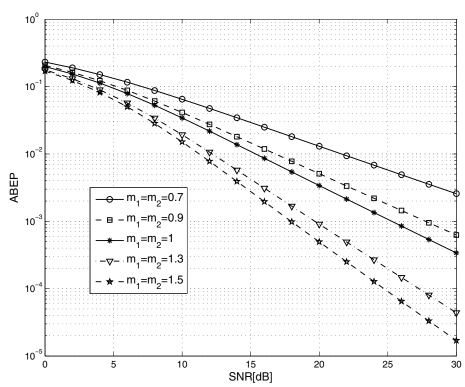

Fig. 1. Average BEP for BPSK in a dual-hop AF transmission over Nakagami-m fading with different values of m.

V. Conclusion

We have derived closed-form ABEP expressions for several modulation schemes of CSI-assisted AF dual-hop relay link operating in independent and non-identical Nakagami-m fading channels. This analysis is useful in that it allows investigation of the AF relaying performance subject to different fading conditions both for source to relay and relay to destination links.

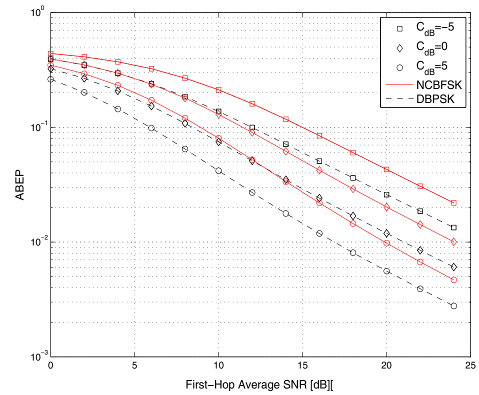

Fig. 2. Average BEP for binary DPSK and binary NCSK in a dual-hop CSI-assisted relaying system over Nakagami-m fading channel with $m_1 = 0.7$ and $m_2 = 1.5$ with balanced and unbalanced links.

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