REGULARIZED SEMI-BLIND ESTIMATOR OVER MIMO-OFDM SYSTEMS

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1. ABSTRACT

In semi-blind channel estimation techniques, the choice of the regularizing parameter that weights the blind criterion when linearly combined to the training-based least square criterion has a great impact on channel estimation performance. If a scalar regularization is considered, it has been noted that the optimal value of the regularizing factor has no closed-form expression. In a recent work, we proved that by using a regularization matrix instead, we not only enhance the performance but also can determine a closed-form expression for the optimal regularizing matrix that minimizes the asymptotic mean-square-error of the channel estimate. In this paper, we generalize our work to the context of Multiple-Input-Multiple-Output-Orthogonal-Frequency-Division-Multiplexing (MIMO-OFDM). As an application, we propose to make a performance comparison between linear prediction and subspace semi-blind estimators. In particular, we assess by simulations the accuracy of the derived results and investigate the Bit Error Rate performance as well as the impact of channel overmodeling.

Key words: semi-blind equalization, MIMO-OFDM, regularization, asymptotic performance.

2. INTRODUCTION

Multiple-input multiple-output orthogonal-frequency division multiplexing (MIMO-OFDM) has been proposed as a strong candidate for future generation wireless communication systems. Using multiple transmit and receive antennas, a MIMO-OFDM system can achieve high data rates without increasing the power or the bandwidth as compared to a single-antenna and/or a single-carrier system, provided that an accurate channel estimate is available at the receiver side [1].

In MIMO-OFDM systems, a variety of methods have been applied for getting an accurate estimate of the channel. They can be classified into three classes, namely, training-based methods, blind methods and semi-blind methods. Training-based methods rely on the periodic transmission of known symbols, entailing the reduction of the system bandwith efficiency. On the other hand, blind methods, do not require any training symbols, at the expense of a high computational complexity. Semi-blind methods emerged as new promising techniques which can allow significant reduction in the number of training symbols while keeping a good quality of the channel estimate.

Regularized semi-blind estimators have been proposed in many previous works. They are essentially based on combining linearly the training sequence-based criterion with the blind criterion. A weighting factor (regularizing constant) is usually employed for trade-off between the Least-Square (LS) and the blind criteria. Given its impact on the channel estimation performance, the regularizing factor has been proposed to be set in such a way that it minimizes the asymptotic Mean-Square-Error (MSE), [2]. Since the optimal regularizing constant has no closed-form expression, [2] proposed to employ iterative algorithms that converge to the optimal solution. On the other hand, [3] proposed to evaluate the asymptotic MSE at finite discrete possible values for the regularizing constant and keep thereafter the value that exhibits the least channel estimation error. In [4], an explicit formula was given by assuming that the minimization of the semi-blind cost can be transformed into a Weighted-Least-Square minimization problem (WLS) [5].

Recently, we proposed in [6] to employ a regularizing matrix instead of a regularizing constant. Interestingly, we proved in this case that a closed-form expression for the optimal regularizing matrix exists. Moreover, we found that the proposed scheme asymptotically outperforms all the previously aforementioned techniques. In this paper, we propose to generalize our work to the context of MIMO-OFDM. We particularly derive the optimal regularizing matrices expressions in the case of subspace and linear prediction techniques. We assess the accuracy of our theoretical derivations and investigate practical aspects such as channel overmodeling, and the impact of the number of pilot symbols on the mean square error. Finally, we analyze the Bit Error Rate (BER) performance in a turbo decoding context.

3. CHANNEL ESTIMATION TECHNIQUES

3.1. Blind channel estimation

In this paper, we consider a MIMO-OFDM system with N_t transmitting antennas and N_r receiving antennas.

Let $\mathbf{x}(n,k) = [x_1(n), \cdots, x_{N_t}(n)]$ be the n-th transmitted sample of the k-th OFDM symbol after the IDFT module and $\mathbf{y}(n,k) = [y_1(n,k), \cdots, y_{N_r,k}(n)]$ be the received signal before the DFT module. Assuming that the channel length L is less than the cyclic prefix length μ, the linear convolution between the channel and the transmitted signal is transformed into a circular convolution as follows:

$$\mathbf{y}(n,k) = \sum_{l=0}^{L} \mathbf{H}_l \mathbf{x}(n-l,k) + \mathbf{v}(n,k), \quad n = 0, \cdots, K-1,$$

where $\mathbf{H}_l$ is the $N_r \times N_t$ channel response matrix at time l, and $\mathbf{v}(n,k)$ is the noise vector, K represents the number of subcarriers. Moreover, if $n < 0$, due to the circular convolution, the value $\mathbf{x}(n,k)$ is set to $\mathbf{x}(K+n,k)$. Let $\mathbf{H} = [\mathbf{H_0}^{\mathrm{T}}, \cdots, \mathbf{H}_L{}^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{h} = \mathrm{vec}(\mathbf{H})$. Stacking $M+1$ observations $\mathbf{y}(n,k)$ in the $N_r(M+1)$ vector, $\mathbf{y}_M(n,k) = [\mathbf{y}(n,k), \cdots, \mathbf{y}(n-M,k)]$, we get for $M \le n \le K-1$

$$\mathbf{y}_M(n,k) = \boldsymbol{\mathcal{I}}_M(\mathbf{H})\mathbf{x}_L(n,k) + \mathbf{v}_M(n,k),$$

where $\mathbf{v}_M(n,k) = [\mathbf{v}^{\mathrm{T}}(n,k), \cdots, \mathbf{v}^{\mathrm{T}}(n-M,k)]^{\mathrm{T}}$ and $\boldsymbol{\mathcal{I}}_M(\mathbf{H})$ is the $(M+1)N_r \times (M+L+1)N_t$ block Toeplitz matrix with the first block row given by $[\mathbf{H}(0), \cdots, \mathbf{H}(L), \mathbf{0}, \cdots, \mathbf{0}]$.

Assuming that $\mathbb{E}|\mathbf{x}(n,k)|^2 = 1$, the covariance matrix of the received signal can be expressed as:

$$\mathbf{R}_M = \mathbb{E}\mathbf{y}_M(n,k)\mathbf{y}_M^{\mathrm{H}}(n,k) = \boldsymbol{\mathcal{I}}_M(\mathbf{H})\boldsymbol{\mathcal{I}}_M{}^{\mathrm{H}}(\mathbf{H}) + \sigma^2 \mathbf{I}_{(M+1)N_r},$$

where σ^2 denotes the noise variance.

It is estimated in practice as:

$$\widehat{\mathbf{R}}_M = \frac{1}{(K-M)n_g} \sum_{n=M}^{K-1} \sum_{k=0}^{n_g-1} \mathbf{y}_M(n,k)\mathbf{y}_M^{\mathrm{H}}(n,k).$$

Most blind channel estimation techniques evaluate the channel up to a matrix ambiguity by solving the following quadratic form minimization problem:

$$\min_{\|\mathbf{h}\|=1} \mathbf{h}^{\mathrm{H}} \widehat{\mathbf{B}}_K^{\mathrm{H}} \widehat{\mathbf{B}}_K \mathbf{h}, \tag{1}$$

where $\widehat{\mathbf{B}}_K$ is an estimated matrix of $\mathbf{B}$, $\mathbf{B}$ being the matrix that depends on the considered blind estimation technique.

3.2. Least square channel estimation

We assume that the channel estimation at the receiver side is conducted over n_g OFDM symbols, each OFDM symbol containing k_p pilot samples (i.e pilot subcarriers). Obviously, the least square channel estimation is possible only when $n_p = n_g k_p \ge (L+1)N_t$ where $L+1$ is the maximum length of all channels.

For MIMO-OFDM systems, optimal pilot sequences and optimal pilot placement of the pilot tones with respect to the MSE of the least-square channel estimate were derived in [7].

We design our pilot sequence according to [7] as follows:

Let $\widetilde{\mathbf{y}}_i(k)$ denotes the $k_p \times 1$ frequency domain vector of pilot samples received at time k by the i-th antenna and let $\widetilde{\mathbf{y}}_i = [\widetilde{\mathbf{y}}_i^{\mathrm{T}}(0), \cdots, \widetilde{\mathbf{y}}_i^{\mathrm{T}}(n_g-1)]^{\mathrm{T}}$. Then $\widetilde{\mathbf{y}}_i$ satisfies:

$$\widetilde{\mathbf{y}}_i = \mathbf{A}\widetilde{\mathbf{h}}_i + \widetilde{\mathbf{v}}_i,$$

where $\mathbf{A}$ is a $n_p \times (L+1)N_t$ matrix that depends on the pilot symbols and chosen to be orthogonal, $\mathbf{A}^{\mathrm{H}}\mathbf{A} = n_p \mathbf{I}_{(L+1)N_t}$, $\widetilde{\mathbf{v}}_i$ is the noise vector with respect to the ith receiving antenna and $\widetilde{\mathbf{h}}_i$ is the channel vector response associated to the i-th receiving antenna given by:

$$\widetilde{\mathbf{h}}_i = [h_{i,1}(0), \cdots, h_{i,1}(L), \cdots, h_{i,N_t}(0), \cdots, h_{i,N_t}(L)]^{\mathrm{T}}.$$

Defining

$$\begin{aligned} \mathbf{Y} &= [\widetilde{\mathbf{y}}_1, \cdots, \widetilde{\mathbf{y}}_{N_r}], \\ \widetilde{\mathbf{H}} &= \left[\widetilde{\mathbf{h}}_1, \cdots, \widetilde{\mathbf{h}}_{N_r}\right], \\ \widetilde{\mathbf{V}} &= [\widetilde{\mathbf{v}}_1, \cdots, \widetilde{\mathbf{v}}_{N_r}], \end{aligned}$$

we have

$$\mathbf{Y} = \mathbf{A}\widetilde{\mathbf{H}} + \widetilde{\mathbf{V}}.$$

Let $\widetilde{\mathbf{h}} = \mathrm{vec}(\widetilde{\mathbf{H}})$ [1]. The least-square estimate minimizes the following criterion:

$$\min_{\widetilde{\mathbf{h}}} \|\mathrm{vec}(\mathbf{Y}) - \mathbf{I}_{N_r} \otimes \mathbf{A}\widetilde{\mathbf{h}}\|^2 = \min_{\mathbf{h}} \|\mathrm{vec}(\mathbf{Y}) - \widetilde{\mathbf{A}}\mathbf{h}\|^2, \tag{2}$$

where $\widetilde{\mathbf{A}} = (\mathbf{I}_{N_r} \otimes \mathbf{A})\,\mathbf{E}$ and $\mathbf{h} = \mathrm{vec}(\mathbf{H})$, $\mathbf{E}$ being the permutation matrix that transforms $\widetilde{\mathbf{h}}$ into $\mathbf{h}$ and $\otimes$ represents the Kronecker product.

3.3. Semi-blind channel estimation

To solve the intrinsic indeterminations of blind channel estimation techniques and improve their performance, [2] to combine linearly the training sequence criterion (2) with the blind criterion (1), thus leading to the following cost function :

$$C(\mathbf{f}, \alpha) = \|\mathrm{vec}(\mathbf{Y}) - \widetilde{\mathbf{A}}\mathbf{f}\|^2 + \alpha K n_g \mathbf{f}^{\mathrm{H}} \widehat{\mathbf{B}}_K^{\mathrm{H}} \widehat{\mathbf{B}}_K \mathbf{f}$$

where α is a regularizing constant. The semi-blind estimator is then given by [2]:

$$\widehat{\mathbf{h}}_\alpha(\mathbf{A}) = \left(\widetilde{\mathbf{A}}^{\mathrm{H}}\widetilde{\mathbf{A}} + \alpha K n_g \widehat{\mathbf{B}}_K^{\mathrm{H}} \widehat{\mathbf{B}}_K\right)^{-1} \widetilde{\mathbf{A}}^{\mathrm{H}} \mathrm{vec}(\mathbf{Y}).$$

[1] Note that $\tilde{\mathbf{h}}$ and $\mathbf{h}$ are the same up to a permutation.

It was shown in [2] that the choice of the regularizing constant has a great impact on the channel estimation error. Besides, no closed-form expression for the regularizing constant that minimizes the estimation MSE was found.

In [6], we proposed to use a regularizing matrix instead of a regularizing constant. Particularly, we have shown that for SIMO systems, once the blind criterion is expressed under a quadratic form, and under mild assumptions, the optimal regularizing matrix has a closed-form expression. In this paper, we propose to extend this work to MIMO-OFDM systems. We consider to minimize the following cost function:

$$C(\mathbf{f},\mathbf{\Lambda}) = \|\mathrm{vec}(\mathbf{Y}) - \widetilde{\mathbf{A}}\mathbf{f}\|^2 + (Kn_g)\mathbf{f}^{\mathrm{H}}\widehat{\mathbf{P}}_K\mathbf{\Lambda}\widehat{\mathbf{P}}_K\mathbf{f},$$

where $\widehat{\mathbf{P}}_K$ is the estimate of $\mathbf{P}$, the orthgonal projector on the space spanned by $\mathbf{B}_K$. The semi-blind channel estimate is therefore given by:

$$\widehat{\mathbf{h}}_{\mathbf{\Lambda}} = \left(\widetilde{\mathbf{A}}^{\mathrm{H}}\widetilde{\mathbf{A}} + (Kn_g)\widehat{\mathbf{P}}_K\mathbf{\Lambda}\widehat{\mathbf{P}}_K\right)^{-1}\widetilde{\mathbf{A}}^{\mathrm{H}}\mathrm{vec}(\mathbf{Y}).$$

In [6], it was shown that under mild assumptions, the optimal regularizing matrix is given by:

$$\mathbf{\Lambda}_{\mathrm{op}} = \left(\mathbf{B}^{\#}\mathbf{\Sigma}_{\infty}\mathbf{B}^{\#}\right)^{\#}. \tag{3}$$

where

$$\mathbf{\Sigma}_{\infty} = \frac{1}{\sigma^2}\lim_{\substack{Kn_g\to\infty \\ \frac{Kn_g}{n_p}\to\gamma}} Kn_g\mathrm{Cov}(\widehat{\mathbf{B}}_K\mathbf{h}).$$

4. SEMI-BLIND CHANNEL ESTIMATION BASED ON SUBSPACE (SS) AND LINEAR PREDICTION (LP) BLIND CRITERION

In this section, we briefly review the blind channel identification methods based on LP [8] and SS [9] criteria. We also provide in each case an expression for the semi-blind channel estimate.

4.1. Subspace blind criterion

Based on the eigenvalues of the covariance matrix, one can perform the decomposition into signal and noise subspaces. The signal subspace is spanned by eigenvectors corresponding to the largest $(M+L+1)N_t$ eigenvalues, whereas the noise subspace is spanned by the remaining eigenvalues which are all equal to the noise variance σ^2. Denote by $\mathbf{\Pi}_{\mathrm{sub}}$ the orthogonal projection matrix on the noise subspace. Then we have:

$$\mathbf{\Pi}_{\mathrm{sub}}\boldsymbol{\mathcal{I}}_M(\mathbf{H}) = 0. \tag{4}$$

Splitting the projection matrix $\mathbf{\Pi}_{\mathrm{sub}}$ into $M+1$ matrices such that $\mathbf{\Pi}_{\mathrm{sub}} = [\mathbf{\Pi}_0,\cdots,\mathbf{\Pi}_M]$, where each matrix $\mathbf{\Pi}_i$ is $(M+1)N_r \times N_r$, then (4) can be rewritten as:

$$\boldsymbol{\mathcal{D}}(\mathbf{\Pi}_{\mathrm{sub}})\begin{bmatrix}\mathbf{H}_0\\ \vdots \\ \mathbf{H}_L\end{bmatrix} = 0,$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{\Pi}_{\mathrm{sub}}) = \begin{pmatrix} \mathbf{\Pi}_0 & & 0 \\ \vdots & \ddots & \\ \mathbf{\Pi}_M & & \mathbf{\Pi}_0 \\ & \ddots & \vdots \\ 0 & & \mathbf{\Pi}_M \end{pmatrix},$$

thus giving,

$$(\mathbf{I}_{N_t}\otimes\boldsymbol{\mathcal{D}}(\mathbf{\Pi}_{\mathrm{sub}}))\,\mathbf{h} = \mathbf{B}_{\mathrm{sub}}\mathbf{h} = 0. \tag{5}$$

Let $\mathbf{P}_{\mathrm{sub}}$ be the orthogonal projector onto the column range space of $\boldsymbol{\mathcal{D}}(\mathbf{\Pi}_{\mathrm{sub}})$, and $\widehat{\mathbf{P}}_{\mathrm{sub}}$ its estimate, then the semi-blind estimator based on the subspace criterion is given by:

$$\widehat{\mathbf{h}}_{\mathrm{sub}} = \left(\widetilde{\mathbf{A}}^{\mathrm{H}}\widetilde{\mathbf{A}} + Kn_g\left(\mathbf{I}_{N_t}\otimes\widehat{\mathbf{P}}_{\mathrm{sub}}\right)\mathbf{\Lambda}_{\mathrm{sub}}\left(\mathbf{I}_{N_t}\otimes\widehat{\mathbf{P}}_{\mathrm{sub}}\right)\right)^{-1}\widetilde{\mathbf{A}}^{\mathrm{H}}\mathrm{vec}(\mathbf{Y})$$

where $\mathbf{\Lambda}_{\mathrm{sub}}$ is the regularizing matrix for this semi-blind criterion.

4.2. Linear prediction (LP) criterion

The LP approach is based on the observation that if the channels have no zeros in common, then a $N_r \times N_r$ polynomial matrix $\mathbf{P}(z) = \mathbf{I}_{N_r} + \sum_{i=0}^{M}\mathbf{P}_i z^{-i}$ exists such that

$$\mathbf{P}(z)\sum_{i=0}^{L}\mathbf{H}_i z^{-i} = \mathbf{H}(0),$$

or equivalently:

$$\delta(\mathbf{P})\begin{bmatrix}\mathbf{H}(0)\\ \vdots \\ \mathbf{H}(L)\end{bmatrix} = \begin{bmatrix}\mathbf{H}(0)\\ \mathbf{0} \\ \vdots \\ \mathbf{0}\end{bmatrix}$$

where $\delta(\mathbf{P})$ is the $(L+M)N_r \times (L+1)N_r$ matrix given by:

$$\delta(\mathbf{P}) = \begin{pmatrix} \mathbf{I}_{N_r} & & & \mathbf{0} \\ \mathbf{P}_1 & \ddots & & \\ \vdots & \ddots & \ddots & \\ \mathbf{P}_M & \vdots & \ddots & \mathbf{I}_{N_r} \\ & \ddots & \vdots & \mathbf{P}_1 \\ & & \ddots & \vdots \\ \mathbf{0} & & & \mathbf{P}_M \end{pmatrix}.$$

The linear predictor $\mathbf{P} = [\mathbf{P}_1,\cdots,\mathbf{P}_M]$ and the prediction error $\mathbf{D} = \mathbf{H}_0\mathbf{H}_0^{\mathrm{H}}$, can be identified by the following Yule-Walker equations:

$$\begin{cases} [\mathbf{P}_1,\cdots,\mathbf{P}_M] &= -\left[\overline{\mathbf{R}}_1,\cdots,\overline{\mathbf{R}}_M\right]\left(\mathbf{R}_{M-1} - \sigma^2\mathbf{I}\right)^{\#} \\ \mathbf{D} &= \mathbf{R}_0 + \sum_{i=1}^{P}\mathbf{P}_i\overline{\mathbf{R}}_i^{\mathrm{H}} \end{cases}$$

where

$$\begin{aligned} \mathbf{R}_{M-1} &= \mathbb{E}\left[\mathbf{y}_{M-1}(n,k)\mathbf{y}^{\mathrm{H}}_{M-1}(n,k)\right], \\ \overline{\mathbf{R}}_i &= \mathbb{E}\left[\mathbf{y}(n,k)\mathbf{y}^{\mathrm{H}}(n-i,k)\right]. \end{aligned}$$

Denote by $\mathbf{\Pi}_{\mathrm{lin}}$ the orthogonal projector on the null column space spanned by $\mathbf{D}$. Therefore,

$$\mathbf{\Pi}_{\mathrm{lin}} \otimes \boldsymbol{\delta}(\mathbf{P}) \begin{bmatrix} \mathbf{H}(0) \\ \vdots \\ \mathbf{H}(L) \end{bmatrix} = \mathbf{0},$$

thus giving:

$$\mathbf{I}_{N_t} \otimes (\mathbf{\Pi}_{\mathrm{lin}} \otimes \boldsymbol{\delta}(\mathbf{P}))\,\mathbf{h} = \mathbf{B}_{\mathrm{lin}}\mathbf{h} = \mathbf{0}.$$

Let $\mathbf{P}_{\mathrm{lin}}$ be the projector on the column range space of $\mathbf{\Pi}\otimes\boldsymbol{\delta}(\mathbf{P})$, and $\widehat{\mathbf{P}}_{\mathrm{lin}}$ its estimate, then the semi-blind estimator based on the LP criterion is given by:

$$\widehat{\mathbf{h}}_{\mathrm{lin}} = \left(\widetilde{\mathbf{A}}^{\mathrm{H}}\widetilde{\mathbf{A}} + Kn_g\left(\mathbf{I}_{N_t}\otimes \widehat{\mathbf{P}}_{\mathrm{lin}}\right)\mathbf{\Lambda}_{\mathrm{lin}}\left(\mathbf{I}_{N_t}\otimes \widehat{\mathbf{P}}_{\mathrm{lin}}\right)\right)^{-1}\widetilde{\mathbf{A}}^{\mathrm{H}}\mathrm{vec}(\mathbf{Y}).$$

5. ASYMPTOTIC ERROR AND CLOSED-FORM EXPRESSION FOR THE REGULARIZING MATRIX

Following the same lines as in [6], we can prove that, under some mild assumptions, the asymptotic covariance matrix of the estimated error $\delta\mathbf{h} = \widehat{\mathbf{h}} - \mathbf{h}$ is given by:

$$\begin{aligned} \mathrm{Cov}(\delta\mathbf{h}) &= \lim_{\substack{Kn_g\to\infty \\ \frac{Kn_g}{n_p}\to\gamma}} n_p\mathbb{E}\left(\delta\mathbf{h}\delta\mathbf{h}^{\mathrm{H}}\right) \\ &= \sigma^2\mathbf{M}^{-1}\left(\mathbf{I}_{N_rN_t(L+1)} + \gamma\mathbf{C}\right)\mathbf{M}^{-1} \end{aligned}$$

where

$$\begin{aligned} \mathbf{M} &= \left(\mathbf{I}_{(L+1)N_rN_t} + \gamma\left(\mathbf{I}_{N_t}\otimes\mathbf{P}\right)\mathbf{\Lambda}\left(\mathbf{I}_{N_t}\otimes\mathbf{P}\right)\right) \\ \mathbf{C} &= \left(\mathbf{I}_{N_t}\otimes\mathbf{P}\right)\mathbf{\Lambda}\mathbf{B}^{\#}\mathbf{\Sigma}_{\infty}\mathbf{B}^{\#}\mathbf{\Lambda}\left(\mathbf{I}_{N_t}\otimes\mathbf{P}\right) \end{aligned}$$

and $\mathbf{P} = \mathbf{P}_{\mathrm{sub}}$ (resp. $\mathbf{P} = \mathbf{P}_{\mathrm{lin}}$), $\mathbf{B} = \mathbf{B}_{\mathrm{sub}}$ (resp. $\mathbf{B} = \mathbf{B}_{\mathrm{lin}}$) and $\mathbf{\Sigma}_{\infty} = \mathbf{\Sigma}_{\infty,\mathrm{sub}}$ (resp. $\mathbf{\Sigma}_{\infty} = \mathbf{\Sigma}_{\infty,\mathrm{lin}}$) to refer to the SS-based semi-blind estimator (resp. LP-based semi-blind estimator).

Let $\mathbf{J}_p^{\tau} \triangleq \begin{bmatrix} \vdots & \mathbf{I}_{p-\tau} \\ \mathbf{0}_{\tau} & \cdots \end{bmatrix}$ for $\tau \geq 0$ and $\mathbf{J}_p^{\tau} \triangleq \left(\mathbf{J}_p^{-\tau}\right)^{\mathrm{T}}$ if $\tau < 0$.

For the LP-based estimator, we can prove that $\mathbf{\Sigma}_{\infty}$ is given by:

$$\begin{aligned} \mathbf{\Sigma}_{\infty,\mathrm{lin}} = \mathbf{I}_{N_t}\otimes\mathrm{diag}\Big(&\overline{\mathbf{P}}_{\mathrm{lin}}\overline{\mathbf{P}}_{\mathrm{lin}}{}^{\mathrm{H}}, \left(\mathbf{I}_{M+L-2}\otimes\overline{\mathbf{P}}_{\mathrm{lin}}\right)\mathcal{J}_{\mathrm{lin}} \\ &\left(\mathbf{I}_{M+L-2}\otimes\overline{\mathbf{P}}_{\mathrm{lin}}{}^{\mathrm{H}}\right)\Big) + \mathcal{O}(\sigma^2) \end{aligned}$$

where:

$$\begin{aligned} \overline{\mathbf{P}}_{\mathrm{lin}} &= \mathbf{\Pi}_{\mathrm{lin}}\left[\mathbf{I}_{N_r}\ \ \mathbf{P}\right] \\ \mathcal{J}_{\mathrm{lin}} &= \left(\sum_{k=-M}^{M}\mathbf{J}^{k}_{M+L-2}\otimes\mathbf{J}^{k}_{M+1}\otimes\mathbf{I}_{N_r}\right). \end{aligned}$$

For the SS-based semi-blind estimator, $\mathbf{\Sigma}_{\infty}$ is given by:

$$\mathbf{\Sigma}_{\infty,\mathrm{sub}} = \mathbf{I}_{N_t}\otimes\left(\sum_{k=-M}^{M}\mathbf{J}^{k}_{M+1}\otimes\mathbf{\Pi}_{\mathrm{sub}}\left(\mathbf{J}^{k}_{M+1}\otimes\mathbf{I}_{N_r}\right)\mathbf{\Pi}_{\mathrm{sub}}\right) + \mathcal{O}(\sigma^2).$$

Note that if we neglect the terms in $\mathcal{O}(\sigma^2)$, we end-up with expressions that are independent from the system parameters, $\mathbf{h}$ and σ^2. Hence, the implementation of the optimal weighting matrix in (3) can be achieved without resorting to iterative techniques.

6. SIMULATIONS

In all the simulations described below, we consider a MIMO-OFDM system with $N_t = 2$ transmit antennas and $N_r = 4$ receive antennas. The length of the cyclic prefix is 20, and the block size M is equal to 10. We assume also a Rayleigh channel model with a $L+1 = 5$ tap MIMO-FIR filter where each tap is represented by a 2×4 random matrix whose elements are i.i.d complex Gaussian variables with zero mean and variance equal to $\frac{1}{L+1}$. We also assume that 16 subcarriers are used for pilot transmission.

The empirical MSE is given by:

$$\mathrm{MSE} = \frac{1}{N_T}\sum_{k=1}^{N_T}\|\widehat{\mathbf{h}}_k - \mathbf{h}_k\|,$$

where N_T is the total number of Monte Carlo iterations.

6.1. Accuracy of the channel estimation error

In this experiment, we set the number of the subcarriers K to 2048. We estimate empirically the MSE over $N_T = 100$ iterations. Fig. 1 displays in the same graph the empirical MSE as well as the theoretical MSE for the linear prediction and subspace-based semi-blind estimators. At low SNR, we observe that the theoretical MSE tends to underestimate the real MSE since it does not take into account the term of order σ^2 in the expression of $\mathbf{\Sigma}_{\infty}$. However at moderate and high SNR, we obtain a good match between theoretical and empirical results.

6.2. Impact of channel overmodeling

In this experiment, the channel estimation performance in terms of MSE is investigated. The simulation is undertaken based on 100 Monte Carlo runs of the transmission of $n_g = 4$ OFDM symbols on $K = 512$ subcarriers, of which $n_p = 16$

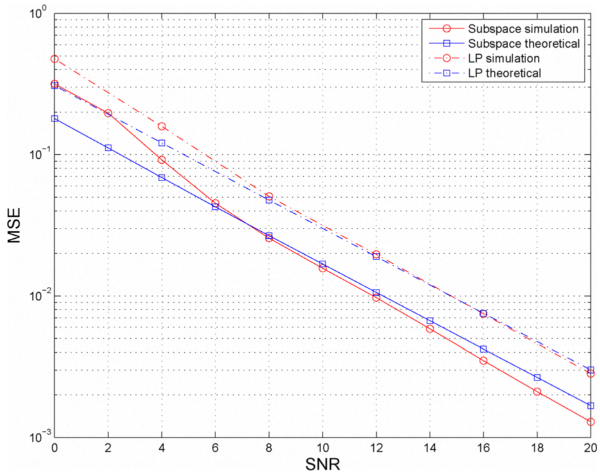

Fig. 1. Theoretical and empirical MSE versus SNR

subcarriers are used for training. The channel is set to be the same for all the SNR range. We propose to investigate the impact of the over-modeling of the channel on the performance of the semi-blind estimators. We note that if the estimated channel order $\widehat{L}$ is equal to L, the semi-blind SS estimator outperforms the LP-based estimator. But, once $\widehat{L}$ is greater than L, the LP-based estimator becomes better, as we can see on fig. 2.

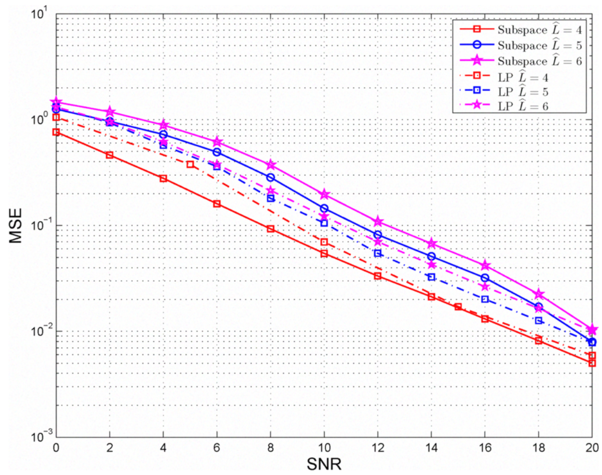

Fig. 2. Impact of channel overmodeling.

6.3. Mean square error with respect to the number of pilots

In this section, we investigate the impact of the number of pilots on the mean square error. Fig. 3 compares the MSE of the least square channel estimator with that of the semi-blind estimators, when the number of carriers K is set to 256 and the SNR is set to 10 dB. We note that for equal MSE both the

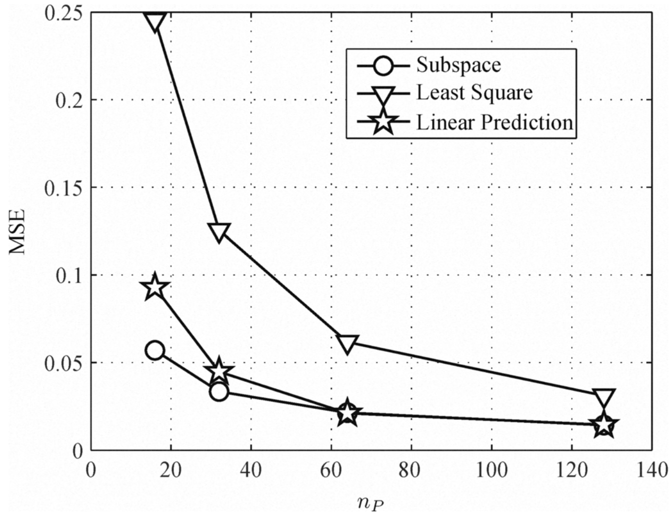

Fig. 3. Impact of the number of pilots on the mean square error

linear prediction and subspace estimator, employ at least 20 pilot symbols less than the least square estimator.

6.4. Semi-blind estimation with iterative decoding

At the receiver side, we perform iterative symbol detection and channel decoding. The receiver consists of a MIMO detector and a SISO channel decoder that exchange extrinsic soft information with each other, so as to maximize the aposteriori probabilities. We set the number of iterations between the MIMO detector and the SISO decoder to 2 [10]. We assume that the binary information data are encoded by a rate $1/2$ NRNSC code with constraint length set to 5 defined in octal form by $(037\ 021)$. Throughout our simulations, we assume that each frame is composed of 4 OFDM symbols with $K = 512$ subcarriers. We also consider $16 - QAM$ constellations with Gray labeling and we assume that the channel length has been over-estimated, $\widehat{L} = 6$. Fig. 4 displays the Bit Error Rate for the Least square and the two semi-blind based schemes. We note that both semi-blind schemes achieve almost the same performance and outperform the least-square based scheme by about 1 dB.

7. CONCLUSION

In this paper, we generalized the matrix-regularized semi-blind estimation technique to the context of MIMO-OFDM systems. As an application, we investigated the performance of subspace- and linear prediction-based estimators. A closed-form expression for the optimal regularizing matrix as well as

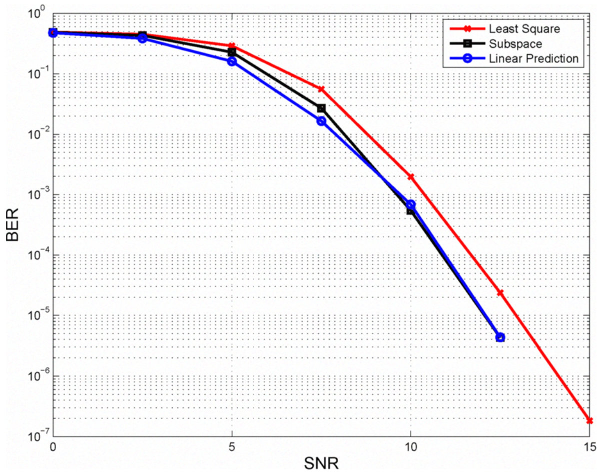

Fig. 4. BER versus SNR

the asymptotic error have been provided. We also provided simulations which strongly support our asymptotic derivations and illustrate the significant gain in terms of data throughput when using our semi-blind methods as compared to the non-blind least-squares technique.

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