EM Algorithm for Non-Data-Aided SNR Estimation of Linearly-Modulated Signals over SIMO Channels

Mohamed Ali Boujelben, Faouzi Bellili, Sofiène Affes, and Alex Stéphenne
INRS-EMT, 800, de la Gauchetière Ouest, Bureau 6900, Montreal, Qc, H5A 1K6, Canada
Emails : boujelben@emt.inrs.ca, bellili@emt.inrs.ca, affes@emt.inrs.ca, stephenne@ieee.org

Abstract— **In this paper, we address the problem of non-data-aided SNR estimation in wireless SIMO channels. We derive the per-antenna ML SNR estimator using the expectation-maximization (EM) algorithm under constant channels and additive white Gaussian noise (AWGN). The new method is valid for any arbitrary constellation. It is NDA and, therefore, does not impinge on the hole throughput of the system. We obtain two non linear vector equations which are tackled by a less complex approach based on the EM algorithm. The noise components are assumed to be spatially uncorrelated over all the antenna elements and temporally white with equal power. Besides, in order to evaluate our EM-ML SNR estimator, we derive the Cramér-Rao lower bound (CRLB) in the DA case. Monte Carlo simulations show, that our new estimator offers, a substantial performance improvement over the SISO ML SNR estimator due to the optimal usage of the mutual information between the antenna branches, and that it reaches the derived DA CRLBs. To the best of our knowledge, we are the first to derive the ML per-antenna SNR estimators as well as the CRLBs in the NDA and the DA case, respectively, both over SIMO channels.**

I. Introduction

Many modern communication systems require accurate SNR estimates for the optimal usage of radio resources [1, 2, 3]. SNR estimators may be divided into two major categories, data aided (DA) and non-data-aided (NDA). DA methods use the knowledge of the transmitted symbols to facilitate the estimation process. NDA methods base the estimation only on the received signals.

In both cases, SNR estimates can be obtained from the inphase and quadrature (I/Q) components of the received signal or simply from its magnitude (envelope). They are, respectively, referred to as I/Q-based and envelope-based SNR estimators. So far, for linearly-modulated signals over flat fading channels in single input single output (SISO) transmissions, various SNR estimation techniques have been reported in the literature. These include the maximum-likelihood (ML) envelope-based estimator [4] and the ML I/Q-based estimator [5, 6]. In both cases, the analytical derivation of the ML estimator was recognized to be mathematically intractable, and the numerical computations of the ML SNR estimates were carried out using the iterative expectation-maximization (EM) algorithm [7]. On the other hand, it has been recently shown [8] that the exploitation of the mutual information offered by SIMO systems can lead to remarkable improvements in SNR estimation with moment-based estimators. However, contrarily to the I/Q ML-based estimators, it is well known that the moment-based SNR estimators do not exploit the whole information carried by the received signal.

Work supported by a Canada Research Chair in Wireless Communications, the Cooperative Research and Development program of NSERC, and Ericsson Canada.

Motivated by these facts, a maximum-likelihood SNR estimator, which uses the entire information (I/Q-based estimator) is developed in this paper. Unfortunately, it turns out that the analytical derivation of the ML solution is fairly complex from the computational point of view. Therefore, we resort to the EM algorithm [7] which has been so far widely used in signal parameter estimation to numerically find the ML estimates. In order to assess the absolute performance of our estimator, we derive a CRLB for SIMO channels in the DA case. The simulated estimator performances are then compared to the CRLBs over complex additive white Gaussian noise (AWGN) channels and shown to coincide over a wide SNR range.

The structure of the rest of this paper is as follows. In section II, we introduce the equivalent baseband model of the signal. The EM-based ML SNR estimator is developed in section III. The CRLB evaluation in the DA case is developed in section IV. The simulation results are presented in section V and some concluding remarks are drawn out in section VI.

II. System Model

We consider an array of N_a receiving antenna elements. The channel is supposed to be with constant gain coefficients $\{S_i\}_{i=1,2,\ldots,N_a}$ over the observation interval. We assume that the same noise power is experienced over all the antenna elements. Assuming an ideal receiver with perfect synchronization, the received signal at the output of the matched filter, on the i^{th} antenna element, is given by :

$$y_i(n) = S_i \ a(n) \ e^{j\phi_i} + w_i(n), \ n = 1, 2, \ldots, N, \quad (1)$$

where, at time index n, $a(n)$ is the transmitted symbol and $y_i(n)$ is the corresponding received sample on the i^{th} antenna element. The noise components $w_i(n)$ are modeled by zero-mean Gaussian random variables with independent real and imaginary parts, each of variance σ^2 and N is the length of the observation window. Moreover, the transmitted symbols are assumed to be independent and identically distributed (iid) and drawn from any linear M-ary constellation. ϕ_i accounts for any non-random phase shift introduced by the channel.

978-1-4244-4148-8/09/$25.00 ©2009 Crown
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

Based on the N received samples, the true per-antenna SNRs that we wish to estimate are given by :

$$\rho_i = \frac{S_i^2}{2\sigma^2},\ i = 1, 2, \ldots, N_a. \quad (2)$$

From (2), we see that there are $N_a + 1$ parameters which are involved in the derivation of the ML per-antenna SNR estimators and CRLB, $\{S_i\}_{i=1,2,\ldots,N_a}$ and σ^2.

The received signals, on all the antenna elements, at a given instant n can be written in the following vectorial form :

$$\boldsymbol{y}(n) = [y_1(n), y_2(n), \ldots, y_{Na}(n)]^T,\ n = 1, 2, \ldots, N, \quad (3)$$

where $[.]^T$ stands for the transpose operator. Then, considering the entire observation window, the received signal can be written in the following matrix form :

$$\boldsymbol{Y} = [\boldsymbol{y}(1), \boldsymbol{y}(2), \ldots, \boldsymbol{y}(N)]. \quad (4)$$

Assuming iid received samples, it can be shown that the probability density function (pdf) of the received vector $\boldsymbol{y}(n)$ parametrized by $\boldsymbol{\theta}$ is given by :

$$Pr[\boldsymbol{y}(n); \boldsymbol{\theta}] = \sum_{k=1}^{M} P[\boldsymbol{y}(n)|a_k \in \tilde{C}; \boldsymbol{\theta}]\ P[a_k], \quad (5)$$

where $\tilde{C}$ is the alphabet of any constellation and $\boldsymbol{\theta} = [S_1, S_2, \ldots, S_{N_a}, \sigma^2]^T$ is the vector of the unknown parameters involved in the different per-antenna SNRs that we wish to estimate. Then, using (1), one can write :

$$\Pr[\boldsymbol{y}(n)|a_k \in \tilde{C}; \boldsymbol{\theta}] = \prod_{i=1}^{Na} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|y_i(n) - S_i e^{j\phi_i} a_k|^2}{2\sigma^2}\right), \quad (6)$$

where $|.|$ stands for the norm of any complex number.

III. ITERATIVE EXPECTATION MAXIMIZATION (EM) ALGORITHM FOR PER-ANTENNA SNR ESTIMATION IN SIMO SYSTEMS

In the following, an SNR estimator based on the expectation-maximization (EM) algorithm [7] is developed with a significantly lower computational load than other numerical or analytical ML estimators. Conditioned on an observation interval of N independent samples $\boldsymbol{y}(n)$ and an estimate $\boldsymbol{\theta}^{(p-1)}$ of the parameter vector $\boldsymbol{\theta}$ (computed in the step $p-1$ of the iterative procedure) the EM algorithm is derived in the sequel.

In fact, it is known that a_k can take one of the M different values and when they are assumed iid, we have :

$$Pr[a_k \in \tilde{C}] = \frac{1}{M}, \quad (7)$$

where M is the modulation order. From (6), we can write :

$$Pr[\boldsymbol{y}(n)|a_k \in \tilde{C}; \boldsymbol{\theta}] = \prod_{i=1}^{Na} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|y_i(n)|^2 + S_i^2|a_k|^2 - 2S_i\Re\{y_i(n)^* e^{j\phi_i} a_k\}}{2\sigma^2}\right), \quad (8)$$

where $(.)^*$ indicates complex conjugation and $\Re\{.\}$ stands for the real part of any complex number. The expectation step (E-step) of the suggested EM algorithm is established as follows :

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(p-1)}) = \sum_{n=1}^{N} E_a\left\{\ln\left(Pr[\boldsymbol{y}(n)|a_k \in \tilde{C}]\right)|\boldsymbol{\theta}^{(p-1)}, y(n)\right\}, \quad (9)$$

where $E_a\{.\}$ denotes expectation with respect to $a \in \tilde{C}$. From (8), the log-likelihood function is given by :

$$\ln(Pr[\boldsymbol{y}(n)|a_k \in \tilde{C}; \boldsymbol{\theta}]) = -N_a \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N_a}(|y_i(n)|^2 + S_i^2|a_k|^2 - 2S_i\Re\{y_i(n)^* e^{j\phi_i} a_k\}). \quad (10)$$

The E-step is simply obtained as :

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(p-1)}) = -N_a N \ln(2\pi\sigma^2) - \frac{N}{2\sigma^2}\sum_{i=1}^{N_a}(M_2^i + S_i^2 A^{(p)} - 2S_i B_i^{(p)}) \quad (11)$$

where M_2^i is the second moment of the received signal on the i^{th} antenna element, $A^{(p-1)}$ and $B_i^{(p-1)}$ are defined as follows :

$$A^{(p)} = \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{M}|a_k|^2 P_{k,n}^{(p)}, \quad (12)$$

$$B_i^{(p)} = \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{M}\Re\{y_i^*(n) e^{j\phi_i} a_k\} P_{k,n}^{(p)}, \quad (13)$$

with :

$$P_{k,n}^{(p)} = Pr[a_k|\boldsymbol{y}(n); \boldsymbol{\theta}^{(p-1)}], = \frac{Pr[\boldsymbol{y}(n)|a_k; \boldsymbol{\theta}^{(p-1)}] Pr[a_k]}{Pr[\boldsymbol{y}(n)|\boldsymbol{\theta}^{(p-1)}]}. \quad (14)$$

Since the denominator and $Pr[a_k]$ do not depend on the index k, it can be replaced by a normalization factor v_n. Therefore, we can calculate this probability using $\sum_{k=1}^{M} P_{k,n}^{(p)} = 1$ and considering only $Pr[y(n)|a_k \in \tilde{C}; \boldsymbol{\theta}^{(p-1)}]$. The maximization step (M-step) is used to update the estimates. Since the $N_a + 1$ parameters, $\{S_i\}_{i=1,2,\ldots,N_a}$ and σ^2, are not interrelated, the optimization equations follow :

$$\left[\frac{\partial Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(p-1)})}{\partial S_i}\right]|_{S_i = S_i^{(p)}} = 0, \quad (15)$$

$$\left[\frac{\partial Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(p-1)})}{\partial \sigma^2}\right]|_{\sigma^2 = \sigma^{2(p)}} = 0. \quad (16)$$

setting the partial derivatives of (15) and (16) to zero and resolving them to $\{S_i\}_{i=1,2,\ldots,N_a}$ and σ^2, the p^{th} estimate parameter $\boldsymbol{\theta}^{(p)}$ appears as :

$$\widehat{S_i}^{(p)} = \frac{B_i^{(p)}}{A^{(p)}}, \quad (17)$$

$$\widehat{\sigma^2}^{(p)} = \frac{1}{2N_a}\sum_{i=1}^{N_a}\left(M_2^i - \frac{B_i^{(p)^2}}{A^{(p)}}\right). \quad (18)$$

The initial entries to our algorithm are $P_{i,k}^{(0)} = \frac{1}{M}$. With this initialization, it is easily verified, that in the majority of cases the value of $B_i^{(0)} \to 0$, which makes the EM approach fail. To circumvent this problem, we calculate $S_i^{(0)}{}_{i=1,2,\ldots N_a}$ and $\sigma^{2^{(0)}}$ in the initialization step differently from the ones calculated in the iteration steps. We added a norm operator to $\Re\{y_i^*(n)e^{j\phi_i}a_k\}$ so that $B_i^{(0)}$ does not approach zero. Using this initialization, we can ensure that the N_a+1 parameter estimates go near their true values.

The algorithm presented in this paper can now be summarized as follows :
Initialization :

$$\begin{aligned} P_{k,n}^{(0)} &= \frac{1}{M}, \\ \widehat{S_i}^{(0)} &= \frac{\sum_{n=1}^{N}\sum_{k=1}^{M}|\Re\{y_i^*(n)e^{j\phi_i}a_k\}|P_{k,n}^{(0)}}{\sum_{n=1}^{N}\sum_{k=1}^{M}|a_k|^2 P_{k,n}^{(0)}}, \\ \widehat{\sigma^2}^{(0)} &= \frac{1}{2N_a}\sum_{i=1}^{N_a}(M_2^i - \\ &\quad \frac{\left(\sum_{n=1}^{N}\sum_{k=1}^{M}|\Re(y_i^*(n)e^{j\phi_i}a_k)|P_{k,n}^{(0)}\right)^2}{\sum_{n=1}^{N}\sum_{k=1}^{M}|a_k|^2 P_{k,n}^{(0)}}). \end{aligned} \quad (19)$$

Iteration : $p = 1,2,\ldots P$

$$\begin{aligned} P_{k,n}^{(p)} &= \frac{Pr[\boldsymbol{y}(n)|a_k \in \tilde{C};\boldsymbol{\theta}^{(p-1)}]}{\sum_{k=1}^{M} Pr[\boldsymbol{y}(n)|a_k \in \tilde{C}]} \\ A^{(p)} &= \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{M}|a_k|^2 P_{k,n}^{(p)}, \end{aligned}$$

for $i = 1,2,\ldots \quad N_a$

$$\begin{aligned} B_i^{(p)} &= \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{M}\Re\{y_i^*(n)e^{j\phi_i}a_k\}P_{k,n}^{(p)}, \\ \widehat{S_i}^{(p)} &= \frac{B_i^{(p)}}{A^{(p)}}, \end{aligned}$$

end

$$\widehat{\sigma^2}^{(p)} = \frac{1}{2N_a}\sum_{i=1}^{N_a}\left(M_2^i - \frac{(B_i^{(p)})^2}{A^{(p)}}\right), \quad (20)$$

Results :

$$\hat{\rho}_i = \frac{\widehat{S_i}^2}{2\widehat{\sigma^2}}, \; i = 1,2,\ldots N_a. \quad (21)$$

IV. Derivation of the SIMO SNR CRLBs in the DA case

In this section, based on the assumptions made so far, we will derive the Cramér-Rao bounds for the SNR estimates in the DA case (where all the transmitted symbols are assumed to be perfectly known to the receiver) of any linearly-modulated signal when it is AWGN-corrupted. To do so, we consider the following parameter vector :

$$\begin{aligned} \boldsymbol{\alpha} &= [\alpha_1, \alpha_2, \ldots, \alpha_{N_a+1}]^T, \\ &= [\rho_1, \rho_2, \ldots, \rho_{N_a}, \sigma^2]^T. \end{aligned} \quad (22)$$

The CRLB of the i^{th} element in $\boldsymbol{\alpha}$ is provided by :

$$\text{CRLB}_{\text{DA}}(\alpha_i) = [\boldsymbol{I}^{-1}(\boldsymbol{\alpha})]_{i,i}, \quad (23)$$

where $\boldsymbol{I}$ denotes the Fisher information matrix (FIM) given as :

$$[\boldsymbol{I}(\boldsymbol{\alpha})]_{ij} = -\,\mathrm{E}\left\{\frac{\partial^2 \ln P[\boldsymbol{Y};\boldsymbol{\alpha}]}{\partial\alpha_i\partial\alpha_j}\right\}. \quad (24)$$

We can easily verify that $\mathrm{E}\left\{\frac{\partial^2 \ln P[\boldsymbol{Y};\boldsymbol{\theta}]}{\partial\rho_i\partial\rho_j}\right\} = 0$, for $i \neq j$. For ease of notation, from now on we will use the following ones :

$$c = -E\left\{\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial{\sigma^2}^2}\right\}, \quad (25)$$

$$a_i = -E\left\{\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial\rho_i^2}\right\}, \quad 1 \le i \le N_a+1 \quad (26)$$

$$b_i = -E\left\{\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial\rho_i\partial\sigma^2}\right\}, \quad 1 \le i \le N_a+1. \quad (27)$$

It should be noted that c, a_i and b_i are the elements of the FIM, in the DA scenario, that would be obtained if we were to receive, on the i^{th} antenna elements, the corresponding N samples $\{y_i(n)\}_{n=1,2,\cdots,N}$. Using c, a_i and b_i, the FIM can be written as :

$$\boldsymbol{I}(\boldsymbol{\alpha}) = \begin{pmatrix} a_1 & 0 & 0 & \cdots & 0 & b_1 \\ 0 & a_2 & 0 & \cdots & 0 & b_2 \\ 0 & 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 0 & a_{N_a} & b_{N_a} \\ b_1 & b_2 & \cdots & \cdots & b_{N_a} & c \end{pmatrix}, \quad (28)$$

where $\boldsymbol{I}(\boldsymbol{\alpha})$ is $((N_a+1)\times(N_a+1))$ matrix. However, we need to invert the matrix $\boldsymbol{I}(\boldsymbol{\alpha})$ before being able to find the CRLBs for the DA SNR estimates. In fact, resolving for X the linear equations system $\boldsymbol{I}(\boldsymbol{\alpha})X = B$, where X and B are any $((N_a+1)\times 1)$ column vectors, we establish the inverse $\boldsymbol{I}^{-1}(\boldsymbol{\alpha})$ of $\boldsymbol{I}(\boldsymbol{\alpha})$ as given by (29) at the top of the next page.

Injecting the expression of c, a_i and b_i, defined in (25)-(27), in the i^{th} diagonal element of $\boldsymbol{I}^{-1}(\boldsymbol{\alpha})$, we get the CRLB

978-1-4244-4148-8/09/$25.00 ©2009 Crown
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

$$\boldsymbol{I}^{-1}=\frac{1}{c-\sum\limits_{l=1}^{N_a}\frac{b_l^2}{a_l}}\begin{pmatrix} \frac{c}{a_1}-\sum\limits_{\substack{l=1\\l\neq 1}}^{N_a}\frac{b_l^2}{a_1 a_l} & \frac{b_1 b_2}{a_1 a_2} & \cdots & \frac{b_1 b_{N_a}}{a_1 a_{N_a}} & -\frac{b_1}{a_1} \\ \frac{b_2 b_1}{a_2 a_1} & \frac{c}{a_2}-\sum\limits_{\substack{l=1\\l\neq 2}}^{N_a}\frac{b_l^2}{a_2 a_l} & \ddots & \vdots & -\frac{b_2}{a_2} \\ \vdots & \ddots & \ddots & \frac{b_{N_a-1} b_{N_a}}{a_{N_a-1} a_{N_a}} & \vdots \\ \frac{b_{N_a} b_1}{a_{N_a} a_1} & \cdots & \frac{b_{N_a} b_{N_a-1}}{a_{N_a} a_{N_a-1}} & \frac{c}{a_{N_a}}-\sum\limits_{\substack{l=1\\l\neq N_a}}^{N_a}\frac{b_l^2}{a_{N_a} a_{N_a}} & -\frac{b_{N_a}}{a_{N_a}} \\ -\frac{b_1}{a_1} & -\frac{b_2}{a_2} & \cdots & -\frac{b_{N_a}}{a_{N_a}} & 1 \end{pmatrix}. \quad (29)$$

for the DA SNR estimates over the i^{th} antenna element as follows :

$$\mathrm{CRLB}_{\mathrm{DA}}(\rho_i)=\frac{\left(\frac{E\left\{\frac{\partial^2 A}{\partial \sigma^{2^2}}\right\}}{E\left\{\frac{\partial^2 A}{\partial \rho_i^2}\right\}}-\sum_{k=1,k\neq i}^{N_a}\frac{\left(E\left\{\frac{\partial^2 A}{\partial \rho_k \partial \sigma^2}\right\}\right)^2}{E\left\{\frac{\partial^2 A)}{\partial \rho_i^2}\right\}E\left\{\frac{\partial^2 A}{\partial \rho_k^2}\right\}}\right)}{-E\left\{\frac{\partial^2 A}{\partial \sigma^{2^2}}\right\}+\sum_{k=1}^{N_a}\frac{\left(E\left\{\frac{\partial^2 A}{\partial \rho_k \partial \sigma^2}\right\}\right)^2}{E\left\{\frac{\partial^2 A}{\partial \rho_k^2}\right\}}} \quad (30)$$

where A is the log-likelihood function straightforwardly given by :

$$\begin{aligned} A &= \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)=\sum_{n=1}^{N}\sum_{i=1}^{N_a}-\ln(2\pi\sigma^2) \\ &- \frac{|\boldsymbol{y}_i(n)|^2}{2\sigma^2}-\rho_i|a_n|^2+2\sqrt{\frac{\rho_i}{2\sigma^2}}\Re\{y_i(n)^* a_n\}. \quad (31)\end{aligned}$$

The required partial derivatives of (31) with respect to different parameters are :

$$\begin{aligned} \frac{\partial^2 A}{\partial \sigma^{2^2}} &= \frac{NN_a}{\sigma^4}-\sum_{i=1}^{N_a}\sum_{n=1}^{N}\frac{|y_i(n)|^2}{\sigma^6} \\ &+ \sum_{i=1}^{N_a}\sum_{n=1}^{N}\sqrt{\frac{\rho_i}{2}}\frac{3}{2\sigma^5}C_i(n), \quad (32) \\ \frac{\partial^2 A}{\partial {\rho_i}^2} &= -\sum_{n=1}^{N}\frac{C_i(n)}{2\sqrt{2\sigma^2\rho_i^3}}, \quad (33) \\ \frac{\partial^2 A}{\partial \rho_i \partial \sigma^2} &= -\sum_{n=1}^{N}\frac{1}{2\sqrt{2\rho_i}\sigma^3}C_i(n), \quad (34)\end{aligned}$$

where $C_i(n)=\Re\{y_i(n)^* a_n\},\ i=1,2,\ldots,N_a$. The corresponding expected values of the partial derivatives are :

$$\begin{aligned} E\left\{\frac{\partial^2 A}{\partial \sigma^{2^2}}\right\} &= -\sum_{i=1}^{N_a}\frac{N(2+\rho_i)}{2\sigma^4}, \quad (35) \\ E\left\{\frac{\partial^2 A}{\partial {\rho_i}^2}\right\} &= -\frac{N}{2\rho_i},\ i=1,2,\ldots,N_a, \quad (36) \\ E\left\{\frac{\partial^2 A}{\partial \rho_i \partial \sigma^2}\right\} &= -\frac{N}{2\sigma^2},\ i=1,2,\ldots,N_a. \quad (37) \\ & \quad (38)\end{aligned}$$

Finally, injecting (35) - (37) in (30), we obtain :

$$\mathrm{CRLB}_{\mathrm{DA}}(\rho_i)=\frac{\rho_i^2+2N_a\rho_i}{N_a N}. \quad (39)$$

V. SIMULATION RESULTS

In this section, we will assess the performance of our ML SNR estimator. Monte-Carlo simulations will be run over 5000 realizations. The normalized mean square error (NMSE), defined in (40), will be used as a performance measure for the different estimators :

$$\mathrm{NMSE}(\rho_i)=\frac{E\{(\hat{\rho}_i-\rho_i)^2\}}{\rho_i^2}. \quad (40)$$

The NMSE can be compared to the normalized DA CRLB ($\mathrm{NCRLB_{DA}}$) defined as follows :

$$\mathrm{NCRLB}_{\mathrm{DA}}(\rho_i)=\frac{\mathrm{CRLB}_{\mathrm{DA}}(\rho_i)}{\rho_i^2}. \quad (41)$$

In the following, the EM-based NDA SNR estimator will be evaluated for two constellations, QPSK and 16-QAM, to serve as representative examples of both the constant-envelope and non-constant modulus constellations, respectively.

To begin with, in Fig. 1, we focus on the impact of varying the observation window length N on the performance of our proposed EM-based ML SNR estimator. It can be seen that it provides sufficiently accurate SNR estimates even, for low SNR values (2 dB), and even if few received samples are used in the estimation process. On the other hand, despite the fact that increasing N has a substantial impact on the estimation accuracy, the relative performance improvement with respect to N, for our estimator for the different considered antenna-array sizes and modulation orders, is the same. Consequently, from now on, any conclusion on the relative performance of the estimator will be assumed to hold regardless of the observation interval length (this was validated empirically over multiple simulations).

Fig. 2 shows the NMSE of our SIMO ML estimator measured on the first antenna branch for $16-$QAM constellation. It should be mentioned that for $N_a=1$ our SIMO per-antenna estimator reduces simply to the SISO ML estimator

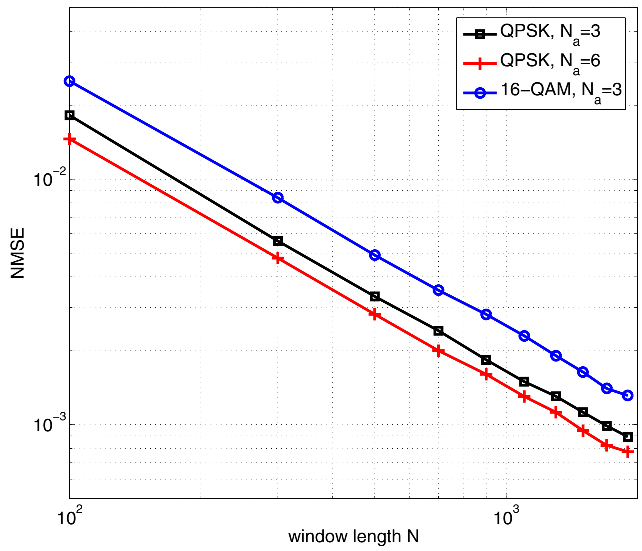

Fig. 1. NMSE of the SNR estimates for different observation window lengths, SNR = 2 dB.

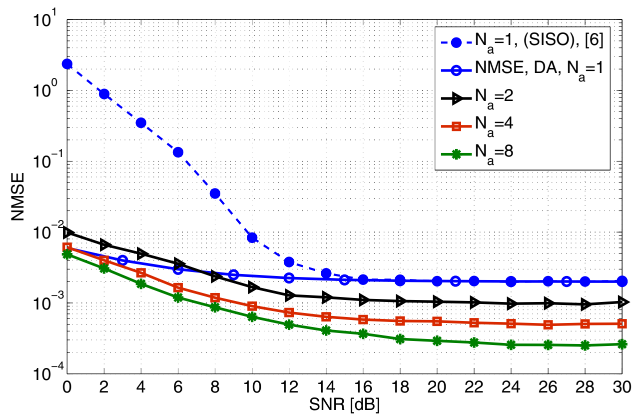

Fig. 2. NMSE of the SNR estimates for different numbers of antennas, $N = 512$, 16−QAM.

recently developed in [6]. It is clearly seen that the use of a receiving antenna array improves the achievable performance of the ML SNR estimator especially in the low SNR region. Furthermore, the estimation accuracy can also be substantially improved for high SNR values by increasing the number of receiving antenna elements N_a, due to the fact that more information is available at the receiver. This observation can be better illustrated in Fig. 3.

In fact, Fig. 3 depicts the NMSE of our SIMO NDA SNR estimator for 16−QAM constellation versus the NCRLBs in the DA case for different antenna-array sizes (i.e, $N_a = 2, 4, 8$). It can be noticed that, at high SNR values, the performance of our NDA SNR estimator is close to the NCRLB_{DA}. This means that, in this SNR region, our new NDA SNR estimator exhibits performances equivalent to those that could be achieved if the transmitted data were perfectly known at the

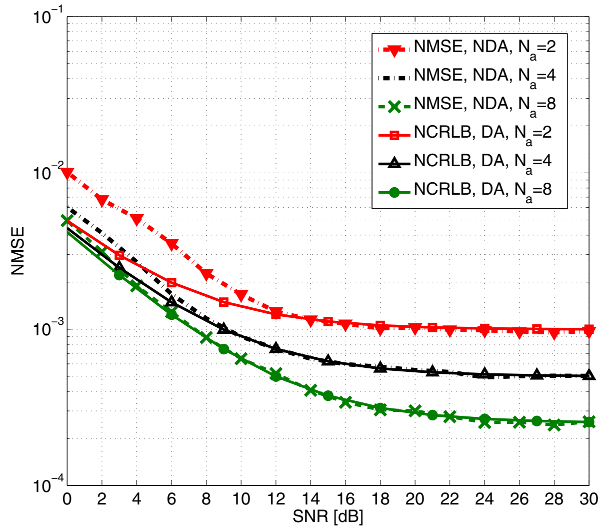

Fig. 3. Comparison between the NCRLB and the NMSE for 16−QAM constellation, $N = 512$.

receiver. Moreover, for low SNR values, our NDA estimator approaches the NCRLB_{DA} as the number of antenna elements increases. This is due to the optimal exploitation of the increasing mutual information across the antenna branches.

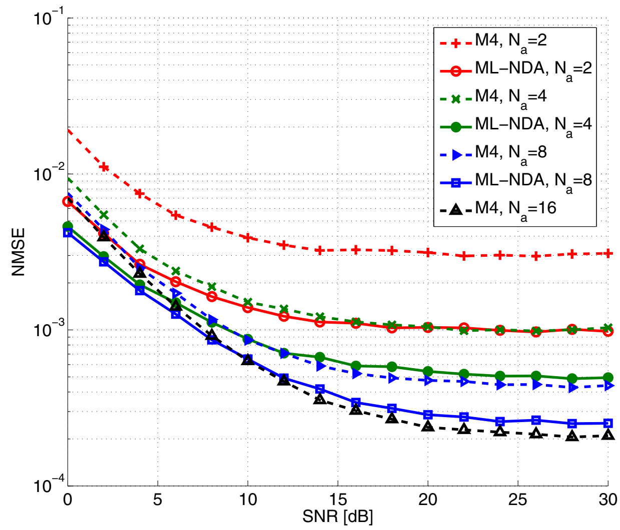

Fig. 4. NMSE of the SNR estimates for different numbers of antennas, $N = 512$, QPSK.

Fig. 4 shows the NMSE of our EM-based ML estimator and the recently derived moment-based M4 estimator [8], measured on the first antenna branch for QPSK constellation and different antenna-array sizes (i.e, $N_a = 2, 4, 8$). We also plot the NMSE of the ML estimator derived for SISO channels [6].

We can see that, for a given N_a, our EM-based ML estimator outperforms the M4 estimator [8] over the entire SNR region. This is hardly surprising, since I/Q-based approaches yield in general a better performance than envelope-based approaches because the former do not discard the phase information.

Fig. 5 illustrates another scenario in which we simulate a SIMO model with 4 antenna branches. We assume that we have the same noise power $2\sigma^2$ on the first two antenna elements, but different noise powers on the third and fourth antenna branches, $2\sigma_3^2$ and $2\sigma_4^2$, respectively.

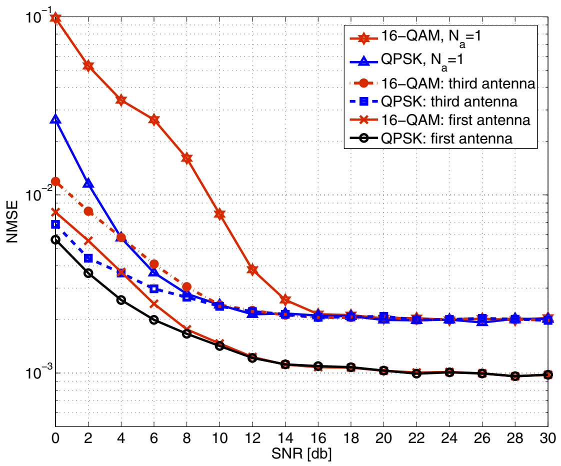

Fig. 5. NMSE of the SNR estimates for the first and third antenna, $N = 512$, QPSK, $16-$QAM.

We can see that the performance of the estimator on the first antenna element is better than the one that could be achieved on the third antenna element. This is basically due to the optimal exploitation of the mutual information (the first and second antenna elements have the same noise power). In addition, we can notice that the NMSE of the third antenna element is lower than the NMSE of the SISO ML estimator, presented in Figs. 4 and 2, especially in the low SNR region. This improvement can be also explained by the use of the mutual information between all the antenna branches.

VI. Conclusion

In this paper, a new EM-based ML SNR estimator for the SIMO channels and for any arbitrary modulation was presented. Our EM iterative algorithm assumes that the noise components on all the antenna elements can be adequately modeled by complex Gaussian variables, of equal average power, which are temporally and spatially white (i.e, uncorrelated across antenna elements). The proposed new SIMO SNR estimator is shown to exhibit remarkable performance improvements compared to the SISO ML SNR estimator. This is due to the optimal exploitation of the mutual information between the antenna elements. The Cramér-Rao lower bounds for per-antenna SNR estimates of linearly-modulated signals, in SIMO configurations, were also derived in closed-form expression in the DA case. We saw that, for relatively high SNR values, our new ML SNR estimator exhibits performances equivalent to those that could be achieved if the transmitted data were perfectly known. To the best of our knowledge, we are the first to derive the ML per-antenna SNR estimators as well as the CRLBs in the NDA and the DA case, respectively, both over SIMO channels.

References

[1] A. Ramesh, A. Chockaligam, and L. B. Milstein, "SNR estimation in generalized fading channels and its application to turbo decoding," *IEEE International Conference on Communications*, ICC'01, vol. 4, pp. 1094-1098, 2001.

[2] A. Das and M. Miller, "Remote non-linearity detection via burst power dithering and EM based SNR estimation," *International Workshop on Satellite and Space Communications*, IWSSC, Sept. 2007.

[3] F. Bellili, A. Stéphenne and S. Affes, "SNR estimation of QAM-modulated transmissions over time-varying SIMO channels," *IEEE International Symposium on Wireless Communication Systems. ISWCS'08.* pp. 199-203, 2008.

[4] R. Lopez-Valcarce, C. Mosquera, and W. Gappmair, "Iterative envelope-based SNR estimation for nonconstant modulus constellations,"in Proc. *IEEE SPAWC'07*, pp. 1-5, June 2007.

[5] A. Das (Nandan), "NDA SNR estimation : CRLBs and EM based estimators," TENCON 2008. *IEEE Region 10 Conference*, Nov. 2008.

[6] W. Gappmair, R. López-Valcarce and C. Mosquera, "ML and EM algorithm for non-data-aided SNR estimation of linearly modulated signals," *6th International Symposium on Communication Systems, Networks and Digital Signal Processing, CNSDSP'08.* pp. 530-534, July 2008.

[7] T. K. Moon, "The expectation-maximization algorithm," *IEEE Signal Processing* Mag. vol. 13, pp. 47−60, Nov. 1996.

[8] A. Stéphenne, F. Bellili and S. Affes, "Moment-based SNR estimation for SIMO wireless communication systems using arbitrary QAM," *Proc. of 41st Asilomar Conference on Signals, Systems and Computers,* pp. 601-605, Nov. 2007.

978-1-4244-4148-8/09/$25.00 ©2009 Crown
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.