ML Estimator Based on the EM Algorithm for Subcarrier SNR Estimation in Multicarrier Transmissions

Jean-Guy Descure[1,2], Faouzi Bellili[1], and Sofiène Affes[1]
[1] INRS-EMT, 800, de la Gauchetière Ouest, Bureau 6900, Montreal, Qc, H5A 1K6, Canada
[2] ENST PARIS, 46, rue Barrault, 75013, Paris, France
Emails: descure@emt.inrs.ca, bellili@emt.inrs.ca, affes@emt.inrs.ca

***Abstract*—In this paper, considering multicarrier transmissions, we present a maximum likelihood estimator of the subcarrier signal-to-noise ratio (SNR) based on the expectation-maximization (EM) algorithm. This new estimator is applicable to any linearly-modulated signal. It is a non-data-aided (NDA) method since no *a priori* knowledge is assumed about the transmitted data. The channel gains and phase distortions on the different subcarriers are assumed to be constant during the observation window, and the signal is assumed to be corrupted by additive white Gaussian noise (AWGN). The performances of our estimator are empirically assessed using Monte-Carlo simulations, showing that the new algorithm reaches the corresponding Cramér-Rao lower bounds (CRLBs) over a wide SNR range.**

I. Introduction

Modern communication systems often require accurate knowledge of the SNR which is a key parameter that should be estimated in many applications, such as adaptive modulation [1] or bit allocation strategies [2].

In multi-carrier transmissions, such as orthogonal frequency division multiplexing (OFDM), the majority of the previously introduced algorithms estimate the average overall the subcarriers [3]. However, estimating the per-carrier SNR is also required. For instance, the knowledge of the subcarrier SNR allows the system to adapt its modulation tone by tone, instead of using the same modulation scheme on all the subcarriers, which would be then limited by the ones with the poorest channel conditions.

Roughly speaking, SNR estimators are mainly categorized in two major categories: data-aided (DA) and non-data-aided (NDA) estimators. In contrast to DA SNR estimators, which assume perfect or partial *a priori* knowledge of the transmitted symbols, NDA techniques base the estimation process on the received samples only. DA SNR estimators are easier to derive, but they have the major drawback of limiting the whole throughput of the system due to the transmission of known symbols.

Furthermore, SNR estimators which are based on the magnitude of the received samples only are called envelope-based SNR estimators. However, these estimators do not use the whole information carried by the received samples. In fact, when using the inphase and quadrature (I/Q) components, more accurate SNR estimators can be derived and they are referred to as I/Q-based estimators.

Work supported by a Canada Research Chair in Wireless Communications, the Cooperative Research and Development program of NSERC, and Ericsson Canada.

In parameter estimation, the analytical NDA maximum-likelihood (ML) estimator is usually recognized to be mathematically not tractable or very tedious to derive. However, the NDA ML estimates can be numerically computed, even if no closed-form solution is available. In this paper, we derive the NDA ML I/Q-based subcarrier SNR estimator by applying the EM algorithm, which has already been used for the estimation of many channel parameters [4], [5].

To the best of our knowledge, the presented estimator is the first NDA per-carrier SNR estimator in multicarrier transmissions that exploits cross information about the noise power between the different subcarriers. Instead of calculating an overall SNR, as it is usually done [6], our estimator calculates the SNR on each subcarrier. The performances of the proposed estimator are compared to the one presented in [7], which is a recent DA SNR estimator that achieves good performances particularly for SNR values beyond 10 dB and can be used carrier per carrier especially in OFDM systems. It will be shown by computer simulations that our ML per-carrier SNR estimator, even though NDA, outperforms this estimator over the entire SNR range.

The rest of the paper is organized as follows. The system model is exposed in section II. In section III, the EM algorithm for the subcarrier SNR estimation in multicarrier transmissions is derived. The simulation results are presented in section IV and some concluding remarks are finally drawn out in section V.

II. System Model

We consider a traditional digital communication system broadcasting and receiving any multicarrier signal. The channel gains coefficients and phases $\{S_k\}_{k=1,2,\ldots,K}$ and $\{\phi_k\}_{k=1,2,\ldots,K}$, respectively, are supposed to remain constant over the observation window, for all the K subcarriers. We assume also that we receive on the k^{th} subcarrier, an AWGN-corrupted signal with noise power $2\sigma_k^2$. Assuming perfect synchronization, the received signal at the input of the matched filter can be modeled as follows:

$$y_k(n) = S_k e^{j\phi_k} a_k(n) + w_k(n), \quad k = 1, 2, ...K, \quad n = 1, 2, ...N \tag{1}$$

where, on the k^{th} tone and at time index n, $a_k(n)$ is the transmitted symbol and $y_k(n)$ is the corresponding received sample. $\{w_k(n)\}_{k=1,2,...,K, n=1,2,...,N}$ are the additive white noise components, which are modelled by complex zero-mean Gaussian random variables with independent real and imaginary parts, each of variance σ_k^2. N is the number of the received samples during the transmission interval and j is the imaginary number that verifies $j^2 = -1$. We denote by $\mathcal{C} = \{c_1, c_2, ..., c_M\}$ the alphabet of size M from which the transmitted symbols are drawn, and the constellation energy is supposed to be normalized to one, i.e., $\mathrm{E}\{|a_k(n)|^2\} = 1$, for $k = 1, 2, ..., K$. Moreover, we assume that a subset of the subcarriers experience the same noise power $2\sigma^2$ (say, without loss of generality, the first L subcarriers, i.e., $\sigma_k^2 = \sigma^2$, $k = 1, 2, ..., L$).

Based on the NK received samples, the subcarrier true SNRs (in dB) that we wish to estimate are defined as:

$$\rho_k = \begin{cases} 10\log_{10}\left(\frac{S_k^2}{2\sigma^2}\right), & k = 1, 2, ...L, \\ 10\log_{10}\left(\frac{S_k^2}{2\sigma_k^2}\right), & k = L+1, L+2, ...K. \end{cases} \tag{2}$$

III. Subcarrier NDA ML SNR Estimator using the Expectation Maximization Algorithm

In the following, a NDA SNR estimator based on the EM algorithm is presented. Roughly speaking, the EM algorithm [8] can be described as follows. At the i^{th} iteration, it runs in two steps. The E-Step computes the log-likelihood function conditioned by the parameter estimated at the $(i-1)^{th}$ iteration. The M-Step finds the value of the parameter that maximizes the function computed at the E-Step :

$$\mathrm{E-Step}: Q\left(\mathbf{\Theta}^{(i)}, \mathbf{\Theta}^{(i-1)}\right) = E\left[\mathcal{L}\left(\mathbf{\Theta}^{(i)}|\mathbf{Z}\right)|\mathbf{\Theta}^{(i-1)}, \mathbf{Y}\right],$$

$$\mathrm{M-Step}: \mathbf{\Theta}^{(i)} = \arg\max Q\left(\mathbf{\Theta}^{(i)}, \mathbf{\Theta}^{(i-1)}\right),$$

with the following notations :

$$\mathbf{y}_k = [y_k(1), y_k(2), ..., y_k(N)]^T, \quad k = 1, 2, ..., K, \tag{3}$$

$$\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_K], \tag{4}$$

$$\mathbf{a}_k = [a_k(1), a_k(2), ..., a_k(N)]^T, \quad k = 1, 2, ..., K, \tag{5}$$

$$\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_K], \tag{6}$$

$$\mathbf{z}_k = [\mathbf{y}_k; \mathbf{a}_k], \quad k = 1, 2, ..., K, \tag{7}$$

$$\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_K], \tag{8}$$

$$\mathcal{L}\left(\mathbf{\Theta}^{(i)}|\mathbf{Z}\right) = \ln\left\{\Pr\left[\mathbf{Z}|\mathbf{\Theta}^{(i)}\right]\right\}. \tag{9}$$

Actually, the EM algorithm has been recently derived for SNR estimation, but in single-carrier transmissions [4], assuming perfect knowledge about the channel phase. Our estimator is more general and does not require any knowledge about the introduced phase distortions $\{\phi_k\}_{k=1,2,...,K}$. Therefore, the parameter vector to be estimated in the presence of unknown subcarriers phases is :

$$\mathbf{\Theta} = \left[S_1, S_2, ..., S_K, \sigma^2, \sigma_{L+1}^2, \sigma_{L+2}^2, ..., \sigma_K^2, \phi_1, \phi_2, ..., \phi_K\right]. \tag{10}$$

Using the notations adopted in [4], we can write the likelihood function on the k^{th} subcarrier as follows :

$$L\left(\mathbf{\Theta}_k|\mathbf{z}_k\right) = \prod_{n=1}^{N}\prod_{m=1}^{M}\{\Pr\left[a_k(n) = c_m\right] \times \Pr\left[y_k(n)|a_k(n) = c_m, \mathbf{\Theta}_k\right]\}^{x_{n,m}(k)}, \tag{11}$$

where $x_{n,m}(k)$ is the k^{th} subcarrier indicator that equals 1 if $a_k(n) = c_m$ and 0 otherwise for $k = 1, 2, ..., K$, $n = 1, 2, ..., N$ and $m = 1, 2, ..., M$. $\mathbf{\Theta}_k$ is the parameter vector to be estimated on the k^{th} subcarrier: $\mathbf{\Theta}_k = \left[S_k, \sigma_k^2, \phi_k\right]$. Hence, the global log-likelihood function is given by :

$$\mathcal{L}\left(\mathbf{\Theta}|\mathbf{Z}\right) = \sum_{k=1}^{K}\mathcal{L}\left(\mathbf{\Theta}_k|\mathbf{z}_k\right). \tag{12}$$

It follows that :

$$Q\left(\mathbf{\Theta}^{(i)}, \mathbf{\Theta}^{(i-1)}\right) = \sum_{k=1}^{K} Q_k\left(\mathbf{\Theta}_k^{(i)}, \mathbf{\Theta}_k^{(i-1)}\right), \tag{13}$$

and from (11), we have :

$$Q_k(\mathbf{\Theta}_k^{(i)}, \mathbf{\Theta}_k^{(i-1)}) = \sum_{n=1}^{N}\sum_{m=1}^{M}\left\{x_{n,m}^{(i)}(k)\left(\ln(\sigma_k^2) + \frac{|y_k(n) - S_k e^{j\phi_k} c_m|^2}{2\sigma_k^2}\right)\right\}, \tag{14}$$

with :

$$x_{n,m}^{(i)}(k) = E\left[x_{n,m}(k)|y_k(n), \mathbf{\Theta}_k^{(i-1)}\right] = \frac{\exp\left\{\frac{-\left|y_k(n) - S_k^{(i-1)} e^{j\phi_k^{(i-1)}} c_m\right|^2}{2\sigma_k^{2(i-1)}}\right\}}{\sum_{p=1}^{M}\exp\left\{\frac{-\left|y_k(n) - S_k^{(i-1)} e^{j\phi_k^{(i-1)}} c_p\right|^2}{2\sigma_k^{2(i-1)}}\right\}}. \tag{15}$$

The components of $\mathbf{\Theta}$ are independent and the maximum of Q is then simply given by differentiating Q with respect to each component of $\mathbf{\Theta}$ and setting each partial derivative to zero :

$$\frac{\partial Q}{\partial\phi_k} = \sum_{n}^{N}\sum_{m}^{M}\frac{-jS_k x_{n,m}^{(i)}(k)}{2\sigma_k^2}\left[e^{j\phi_k} c_m y_k(n)^* - e^{-j\phi_k} c_m^* y_k(n)\right],$$

$$\frac{\partial Q}{\partial\phi_k} = 0 \Longrightarrow \phi_k^{(i)} = \frac{1}{2}\arg\left(\frac{\sum_n^N\sum_m^M x_{n,m}^{(i)}(k) y_k(n)}{\sum_n^N\sum_m^M x_{n,m}^{(i)}(k) y_k(n)^*}\right).$$

We finally obtain the following results. For $k = 1, 2, ..., K$, we have :

$$\phi_k^{(i)} = \arg\left(\sum_{n}^{N}\sum_{m}^{M} x_{n,m}^{(i)}(k) y_k(n)\right), \tag{16}$$

$$S_k^{(i)} = \frac{\sum_n^N\sum_m^M x_{n,m}^{(i)}(k)\left(c_m y_k(n)^* e^{j\phi_k^{(i)}} + c_m^* y_k(n) e^{-j\phi_k^{(i)}}\right)}{2\times\sum_n^N\sum_m^M x_{n,m}^{(i)}(k)|c_m|^2}, \tag{17}$$

and for $k = L+1, L+2, ...K$, we have :

$$\sigma_k^{2(i)} = \frac{\sum_n^N \sum_m^M x_{n,m}^{(i)}(k) \left| y_k(n) - S_k^{(i)} e^{j\phi_k^{(i)}} c_m \right|^2}{2N}, \quad (18)$$

$$\sigma^{2(i)} = \frac{\sum_{l=1}^L \sum_n^N \sum_m^M x_{n,m}^{(i)}(l) \left| y_l(n) - S_l^{(i)} e^{j\phi_l^{(i)}} c_m \right|^2}{2LN}. \quad (19)$$

Note that these results are applicable to any constellation and in any multicarrier transmission scheme provided that the subchannels are assumed to be independent, such as OFDM signaling. Moreover, the channel phases are assumed to be completely unknown, contrarily to the algorithm presented in [4] which is only applicable when there is no channel distortion phases or when they are perfectly recovered at the receiver. It should be mentioned also that our estimator does not provide very accurate phase estimates $\{\widehat{\phi}_k\}_{k=1,2,...,K}$. However, the ambiguity on the estimation of the phases is reduced and the SNR estimates $\{\widehat{\rho}_k\}_{k=1,2,...,K}$ are still sufficiently accurate. These points will be explained and discussed in the following section.

IV. Simulation Results

In this section, we include some graphical representations of the estimated mean square error (MSE) of the subcarrier SNR estimates on the first tone, given by :

$$MSE = E\left[(\rho_1 - \widehat{\rho}_1)^2\right], \quad (20)$$

which will serve as a representative case for all the first L subcarriers experiencing the same noise power σ^2. The MSE of the subcarrier SNR estimates on one of the $K-L$ remaining subcarriers are exactly those which are obtained with a traditional single-carrier system, and they are not included in this paper as they have been recently presented in [4], [9]. All the simulations were run over 2000 Monte-Carlo simulations, and with initial values of $\mathbf{\Theta}$: $S_k^{(0)} = 1, \sigma_k^{2(0)} = 1$ and $\phi_k^{(0)} = 2\pi$, for $k = 1, 2, ..., K$.

We examine in Figs 1 to 5, the performance behaviour of our SNR estimator on the first subcarrier, as being hypothetically estimated by a traditional single-carrier system.

Figs. 1 and 2 show the relevance of taking into account the channel phase in the parameter vector to be estimated. In fact, considering the distortion phase as an additional unknown parameter, our estimator provides indeed more accurate SNR estimates at low SNR values and therefore outperforms the ML NDA estimator presented by Das in [4].

We notice also as it has been pointed out in [4] that the estimates returned by the EM algorithm at low SNR values ($\rho_1 < 2$ dB) are sometimes very far from the true values. These inaccuracies are, in fact, related to the behavior of the log-likelihood function for these SNR values. This function is indeed flat around its maximum in this SNR region, and even a small deviation from the maximum of the log-likelihood function results in large deviations from the coordinates of this maximum. On the other hand, we notice that the returned SNR estimates are more accurate when the channel phase is

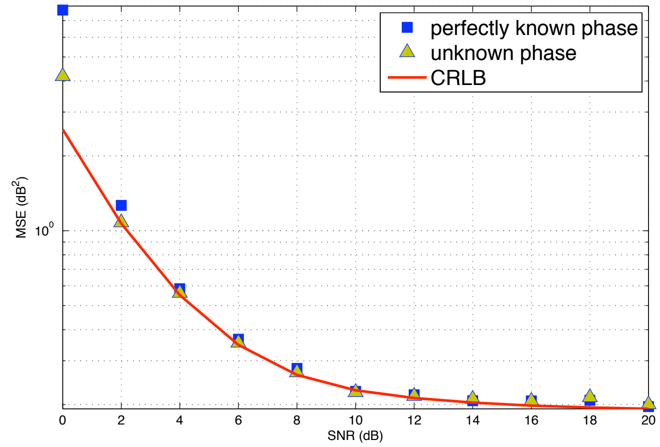

Fig. 1. MSE of the SNR estimates in single-carrier system, $N = 100$, QPSK.

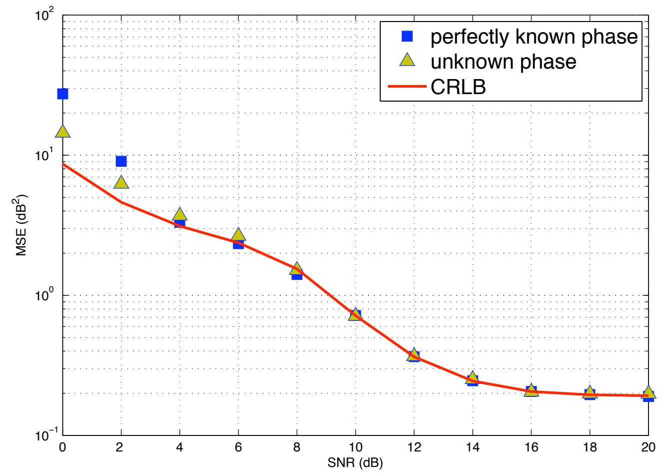

Fig. 2. MSE of the SNR estimate in single-carrier system, $N = 100$, 16-QAM.

assumed completely unknown to the receiver, compared to those obtained when the phase is assumed perfectly known [4].

This result appears to be aberrant at first sight since we have K additive nuisance parameters $\{\phi_k\}_{k=1,2,...,K}$ which should normally result in less accurate estimates of all the parameters. Besides, the obtained result, in the low SNR region, can also be explained by examining the behavior of the log-likelihood shape toward these new nuisance parameters. Indeed, Figs. 3, 4 and 5 depict the log-likelihood function for QPSK at SNR = 2dB, as a function of the estimates of S_1, σ^2 and ϕ_1, respectively projected on the three hyperplanes $\widehat{S_1} = 1$, $\widehat{\sigma}^2 = 0.315$ and $\widehat{\phi_1} = \pi/4$. First, we can see that there are exactly four maxima, located in each of the four quadrants ($[0, \pi/2], [\pi/2, \pi], [\pi, 3\pi/2]$ and $[3\pi/2, 2\pi]$), which could affect the SNR estimation, since the EM algorithm finds iteratively a local maximum, whose coordinates are the desired estimates. This explains why the estimate of ϕ_1 is not reliable, as it depends on its choosen initial value $\phi_1^{(0)}$. Actually, its estimate $\widehat{\phi_1}$ is admissible only if the initial value $\phi_1^{(0)}$ is in the same quadrant where the true value ϕ_1 lies, i.e., if $\phi_1 \in [0, \pi/2]$, $\widehat{\phi_1}$ is reliable if $\phi_1^{(0)} \in [0, \pi/2]$ and is not reliable if $\phi_1^{(0)} \in [\pi/2, \pi]$, $\phi_1^{(0)} \in [\pi, 3\pi/2]$ or $\phi_1^{(0)} \in [3\pi/2, 2\pi]$. Fortunatly, these four maxima have all the same coordinates, with respect to S_1 and σ_1^2, which ensures that the SNR estimates remain reliable even if the phase estimates are sometimes very far from the true values. Actually, assuming i.i.d. received samples, i.e., $\Pr(c_i) = \frac{1}{M}$, for $i = 1, 2, ..., M$, the likelihood function for each received sample $y_k(n)$, $k = 1, 2, ...K$, $n = 1, 2, ...N$, is given by :

$$\begin{aligned} P\left[y_k(n)|\boldsymbol{\Theta}_k\right] &= \sum_{c_i \in \mathcal{C}} \frac{\exp\left\{-\frac{\left|y_k(n)-S_k c_i e^{j\phi_k}\right|^2}{2\sigma^2}\right\}}{2\pi M \sigma^2}, \\ &= \sum_{\widetilde{c_i} \in \widetilde{\mathcal{C}}} \frac{\exp\left\{-\frac{\left|y_k(n)-S_k \widetilde{c_i} e^{j\Omega} e^{j\phi_k}\right|^2}{2\sigma^2}\right\}}{2\pi M \sigma^2}, \end{aligned} \tag{21}$$

where $\widetilde{c_i} = c_i e^{-j\Omega}$, for $i = 1, 2, ..., M$, and Ω being any angle that leaves the constellation globally invariant. Therefore, the new constellation alphabet $\widetilde{\mathcal{C}}$ is exactly the original constellation alphabet $\mathcal{C}$, i.e., $\widetilde{\mathcal{C}} = \mathcal{C}$. Then, one can write :

$$P[y_k(n)|\boldsymbol{\Theta}_k] = \sum_{c_i \in \mathcal{C}} \frac{\exp\left\{-\frac{\left|y_k(n)-S_k c_i e^{j(\phi_k+\Omega)}\right|^2}{2\sigma^2}\right\}}{2\pi M \sigma^2}. \tag{22}$$

The log-likelihood function is then periodic, its period being equal to the smallest value of Ω, which will be denoted by $\Omega_{\mathcal{C}}$ in the following. The ambiguity on the estimation of the phases is therefore reduced to a multiple of $\Omega_{\mathcal{C}}$, depending on the constellation alphabet $\mathcal{C}$, and does not affect the SNR estimation. For a QAM constellation, $\Omega_{\mathcal{C}} = \frac{\pi}{2}$, and for a M-PSK constellation, $\Omega_{\mathcal{C}} = \frac{2\pi}{M}$. For example, for a QPSK constellation, $\Omega_{\mathcal{C}} = \pi/2$, and the possible ML estimates $\widehat{\phi_1}$ of ϕ_1 are the following if $\phi_1 \in [0, \pi/2]$:

$$\widehat{\phi_1} = \phi_1 \quad , \text{ if } \phi_1^{(0)} \in [0, \pi/2], \tag{23}$$

$$\widehat{\phi_1} = \phi_1 + \frac{\pi}{2} \quad , \text{ if } \phi_1^{(0)} \in [\pi/2, \pi], \tag{24}$$

$$\widehat{\phi_1} = \phi_1 + \pi \quad , \text{ if } \phi_1^{(0)} \in [\pi, 3\pi/2], \tag{25}$$

$$\widehat{\phi_1} = \phi_1 + \frac{3\pi}{2} \quad , \text{ if } \phi_1^{(0)} \in [3\pi/2, 2\pi]. \tag{26}$$

Moreover, it can be seen that the slopes around the maxima of the log-likelihood function are flatter in Fig. 3 (which is exactly the shape of the log-likelihood function when the channel phase is assumed to be perfectly known) than in Figs. 4 and 5, where the phase is assumed to be completely unknown. Consequently, the error risk on the estimates at low SNR values is reduced by taking into account the channel phase as a coordinate in the search of a maximum of the log-likelihood function.

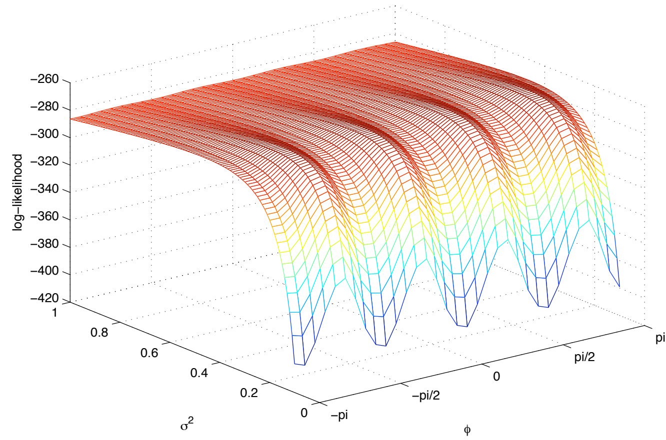

Fig. 3. Log-likelihood function in the hyperplane $\widehat{S_1} = 1$.

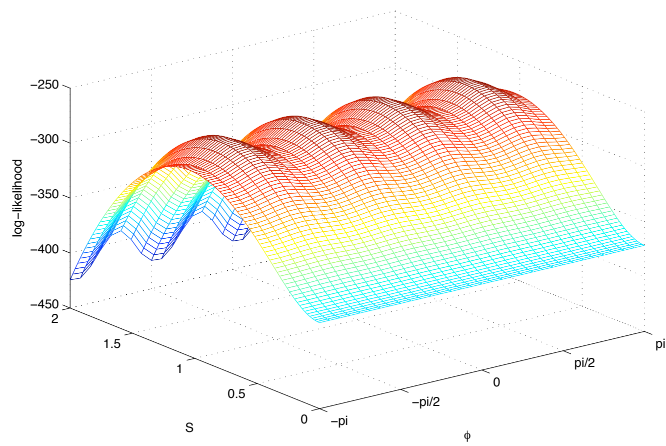

Fig. 4. Log-likelihood function in the hyperplane $\widehat{\sigma^2} = 0.315$.

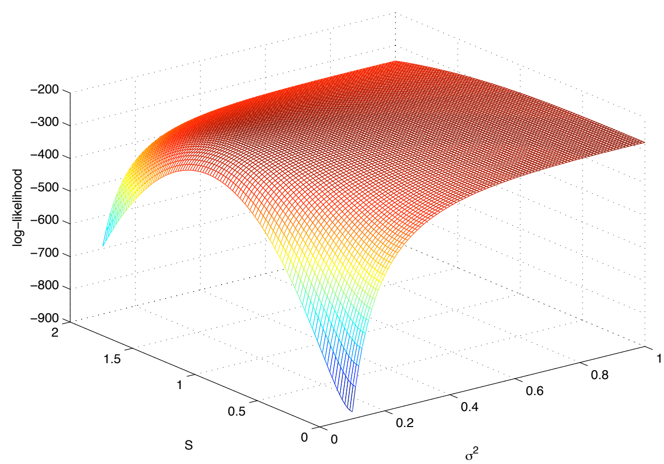

Fig. 5. Log-likelihood function in the hyperplane $\widehat{\phi_1} = \pi/4$.

In Fig. 6, the MSE of the SNR estimates in a multicarrier system are plotted for different numbers of subcarriers experiencing the same noise power $2\sigma^2$. We see that, as expected, the SNR estimates become more accurate as the number of the subcarriers experiencing the same noise power σ^2 increases, which can be explained by the use of the mutual information about σ^2. In this figure, the MSE of the DA phase-based SNR estimator presented in [7], which assumes perfect *a priori* knowledge of all the transmitted symbols, is also plotted for comparison purposes. It can be seen that our blind ML estimator clearly outperforms the phase-based estimator, despite the fact that the latter relies on the perfect knowledge of the transmitted symbols.

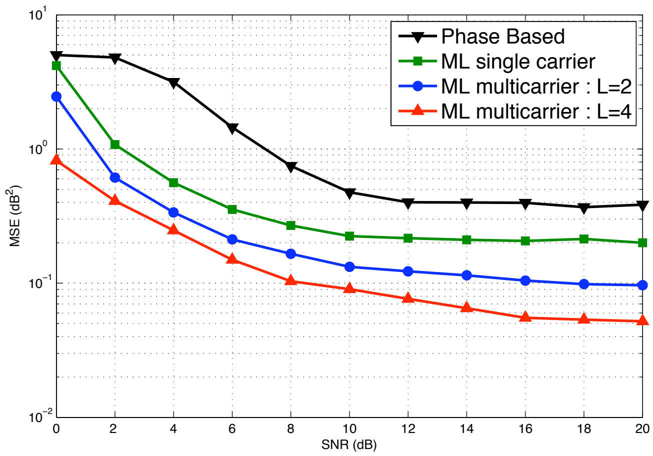

Fig. 6. Comparison of MSE for different values of L. $N = 100$, QPSK.

In Fig. 7, for QPSK signals and with 4 carriers experiencing the same noise power $2\sigma^2$, we compare the performance of our ML estimator with the corresponding CRLB, recently derived in [10]. It can be clearly seen that our estimator reaches the CRLB over the entire considered SNR region.

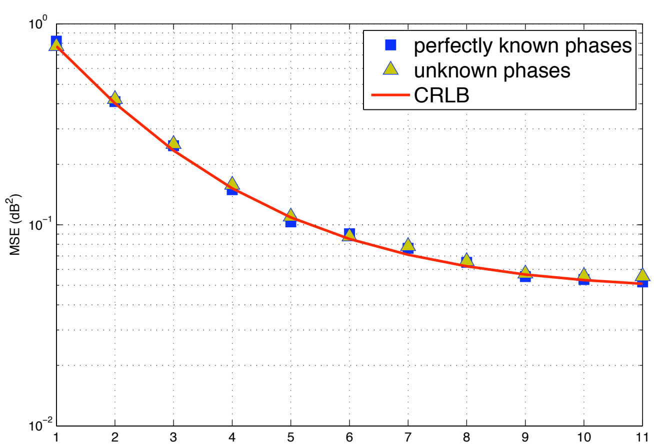

Fig. 7. MSE of the SNR estimates for the first subcarrier, $L = 4$. $\{\rho_{l+1} = \rho_k + 2\}_{l=1,2,\ldots,L-1}$, $N = 100$, QPSK.

V. Conclusion

In this paper, we presented a ML subcarrier SNR estimator based on the EM algorithm. This SNR estimator reaches the CRLB for a wide of SNR values. It is a NDA technique which does not require any *a priori* information about the transmitted symbols and therefore it does not reduce the throughput of the system. In the special case of a single-carrier system, this estimator, which assumes the channel phase to be completely unknown, outperforms at low SNR values, the existing ML estimators that assume the channel phase to be perfectly known at the receiver [4], [9]. For multicarrier systems, it also outperforms the recently introduced DA per-carrier SNR estimator [7]. The accuracy of this estimator also increases with the number of the subcarriers experiencing the same noise power, since the mutual information between the different subcarriers is fully exploited.

References

[1] S. Sampei and H. Harada, "System Design Issues and Performances Evaluations for Adaptive Modulation in New Wireless Access Systems", *Proceedings of the IEEE*, vol. 95, pp. 2456-2471, Dec. 2006.

[2] A. M. Wyglinski, F. Labeau, and P. Kabal, "An Efficient Bit Allocation Algorithm for Multicarrier Modulation", *IEEE Wireless Communications and Networking Conference WCNC'04*, vol. 2, pp. 1194-1199, Mar. 2004

[3] D. Athanasios and K. Grigorios, "BPSK SNR Estimation Algorithms for HiperLAN/2 Transceiver in AWGN Channel", *7th International Conference on Advanced Communication Technology*, vol. 1, pp. 196-200, 2005.

[4] A. Das (Nandan), "NDA SNR Estimation: CRLBs and EM Based Estimators", *IEEE TENCON*: Hyderabad, India, Nov. 2008.

[5] C. H. Aldana, A. A. Salvekar, J. Tellado, and J. Cioffi, "Accurate Noise Estimates in Multicarrier Systems", *IEEE VTS-Fall VTC*, vol. 1, pp. 434-438, 2000.

[6] F.-X. Socheleau, A. Aissa-El-Bey and S. Houcke, "Non Data-Aided SNR Estimation of OFDM Signals", *IEEE Communications Letters*, Nov. 2008.

[7] I. Trachanas and N. J. Fliege, "A Novel Phase Based SNR Estimation Method for Constant Modulus Constellations", *3rd International Symposium on Communications, Control and Signal Processing ISCCSP'08*, pp. 1179-1183, Mar. 2008.

[8] A. P. Dempster, N. M. Laird and D. B. Rubin, "Maximum Likelihood from Incomplete Data via the EM Algorithm", *Journal of the Royal Statistical Society. Series B (Methodological)*, vol. 39, pp. 1-38, 1977.

[9] W. Gappmair, R. Lopez-Valcarce and C. Mosquera, "ML and EM Algorithm for Non-Data-Aided SNR Estimation of Linearly Modulated Signals", *IEEE 6th International Symposium on Communication Systems, Networks and Digital Signal Processing*, pp. 530-534, July 2008.

[10] F. Bellili, A. Stéphenne and S. Affes, "Cramér-Rao Bounds for SNR Estimates in Multicarrier Transmissions', *IEEE Communications and Networking Conference WCNC'09*, 2009.