Cramér-Rao Bounds for SNR Estimates in Multicarrier Transmissions

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Abstract— **Considering orthogonal frequency division multiplexing (OFDM) transmissions, we derive analytical expressions for the Cramér-Rao bounds for the subcarrier signal-to-noise ratio (SNR) estimates. The channel coefficients of the different subcarriers are assumed to be constant over the observation interval and the received signal is assumed to be corrupted by additive white Gaussian noise (AWGN). We will show that exploiting the mutual information between the different tones improves the achievable performance of subcarrier SNR estimators.**

I. Introduction

Parameter estimation has reached a mature state in OFDM communication systems [1]. However, not very much information is available on the SNR estimation despite the fact that it is often a requirement for many applications in such systems. In fact, most of the bit allocation algorithms [2], in multicarrier systems, are based on the *a priori* knowledge of the subcarrier SNR estimates, which are usually computed at the receiver then sent back to the transmitter using feedback. For instance, accurate subcarrier SNR estimates are required for the adaptive bit loading strategies. In fact, many of the multicarrier modulation systems, especially the wireless ones [3], use conventional multicarrier modulation which employs the same constellation size on all the subcarriers. However, the performance of these systems is typically limited by the subcarriers with poorest error performance. One solution to this problem is to perform adaptive "bit loading" where the constellation size varies from one subcarrier to another according to their SNR values. In extreme situations, the subcarriers experiencing poor SNR values can be *nulled* or "turned off".

Roughly speaking, SNR estimators can be categorized in two major categories, the non-data-aided (NDA) and data-aided (DA) estimators. Unlike DA SNR estimators which suppose an *a priori* perfect or partial knowledge of the transmitted symbols, NDA estimators base the estimation process only on the received samples.

The performance of any subcarrier SNR unbiased estimator is often statistically assessed in terms of its variance. The latter is usually computed using Monte Carlo computer simulations. A given SNR estimator is said to outperform another one if it has lower variance. But a well-known common lower bound for the variance of any unbiased SNR estimator is the Cramér-Rao lower bound (CRLB) which informs about the achievable performance of SNR estimators. Moreover, when the SNR CRLBs are derived using the inphase and quadrature components of the received samples, they are referred to as I/Q CRLBs. However, when they are computed using the magnitude of the received samples, they are called envelope-based CRLBs.

In single-carrier transmissions over AWGN channels, the CRLBs for the SNR estimates are already available in the literature. Indeed, they are either derived in closed forms or numerically computed [4, 5, 6, 7]. But to the best of our knowledge, they have never been addressed in the case of OFDM transmissions when we are interested in the SNR per subcarrier, taking into account the mutual information between subcarriers. In this paper, always assuming the channel coefficients to be constant over the observation interval across all the subcarriers, we derive the CRLBs for the subcarrier SNR estimates. We will show how the mutual information between the different tones can improve the achievable performance of per-carrier SNR estimators.

The present article will be organized as follows. In section II, we will introduce the system model used throughout this paper. Then, in section III, we will derive the CRLBs for the subcarrier SNR estimation. Finally, before concluding the article, we will present in IV, as an example, some graphical representations of the non-data-aided (NDA) SNR estimation CRLBs for square QAM constellation modulated signals.

II. System Model

We consider a digital communication system broadcasting and receiving any OFDM modulated signal. The channel-gain coefficients $(S_i)_{i=1,2,\cdots,K}$ are assumed to be constant over the observation interval for each of the K subcarriers. We assume also that, on the i^{th} subcarrier, we receive an AWGN-corrupted signal with noise power $2\sigma_i^2$. Assuming an ideal receiver with perfect synchronization, the received signal can be modelled as a complex signal as follows :

$$y_i(n) = S_i\alpha_i(n)e^{j\phi_i} + w_i(n), n = 1, 2, \ldots, N, \quad (1)$$

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where N is the the number of the received symbols on each subcarrier i $(i = 1, 2 \cdots, K)$ and at time index n, $\alpha_i(n)$ is the transmitted symbol, $y_i(n)$ is the corresponding received sample and $w_i(n)$ is an additive white noise component which is modelled by a complex Gaussian random variable with independent real and imaginary parts. The noise components are also assumed to be mutually independent between the different subcarriers. In addition, the transmitted symbols $\{\alpha_i(n)\}_{n=1,2,\cdots,N;i=1,2,\cdots,K}$ are assumed to be independent and identically distributed (iid). The channel is supposed to be slowly time varying so that it can be adequately modelled, over the observation interval, by a constant real gain S_i and a phase ϕ_i that accounts for any constant phase distortion introduced by the channel. Moreover, the constellation energy is supposed to be normalized to one, which means $\mathrm{E}\{|\alpha_i(n)|^2\} = 1$.

Considering the received samples on all the subcarriers, the received signal can be conveniently written in the following matrix form :

$$\boldsymbol{Y} = \begin{pmatrix} y_1(1) & y_2(1) & \cdots & y_K(1) \\ y_1(2) & y_2(2) & \cdots & y_K(2) \\ \vdots & \vdots & \ddots & \vdots \\ y_1(N) & y_2(N) & \cdots & y_K(N) \end{pmatrix}. \quad (2)$$

In this paper, we will consider the most general case where we will assume that we do not have any *a priori* information about the dependence that may exist between the channel coefficients $(S_i)_{i=1,2,\cdots,K}$ across the different subcarriers. Indeed, we suppose that the only available information is $S_i \neq S_j$ for $i \neq j$. However, without loss of generality, we will assume that the first L subcarriers experience the same noise power $2\sigma^2$ and the remaining $K - L$ subcarriers experience different noise powers $(2\sigma_i^2)_{i=L+1,L+2,\cdots,K}$.

Based on the N received samples, the true subcarrier SNRs that we wish to estimate are defined as

$$\rho_i = \begin{cases} \dfrac{S_i^2}{2\sigma^2}, & \text{for } i = 1, 2 \cdots, L \\ \\ \dfrac{S_i^2}{2\sigma_i^2}, & \text{otherwise.} \end{cases} \quad (3)$$

From eq. (3), we see that there are $2K - L + 1$ parameters which are involved in the derivation of the subcarrier SNR CRLBs, σ^2, $(S_i)_{i=1,2,\cdots,K}$ and $(\sigma_i)_{i=L+1,L+2,\cdots,K}$. Hence, we define the following parameter vector :

$$\begin{aligned} \boldsymbol{\theta} &= [\theta_1, \theta_2, \cdots, \theta_{2K-L+1}]^T, & (4) \\ &= [S_1, S_2, S_3, \cdots, S_L, \sigma^2, S_{L+1}, \sigma_{L+1}^2, & (5) \\ &\quad S_{L+2}, \sigma_{L+2}^2, \dots S_K, \sigma_K^2]^T. \end{aligned}$$

Moreover, since, for a clearer interpretation, we usually use the decibel scale, we will consider the following parameter transformations :

$$g_i(\boldsymbol{\theta}) = \begin{cases} 10\log_{10}\left(\dfrac{S_i^2}{2\sigma^2}\right), & \text{for } i = 1, 2 \cdots, L \\ 10\log_{10}\left(\dfrac{S_i^2}{2\sigma_i^2}\right), & \text{for } i = L+1, ..., K, \end{cases} \quad (6)$$

which can be conveniently rewritten in the following K-dimensional parameter transformation

$$\boldsymbol{\rho} = \boldsymbol{g}(\boldsymbol{\theta}) = [g_1(\boldsymbol{\theta}), g_2(\boldsymbol{\theta}), g_3(\boldsymbol{\theta}), \dots, g_K(\boldsymbol{\theta})]. \quad (7)$$

III. Derivation of the subcarrier SNR CRLBs

In this section, based on the assumptions made so far, we will derive the Cramér-Rao bounds for the SNR estimates of any modulated signal (PAM, MPSK, QAM) when it is AWGN-corrupted. Therefore, we denote by $P[\boldsymbol{Y}; \boldsymbol{\theta}]$ the probability density function of $\boldsymbol{Y}$ parameterized by $\boldsymbol{\theta}$. As shown in [8], the covariance matrix, $\boldsymbol{C}_{\hat{\boldsymbol{\rho}}}$, of any unbiased SNR estimator $\hat{\boldsymbol{\rho}} = [\hat{\rho}_1, \hat{\rho}_1, \cdots, \hat{\rho}_K]$, where $\hat{\rho}_i$ is an estimate of ρ_i for $i = 1, 2 \cdots, K$, satisfies

$$\boldsymbol{C}_{\hat{\boldsymbol{\rho}}} - \frac{\partial \mathrm{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \boldsymbol{I}^{-1}(\theta) \frac{\partial \boldsymbol{g}(\boldsymbol{\theta})^T}{\partial \boldsymbol{\theta}} \geq \mathbf{0}, \quad (8)$$

where, taking the expectation $\mathrm{E}\{.\}$ with respect to $\boldsymbol{Y}$, $\boldsymbol{I}(\boldsymbol{\theta})$ is the Fisher information matrix (FIM) given as

$$[\boldsymbol{I}(\boldsymbol{\theta})]_{ij} = -\mathrm{E}\left\{\frac{\partial^2 \ln P[\boldsymbol{Y}; \boldsymbol{\theta}]}{\partial \theta_i \partial \theta_j}\right\}, \quad (9)$$

and $\partial \mathrm{g}(\boldsymbol{\theta})/\partial \boldsymbol{\theta}$ is the $(K \times (2K - L + 1))$ Jacobian matrix [8] given by :

$$\left[\frac{\partial \mathrm{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right]_{ij} = \frac{\partial g_i(\boldsymbol{\theta})}{\partial \theta_j}. \quad (10)$$

Then, using (5), (6) and (7), $\partial \mathrm{g}(\boldsymbol{\theta})/\partial \boldsymbol{\theta}$ simply reduces to the following block matrix :

$$\frac{\partial \mathrm{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \begin{pmatrix} \boldsymbol{\mathcal{G}}_1 & \mathbf{0}_1 \\ \mathbf{0}_2 & \boldsymbol{\mathcal{G}}_2 \end{pmatrix}, \quad (11)$$

where $\mathbf{0}_1$ and $\mathbf{0}_2$ are, respectively, $((K-L) \times (L+1))$ and $(L \times 2(K-L))$ zero matrices and $\boldsymbol{\mathcal{G}}_1$ and $\boldsymbol{\mathcal{G}}_2$ are given as follows :

$$\boldsymbol{\mathcal{G}}_1 = \begin{pmatrix} \frac{\partial g_1(\boldsymbol{\theta})}{\partial S_1} & 0 & \cdots & 0 & \frac{\partial g_1(\boldsymbol{\theta})}{\partial \sigma^2} \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \frac{\partial g_L(\boldsymbol{\theta})}{\partial S_L} & \frac{\partial g_L(\boldsymbol{\theta})}{\partial \sigma^2} \end{pmatrix}, \quad (12)$$

$$\boldsymbol{\mathcal{G}}_2 = \begin{pmatrix} \frac{\partial g_{L+1}(\boldsymbol{\theta}^{(L+1)})}{\partial \boldsymbol{\theta}^{(L+1)}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \frac{\partial g_K(\boldsymbol{\theta}^{(K)})}{\partial \boldsymbol{\theta}^{(K)}} \end{pmatrix}, \quad (13)$$

where it should be noted that $\boldsymbol{\mathcal{G}}_2$ is also a block matrix with elements $\mathbf{0}$ being (1×2) zero vectors and $\boldsymbol{\theta}^{(i)}$ being given by

$$\boldsymbol{\theta}^{(i)} = \begin{cases} [S_i, \sigma^2], & \text{for} \quad i = 1, 2 \cdots, L \\ [S_i, \sigma_i^2], & \text{for} \quad i = L+1, L+2 \cdots, K. \end{cases} \tag{14}$$

Now, we will derive the FIM elements given by (9). In fact, since the transmitted symbols are independent and identically distributed and we have independent noise components between the subcarriers, the log-likelyhood function for the KN observed symbols is

$$\ln\left(P[\boldsymbol{Y};\boldsymbol{\theta}]\right) = \sum_{i=1}^{K} \ln\left(P[\boldsymbol{y}_i;\boldsymbol{\theta}^{(i)}]\right), \tag{15}$$

where $\boldsymbol{\theta}^{(i)}$ is given by (14) and $\boldsymbol{y}_i$ is a vector that contains the received samples on the i^{th} subcarrier :

$$\boldsymbol{y}_i = [y_i(1), y_i(2), \cdots, y_i(N)]. \tag{16}$$

For ease of notation, from now on we will use the following definitions :

$$c = -E\left\{\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\theta}]\right)}{\partial {\sigma^2}^2}\right\}, \tag{17}$$

$$a_i = -E\left\{\frac{\partial^2 \ln\left(P[\boldsymbol{y}_i;\boldsymbol{\theta}^{(i)}]\right)}{\partial S_i^2}\right\}, \quad 1 \leq i \leq K \tag{18}$$

$$b_i = \begin{cases} -E\left\{\dfrac{\partial^2 \ln\left(P[\boldsymbol{y}_i;\boldsymbol{\theta}^{(i)}]\right)}{\partial S_i \partial \sigma^2}\right\}, & 1 \leq i \leq L \\ -E\left\{\dfrac{\partial^2 \ln\left(P[\boldsymbol{y}_i;\boldsymbol{\theta}^{(i)}]\right)}{\partial S_i \partial \sigma_i^2}\right\}, & L+1 \leq i \leq K \end{cases} \tag{19}$$

$$d_i = \begin{cases} -E\left\{\dfrac{\partial^2 \ln\left(P[\boldsymbol{y}_i;\boldsymbol{\theta}^{(i)}]\right)}{\partial {\sigma^2}^2}\right\}, & 1 \leq i \leq L \\ -E\left\{\dfrac{\partial^2 \ln\left(P[\boldsymbol{y}_i;\boldsymbol{\theta}^{(i)}]\right)}{\partial {\sigma_i^2}^2}\right\}, & L+1 \leq i \leq K. \end{cases} \tag{20}$$

(21)

It should be noted that a_i, b_i and d_i are the elements of the FIM that would be obtained if we were to receive, on the i^{th} subcarrier, the corresponding N samples $\{y_i(n)\}_{n=1,2,\cdots,N}$ as being from a traditional single-carrier system. It is also worth noting that, at this stage, we have

$$c = \sum_{i=1}^{L} d_i. \tag{22}$$

On the other hand, using eqs. (9) and (15), it can be shown that the FIM is also a block matrix given by :

$$\boldsymbol{I}(\boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{I}_1 & \mathbf{0}_3 \\ \mathbf{0}_3^T & \boldsymbol{I}_2 \end{pmatrix}, \tag{23}$$

where $\mathbf{0}_3$ is a $((L+1) \times 2(K-L))$ zero matrix and $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ are given by :

$$\boldsymbol{I}_1 = \begin{pmatrix} a_1 & 0 & 0 & \cdots & 0 & b_1 \\ 0 & a_2 & 0 & \cdots & 0 & b_2 \\ 0 & 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 0 & a_L & b_L \\ b_1 & b_2 & \cdots & \cdots & b_L & c \end{pmatrix}, \tag{24}$$

$$\boldsymbol{I}_2 = \begin{pmatrix} \boldsymbol{J}_{L+1}(\boldsymbol{\theta}^{(L+1)}) & \underline{\underline{\mathbf{0}}} & \cdots & \underline{\underline{\mathbf{0}}} \\ \underline{\underline{\mathbf{0}}} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \underline{\underline{\mathbf{0}}} \\ \underline{\underline{\mathbf{0}}} & \cdots & \underline{\underline{\mathbf{0}}} & \boldsymbol{J}_K(\boldsymbol{\theta}^{(K)}) \end{pmatrix}, \tag{25}$$

where it should be noted again that $\boldsymbol{I}_2$ is also a block matrix with elements $\underline{\underline{\mathbf{0}}}$ being (2×2) zero matrices and $\boldsymbol{J}_i(\boldsymbol{\theta}^{(i)})$ being simply given by

$$\boldsymbol{J}_i(\boldsymbol{\theta}^{(i)}) = \begin{pmatrix} a_i & b_i \\ b_i & d_i \end{pmatrix}, \text{for } i = L+1, L+2, \cdots, K \tag{26}$$

Equation (26) represents the (2×2) FIM that would be obtained if we were only considering the N received samples on the i^{th} subcarrier, as being from a traditional single-carrier communication system.

Finally, from (8) and using (11) and (23), the CRLB of the SNR estimator covariance matrix is given by

$$\mathbf{CRLB}(\boldsymbol{\rho}) = \begin{pmatrix} \boldsymbol{\mathcal{G}}_1 \boldsymbol{I}_1^{-1} \boldsymbol{\mathcal{G}}_1^T & \mathbf{0}_4 \\ \mathbf{0}_4 & \boldsymbol{\mathcal{G}}_2 \boldsymbol{I}_2^{-1} \boldsymbol{\mathcal{G}}_2^T \end{pmatrix}, \tag{27}$$

where $\mathbf{0}_4$ is a $(L \times (K-L))$ zero matrix. Moreover, since we are only interested in the subcarrier SNR estimation, the CRLB for SNR estimates on the i^{th} subcarrier, $\text{CRLB}^i(\rho_i)$, is given by the i^{th} diagonal element of the matrix $\mathbf{CRLB}(\boldsymbol{\rho})$, which means :

$$\text{CRLB}^i(\rho_i) = [\mathbf{CRLB}(\boldsymbol{\rho})]_{ii}, \quad \text{for} \quad i = 1, 2, \cdots, K. \tag{28}$$

For the $(K-L)$ last subcarriers, which are experiencing different noise powers, it can be seen that

$$\boldsymbol{\mathcal{G}}_2 \boldsymbol{I}_2^{-1} \boldsymbol{\mathcal{G}}_2^T = \begin{pmatrix} \text{CRLB}^{L+1}(\rho_{L+1}) & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \text{CRLB}^K(\rho_K) \end{pmatrix}, \tag{29}$$

where $\text{CRLB}^i(\rho_i) = \frac{\partial g_i(\boldsymbol{\theta}^{(i)})}{\partial \boldsymbol{\theta}^{(i)}} \boldsymbol{J}_i^{-1}(\boldsymbol{\theta}^{(i)}) \frac{\partial g_i(\boldsymbol{\theta}^{(i)})}{\partial \boldsymbol{\theta}^{(i)}}^T$, for $L+1 \leq i \leq K$, is the CRLB for the SNR estimates that can be achieved on the i^{th} subcarrier by receiving the same corresponding N samples $(\boldsymbol{y}_i)$ as being from a traditional single-carrier system.

$$
\boldsymbol{I}_1^{-1} = \frac{1}{c - \sum_{l=1}^{L} \frac{b_l^2}{a_l}} \begin{pmatrix} \frac{c}{a_1} - \sum_{\substack{l=1 \\ l\neq 1}}^{L} \frac{b_l^2}{a_1 a_l} & -\frac{b_1 b_2}{a_1 a_2} & \cdots & -\frac{b_1 b_L}{a_1 a_L} & -\frac{b_1}{a_1} \\ -\frac{b_2 b_1}{a_2 a_1} & \frac{c}{a_2} - \sum_{\substack{l=1 \\ l\neq 2}}^{L} \frac{b_l^2}{a_2 a_l} & \ddots & \vdots & -\frac{b_2}{a_2} \\ \vdots & \ddots & \ddots & -\frac{b_{L-1} b_L}{a_{L-1} a_L} & \vdots \\ -\frac{b_L b_1}{a_L a_1} & \cdots & -\frac{b_L b_{L-1}}{a_L a_{L-1}} & \frac{c}{a_L} - \sum_{\substack{l=1 \\ l\neq L}}^{L} \frac{b_l^2}{a_L a_l} & -\frac{b_L}{a_L} \\ -\frac{b_1}{a_1} & -\frac{b_2}{a_2} & \cdots & -\frac{b_L}{a_L} & 1 \end{pmatrix}. \tag{30}
$$

However, for the L first subcarriers, which experience the same noise power, we need to invert the matrix $\boldsymbol{I}_1$ before being able to find the CRLBs for the SNR estimates. In fact, resolving for $\boldsymbol{X}$ the linear equations system $\boldsymbol{I}_1\boldsymbol{X} = \boldsymbol{B}$, where $\boldsymbol{X}$ and $\boldsymbol{B}$ are any $((L+1)\times 1)$ column vectors, yields the inverse $\boldsymbol{I}_1^{-1}$ of $\boldsymbol{I}_1$ as given by (30). Thus, by denoting the i^{th} row of $\boldsymbol{\mathcal{G}}$ by $\boldsymbol{g}_i$ and taking the first diagonal elements of the matrix $\mathbf{CRLB}(\boldsymbol{\rho})$, we get the CRLBs for the SNR estimates on the first L subcarriers as follows :

$$
\begin{aligned}
\text{CRLB}^i(\rho_i) &= \boldsymbol{g}_i \boldsymbol{I}_1^{-1} \boldsymbol{g}_i^T \quad i = 1, 2, \cdots, L, && (31) \\
&= \frac{1}{c - \sum_{l=1}^{L} \frac{b_l^2}{a_l}} [A_i + B_i], && (32)
\end{aligned}
$$

where

$$
\begin{aligned}
A_i &= \left(\frac{c}{a_i} - \sum_{\substack{l=1 \\ l\neq i}}^{L} \frac{b_l^2}{a_i a_l} \right)^2 \left(\frac{\partial g_i(\boldsymbol{\theta})}{\partial S_i} \right)^2, && (33) \\
B_i &= \left(\frac{\partial g_i(\boldsymbol{\theta})}{\partial \sigma^2} \right)^2 - 2\frac{b_i}{a_i} \frac{\partial^2 g_i(\boldsymbol{\theta})}{\partial S_i \partial \sigma^2}. && (34)
\end{aligned}
$$

Recall that $c = \sum_{i=1}^{L} d_i$ and the unknowns $\{a_i\}_{i=1,2,\cdots,L}$, $\{b_i\}_{i=1,2,\cdots,L}$ and $\{d_i\}_{i=1,2,\cdots,L}$ are the FIM elements that can be obtained for each subcarrier over which we receive the corresponding N samples as being from a classical single-carrier system.

Finally, it should be noted that when the channel is assumed constant over time, these unknowns have already been derived in the literature for different modulation types, both for DA and NDA SNR estimates. For example, for QPSK modulated signals, they are given by [6] :

$$
\begin{aligned}
a_i^{\text{NDA}} &= \frac{N\left[1 - f\left(\frac{\rho_i}{2}\right)\right]}{\sigma^2} \quad &;\quad a_i^{\text{DA}} &= \frac{N}{\sigma^2}, \\
b_i^{\text{NDA}} &= \frac{N S_i f\left(\frac{\rho_i}{2}\right)}{\sigma^4} \quad &;\quad b_i^{\text{DA}} &= 0, \\
d_i^{\text{NDA}} &= \frac{N}{\sigma^4} - \frac{N S_i^2}{\sigma^6} f\left(\frac{\rho_i}{2}\right) \quad &;\quad b_i^{\text{DA}} &= \frac{N}{\sigma^4},
\end{aligned} \tag{35}
$$

where

$$
f(t) = \frac{\exp(-t)}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{u^2 \exp\left(-\frac{u^2}{2}\right)}{\cosh(u\sqrt{2t})} du. \tag{36}
$$

These unknowns were also derived for general $2^{2p}-ary$ QAM modulated signals in [9], but the results are not presented here for lack of space. Therefore, the CRLBs for subcarrier SNR estimation, on one of the first subcarriers, can now be directly deduced using (32), (33) and (34) in both DA and NDA scenarios. It should be noted that the CRLBs on one of the first L subcarriers depend on the SNR on the other $L-1$ tones which are experiencing exactly the same noise power. However, the CRLBs on one of the last $K-L$ subcarriers, that do not experience the same noise power, remain unchanged as compared to the CRLBs that can be achieved in single-carrier transmissions.

IV. Simulation Results

In this section, we include some graphical representations of the inphase/quadrature (I/Q) NDA CRLBs for the SNR estimates on one of the first L subcarriers which are assumed to experience the same noise power. In fact, assuming the channel to be complex and constant over the observation interval and only using inphase and quadrature components of the received samples, we have already derived the FIM elements a_i, b_i and d_i for $2^{2p} - ary$ QAM modulated signals in [9]. Therefore, the CRLBs can be directly deduced using (32). The I/Q NDA CRLBs for the remaining $K - L$ subcarriers were not included because they are exactly those which are obtained in a traditional single-carrier system. These are well covered in the literature.

In fact, Figs 1, 2 and 3 depict the I/Q CRLBs for NDA SNR estimates on the first subcarrier when it is being seen as used in a single-carrier and a multicarrier system. We see the CRLBs on this tone that exploit the mutual information between subcarriers are lower than those that can be achieved in a single-carrier system. These improvements on the achievable performance

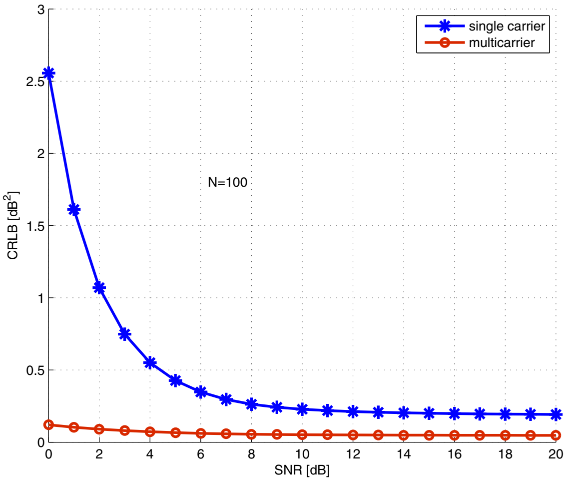

Fig. 1. CRLB for the SNR estimates on the first subcarrier, $L = 4$, $\{\rho_{i+1} - \rho_i = 2 \text{ dB}\}_{i=1,2,\cdots,L-1}$, $N = 100$, 4-QAM.

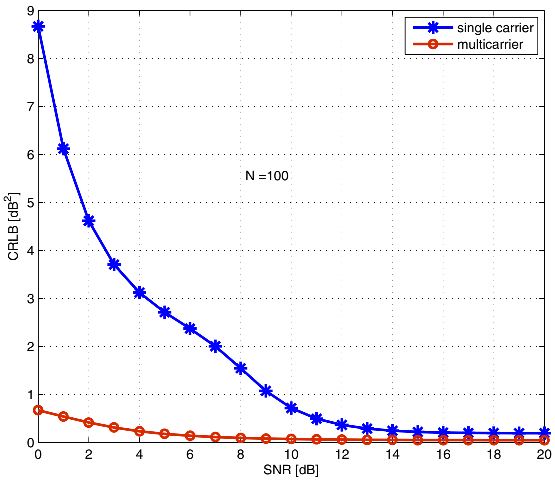

Fig. 2. CRLB for the SNR estimates on the first subcarriers, $L = 4$, $\{\rho_{i+1} - \rho_i = 2 \text{ dB}\}_{i=1,2,\cdots,L-1}$, $N = 100$, 16-QAM.

stem from the improvements on the estimation accuracy of the noise power which is the same on the other $L - 1$ first subcarriers. In fact, the received samples on these tones carry additional information about the noise power. However, the achievable performance of the SNR estimators on the last subcarriers remains unchanged. This is hardly surprising since the noise powers are mutually different on these subcarriers and no *a priori* knowledge is assumed about the dependence of the channel coefficients.

V. Conclusion

Cramér-Rao bounds for SNR estimates of modulated signals, in multicarrier systems, are derived when the signal is corrupted by additive white Gaussian noise.

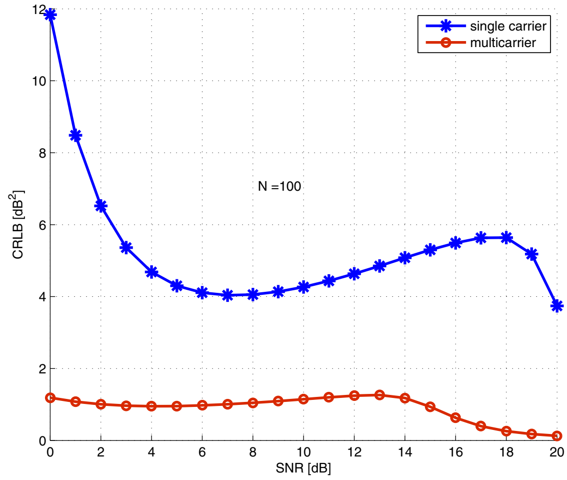

Fig. 3. CRLB for the SNR estimates on the first subcarrier, $L = 4$, $\{\rho_{i+1} - \rho_i = 2 \text{ dB}\}_{i=1,2,\cdots,L-1}$, $N = 100$, 256-QAM.

No *a priori* knowledge is assumed about the dependence that may exist between the channel coefficients for the different subcarriers. It was shown that exploiting the mutual information between the subcarriers leads to an improvement on the achievable performance of the unbiased subcarrier SNR estimators.

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