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Outage Analysis of wireless Systems over Composite Fading/Shadowing Channels with Co-channel Interference

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***Abstract*—This paper presents an outage analysis of wireless systems operating in gamma-shadowed Nakagami-faded environments where the desired signal also suffers from co-channel interference. The interfering signals are also subject to fading and shadowing. Based on the obtained signal to interference ratio (SIR) probability density function (pdf), closed-form expressions for the outage probability are obtained in both cases of statistically identical interferers and multiple interferers with different parameters. The effects on the aforementioned performance metric of the reuse distance and of the combined fading, shadowing and co-channel interference are analyzed subsequently. The newly derived closed-form expressions for the outage probability allow as to assess the effects of the different channel and interference parameters easily.**

***Index Terms*—Co-channel interference, outage probability, Nakagami fading, shadowing.**

I. Introduction

Accurate system planning and performance evaluations need to take into account the presence of channel propagation impairments such as large-scale fading, which arises from shadowing, and small-scale fading, which is due to multipath propagation [1], [2]. In an interference-limited environment, co-channel interference, which is due to the aggressive frequency reuse in neighboring cells, should also be considered as a corruptive effect. Each interfering signal is also subject to multipath and shadow fading and it is necessary to incorporate these effects in assessing the performance of wireless systems. In such an environment, the link performance evaluation depends on many channel parameters. To asses the impact of these different parameters on system evaluation metrics, namely, the outage probability, closed-form and tractable expressions are highly desirable. Nevertheless, difficulties arise when using the conventional compound Nakagami-lognormal channel model since its composite probability density function (pdf) is not in closed form. A three-parameter compound pdf was recently proposed as a substitute to the Nakagami-lognormal model [3], [5]. This model leads to a closed-form solution for the density function of the desired signal power, simplifying the performance analysis. In [6], a gamma-distributed shadowing was proposed as a substitute to the lognormal shadowing and a compound Rayleigh fading gamma shadowing model was presented as the K-distribution. Recently, in [7], the closed-form pdf of the composite Nakagami-m fading gamma shadowing was introduced as the generalized K-distribution.
One of the most important link evaluation and planning parameters is the outage probability, which is the probability that the instantaneous desired signal to interfering signal power ratio is below a specific protection threshold. Outage probability in the presence of co-channel interference has been assessed several times. For fading-only channels, a closed-form expression for the probability of outage was recently presented in [8]. But so far, to the best of our knowledge, no closed-from expressions for the outage probability over composite channels have been obtained. In [9], J. C. Lin et al. conducted an outage analysis for microcellular wireless systems that operate in shadowed Rician/Nakagami fading environments. They provided a closed-form expression for the outage probability in the absence of shadowing for independent identically distributed Nakagami faded interferers, but numerical integration was required to solve the shadowed case. The compound Nakagami-m fading gamma shadowing model was considered by I. M. Kostic in [10]. Nevertheless, the latter did not provide a closed-form expression for the probability of outage, so that the evaluation was done numerically.
This paper derives closed-form expressions for the outage probability of a cellular system operating over a Nakagami fading lognormal shadowing channel in the presence of co-channel interference. A three compound gamma shadowing Nakagami fading model is used to approximate the Nakagami lognormal channel. We allow for the fact that all the interferers have the same mean power, but could have different fading parameters. The new tractable expressions of the outage probability are used to assess the effects of the channel and interference parameters on the system performance.
The remainder of this paper is organized as follows. The next section describes in more details our propagation and co-channel interference models. In section III, the statistical properties of the SIR are assessed and are used to derive closed-form expressions for the probability of outage. The effects of a combined shadowing fading channel and co-channel interference are analyzed subsequently in section IV. The final section summarizes our main results and concludes.

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II. Channel and System Model

In this section, we first outline the models for the different propagation impairments affecting the studied cellular system, and then present our assumptions for the co-channel interference.

A. Channel Model

Mobile radio systems are subject to fast multipath fading due to the combination of randomly delayed reflected, scattered and diffracted signal components. In slowly varying flat fading multipath channels, the envelope Z of the received signal is commonly modeled by a Nakagmi-m distribution [11]

$$p_Z(z) = \frac{2(\frac{m}{\Omega})^m}{\Gamma(m)} z^{2m-1} e^{(-\frac{mz^2}{\Omega})}, \quad z \geq 0, \tag{1}$$

denoted by $Z \sim N(m, \Omega)$, where $\Omega = E(Z^2)$ is the local mean received power, m is the fading severity parameter ($m \geq 1/2$), and $\Gamma(\cdot)$ is the gamma function. Thus, the pdf of the received signal power $A = Z^2$ is a gamma distribution given by

$$p_A(a) = \frac{(\frac{m}{\Omega})^m}{\Gamma(m)} a^{m-1} e^{(-\frac{ma}{\Omega})}. \tag{2}$$

In urban macrocell systems, the link quality also suffers from shadowing caused by the variability associated with large scale environmental obstacles. This induces a fluctuation of the mean power Ω about a constant area mean power P. Empirical studies have shown that Ω has a lognormal distribution

$$p_\Omega(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(\ln(x)-\ln(P))^2}{2\sigma^2}}, \quad x \geq 0, \tag{3}$$

where P is related to the path loss and σ is the shadow standard deviation.
For simple one-slope path-loss model [1-2], we have

$$P = P_t C (d_r/d)^\alpha, \quad d \geq d_r, \tag{4}$$

where P_t is the average transmit power, C is dimensionless and depends on the antenna properties, d_r is a reference distance, d is the distance between the transmitter and the receiver and α is the path-loss exponent ($2 \leq \alpha \leq 4$, for urban macrocell environments). However, the lognormal pdf is often inconvenient when further analysis is required. Therefore, as an approximation to the lognormal pdf, and as was done in [7], we propose to use the gamma pdf, denoted by $G(\lambda, \Omega_s)$, and defined by

$$p_\Omega(x) = \frac{1}{\Gamma(\lambda)} (\frac{\lambda}{\Omega_s})^\lambda x^{\lambda-1} e^{-\frac{\lambda}{\Omega_s}x}, \quad x > 0, \quad \lambda > 0. \tag{5}$$

In (5), the parameter λ inversely reflects the shadowing severity and Ω_s is the gamma shadow mean power. Relations between the parameters of the lognormal pdf and the gamma pdf can be obtained by a moment-matching technique. In [10], the authors obtained

$$\lambda = 1/(e^{\sigma^2} - 1) \quad \text{and} \quad \Omega_s = P\sqrt{\lambda + 1/\lambda}. \tag{6}$$

The standard deviation expressed in decibels, also known as the shadowing spread or 'dB-spread', is given by $\sigma_{dB} = 8.686\ \sigma$.

B. Interference and System Model

We consider a highly spectrally efficient interference-limited scenario in which the noise component is negligible compared to the co-channel interference [12], [13]. Neglecting the noise also makes the analysis more tractable. Thus, the signal to interference plus noise power ratio reduces to the signal to interference power ratio (SIR). Therefore, the desired user SIR, γ, can be written as

$$\gamma = \frac{S_d}{I} = \frac{S_d}{\sum_{i=1}^N S_i}, \tag{7}$$

where S_d is the desired signal power, I is the total interfering power, S_i is the received power level from the ith interferer and N is the number of mutually independent co-channel interferers. Assuming that the transmitted power of all users is the same, we have

$$\begin{aligned} S_d &= Z_d^2 \omega_d, \\ S_i &= Z_i^2 \omega_i, \end{aligned} \tag{8}$$

where $Z_d \sim N(m, \omega_d)$ and $\omega_d \sim G(\lambda, \Omega_d)$ are, respectively, the fading and the shadowing processes affecting the desired user. Similarly $Z_i \sim N(m_i, \omega_i)$ and $w_i \sim G(\lambda_i, \Omega_i)$ are the fading and shadowing processes associated with the ith interfering signal. Since the signal powers of both desired and interfering users experience fluctuations due to multipath fading and shadowing, γ is also a random variable which depends on the distribution of the S_d and S_i values. In gamma-shadowed Nakagami-m faded channels, the pdf of the desired signal power S_d is given by [7]:

$$f_{S_d}(x) = \frac{2(\frac{m\lambda}{\Omega_d})^{\frac{m+\lambda}{2}}}{\Gamma(m)\Gamma(\lambda)} x^{\frac{m+\lambda-2}{2}} K_{\lambda-m}\left(2\sqrt{\frac{m\lambda x}{\Omega_d}}\right). \tag{9}$$

This pdf is commonly known as the generalized K-distribution, denoted by $K(m, \lambda, \Omega_d)$. For $m = 1$, it reduces to the Suzuki distribution [14], which approximates adequately the Raleigh-lognormal model. Similarly, each interfering signal S_i is assumed to be generalized K-distributed and denoted by $S_i \sim K(m_i, \lambda_i, \Omega_i)$. Therefore, to compute the pdf of $I = \sum_{i=1}^N S_i$, we need to compute the pdf of the sum of N generalized K-distributed random variables (RVs), which is unpractical. To simplify the problem, we assume that the mean powers of all interferers are identical, i.e. $\omega_i = \omega_I, i = 1, 2, \ldots, N$. This assumption has been considered by M.S. Alouini et al. in [15] to quantify the area spectral efficiency of interference-limited cellular systems in the presence of both Rayleigh fading and lognormal shadowing, but no closed-form expression for the output SIR was proposed. Other related papers are [16] and [17]; both of which assumed identically distributed interferers and did not consider shadowing. By considering the aforementioned assumption, the authors in [15]-[17] derived upper and lower bounds on the real system performances by assuming that all the interfering users are at the best case or the worst case location for interference. Nevertheless, in practice, there are several wireless systems that could be adequately modeled using the assumption of equal mean power interferers. Indeed, if we consider the case of one MIMO interfering signal disturbing the desired user on the uplink of a communication link, the interfering signals, coming from collocated antennas, are subject to the same shadowing process while they could experience different fading conditions. The same phenomenon could also be encountered in the case of relayed communications. Indeed, when a cluster of users use another mobile as a relay, the latter could receive at the same time a desired signal from another user not collocated with the interfering users. Therefore, the assumption of equal mean powers interferer may be useful in modeling many wireless communication scenarios. In a more general case, when the interfering users have different mean powers, our results may serve as useful real performances approximation.

Assuming equal mean power interferers, the total interference can be written as

$$I = \omega_I \sum_{i=1}^{N} Z_i^2, \tag{10}$$

where $\omega_I \sim G(\lambda_I, \Omega_I)$. The pdf of the interfering signal is then

$$f_I(y) = \int_0^\infty f_{I/w_I}(y/w) f_{w_I}(w) dw, \tag{11}$$

where f_{I/w_I} is the pdf of the interference I given the shadowing w_I. From (10), f_{I/w_I} is equal to the pdf of N squared Nakagami random variables. In our study, we will distinguish two scenarios, namely, statistically identical and statistically non-identical mutually independent interferers.

1) Multiple i.i.d. interferers: The first scenario is when there are N mutually independent interferers assumed to be statistically identical, so that the Z_is are i.i.d. with

$$m_i = m_I, \quad i = 1, 2, ..., N. \tag{12}$$

The assumption (12) holds when all the N interferers are constrained to be equidistant from the receiver. This assumption is convenient for deriving the system performance metrics in the worst case and best case interference configuration. For a MIMO interferer, with closely spaced antennas, interfering signals coming from the different antennas are subject to statistically identical fading processes.

When interferers are i.i.d., $f_I(y)$ is given by [18]

$$f_I(y) = \frac{2(\frac{m_I\lambda_I}{\Omega_I})^{\frac{Nm_I+\lambda_I}{2}}}{\Gamma(Nm_I)\Gamma(\lambda_I)} y^{\frac{Nm_I+\lambda_I-2}{2}} K_{\lambda_I - Nm_I}\left(2\sqrt{\frac{m_I\lambda_I y}{\Omega_I}}\right), \tag{13}$$

where m_I is the fading parameter shared by all interfering signals. This comes from the fact that the sum of N gamma distributed RVs of fading parameter m_I and average power ω is a gamma-distributed RV with fading parameter Nm_I and average power $N\omega$. By averaging over the gamma shadowing distribution, we get the SIR pdf in (13).

2) Multiple non-i.i.d. interferers: The second scenario is when there are N mutually independent interferers assumed to be statistically non identical, so that the Z_is are non-i.i.d. such that equation (12) does not have to hold. This assumption covers more general interference scenarios. It holds when the antennas of the MIMO interfering user are sufficiently distant so that signals coming from the different antennas will travel through different channels with different fading severity parameters.

Recently a closed-form expression for the sum of squared non i.i.d. Nakagami RVs has been derived by G. Karagiannidis et al. in [19]. Using the proposed pdf, the conditional probability of the interference with respect to the shadowing w can be written as

$$f_{I/w}(y/w) = \sum_{i=1}^{N}\sum_{k=1}^{mi} EL(i,k,\{m_q\}_{q=1}^N, \{\tfrac{w}{m_q}\}_{q=1}^N, \{l_q\}_{q=1}^{N-2}) f_{Y_i}(y,k,\tfrac{w}{m_i}), \tag{14}$$

where

$$f_{Y_i}(y, m_i, \eta_i) = \frac{y^{m_i-1}}{\eta_i^{m_i}(m_i-1)!} e^{(-\frac{y}{\eta_i})}, \tag{15}$$

is the Erlang distribution, and EL is a function given in [18]. By inserting (5) and (14) in (11), we get the pdf of the interfering signal using [20, eq. 3.471.9] as

$$f_I(y) = \frac{2\lambda_I^{\frac{\lambda_I}{2}}}{\Gamma(\lambda_I)} \sum_{i=1}^{N}\sum_{k=1}^{mi} \frac{EL(i,k,\{m_q\}_{q=1}^N,\{1/m_q\}_{q=1}^N,\{l_q\}_{q=1}^{N-2})}{(\frac{1}{m_i})^{\frac{k+d+\lambda_I}{2}}(\frac{\lambda_I}{\Omega_I})^{\frac{d-k}{2}}\Gamma(k)} y^{\frac{d+\lambda_I+k-2}{2}} K_{d+\lambda_I-k}\left(2\sqrt{\frac{y\lambda_I m_i}{\Omega_I}}\right), \tag{16}$$

where $d = -\sum_{i=1}^N m_i + \sum_{s=1}^{N-1} m_{s+U(s-i)}$ and $U(\cdot)$ is the well known unit step function defined as $U(x \geq 0) = 1$ and zero otherwise. Finally, it should be noted that the pdf in (14) is only applicable for integer values of m_i. For a notation simplicity reason, we will omit, later on, the argument of the function EL.

III. Outage Analysis

The outage probability is an important performance measure of communication links operating over composite fading/shadowing channels. It is defined as the probability that the output SIR falls below a given threshold y_{th}, i.e.,

$$P_{out} = \int_0^{y_{th}} f_\gamma(y) dy, \tag{17}$$

where $f_\gamma(y)$ is the pdf of the SIR given by $\gamma = S_d/I$ and y_{th} is the system protection ratio which depends on the type of modulation employed and the receiver characteristics. The pdf of the SIR can be derived as

$$f_\gamma(y) = \int_0^\infty f_I(z) f_{S_d}(yz) z dz, \tag{18}$$

and its nth moment is defined as

$$E[\gamma^n] = \int_0^\infty y^n f_\gamma(y) dy. \tag{19}$$

The SIR moments evaluation is a key tool for the amount of fading calculation and for a moment-based performance analysis [22].

$$f_\gamma(y) = \left(\frac{m\lambda}{m_I\lambda_I}\right)^\lambda \left(\frac{\Omega_I}{\Omega_d}\right)^\lambda y^{\lambda-1}\frac{B(Nm_I+\lambda, m+\lambda_I)}{B(\lambda_I,\lambda)B(Nm_I,m)}\,{}_2F_1(\lambda+\lambda_I, Nm_I+\lambda; \lambda_I+Nm_I+m+\lambda; 1-\frac{m\lambda}{m_I\lambda_I}\frac{\Omega_I}{\Omega_d}y), \quad (20)$$

3) Multiple i.i.d. interferers: Substituting (9) and (13) into (18) and changing the variable of integration to $x = \sqrt{z}$, the integral in (18) can be solved with the help of [20, Eq. 6.576.4]. After laborious manipulations, we get the pdf formula in (20), where ${}_2F_1(a, b; c; z)$ is the Gauss Hypergeometric function and $B(\cdot,\cdot)$ is the Beta function [22]. Using [20, eq. 7.811.4], one can easily verify that $\int_0^\infty f_\gamma(y)dy = 1$. Using the same formulas, the nth moment of the SIR γ is derived as

$$E[\gamma^n] = \left(\frac{Nm_I\lambda_I\rho}{m\lambda}\right)^n \frac{\Gamma(n+\lambda)\Gamma(n+m)\Gamma(\lambda_I-n)\Gamma(Nm_I-n)}{\Gamma(m)\Gamma(\lambda)\Gamma(Nm_I)\Gamma(\lambda_I)}, \quad (21)$$

where $\rho = \frac{\Omega_d}{\Omega_I}$ is the average SIR. From (17), and using the transformation [20, Eq. 9.131.2], and the integration formulas given by

$$\int z^{\alpha-1}{}_2F_1(a, b; c; z)dz = \frac{z^\alpha}{\alpha}{}_3F_2(a, b, \alpha; c, \alpha+1; z), \quad (22)$$

the outage probability is obtained after several manipulations as

$$P_{out} = \left(\frac{m\lambda}{m_I\lambda_I\rho}\right)^\lambda y_{th}^\lambda \frac{\Gamma(Nm_I+\lambda)\gamma(m-\lambda)}{\lambda\Gamma(Nm_I+m)B(\lambda,\lambda_I)B(Nm_I,m)}$$
$${}_3F_2(\lambda+\lambda_I, Nm_I+\lambda, \lambda; \lambda_I-m+, \lambda+1; \frac{m\lambda}{m_I\lambda_I\rho}y_{th})$$
$$+\left(\frac{m\lambda}{m_I\lambda_I\rho}\right)^m y_{th}^m \frac{\Gamma(m+\lambda_I)\Gamma(\lambda-m)}{m\Gamma(\lambda_I+\lambda)B(\lambda,\lambda_I)B(Nm_I,m)}$$
$${}_3F_2(Nm_I+m, m+\lambda_I, m; m-\lambda+1, m+1; \frac{m\lambda}{m_I\lambda_I\rho}y_{th}), \quad (23)$$

where ${}_3F_2(a, b, c; a_1, b_1; z)$ is the Generalized Hypergeometric function [22].
Let $c = \frac{m\lambda}{m_I\lambda_I\rho}$. If $cy_{th} \ll 1$, which is usually the case, ${}_3F_2(\cdot,\cdot,\cdot;\cdot,\cdot;cy_{th}) \to 1$. To corroborate this assumption, let us consider the special case $\lambda = \lambda_I = \infty$ (no shadowing). In this case, we have $c = \frac{m}{m_I\rho}$, and upon taking the limit $(\lambda \longrightarrow \infty)$, the first term of (23) evaluates to zero, while the second term, using the asymptotic expression of the gamma function [22], $\Gamma(x) \approx \sqrt{2\pi}x^{x-\frac{1}{2}}e^{-x}/(x \to \infty)$ yields

$$P_{out} \approx \frac{(\frac{m}{m_I\rho}y_{th})^m}{mB(Nm_I,m)}, \quad \lambda = \lambda_I \to \infty, \quad \rho/y_{th} \gg 1. \quad (24)$$

As a check on (24), the special case of Rayleigh fading $(m = m_I = 1)$ would result in $P_{out} \approx y_{th}/\rho$, which agrees with [23, eq. 29] for $\rho/y_{th} \gg 1$.

4) Multiple non-i.i.d. interferers: Substituting (9) and (16) into (18), and making the same calculus steps performed in the case of i.i.d. interferers, we obtain the SIR pdf for the non-i.i.d. case as in (25). Using [20, eq. 7.811.4], we find that the nth moment of the output SIR, in the presence of N non i.i.d interferers, is given by

$$E[\gamma^n] = \left(\frac{\lambda_I\rho}{m\lambda}\right)^n \frac{\Gamma(m+n)\Gamma(\lambda+n)}{\Gamma(\lambda_I)\Gamma(m)\Gamma(\lambda)} \sum_{i=1}^{M}\sum_{k=1}^{mi} EL\lambda_I^d m_i^n \frac{\Gamma(k-n)\Gamma(\lambda_I+d-n)}{\Gamma(k)}. \quad (26)$$

Using [20, Eq. 9.131.2] and the integration formulas in (22), we find that the probability of outage in the presence of N non-i.i.d. interferers is

$$P_{out} = \frac{(m\lambda)^\lambda}{\lambda\Gamma(\lambda_I)\Gamma(m)}y_{th}^\lambda \sum_{i=1}^{M}\sum_{k=1}^{mi}\frac{EL}{\lambda_I^{d+\lambda}m_i^\lambda}\frac{\Gamma(\lambda+d+\lambda_I)\Gamma(m-\lambda)}{B(\lambda,k)}$$
$${}_3F_2(\lambda+d+\lambda_I, \lambda+k, \lambda, \lambda-m+1, \lambda+1, \frac{m\lambda}{m_i\lambda_I\rho}y_{th})$$
$$+\frac{(m\lambda)^m}{m\Gamma(\lambda_I)\Gamma(\lambda)}y_{th}^m \sum_{i=1}^{M}\sum_{k=1}^{mi}\frac{EL}{\lambda_I^{d+m}m_i^m}\frac{\Gamma(m+d+\lambda_I)\Gamma(\lambda-m)}{B(m,k)}$$
$${}_3F_2(m+k, m+d+\lambda_I, m, m-\lambda+1, m+1, \frac{m\lambda}{m_i\lambda_I\rho}y_{th}). \quad (27)$$

As we have done in the case of i.i.d. interferers, the asymptotic representation of the outage probability [i.e., in the absence of shadowing $(\lambda = \lambda_I = \infty)$], is given by

$$P_{out} = \frac{(m\frac{y_{th}}{\rho})^m}{m}\sum_{i=1}^{M}\sum_{k=1}^{mi}\frac{EL}{m_i^m B(m,k)}, \quad \lambda = \lambda_I \to \infty, \quad \rho/y_{th} \gg 1. \quad (28)$$

IV. Numerical results

To gain a better understanding as to how the fading, shadowing and co-channel interference affect the outage probability, some plots are presented in this section for both i.i.d and non i.i.d. interferers. Without loss of generality, we assume identical shadowing statistics for both desired and interfering signals, which is a quite reasonable assumption [9]. In Fig. 1, we plot the SIR pdf for different numbers of co-channel interferers and for different levels of the desired user fading severity. As one can see from these numerical results, the number of co-channel interferers and the desired fading severity are dominant factors in determining the outage probability. Fig. 2 illustrates the effect of shadowing and co-channel interference on the probability of outage in severely faded microcellular environments $(m > m_I)$. We notice that the presence of six i.i.d. co-channel interferers highly increases the probability of outage compared to the case of only one co-channel interferer. To gain more insight into the effects of the shadowing, the curve without a marker reports the outage performance without shadowing $(\lambda \to \infty)$. A general observation is that shadowing degrades the probability of outage. Nevertheless, an important phenomenon to be noticed, for low SIR/y_{th}, is that more severe shadowing can cause lower outage probability. A similar observation has been noted for lognormal shadowing in [9]. In Fig. 3, we show plots of the outage probability in the case of non i.i.d interferers. The observations made before, in the case of i.i.d. co-channel interferers, are also valid in this case.

In Fig. 4, we observe that the system performance is insensitive to changes in the fading severity of interfering signals. This phenomenon demonstrates that the number of interferers and the shadowing spread have the most significant effect on the channel capacity. Therefore, the system performance is

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$$f_\gamma(y) = \frac{(\frac{m\lambda}{\lambda_I \rho})^\lambda}{\Gamma(\lambda_I)\Gamma(m)} y^{\lambda-1} \sum_{i=1}^{M} \sum_{k=1}^{mi} \frac{EL\Gamma(m+d+\lambda_I)}{m_i^\lambda \lambda_I^d} \frac{B(\lambda+d+\lambda_I, m+k)}{B(\lambda,k)} {}_2F_1(\lambda+d+\lambda_I, \lambda+k; m+\lambda+d+\lambda_I+k; 1-\frac{m\lambda}{\lambda_I m_i \rho} y). \tag{25}$$

predominantly affected by the fading parameter of the desired user rather than by the fading parameters of the interferers.

For the purpose of cell planning, it is useful to represent the outage probability as a function of the reuse distance. From (4), and assuming that the received noise power is P_N, we can write the SNR of the desired user, $\Gamma = \Omega/P_N$, as

$$\Gamma = \frac{P_t}{P_N} C(d_r/d)^\alpha \sqrt{\frac{1+\lambda}{\lambda}}. \tag{29}$$

A similar expression is obtained for the interfering signals assuming that all interferers are at a distance d_I from the receiver. The average SIR ρ is consequently shown to be given by

$$\rho = \left(\frac{d_I}{d}\right)^\alpha \sqrt{\frac{(1+\lambda)\lambda_I}{(1+\lambda_I)\lambda}}, \tag{30}$$

where we assume that both the desired and the interfering signals have the same transmit power. The reuse distance, also known as the co-channel reduction factor, is defined as

$$R = \frac{D}{R_c}, \tag{31}$$

where D is the distance between the centers of the nearest neighboring co-channel cells and R_c is the cell radius. Hence, when $\lambda = \lambda_I$, we get $R = \rho^{1/\alpha}$. Fig. 5 shows the probability of outage versus the reuse distance R. As expected, the minimum reuse distance is an increasing function of the number of interferers and the protection ratio. However, the outage performance as a function of the reuse distance seems to be almost the same for i.i.d. or non i.i.d. interferers.

V. CONCLUSION

A three-compound gamma Nakagami model has been considered to approximate a lognormal-shadowed Nakagami faded channel. Through this type of channel, a general outage analysis of wireless systems has been performed. Based on the compound model, we first derived a new closed-form for the probability density function of the signal to interference ratio in the presence of multiple independent interferers with identical or different fading parameters while assuming an equal mean power for all interfering signals. Based on the obtained SIR pdf, we then derived closed-form expressions for the outage probability. The numerical results highlight the notable effect that shadowing, fading and co-channel interference have on the aforementioned performance measure. In particular, the probability of outage is predominantly affected by the fading parameters of the desired user, rather than by the fading parameters of the interferers.

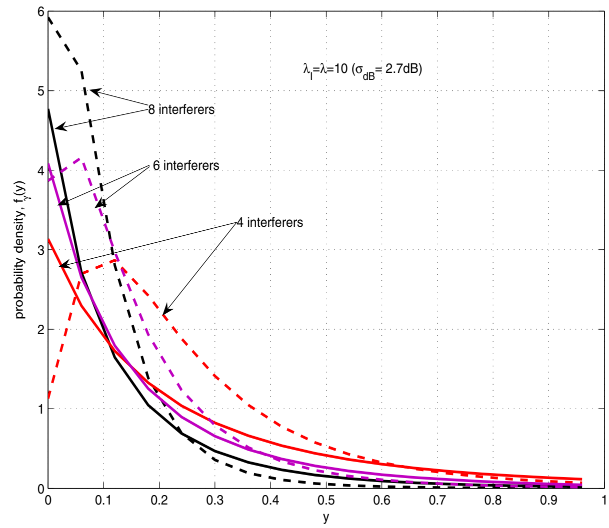

Fig. 1. SIR pdf for different values of the number of non i.i.d. co-channel interferers, for a severely faded desired user ($m = 1$: solid lines) and for a lightly faded desired user ($m = 4$: dashed lines).

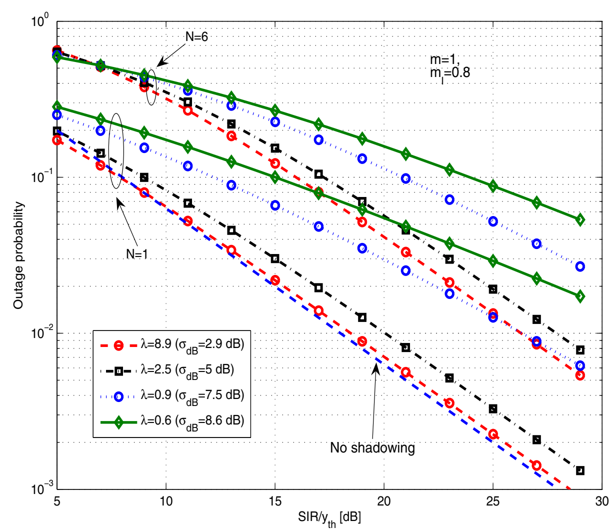

Fig. 2. Outage probability versus the inverse normalized threshold SIR/y_{th} for different shadowing scenarios over a severely faded channel in the presence of $N = 1$ and $N = 6$ i.i.d. co-channel interferers.

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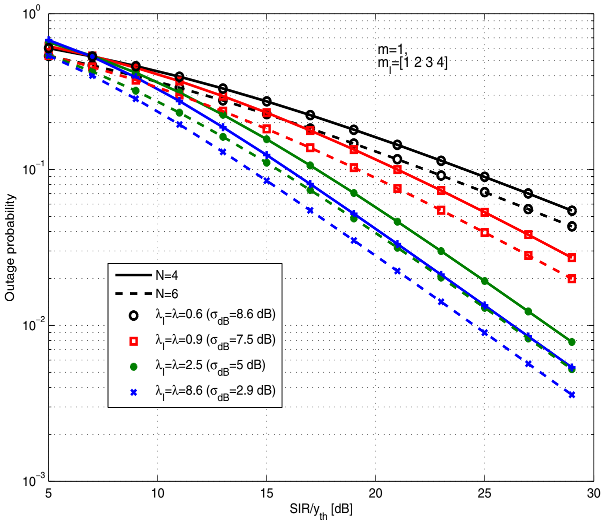

Fig. 3. Outage probability versus the inverse normalized threshold SIR/y_{th} for different shadowing scenarios over a severely faded channel in the presence of $N = 4$ and $N = 6$ non i.i.d. co-channel interferers.

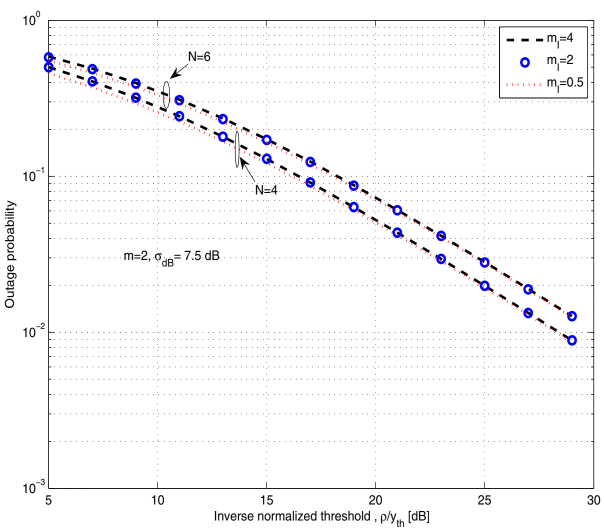

Fig. 4. Outage probability versus the inverse normalized threshold SIR/y_{th} for different interference fading parameters over a severely shadowed channel in the presence of $N = 4$ and $N = 6$ i.i.d. co-channel interferers.

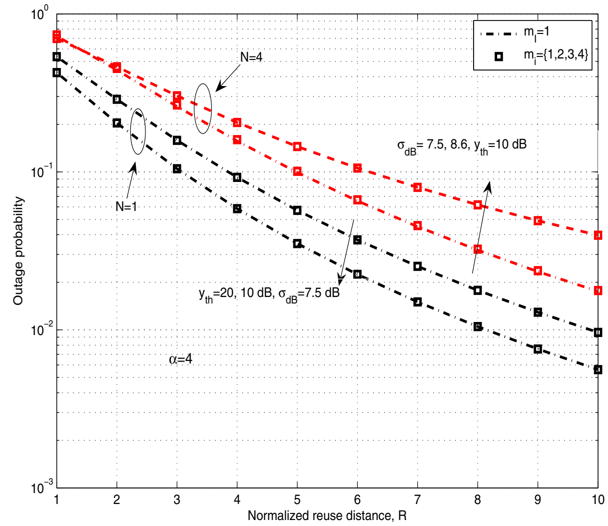

Fig. 5. Outage probability versus the normalized reuse distance for a shadowed Rayleigh faded desired signal and different interfering scenarios.

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