On the Level Crossing Rate and Average Fade Duration of Composite Multipath/Shadowing Channels

Imene Trigui*, Amine Laourine†, Sofiene Affes* and Alex Stéphenne‡
*INRS-EMT, 800, de la Gauchetière Ouest, Bureau 6900, Montreal, H5A 1K6
†Cornell University, School of Electrical and Computer Engeneering, Ithaca, NY
‡Ericsson Canada, 8400, Decarie Blvd, Montreal, H4P 2N2

***Abstract*—In this paper, we consider composite mulipath/shadowing fading channels characterized by Nakagami-m fading with Inverse Gaussian shadowing, and Rice fading with Inverse Gaussian shadowing. These channel models are very convenient for land mobile satellite systems and communications with low mobility or stationary users. From an analytical standpoint, one major advantage of these models is that they lead to closed-from expressions for the envelope probability density function (pdf) and the cumulative density function (CDF). Moreover, in this paper, we derive mathematically tractable new expressions for fundamental channel statistics such as the level crossing rate (LCR) and the average fade duration (AFD). Also, we validate our analytical results by providing several numerical examples showing the LCR and the AFD as a function of the Nakagami-m factor, Rice factor, and shadowing spread.**

I. INTRODUCTION

The statistical characterization of the received signal envelope, in terms of LCR and AFD, in fading and shadowing environments, is very useful in predicting the transmission performances of various modulation schemes. Moreover, the LCR and AFD can carry useful information about the burst error statistics [1], which facilitate the design and selection of error correction techniques [1], or adaptive modulation schemes.

In wireless communications, measurements have shown that the received signal envelope in outdoor microcellular environments is Rician-distributed, since a line-of-sight (LOS) propagation path is commonly present [2]. Rician fading also characterizes the propagation conditions for picocells, which are suitable models for indoor wireless communications. On the other hand, the Nakagami fading model has also been shown to fit well some urban multipath propagation data [3]. In addition to these fading-only models, the received signal envelope may suffer from shadowing which is shown to be log-normally distributed [4]. In such settings, the receiver is subject to the composite multipath/shadowing signal [5]. The resulting composite probability density function (pdf) is unfortunately not in closed form, thereby making the performance evaluation of communication links over these channels cumbersome. Recently, the Inverse Gaussian distribution was proposed as a substitute for the log-normal one [6]. The authors proved that a composite Rayleigh-Inverse Gaussian distribution approximates accurately the Suzuki distribution [7]. The Nakagami-Inverse Gaussian (NIG) distribution (also called G-distribution) was first proposed in [8] in the context of Synthetic Aperture Radar (SAR) image modeling. The Rice-Inverse Gaussian (RIG) model was recently proposed in [9] as a new model for a non-Rayleigh signal amplitude.

So far, expressions for the LCR and AFD have been derived for fading-only models, mainly Rayleigh, Rician and Nakagami channels. In [10,11], the LCR and AFD for Rice-lognormal and Nakagami-lognormal models have also been analyzed. Nevertheless, as said before, performance evaluation over these channels is rather challenging. In this paper, we consider the Nakagami- and Rice-Inverse Gaussian models. For these two channel types, we derive new expressions for the LCR and AFD of the envelope. For the envelope LCR computation, we first have to derive the joint pdf of the Inverse Gaussian (IG) process and its derivative. Once the LCR is obtained, we derive analytical expressions for the cumulative distribution function of the discussed models to get the respective AFD expressions.

The remainder of this paper is organized as follows. In section II, we define the LCR and the AFD of composite channels. In section III, we present the joint pdf of the signal envelope and its derivative for Inverse Gaussian shadowing with Nakagami-m and Rice fading (i.e., NIG and RIG) channels. In sections IV and V, we derive new general expressions for the LCR and the AFD in both cases. Section VI provides some selected numerical results. Section VII concludes the paper while summarizing the main results.

II. DEFINING THE LCR AND AFD OF COMPOSITE CHANNELS

Over a composite multipath/shadowing channel, the envelope $R(t)$ can be modeled as the product of two independent random processes: a fading process $X(t)$ and a shadowing process $Y(t)$. The envelope LCR determines how often the received signal envelope crosses some threshold, usually in a positive-going direction. The LCR is given by [12, eq.(2.106)]:

$$L(R) = \int_0^\infty \dot{r} p_{R\dot{R}}(R, \dot{r}) d\dot{r}, \qquad R \geq 0, \tag{1}$$

978-1-4244-2324-8/08/$25.00 © 2008 IEEE.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2008 proceedings.

where $p_{R\dot{R}}(r,\dot{r})$ is the joint pdf of $R(t)$ and $\dot{R}(t) = dR(t)/dt$, and is given by [13]:

$$p_{R\dot{R}}(r,\dot{r}) = \int_0^\infty \frac{1}{y^2}\left\{\int_{-\infty}^{+\infty} p_{X\dot{X}}(\tfrac{r}{y}, \tfrac{\dot{r}}{y} - \tfrac{\dot{y}r}{y^2}) p_{Y\dot{Y}}(y,\dot{y})d\dot{y}\right\} dy, \quad r \geq 0, -\infty \leq \dot{r} \leq \infty. \tag{2}$$

Similarly, the envelope AFD is the average time when the envelope process is below a given threshold R_{th}. The envelope AFD is given by:

$$A(R_{th}) = \frac{F(R_{th})}{L(R_{th})}, \tag{3}$$

where $F(R) = \int_0^R p(r)dr$ is the cumulative distribution function CDF of the process $R(t)$.

III. Derivation of $p_{X\dot{X}}$ and $p_{Y\dot{Y}}$

A. Derivation of $p_{X\dot{X}}$

1) Nakagami-m Fading: When the fading process $X(t)$ is Nakagami-m distributed, $p_{X\dot{X}}$ is given in [14] as:

$$p_{X\dot{X}}(x,\dot{x}) = \frac{4(\frac{m}{\Omega})^{m+0.5}}{\sqrt{2\pi|\rho''(0)|}\Gamma(m)} x^{2m-1} e^{(-\frac{2m\dot{x}^2}{\Omega|\rho''(0)|} - m\frac{x^2}{\Omega})}, \quad x \geq 0, -\infty \leq \dot{x} \leq \infty. \tag{4}$$

In (4), $\Gamma(\cdot)$ is the Gamma function, $\Omega = E[x^2]$, and m is the fading severity parameter ($m \geq 0.5$). $\rho(\tau)$ is the normalized autocorrelation function of $X(t)$, and $\rho''(\tau)$ denotes the second derivative with respect to τ.

2) Rice Fading: Assuming a symmetrical power spectral density of X, the joint pdf $p_{X\dot{X}}(x,\dot{x})$ of a Rician fading process can be derived as [9]:

$$p_{X\dot{X}}(x,\dot{x}) = \frac{x}{\Omega\sqrt{2\pi|\rho''(0)|\Omega}} e^{-\frac{x^2+\beta^2}{2\Omega}} I_0(\frac{\beta x}{\sqrt{\Omega}}) e^{-\frac{\dot{x}^2}{2\Omega|\rho''(0)|}}. \tag{5}$$

In (5), $\beta^2/2$ is the Rice factor and $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. Note that when the LOS component is absent ($\beta = 0$), equation (5) reduces to the well known Rayleigh pdf.

For equal-amplitude incoming multipath waves with independent phases, the correlation function is given by:

$$\rho(\tau) = J_0(2\pi f_{max}\tau), \tag{6}$$

where $J_0(.)$ is the zero-order Bessel function and f_{max} is the maximum Doppler shift. The corresponding correlation coefficients are calculated as:

$$\begin{aligned} &\rho(0) = 1, \\ &\rho''(0) = -2(\pi f_{max})^2 = -\zeta^2. \end{aligned} \tag{7}$$

B. Derivation of $p_{Y\dot{Y}}$

For an IG process $Y(t)$, to the best of our knowledge, the joint pdf $p_{Y,\dot{Y}}(y,\dot{y})$ is not available in the literature. Let $Y(t)$ be a stationary IG process and denote by $\psi(\tau)$ its normalized underlying covariance function. The bivariate density of this process is, after some manipulations, given by [15]:

$$\begin{aligned} &p_{Y_1Y_2}(y_1,y_2) = \frac{\lambda}{4\pi}\sqrt{\frac{1}{(1-\psi^2)y_1^3y_2^3}} \\ &\{e^{\frac{-\lambda}{2\theta^2(1-\psi^2)}[\frac{y_1-\theta}{\sqrt{y_1}} - \frac{y_2-\theta}{\sqrt{y_2}}]^2 - \frac{\lambda}{\theta^2(1+\psi)}\frac{(y_1-\theta)(y_2-\theta)}{\sqrt{y_1y_2}}} \\ &+ e^{\frac{-\lambda}{2\theta^2(1-\psi^2)}[\frac{y_1-\theta}{\sqrt{y_1}} - \frac{y_2-\theta}{\sqrt{y_2}}]^2 - \frac{\lambda}{\theta^2(1-\psi)}\frac{(y_1-\theta)(y_2-\theta)}{\sqrt{y_1y_2}}}\}, \end{aligned} \tag{8}$$

where θ and λ are, respectively, the mean and the scale parameters. This definition, proposed by Kocherlakota [15], generalizes and corrects the pdf introduced by Wasan [16] in 1986.

For the shadowing process, the covariance function $\psi(\tau)$ is commonly assumed to follow a Gaussian shape [9]:

$$\psi(\tau) = e^{-2(\pi\sigma_s\tau)^2}, \tag{9}$$

where σ_s is related to the 3-dB cutoff frequency f_s according to $f_s = \sigma_s\sqrt{2\ln 2}$. Note that f_s is, in general, much smaller than the maximum Doppler frequency f_{max}. It is therefore convenient to introduce the ratio $n = f_{max}/f_s$.

To compute $P_{Y\dot{Y}}$, we follow the strategy in [17]

$$p_{Y\dot{Y}}(y,\dot{y}) = \lim_{\tau\to 0} \tau p_{Y_1Y_2}(y_1 = y - \dot{y}\tau/2, y_2 = y + \dot{y}\tau/2). \tag{10}$$

As τ approaches 0, we have

$$\begin{aligned} &\psi(\tau) \approx 1 + \frac{\psi''(0)\tau^2}{2} + O(\tau^3) \\ &1 - \psi^2(\tau) \approx -\psi''(0)\tau^2 + O(\tau^3), \end{aligned} \tag{11}$$

where, from (9), we obtain:

$$\psi(0) = 1, \quad \text{and} \quad \psi''(0) = -(2\pi\sigma_s)^2. \tag{12}$$

Since $\psi''(0) \leq 0$, we obtain:

$$\lim_{\tau\to 0} e^{-\frac{\lambda}{\theta^2(1-\psi)}\frac{(y_1-\theta)(y_2-\theta)}{\sqrt{y_1y_2}}} = 0. \tag{13}$$

Consequently,

$$p_{Y\dot{Y}}(y,\dot{y}) = \lim_{\tau\to 0}\frac{\lambda}{4\pi}\frac{\tau}{\sqrt{1-\psi^2}} e^{\frac{\lambda}{8\theta^2\psi''(0)}\frac{\dot{y}^2}{y^3}(y+\theta)^2 - \frac{\lambda}{\theta^2}\frac{(y-\theta)^2}{y}}. \tag{14}$$

Finally, using the fact that $\lim_{\tau\to 0}\frac{\tau}{\sqrt{1-\psi^2}} = \frac{1}{\sqrt{-\psi''(0)}}$, we get:

$$p_{Y\dot{Y}}(y,\dot{y}) = \frac{\lambda}{4\pi\sqrt{\delta}}\frac{1}{y^3} e^{(-\frac{\lambda\dot{y}^2(y+\theta)^2}{8\theta^2\delta y^3} - \frac{\lambda}{\theta^2}\frac{(y-\theta)^2}{y})}, \tag{15}$$

where $\delta = -\psi''(0)$.

IV. Derivation of the Envelope LCR

A. Envelope LCR in Nakagami-m fading Inverse Gaussian Shadowing (NIG) channels

By substituting (4) and (15) in (2), the LCR expression for NIG channels is obtained after some simplifications using [18: eq. 3.323.2, 3.461.3] as:

$$\begin{aligned} L(R) = &\frac{\theta\sqrt{\lambda}\Omega\zeta}{2\pi\Gamma(m)}(\frac{m}{\Omega})^{m-\frac{1}{2}} R^{2m-1}\int_0^{+\infty}\frac{1}{y^{2m+1}(y+\theta)^2} \\ &e^{-\frac{\lambda}{\theta^2}\frac{(y-\theta)^2}{y} - m\frac{R^2}{\Omega y^2}}\sqrt{(y+\theta)^2 y + \frac{16m\theta^2R^2}{\lambda n_c\Omega}} \quad dy, \end{aligned} \tag{16}$$

where $n_c = \frac{n^2}{\ln(2)}$. A closed-form expression of (16) is rather tedious to find. However, it can be readily expressed by a series expansion based on Gauss-Hermite integration [19]. Thus, using a change of variable $z = \frac{\lambda}{\theta^2}y$, the LCR in NIG channels can be written as:

$$L(R) = e^{\frac{2\lambda}{\theta}} \frac{(\frac{\lambda}{\theta^2})^2\Omega\zeta}{2\pi\Gamma(m)} (\frac{m}{\Omega})^{m-\frac{1}{2}} R^{2m-1} \sum_{n=1}^{N} \frac{w_n}{x_n^{2m+1}(x_n+\frac{\lambda}{\theta})^2} e^{-\frac{\lambda^2}{\theta^2 x_n} - m\frac{R^2\lambda^2}{\Omega\theta^4 x_n^2}} \sqrt{(x_n+\frac{\lambda}{\theta})^2 x_n + \frac{16m\lambda^2R^2}{\theta^4 n_c\Omega}} + R_N, \quad (17)$$

where N is the Hermite integration order and R_N is a remainder term that decreases as N increases. In (17), the weights, w_n, and the abscissas, x_n, for N up to 20 are tabulated in [19, Tbl. 25.10]. From (17), we define the Gauss-Hermite representation of the LCR in NIG channels by removing the remainder term R_N.

B. Envelope LCR in Rice fading Inverse Gaussian shadowing (RIG) channels

By substituting (5) and (15) in (2), the LCR expression in RIG channels will be given by:

$$L(R) = \frac{e^{-\frac{\beta^2}{2\Omega}}\theta\sqrt{\lambda\Omega}\zeta}{2\pi} R \int_0^{+\infty} \frac{1}{y^3(y+\theta)^2} e^{-\frac{\lambda}{\theta^2}\frac{(y-\theta)^2}{y} - \frac{R^2}{2y^2\Omega}} \sqrt{\frac{4\theta^2R^2}{\lambda n_c\Omega} + (y+\theta)^2 y} \quad I_0(\frac{R\beta}{y}) \quad dy. \quad (18)$$

Similarly, the LCR above can be accurately calculated using the Laguerre-Gauss Quadrature method.
Substituting $\zeta = \sqrt{2}\pi f_{max}$, (16) and (18) can be normalized with respect to f_{max}, and the formulas become independent of the terminal's velocity.

V. Derivation of the envelope AFD

A. AFD in Nakagami-m fading Inverse Gaussian shadowing (NIG) channels

In order to compute the AFD, we need to derive the CDF of the envelope process. For the NIG model, the envelope pdf $p(r)$ is given by [20]:

$$p(r) = (\sqrt{\frac{\lambda}{2\theta^2}})^{m+\frac{1}{2}} \frac{\sqrt{\frac{\lambda}{2\pi}} 4m^m e^{\frac{\lambda}{\theta}} r^{2m-1}}{\Gamma(m)} \frac{K_{m+\frac{1}{2}}(\sqrt{\frac{2\lambda}{\theta^2}}\sqrt{mr^2+\frac{\lambda}{2}})}{(\sqrt{mr^2+\frac{\lambda}{2}})^{m+\frac{1}{2}}}, \quad (19)$$

where $K_\nu(\cdot)$ is the modified Bessel function of the second kind and order ν [19]. Using the change of variable $x = r^2$, we find that the CDF of a NIG process can be expressed as:

$$F(R) = \frac{A}{2}\left[J_{m,m}(R^2, m, \frac{\lambda}{2}) - J_{m,m}(0, m, \frac{\lambda}{2})\right], \quad (20)$$

where

$$J_{p,q}(x,y,z) = \int x^{q-1} \frac{K_{p+\frac{1}{2}}(b\sqrt{xy+z})}{(\sqrt{xy+z})^{p+\frac{1}{2}}} dx, \quad (21)$$

$A = (\sqrt{\frac{\lambda}{2\theta^2}})^{m+\frac{1}{2}} \frac{\sqrt{\frac{\lambda}{2\pi}} 4m^m e^{\frac{\lambda}{\theta}}}{\Gamma(m)}$ and $b = \sqrt{\frac{2\lambda}{\theta^2}}$.
Using the fact that

$$\frac{dx^{-s}K_s(x)}{dx} = -x^{-s}K_{s+1}(x), \quad (22)$$

and integrating by parts, we obtain:

$$J_{p,q}(x,y,z) = -(q-1)! \sum_{k=0}^{q} \frac{2^k x^{q-k}}{(yb)^k (q-k)!} \frac{K_{p-k+\frac{1}{2}}(b\sqrt{xy+z})}{(\sqrt{xy+z})^{p-k+\frac{1}{2}}}. \quad (23)$$

Consequently, the CDF of the envelope of a NIG process is given by:

$$F(R) = 1 - \frac{e^{\frac{\lambda}{\theta}} m^m \sqrt{\frac{\lambda}{\pi}} (\sqrt{\frac{\lambda}{\theta^2}})^{m+0.5}}{(\sqrt{2})^{m-0.5}} \sum_{k=0}^{m} \frac{2^k R^{2m-2k}}{(m\sqrt{\frac{2\lambda}{\theta^2}})^k (m-k)!} \frac{K_{m-k-\frac{1}{2}}(\sqrt{\frac{2\lambda}{\theta^2}}\sqrt{mR^2+\frac{\lambda}{2}})}{(\sqrt{mR^2+\frac{\lambda}{2}})^{m-k+\frac{1}{2}}}. \quad (24)$$

The AFD of a NIG process is finally obtained by substituting (24) and (16) in (3).

B. Envelope AFD in Rice fading Inverse Gaussian shadowing (RIG) channels

The pdf of the signal envelope in a RIG environment was recently introduced in [9] as:

$$p(r) = \sqrt{\frac{2}{\pi}}\chi^{3/2}\sqrt{\lambda}e^{\frac{\lambda}{\nu}} \frac{r}{(\sqrt{\lambda+r^2})^{3/2}} K_{3/2}(\chi\sqrt{\lambda+r^2}) I_0(\beta r), \quad (25)$$

where $\chi = (\sqrt{\frac{\lambda}{\theta^2}+\beta^2})$. Using an infinite series expansion of $I_0(\cdot)$, the CDF of the RIG signal envelope can be expressed as:

$$F(R) = C\sum_{k=0}^{\infty} \frac{(\frac{\beta}{2})^{2k}}{(k!)^2} \int_0^R \frac{r^{2k+1}}{(\sqrt{\lambda+r^2})^{3/2}} K_{3/2}(\chi\sqrt{\lambda+r^2}) dr, \quad (26)$$

with $C = \sqrt{\frac{2}{\pi}}\chi^{3/2}\sqrt{\lambda}e^{\frac{\lambda}{\nu}}$. The computation of this CDF requires the computation of the infinite series. To efficiently compute the series, we truncate it and derive an upper bound for the truncation error. Consequently, we write:

$$F(R) = F_{N_t}(R) + F_\infty(R), \quad (27)$$

where $F_{N_t}(R)$ is the expression in (27) with the infinite series truncated at the N_tth term (i.e., $k = N_t$) and $F_\infty(R)$ is the truncation error resulting from truncating the infinite series in (27). It can be easily seen that:

$$F_\infty(R) < \frac{C}{(N_t+1)!} \int_0^R \left[e^{(\frac{r\beta}{2})^2} - \sum_{k=0}^{N_t} \frac{(\frac{\beta r}{2})^{2k}}{(k!)}\right] \frac{r}{(\sqrt{\lambda+r^2})^{3/2}} K_{3/2}(\chi\sqrt{\lambda+r^2}) dr. \quad (28)$$

And so, an upper bound of $F_\infty(R)$ is given by:

$$F_{sup}(R) = \frac{C}{(N_t+1)!} g, \quad (29)$$

where, using a change of variable $z = r^2$, we have

$$g = \frac{1}{2}\int_0^{R^2} e^{\frac{\beta^2}{4}z} \frac{K_{3/2}(\chi\sqrt{\lambda+z})}{(\sqrt{\lambda+z})^{3/2}} dz. \quad (30)$$

Integrating by parts and using [18, eq. 2.33], we find that

$$g = \frac{1}{\chi}\left[\frac{K_{1/2}(\chi\sqrt{\lambda+R^2})}{(\sqrt{\lambda+R^2})^{1/2}} + \frac{K_{1/2}(\chi\sqrt{\lambda})}{(\sqrt{\lambda})^{1/2}}\right] + \frac{\pi}{\beta\sqrt{2\chi}} e^{-\frac{\beta^2}{4}\lambda} \left[e^{-\frac{(\lambda+R^2)^2}{\beta^2}} erfi(\sqrt{\frac{\beta^2}{4}(\lambda+R^2)} + \frac{\chi}{\beta}) - erfi(\frac{\chi}{\beta})\right], \quad (31)$$

where $erfi(\cdot)$ is the imaginary error function. Since g is not a function of N_t, $F_{sup}(R)$ will tend to zero as N_t increases, showing that the remainder of the series will converge to zero. Consequently, using (24), the envelope CDF for a RIG process is given by:

$$F(R) \approx C \sum_{k=0}^{N_t} \frac{(\frac{\beta}{2})^{2k}}{k!} \left[\frac{2^k}{\chi^{k+1}} \frac{K_{k-\frac{1}{2}}(\chi\sqrt{\lambda})}{(\sqrt{\lambda})^{1/2-k}} - \sum_{j=1}^{k+1} \frac{2^{j-1} R^{2(k+2-j)}}{\chi^j (k+1-j)!} \frac{K_{\frac{3}{2}-j}(\chi\sqrt{\lambda+R^2})}{(\sqrt{\lambda+R^2})^{\frac{3}{2}-j}} \right]. \tag{32}$$

The envelope AFD of a RIG process is obtained by substituting (32) and (18) in (3).

VI. Simulation Results

The following analysis is conducted in two different shadowing scenario according to Loo's model (c.f. [5] and references therein), namely, infrequent light shadowing and frequent heavy shadowing. The corresponding mean and standard deviation of the lognormal shadowing are equal, respectively, to ($\mu = 0.115$, $\sigma = 0.115$) and ($\mu = -3.9$, $\sigma = 0.8$). Other values are considered to show that our approach holds in more general cases. The corresponding Inverse Gaussian mean θ and scalar shape parameter λ are obtained using a moment matching technique given by:

$$\lambda = \frac{e^{\mu}}{2\sinh(\frac{\sigma^2}{2})}, \quad \text{and} \quad \theta = e^{\mu + \frac{\sigma^2}{2}}. \tag{33}$$

Note that, for all the following simulations, the Laguerre-Gauss Quadrature method was employed with $N = 15$.

A. Analysis in Nakagami-m Inverse Gaussian (NIG) channels

Figure 1 shows the LCR envelope, normalized to the maximum Doppler shift f_{max}, of a NIG process, for different values of m, in frequent heavy and infrequent light shadowing environments. The LCR in (16) was evaluated using the software Mathematica. The obtained results were superimposed to those obtained using the Laguerre-Gauss Quadrature method [19]. With $N = 15$, it can be easily seen that the Laguerre-Gauss Quadrature method provides a good approximation of the LCR expression in (16). According to Figure 1, we observe that a smaller m or a larger σ (heavier shadowing) leads to a larger LCR. This indicates that when the instantaneous signal is exposed to a heavier fading or shadowing, the envelope crosses the threshold R_{th} more frequently. We see also that the effect of the variation of m on the LCR is more pronounced compared to the effect of σ. Indeed, since m characterizes the fast fading of the instantaneous signal envelope $X(t)$, it has a more noticeable effect on the LCR compared to σ, which only characterizes the spread of the slowly varying shadowing component $Y(t)$.
In Figure 2, we show the AFD normalized to f_{max} for NIG channels. We observe that the effect of m and σ is more pronounced for $R_{th} \geq 0$ dB.

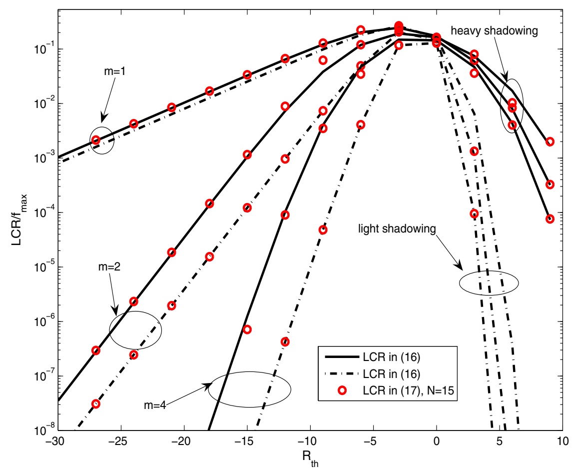

Fig. 1. Envelope LCR for NIG channels in heavy shadowing ($\mu = -3.9$, $\sigma = 0.8$) and infrequent light shadowing ($\mu = 0.115$, $\sigma = 0.115$) environments, $n_c = 10, \Omega = 1$.

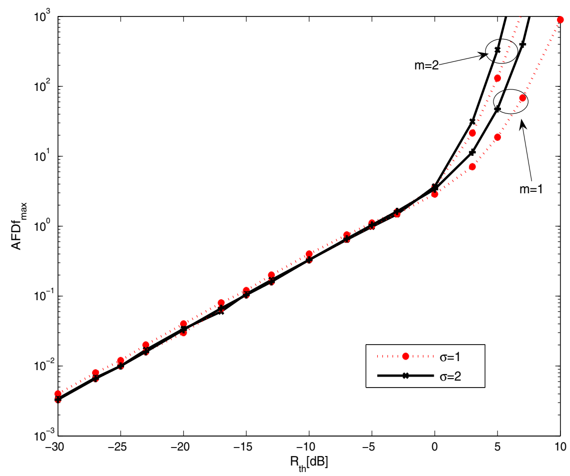

Fig. 2. Envelope AFD for NIG channels, $n_c = 10, \Omega = 1$.

B. Analysis in Rice Inverse Gaussian (RIG) channels

Figure 3 shows the envelope LCR in frequent heavy and infrequent light shadowing environments with different values of β. As expected, it can be noted that the presence of a strong LOS component has more effect when the shadowing is heavy. In Figure 4, in order to verify the accuracy of the series truncation, we have plotted the CDF in (26) and (32). It can be seen there that the effect of the error truncation is negligible. In Figure 5, using the CDF values obtained form (32), we calculate the envelope AFD of a RIG process as a function of the shadowing spread. For $R_{th} < 0$ dB, we observe a larger AFD for more severely shadowed signals. This is as expected because, with heavier shadowing, the received signal is severely attenuated and tends to drop below the receiver threshold more frequently.

VII. Conclusion

Composite multipath/shadowing fading environments are frequently encountered in several propagation scenarios. In

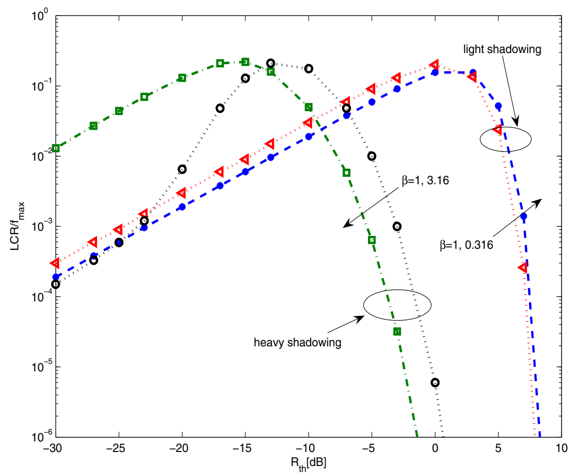

Fig. 3. Envelope LCR for a RIG process in infrequent light shadowing ($\mu = 0.115$, $\sigma = 0.115$) and frequent heavy shadowing ($\mu = -3.9$, $\sigma = 0.8$) channels, $n_c = 10, \Omega = 1$.

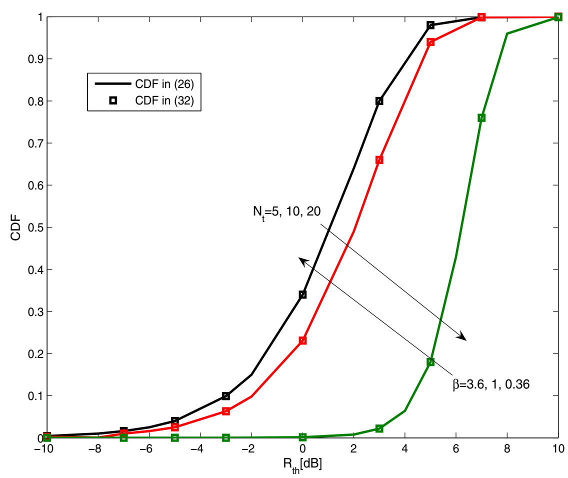

Fig. 4. CDF of a RIG process in infrequent light shadowing environment.

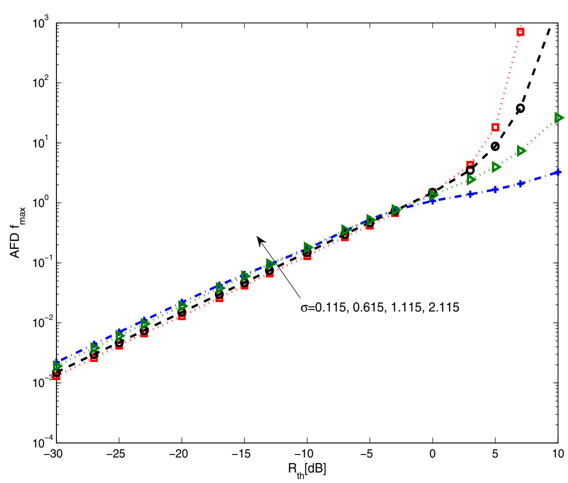

Fig. 5. Envelope AFD of a RIG process for various shadowing spread values, $n_c = 10, \Omega = 1, \beta = 1$.

this paper, we have studied two particular composite channels, namely, the Nakagami-m Inverse Gaussian channel and the Rice Inverse Gaussian channel. We have derived expressions for the joint pdf of the Inverse Gaussian process and its derivative, the envelope level crossing rate, and the average fade duration for both the aforementioned channel models. We also validated the proposed formulas by providing numerical results with various shadowing strength, Nakagami-m and Rice factors.

REFERENCES

[1] J. M Morris and J. L. Chang, "Busrt error statistics of simulated Viterbi decoded BFSK and high-rate punctured codes on fading and scintillating channels," *IEEE Trans. Commun.,* vol. 43, no. 1, pp. 695-700, Feb. 1995.

[2] S. Tsai, "Markov characterization of the HF channel", *IEEE Trans. on Communication,* vol. 17, no. 1, pp. 24-32, Feb. 1969.

[3] M.J. Ho and G.L. Struber, "Co-channel interference of microcellular systems on shadowed Nakagami fading channels," *Proc. IEEE Veh. Technol. Conf. (VTC93),* Secaucus, NJ, pp. 568-571, May 1993.

[4] F. Hansen and F.I. Mano, "Mobile fading-Rayleigh and lognormal superimposed," *IEEE Trans. Vehic. Tech.,* vol. 26, no. 4, pp. 332-335, Nov. 1977.

[5] A. Abdi, W.C. Lau, M.-S. Alouini, and M. Kaveh, "A new simple model for land mobile satellite channels: First and second order statistics," *IEEE Trans. on Wireless Commun.,* vol. 2, no. 2, pp. 519-528, May 2003.

[6] Karmeshu and R. Agrawal, "On efficacy of Rayleigh-inverse Gaussian distribution over K-distribution for wireless fading channels," *Wireless Communications and Mobile Computing,* vol. 7, no. 1, pp. 1-7, Jan. 2007.

[7] H. Suzuki, "A statistical model for urban multipath propagation," *IEEE Trans. Commun.,* vol. 25, no. 4, pp. 673-680, July 1970.

[8] A.C. Frery, H.-J. Muller, C. C. F. Yanasse, and S. J. S. SantAnna, "A model for extremely heterogeneous clutter," *IEEE Trans. on Geoscience and Remote Sensing,* vol. 35, no. 3, pp. 648-659, May 1997.

[9] T. Eltoft, "The Rician inverse Gaussian distribution: a new model for non Rayleigh signal amplitude statistics," *IEEE Trans. on Image Processing,* vol. 14, no. 11, pp. 1722-1735, Nov. 2005.

[10] M. Patzold, U. Killat, and F. Laue, "An extended Suzuki model for land mobile satellite channels and its statisticals properties," *IEEE Trans. on Vehicular Tech.,* vol. 47, no. 4, pp. 617-630, May 1998.

[11] T. Tjhung and C. Chai, "Fade statistics in Nakagami-Lognormal channels," *IEEE Trans. on Communication,* vol. 47, no. 12, pp. 1769-1772, Dec. 1999.

[12] G. L. Stuber, *Principles of Mobile Communications.* Boston, MA: Kluwer, 1996.

[13] A. Krantzik and D. Wolf, "Distribution of the fading intervals of modified Suzuki process," in *Signal Processing V: Theories and Applications,* L. Torres, E. Masgrau, and M. A. Lagunas, Eds. Amsterdam, The Netherlands: Elsevier Sciences, pp. 361-364, 1990.

[14] M. D. Yacoub, "The α-μ distribution: a physical fading model for the Stacy distribution," *IEEE Trans. on Vehicular Tech.,* vol. 56, no. 1, Jan. 2007.

[15] S. Kocherlakota, "The bivariate inverse Gaussian distribution: An introduction," *Communications in Statistics Theory and Method*, pp. 1081-1112, 1986.

[16] M. L. Wasan, *On the inverse Gaussian process.* Skand. Actuar. vol. 51, pp. 69-96, 1968.

[17] L. Primak and V. Ya. Kontorovich, "On the second order statistics of generalized Gamma process," *Proc. Personal Indoor and Mobile Radio Communications (PIMRC'06)*, pp. 1-5, 2006.

[18] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products,* 5th ed. San Diego, CA: Academic, 1994.

[19] M. Abramowitz, I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, Mathematical Tables.* New York: Dover, 1968.

[20] A. Laourine, M.-S. Alouini, S. Affes, and A. Stéphenne, "On the Performance Analysis of Composite Multipath/Shadowing Channels Using the G-distribution", Proc. of IEEE ICC'2008, Beijing, China, pp. 1333-1338, May 19-23, 2008.