ML-BASED JOINT ESTIMATION OF FREQUENCY AND SAMPLING CLOCK OFFSETS FOR OFDM SYSTEMS

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ABSTRACT

In this paper, we present a joint algorithm to estimate the fine symbol timing and carrier frequency offsets of wireless orthogonal frequency division multiplexing (OFDM) signals. To jointly estimate synchronization parameters using the maximum likelihood (ML) criterion, we propose to transmit a special pilot symbol. By using a periodic training sequence, we convert the problem of obtaining the ML solution from searching exhaustively over the entire uncertainty range to that of solving a polynomial, thereby greatly reducing the computational load. With the proposed orthogonal and periodic training sequence, we obtain a closed-form expression for the synchronization parameters, hence greatly simplifying the algorithm complexity. Simulations demonstrate that the joint estimation method provides better accuracy than existing joint and separate sampling clock and carrier frequency offsets estimation algorithms.

1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) [1] has gained increased interest in the last few years due to its spectral efficiency and robustness to multipath channels. A shortcoming of OFDM systems is their sensitivity to the carrier frequency offset (CFO) [2] and the sampling clock offset (SCO) [3]. As the subcarriers are closely spaced over the channel bandwidth, the frequency offset must be kept within a small fraction of the subcarrier distance to avoid severe bit error rate degradations. Moreover, as soon as the number of samples per OFDM symbol (or equivalently the number of sub-carriers) becomes large, the frequency offset between the transmitter's sampling clock and the receiver's sampling clock has to be considered too. For example, for the wireless digital video broadcasting system DVB-T [4], which uses a high number of subcarriers, the sampling clock offset is known to be an important issue [5].

Both carrier frequency and sampling clock offsets destroy the orthogonality between the subcarriers which will generate inter carrier interference (ICI). The ICI behaves like an additional noise leading to system performance degradation. There have been many carrier frequency offset estimation schemes for OFDM signals (e.g., [6]-[9]). These systems assume perfect timing synchronization. However, in practice, this is generally not a valid assumption, because sampling clock and carrier frequency offsets are inseparable in received OFDM signals. Moreover, research devoted to the estimation of the sampling clock offset is rather scarce [10]. Joint estimation algorithms, in general, are more difficult to design than separate estimation algorithms. Hence, algorithms devoted to joint frequency offset and sampling clock offset estimation are very few [10].

Data-aided algorithms using training sequences (TS) and pilot symbols are usually deployed to achieve high estimation accuracy [6]-[9]. In this paper, a special pilot symbol is designed to jointly estimate both sampling clock and carrier frequency offsets over multipath channels using an ML-based approach. The proposed synchronization technique can achieve more accurate results than existing joint estimators under the same channel model conditions, while its computational complexity is drastically reduced due to the use of a periodic [9] or a periodic and orthogonal TS. This approach is very attractive in terms of enhancing estimation accuracy without introducing high computational complexity.

The rest of this paper is organized as follows. We introduce the OFDM signal model over multipath channels in Section 2. The ML-based estimation is then explained in detail in Section 3, where two different methods for ML resolution are allowed thanks to the use of a specially structured training sequence. Simulation results are provided in Section 4. Finally, we conclude our paper in Section 5.

2. SYSTEM MODEL

We consider an OFDM system with N subcarriers operating over a multipath channel h spanning L_h samples. For the sake of clarity, we adopt the following simplifying notation

$$x(u) = x[uT_e], \forall u \in \mathbb{R}, \tag{1}$$

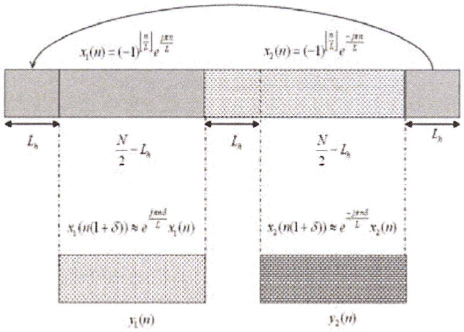

Fig. 1. Pilot symbol structure.

where $x[t]$ is the transmitted signal and T_e is the sampling period at the receiver. In the presence of the CFO and the SCO, the sampled received signal is given by

$$y(n) = e^{j2\pi\epsilon(1+\delta)n} \sum_{l=0}^{L_h-1} h_l x(n(1+\delta) - l) + v(n), \quad (2)$$

where δ is the relative sampling frequency offset normalized by the sampling frequency interval. The frequency offset is given by $\epsilon = \epsilon_n/N$, where ϵ_n is the relative carrier frequency offset normalized by the inter-carrier spacing and $v(n)$ is a zero-mean complex Gaussian noise. The relative sampling clock offset, δ, is usually less than 10^{-4}. We can therefore ignore the term $2\pi\epsilon\delta n$. Unless an iterative estimation is performed, the joint estimation of ϵ and δ is unpractical. However, an iterative estimation seems to be a suboptimal method since it depends on the initialization values and the exit condition. In this paper, we propose to transmit a special pilot symbol, depicted in figure 1, which will allow to jointly estimate the two synchronization parameters.

In figure 1, x_1 and x_2 are two periodic signals with period L selected to be a power of 2. When L divides L_h, which is an assumption made in this paper [1], x_1 and x_2 are composed of $r = \frac{N-2L_h}{2L}$ identical sub-blocks of length L. The rationale behind adopting this sequence structure is the fact that since $N\delta \ll 1$, we can write

$$x_{\{1,2\}}(n(1+\delta)) \approx e^{\pm j\pi\frac{n\delta}{L}} x_{\{1,2\}}(n). \quad (3)$$

At the receiver, y_1 and y_2 must have the same length. Thus, beginning from the first sample, all samples preceded by $L_h - 1$ precursors and not forming a set of L_h/L identical sub-blocks, are discarded. Consequently, the desired received signal is given by:

$$y_1(n) = e^{j2\pi(\epsilon+\frac{\delta}{2L})n} \sum_{l=0}^{L_h-1} h_l x_1(n-l) + v_1(n), \quad n = 0, 1, ..., rL-1, \quad (4)$$

and

$$y_2(n) = e^{j2\pi(\epsilon-\frac{\delta}{2L})n} \sum_{l=0}^{L_h-1} h_l x_2(n-l) + v_2(n), \quad n = rL + L_h, ..., N - L_h - 1. \quad (5)$$

The received signals y_1 and y_2 will be used to estimate, respectively, $\alpha_1 = \epsilon + \frac{\delta}{2L}$ and $\alpha_2 = \epsilon - \frac{\delta}{2L}$. It is straightforward to show that

$$\hat{\epsilon} = \frac{\hat{\alpha_1} + \hat{\alpha_2}}{2}, \quad (6)$$

and

$$\hat{\delta} = L(\hat{\alpha_1} - \hat{\alpha_2}), \quad (7)$$

where variables with a circumflex accent indicate estimates. To guarantee a high accuracy, we choose the ML approach as an estimation method.

3. ML-BASED JOINT ESTIMATION

At the receiver, both y_1 and y_2 will undergo an ML processing to respectively estimate α_1 and α_2. To avoid repetition, we confound notations related to the first and second part of the transmitted symbol and all variables related to them, by simply omitting to write the index 1 or 2. Thus, the received signals (equations (3) and (4)) can be written in a matrix form as

$$\mathbf{y} = \mathbf{C}_\alpha \mathbf{X}\mathbf{h} + \mathbf{v}, \quad (8)$$

where $\mathbf{C}_\alpha$ is the diagonal matrix

$$\mathbf{C}_\alpha = diag(1, e^{j2\pi\alpha}, ..., e^{j2\pi(rL-1)\alpha}), \quad (9)$$

and $\mathbf{X}$, is the $rL \times L_h$ matrix with entries given by

$$[\mathbf{X}]_{i,j} = x(i-j), 0 \leq i \leq rL-1, 0 \leq j \leq L_h - 1. \quad (10)$$

Finally, $\mathbf{y}$ and $\mathbf{v}$ are the $rL \times 1$ matrices with entries $[\mathbf{y}]_l = y(l)$ and $[\mathbf{v}]_l = v(l)$, and $\mathbf{h}$ is the $L_h \times 1$ channel matrix.
Since the additive noises are Gaussian distributed, the log-likelihood function for $\mathbf{h}$ and α takes the form

$$\Gamma(\mathbf{h}, \alpha) = \| \mathbf{y} - \mathbf{C}_\alpha \mathbf{X}\mathbf{h} \|^2, \quad (11)$$

where $\| . \|$ denotes the Euclidian matrix norm. By substituting the expression of h that minimizes $\Gamma(\mathbf{h}, \alpha)$, we get

$$\Gamma(\alpha) = \mathbf{y}^H\mathbf{y} - \mathbf{y}\mathbf{C}_\alpha^H \mathbf{R}\mathbf{C}_\alpha \mathbf{y}^H. \quad (12)$$

Here, $\{.\}^H$ denotes the conjugate transpose operation and $\mathbf{R}$ is an Hermitian matrix given by $\mathbf{R} = \mathbf{X}(\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H$.
We aim at finding α that minimizes $\Gamma(\alpha)$. Thus we have

$$\{\hat{\alpha}\} = \arg\max_\alpha \Psi(\alpha), \quad (13)$$

where

$$\Psi(\alpha) = \mathbf{y}\mathbf{C}_\alpha^H \mathbf{R}\mathbf{C}_\alpha \mathbf{y}^H. \quad (14)$$

Due to the periodicity of x_1 and x_2, matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ (notation confounded to $\mathbf{X}$) with respective entries $x_{1_{i,j}} = x_1(i-j)$ and $x_{2_{i,j}} = x_2(i-j)$ (notation confounded to $x_{i,j}$) may be written as

$$\mathbf{X} = [\mathbf{X}'^T, \mathbf{X}'^T, ..., \mathbf{X}'^T]^T, \quad (15)$$

[1] If not, we can pad the channel with zero coefficient paths.

where $\mathbf{X}'$ is an $L_h \times L_h$ circulant matrix with elements

$$[\mathbf{X}']_{i,j} = x_{|i-j|_L}, \qquad 0 \leq i,j \leq L_h - 1, \qquad (16)$$

with $|i-j|_L$ expressing $i-j$ modulo L. When $L < L_h$, $\mathbf{X}^H\mathbf{X}$ is not a full rank matrix, thus non invertible. To ensure the invertibility of $\mathbf{X}^H\mathbf{X}$, we only consider the case where $L_h = pL$. Indeed, when $L_h = pL$, (7) can be written as

$$\mathbf{y} = \mathbf{C}_\alpha \mathbf{X}_t \sum_{i=0}^{p-1} \mathbf{h}_i + \mathbf{v}, \qquad (17)$$

where $\mathbf{X}_t$ is an $rL \times L$ matrix given by

$$\mathbf{X}_t = [\mathbf{X}_t'^T, \mathbf{X}_t'^T, ..., \mathbf{X}_t'^T]^T, \qquad (18)$$

and $\mathbf{X}_t'$ is an $L \times L$ circulant matrix with entries

$$[\mathbf{X}_t']_{i,j} = x_{|i-j|_L}, \qquad 0 \leq i,j \leq L-1. \qquad (19)$$

In (16), $\mathbf{h}_i$ is an $L \times 1$ matrix with entries

$$h_i(j) = h(j+iL), \quad 0 = 1, ..., L-1. \qquad (20)$$

Consequently,

$$\mathbf{y} = \mathbf{C}_\alpha \mathbf{X}_t \mathbf{h}' + \mathbf{v}, \qquad (21)$$

where $\mathbf{h}' = \sum_{i=0}^{p-1} \mathbf{h}_i$. The Hermitian matrix $\mathbf{R}_t = \mathbf{X}_t(\mathbf{X}_t^H\mathbf{X}_t)^{-1}\mathbf{X}_t^H$ is such that for $(i > j)$ we have:

$$[\mathbf{R}_t]_{i,j} = \begin{cases} \gamma, & \text{if } i-j = L, 2L, ..., rL, \\ 0, & \text{otherwise,} \end{cases} \qquad (22)$$

where γ is a constant. Notice that we could use $x(n) = e^{j2\pi\frac{n}{L}}$ to guarantee the approximation in (2). Nevertheless, for this sequence structure, the matrix $\mathbf{R}_t$ does not exist.

Since L is a power of 2, only channels with even lengths are involved. Nevertheless, when L_h is odd, we can pad the channel with zero coefficient paths.

By taking the derivative of $\Psi(\alpha)$ with respect to α and setting it equal to zero at $\hat{\alpha}$, we get, after some calculations:

$$Im\left\{\sum_{k=1}^{r-1}\left(\sum_{l=kL}^{rL-1} kLy^*(l)y(l-kL)\right)e^{j2\pi\hat{\alpha}kL}\right\} = 0. \quad (23)$$

Since we deal with training signals with small period ($r \gg 2$), an exhaustive search is required to solve the ML estimator. To avoid this search, we solve the problem by polynomial rooting.

3.1. ML estimator based on polynomial rooting

Let P be a polynomial of degree $r-1$ written as:

$$P(z) = \sum_{k=1}^{r-1} a_k z^k, \qquad (24)$$

with $a_k = \sum_{l=kL}^{rL-1} kLy^*(l)y(l-kL)$. We notice that the solution of (20) is a nonzero root of:

$$\Lambda(z) = P(z) - P^*(z). \qquad (25)$$

Polynomial rooting was also employed in [11], to estimate the frequency offset of OFDM systems. The desired root was chosen on the unit circle to minimize an objective function. We use here a different approach.

In the absence of noise, we suppose that the received signal still has a constant modulus [2] and we show that:

$$a_k = \sigma^2 k(r-k)e^{-j2\pi\alpha kL}, \qquad (26)$$

where σ^2 is a constant. Evaluating (24) on the unit circle, $z = e^{j2\pi\beta L}$, we obtain:

$$\Lambda(z) = \sum_{k=1}^{r-1} \sigma^2 k(r-k)\sin(2\pi kL(\alpha-\beta)). \qquad (27)$$

Hence, $\Lambda(z) = 0$ if $\beta = \hat{\alpha} - i/Lr, i = 0, ..., r-1$. We detect the desired root $s = e^{j2\pi\hat{\alpha}L}$ for $i = 0$. Note that, when r is even, $-s$ is also a solution of (26) for $i = r/2$. Based on this, s can be detected by solving the polynomial $z^{r-1}\Lambda(z)$ after canceling out its odd coefficients. Moreover, since we deal with systems deploying a large number of subcarriers and a periodic training sequence with a small period, we can assert that the desired root s corresponds to the non-null root having the minimal phase in terms of absolute value. Consequently, α_1 (and α_2) are estimated as :

$$\hat{\alpha} = \frac{arg(s)}{2\pi L}. \qquad (28)$$

Note that, since L is small compared to N, the rth sub-block can be omitted during the ML processing, when r is odd, without altering significantly the estimation accuracy.

Since the complexity of this method is rather high due to the polynomial rooting process, we propose to simplify the estimation process by using a periodic and orthogonal TS.

3.2. ML estimator using an orthogonal TS

A periodic training sequence $x(n), n = 0, 1..., L-1$, is said to be orthogonal when the matrix $\mathbf{X}_t'$ in (18) is such that $\mathbf{X}_t'^H\mathbf{X}_t' = \sigma_x^2\mathbf{I}_d$, where σ_x^2 is the mean power of the transmitted symbols and $\mathbf{I}_d$ is the identity matrix.

In [12], the ML estimator complexity was greatly decreased by the use of an orthogonal training sequence to estimate the CFO for OFDM systems. Thus, it would be worth trying to find the orthogonal sequence that allows the joint estimation of the SCO and the CFO in a single step. This sequence will be presented later.

Based on the pilot symbol reception scheme given in (3) and (4), and the orthogonality property of the transmitted TS, we can write, in the absence of noise

$$\sum_{l=kL}^{rL-1}\left\{\sum_{l_1=0}^{L_h-1} h_{l_1}x(l-l_1)\right\}^* \left\{\sum_{l_2=0}^{L_h-1} h_{l_2}x(l-kL-l_2)\right\} = (rL-kL)\sigma_x^2||\mathbf{h}||^2. \qquad (29)$$

[2] We will show in the simulations section that the violation of this restrictive assumption does not prevent the accurate estimation of the parameters.

Thus the estimator can be written as

$$Im\left\{\sum_{k=1}^{r-1} kL(rL-kL)\sigma_x^2||h||^2 e^{-j2\pi\alpha kL}e^{j2\pi\hat{\alpha}kL}\right\}=0, \tag{30}$$

or

$$Im\left\{\begin{array}{c}\sum_{k=1}^{(r-1)/2} 2kL(rL-kL)\sigma_x^2||h||^2 \\ \cos(\pi(\hat{\alpha}-\alpha)(rL-2kL))e^{j\pi(\hat{\alpha}-\alpha)rL}\end{array}\right\}=0. \tag{31}$$

For small α, we have:

$$\cos(\pi(rL-2kL)(\hat{\alpha}-\alpha))\simeq 1. \tag{32}$$

The estimator is therefore simplified and becomes

$$Im\left\{\sum_{k=1}^{r-1}\sum_{l=kL}^{rL-1} kLy^*(l)y(l-kL)e^{j\pi\hat{\alpha}rL}\right\}\simeq 0, \tag{33}$$

for small frequency offsets and high SNR. Since emerging OFDM systems deploy an important number of subcarriers, (30) holds. Finally, we estimate α_1 (and α_2) as:

$$\hat{\alpha}=-\frac{1}{\pi rL}arg\left\{\sum_{k=1}^{r-1}\sum_{l=kL}^{rL-1} kLy^*(l)y(l-kL)\right\}. \tag{34}$$

A very known orthogonal TS is the perfect Zadoff-Chu [13] sequence:

$$x(n)=\begin{cases} e^{j\pi n^2 z/L}, & \text{if } L \text{ is odd} \\ e^{j\pi n(n+1)z/L}, & \text{if } L \text{ is even,}\end{cases} n=1,2,...,L, \tag{35}$$

where z is an integer relatively prime to L. This sequence, even if orthogonal, does not enable a joint estimation of ϵ and δ, since ϵ varies with n while δ varies with n^2. For that reason, we propose to design a new periodical and orthogonal sequence written as

$$x(n)=(-1)^{\lfloor\frac{n}{L}\rfloor}e^{j(\pi\frac{n}{L}+\Theta_L(n))}, \tag{36}$$

with $\Theta_L(n(1+\delta))\approx\Theta_L(n)$. For $L=2$, the periodic sequence given in section 1 is also orthogonal. Thus, $\Theta_2(n)=0$. For $L=4$, we find that

$$\Theta_4(n)=\frac{\pi}{2}(-1)^{\lfloor|n|_2\rfloor}sign(1-(-1)^{\lfloor\frac{n}{2}\rfloor}). \tag{37}$$

For $L=8$,

$$\Theta_8(n)=sign(|\lfloor\tfrac{n}{2}\rfloor|_4)(-\tfrac{3\pi}{4}((-1)^{\lfloor\frac{(n+1)}{4}\rfloor}|n|_2+\lfloor\tfrac{|n|_8}{4}\rfloor- \lfloor\tfrac{|n|_8}{5}\rfloor)-\tfrac{\pi}{4}((-1)^{\lfloor\frac{(n+2)}{4}\rfloor}|n+1|_2+\lfloor\tfrac{|n|_8}{5}\rfloor-\lfloor\tfrac{|n|_8)}{6}\rfloor)). \tag{38}$$

Sequence design, being done empirically, is rather tedious for larger values of L.

4. SIMULATION RESULTS

We consider an OFDM system with $N=512$ subcarriers, operating over an eight-path channel. The time domain channel coefficients are independent zero-mean Gaussian random variables. The transmitted TS spans a period $L\in\{2,4,8\}$. All simulation results are obtained by averaging over 1000 independent trials.

A comparative study is carried out with a joint estimation algorithm developed by Liu and Chong (L&C) in [10]. Without impacting performance, we first adapt the L&C method to the case for which a pilot symbol containing $Q=N/L$ identical sub-blocks is transmitted. After an FFT processing of each sub-block, the difference of rotated phases between two adjacent sub-blocks is given by:

$$\begin{aligned}\Phi_p(k)&=\angle Y_{p+1,k}-\angle Y_{p,k}\\ &=2\pi\epsilon L+2\pi k\delta+v_p(k) \quad p=1...Q-1,\\ &\qquad\qquad\qquad\qquad\qquad k=0...L-1,\end{aligned} \tag{39}$$

where $Y_{p,k}$ is the k-th sample of the p-th sub-block at the output of the FFT and v_p is an additive Gaussian noise including ICI. Using a least square estimation, we obtain:

$$\hat{\epsilon}=\frac{\sum_{p=1}^{Q-1}\sum_{k=0}^{L-1}\Phi_p(k)}{2\pi L^2(Q-1)}, \tag{40}$$

$$\hat{\delta}=\frac{\sum_{p=1}^{Q-1}\sum_{k=0}^{L-1}k\Phi_p(k)}{2\pi(Q-1)\sum_{k=0}^{L-1}k^2}. \tag{41}$$

Moreover, frequency offset estimation performance is compared to that obtained by M&M in [9].

In figures 2 and 3, we illustrate results of the joint estimation of $\epsilon=\frac{0.5}{N}$ and $\delta=5\times10^{-5}$ versus SNR for different values of the TS period L. Frequency offset estimation using method A (method in section 3.1) provides better accuracy than the M&M method (P is the number of identical sub-blocks per symbol and H is a design parameter less than or equal to $P-1$), especially at low SNR. At high SNR the two methods give similar results. The ML method clearly outperforms the L&C method which suffers from ICI as suggested by the saturation at high SNR. The SCO estimator achieves a gain of almost 7 dB when decreasing L from 8 to 2.

Figure 4 shows that the fine timing estimator offers reliable and accurate performance even when δ increases. In this figure, we superimpose, for method A and method B (method of section 3.2), the performance curves in terms of sampling clock offset estimation MMSE versus δ, at an SNR of = 20 dB. We assert that the two methods give similar estimation accuracy (which is also confirmed in figure 2 versus SNR). This makes method B very attractive, since it is less demanding from a complexity standpoint.

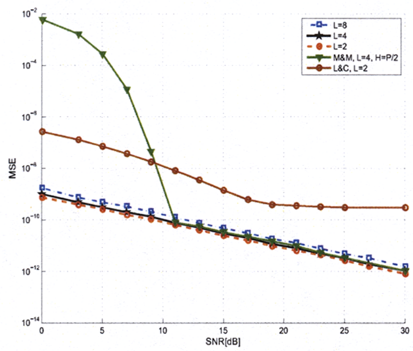

Fig. 2. Frequency offset estimation performance versus SNR.

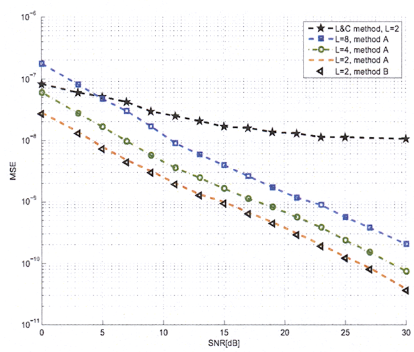

Fig. 3. Sampling clock offset estimation performance versus SNR.

5. CONCLUSION

Based on an ML approach, we have developed a new technique for joint estimation of the sampling clock offset and the carrier frequency using an appropriate training sequence (TS). By the use of a periodic TS, we reduce the ML estimation complexity from that of an exhaustive search over a continuum, to that of solving a polynomial. Nevertheless, this method is numerically complex, mainly when the number of subcarriers increases. Therefore, we use orthogonal and periodic TS to obtain a simpler formulation of the ML estimator. Our method outperforms existing ones over a considerable range of SNR. It uses only one OFDM symbol and allows for the estimation of the synchronization parameters in a single step.

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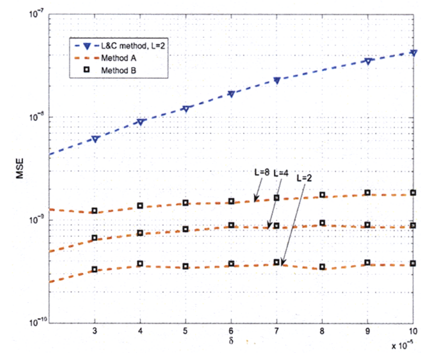

Fig. 4. Sampling clock offset estimation performance versus δ.

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