Moment-Based SNR Estimation for SIMO Wireless Communication Systems Using Arbitrary QAM

Alex Stéphenne[1,2], Faouzi Bellili[3] and Sofiène Affes[1]
[1]INRS-EMT, 800, de la Gauchetière Ouest, Bureau 6900, Montreal, H5A 1K6
[2] Ericsson Canada, 8400, Decarie Blvd, Montreal, H4P 2N2
[3] Tunisia Polytechnic School, INRS-EMT
Emails : bellili@emt.inrs.ca, stephenne@ieee.org, affes@emt.inrs.ca

***Abstract*— A new method for signal-to-noise ratio (SNR) estimation is considered when multiple receiving antenna elements receive quadrature amplitude modulation (QAM) signals in complex additive white gaussian noise (AWGN) spatially and temporally white (uncorrelated between antenna elements). In this paper, we also present the extension of other existing methods to the Single Input Multiple Output (SIMO) configuration. The procedure is non-data-aided (NDA) since it is a moment-based method and does not require, therefore, the a priori knowledge or the detection of the transmitted symbols. Monte Carlo simulations are used to estimate the normalized root mean square error (NRMSE) as a measure of performance. The new method is shown to outperform the best NDA moment-based SNR estimation methods, namely the $\mathrm{M_2M_4}$ and Gao's methods even when they are extended to the SIMO configuration.**

I. Introduction

Many techniques for optimal usage of radio resources are nowadays based on the knowledge of the signal-to-noise ratio. For instance, SNR estimates can be necessary for power control, equalization, handoff and dynamic allocation of resources.

The first basic SNR estimation algorithms were introduced in the 1960's [1], [2]. And since then, researchers have worked on improving performance of existing estimators or finding new ones. Estimation methods can be categorized as data-aided (DA) and non-data-aided (NDA). In fact, DA methods use the knowledge of transmitted symbols to facilitate the estimation. However, NDA methods base the estimation only on the received signals.

Various estimation techniques have been introduced during the last few decades, including maximum-likelihood (ML)-based [3], [4] and moment-based methods [5]. But, to the best of our knowledge, the best two existing NDA methods that are applicable for non-constant envelope modulation are the $\mathrm{M_2M_4}$ method presented in [6], and the method presented in [7]. Both are SISO and must be used antenna per antenna. Hence, we will present in this paper enhanced versions of these methods for the SIMO configuration, in complex additive white gaussian noise (AWGN) spatially and temporally white (uncorrelated between antenna elements). We will show that the use of an array of antennas can lead to a remarkable increase in the estimation accuracy. But the main contribution presented in this paper will be the development of a new moment-based SNR estimation method for the SIMO configuration. The superiority of this new method will be shown both against the $\mathrm{M_2M_4}$ method and the method introduced in [7].

In this paper, we will begin by introducing the main NDA moment-based SNR estimation methods and extend them to the SIMO configuration. Then, we will present a new SIMO moment-based SNR estimation method to finish with a performance comparison between these different methods.

II. System model

We consider a digital communication system over a frequency-flat SIMO fading channel. The time variations of the channel are considered to be relatively slow so that it can be considered constant over the observation interval. We also assume that the noise components at all the N_a antenna elements are modeled by complex Gaussian variables, of equal average power, which are temporally and spatially white (uncorrelated between antenna elements). This can be a valid assumption in practice, for example when the noise is associated with a large number of interference sources or interference propagation paths. The input-output baseband relationship is given by :

$$\mathbf{y}(n) = \mathbf{h}x(n) + \mathbf{w}(n), \tag{1}$$

where

$$\mathbf{y} = [y_0(n), y_1(n) \dots y_{N_a-1}(n)]^T, \tag{2}$$

$$\mathbf{h} = [h_0, h_1 \dots h_{N_a-1}]^T, \tag{3}$$

$$\mathbf{w} = [w_0(n), w_1(n) \dots w_{N_a-1}(n)]^T. \tag{4}$$

The superscript T stands for the transpose operator, $x(n)$ is the n^{th} transmitted quadrature amplitude modulated symbol, $y_i(n)$ and $w_i(n)$ are, respectively, the received signal and the complex AWGN component, with zero mean and variance $N = 2\sigma^2$, at the i^{th} antenna element. Assumed to be constant and unknown to the receiver, h_i is the channel coefficient for the antenna element i. Without loss of generality, the power of the constellation is assumed to be normalized to 1. Given the vectorial samples $\mathbf{y}(n)$ only, for $n = 1, 2 \dots K-1$, our purpose is to estimate the SNR at each antenna element i which is given by :

$$\rho_i = \frac{|h_i|^2}{N} = \frac{P_i}{N}, \tag{5}$$

where $P_i = |h_i|^2$ is the signal power at antenna element i.

978-1-4244-2110-7/08/$25.00 ©2007 IEEE

III. Existing methods and SIMO extensions

In this section we present the classical M_2M_4 method and the method presented in [7] and their direct extension to the SIMO channel configuration.

A. M_2M_4 method and its SIMO extension

1) M_2M_4 method: The M_2M_4 method is a SISO method. Therefore, we consider, for the time being, the antenna element i. The M_2M_4 method uses the 2^{nd} and the 4^{th} order moments which are, respectively, defined as :

$$M_{2;i} = E[y_i(n)y_i(n)^*], \tag{6}$$

$$= P_i + N, \tag{7}$$

$$M_{4;i} = E[(y_i(n)y_i(n)^*)^2], \tag{8}$$

$$= K_a P_i^2 + 4P_i N_i + K_w N_i^2, \tag{9}$$

where E denotes expectation and K_a and K_w are, respectively, the signal and noise kurtosis which are given by :

$$K_a = \frac{E[|x(n)|^4]}{(E[|x(n)|^2])^2} \quad \text{and} \quad K_w = \frac{E[|w(n)|^4]}{(E[|w(n)|^2])^2}. \tag{10}$$

$K_w = 2$ for a complex noise and K_a depends on the modulation type. Solving for P_i and N_i from eqs. (7) and (9), one obtains :

$$P_i = \frac{1}{K_a + K_w - 4}\Big[M_{2;i}(K_w - 2) - \sqrt{(4 - K_a K_w)M_{2;i}^2 + M_{4;i}(K_a + K_w - 4)}\,\Big]. \tag{11}$$

$$N_i = M_{2;i} - P_i. \tag{12}$$

In practice, $M_{2;i}$ and $M_{4;i}$ are computed using

$$\widehat{M}_{2;i} = \frac{1}{K}\sum_{n=0}^{K-1}\left[y_i(n)y_i(n)^*\right], \tag{13}$$

$$\widehat{M}_{4;i} = \frac{1}{K}\sum_{n=0}^{K-1}\left[y_i(n)y_i(n)^*\right]^2. \tag{14}$$

Estimates of P_i and N_i are then given by :

$$\widehat{P}_{i;M_2M_4} = \frac{1}{K_a + K_w - 4}\Big[\widehat{M}_{2;i}(K_w - 2) - \sqrt{(4 - K_a K_w)\widehat{M}_{2;i}^2 + \widehat{M}_{4;i}(K_a + K_w - 4)}\Big], \tag{15}$$

$$\widehat{N}_{i;M_2M_4} = \widehat{M}_{2;i} - \widehat{P}_{i;M_2M_4}. \tag{16}$$

It should also be known that $\widehat{P}_i$ was set to zero if it was not real-valued and $\widehat{N}_i$ was also set to zero if its computed value was negative. These corrections are required because the estimated moments can be relatively noisy. Finally, estimates of the SNR over each antenna element i are obtained using

$$\widehat{\rho}_{i;M_2M_4} = \frac{\widehat{P}_{i;M_2M_4}}{\widehat{N}_{i;M_2M_4}}. \tag{17}$$

2) SIMO extension of the M_2M_4 method: The M_2M_4 method was developed for SISO channels, and it is relatively simple to modify so that it exploits the presence of multiple antenna elements in a SIMO scenario when the noise components at all the N_a antenna elements can be modeled by complex Gaussian variables, of equal average power, which are temporally and spatially white (uncorrelated between antenna elements). To do so, $\widehat{N}_i$ is computed over each antenna using (16). Then, one should average over all antenna elements to have more accurate values of the noise power, which is then used to compute the power estimates. We therefore use

$$\widehat{N}_{M_2M_4\_SIMO} = \frac{1}{N_a}\sum_{i=0}^{N_a-1}\widehat{N}_{i;M_2M_4}, \tag{18}$$

$$\widehat{P}_{i;M_2M_4\_SIMO} = \max\left(0, \widehat{M}_{2;i} - \widehat{N}_{M_2M_4\_SIMO}\right). \tag{19}$$

The SNR estimate over each antenna element i is then

$$\widehat{\rho}_{i;M_2M_4\_SIMO} = \frac{\widehat{P}_{i;M_2M_4\_SIMO}}{\widehat{N}_{M_2M_4\_SIMO}}. \tag{20}$$

B. Gao's method and its SIMO extension

To simplify, we choose to refer to the approach presented in [7] as "Gao's method". We will present in this section Gao's method and its extension to the SIMO configuration of our channel.

1) Gao's method as presented in [7]: Assume that $x(n)$ comes from a constellation that has Q different amplitudes $A_1, A_2, \ldots A_Q$ with Q different probabilities $p_1, p_2, \ldots p_Q$. As explained in [7], the probability density function (pdf) of $|y_i(n)|$ is a mixed Ricean distribution, given by

$$f_{|Y_i(n)|}(|y_i(n)|) = \sum_{q=1}^{Q}\left[p_q\frac{|y_i(n)|}{\sigma^2}exp(-\rho_i A_q^2 - \frac{|y_i(n)|}{\sigma^2})\times I_0(|y_i(n)|\sqrt{\frac{2\rho_i A_q^2}{\sigma^2}})\right]. \tag{21}$$

Since (A_i, p_i) are known for $i = 1, 2 \ldots Q$, the SNR estimation boils down to the estimation of the Ricean factor ρ from a mixed Ricean distribution. The average energy of the transmitted symbols are supposed to be normalized to 1 so that $\sum_{q=1}^{Q} p_i A_i^2 = 1$ and the k^{th} moment of the mixed Ricean distribution is then given by

$$M_k(\sigma^2, \rho_i) = E[|y_i(n)|^k] \tag{22}$$

$$= \sum_{q=1}^{Q}\left[p_q(2\sigma^2)^{\frac{k}{2}}\Gamma(\frac{K}{2}+1)\times e^{-\rho_i A_q^2}{}_1F_1(\frac{K}{2}+1;1;\rho_i A_q^2)\right], \tag{23}$$

where ${}_1F_1(.)$ and $\Gamma(.)$ are, respectively, the confluent hypergeometric and the gamma functions. From (23), it is obvious that the moments depend on two unknown parameters which are the SNR, ρ_i, and the noise variance $2\sigma^2$. Hence, a moment-based SNR estimator requires estimates of at least two different moments. For $k \neq l$, we define the following function of ρ_i :

$$f_{k,l}(\rho_i) = \frac{M_k^l(\sigma^2, \rho_i)}{M_l^k(\sigma^2, \rho_i)}, \quad (24)$$

which no longer depends on σ but only on the SNR ρ_i. Therefore, we can construct moment-based estimators of ρ_i which can be expressed as

$$\rho_{i;Gao_{k,l}} = f_{k,l}^{-1}\left(\frac{\widehat{M}_{k;i}^l}{\widehat{M}_{l,i}^k}\right), \quad (25)$$

where $\widehat{M}_{k;i} = \frac{1}{K}\sum_{n=0}^{K-1}|y_i(n)|^k$. Although the analytical inversion of $f_{k;l}(.)$ is often not tractable, one can implement these estimators by a lookup table.

2) SIMO extension of Gao's method: We use the same idea as for the extension of the M_2M_4 method. In fact, we can simply note from (5) and (7) that

$$\rho_i = \frac{M_{2;i} - N_i}{N_i}, \quad (26)$$

so that

$$N_i = \frac{M_{2;i}}{\rho_i + 1}. \quad (27)$$

Then averaging over all the antenna elements, we can obtain a more accurate estimate of the noise power by computing

$$\widehat{N}_{Gao\_SIMO_{k,l}} = \frac{1}{N_a}\sum_{i=0}^{Na-1}\frac{\widehat{M}_{2;i}}{\widehat{\rho}_{Gao_{k,l}} + 1}. \quad (28)$$

Then we obtain an estimate of each antenna element SNR by injecting (28) in (26) so that

$$\widehat{\rho}_{i;Gao\_SIMO_{k,l}} = \max\left(0, \frac{\widehat{M}_{2;i} - \widehat{N}_{Gao\_SIMO_{k,l}}}{\widehat{N}_{Gao\_SIMO_{k,l}}}\right). \quad (29)$$

IV. New SIMO SNR Estimation Method

Since the method presented in this section depends on a 4^{th} order moment, we will simply refer to it as the M_4 method. It is mainly based on the following 4^{th} order moment :

$$\bar{M}_{4;i,k} = E[y_i(n+1)y_i(n)^* y_k(n+1)^* y_k(n)], \quad (30)$$

$$= \begin{cases}(P_i + N)^2, & \text{if } i = k\\ P_i P_k, & \text{otherwise.}\end{cases} \quad (31)$$

One can immediately see that we can write in matrix form

$$\bar{\mathbf{M}}_4 = E\left\{[\mathbf{y}(n+1)\odot\mathbf{y}(n)^*][\mathbf{y}(n+1)\odot\mathbf{y}(n)^*]^H\right\}, \quad (32)$$

$$= \begin{pmatrix}(P_0+N)^2 & P_0P_1 & \cdots & P_0P_{N_a-1}\\ P_1P_0 & (P_1+N)^2 & \cdots & P_1P_{N_a-1}\\ \vdots & \vdots & \ddots & \vdots\\ P_{N_a-1}P_0 & P_{N_a-1}P_1 & \cdots & (P_{N_a-1}+N)^2\end{pmatrix} \quad (33)$$

where $\odot$ denotes the Schur-Hadamard matrix product, and the superscript H stands for the Hermitian operator . From (31), we have

$$P_i = \sqrt{\bar{M}_{4;i,i}} - N. \quad (34)$$

Substituting (34) in the off-diagonal elements of the matrix $\bar{\mathbf{M}}_4$, we obtain

$$\bar{M}_{4;i,k} = \left(\sqrt{\bar{M}_{4;i,i}} - N\right)\left(\sqrt{\bar{M}_{4;k,k}} - N\right), \text{for i} \neq k. \quad (35)$$

Resolving (35) and taking the negative root, one can find

$$N = \frac{\sqrt{\bar{M}_{4;i,i}} + \sqrt{\bar{M}_{4;k,k}} - \sqrt{\left(\sqrt{\bar{M}_{4;i,i}} - \sqrt{\bar{M}_{4;k,k}}\right)^2 + 4\bar{M}_{4;i,k}}}{2}. \quad (36)$$

In practice, $\bar{M}_{4;i,k}$, which should be real and non-negative, is unknown and should be estimated by simple sample averaging. The estimate is therefore given by

$$\widehat{M}_{4;i,k} = \frac{\max\left(0, \sum_{n=0}^{K-2}\Re[y_i(n+1)y_i^*(n)y_k^*(n+1)y_k(n)]\right)}{K-1}, \quad (37)$$

where $\Re(.)$ returns simply the real part of any complex argument. As mentioned before, (36) is valid only for all $i \neq k$ antenna element pairs, and there are $\frac{1}{2}N_a(N_a - 1)$ such pairs. Hence, to obtain a more accurate value of the noise power estimate $\widehat{N}$, we can average over all the pairs and use the following expression

$$\widehat{N}_{M_4} = \frac{1}{N_a(N_a-1)}\sum_{i=0}^{N_a-1}\sum_{k>i}^{N_a-1}\left[\sqrt{\widehat{\bar{M}}_{4;i,i}} + \sqrt{\widehat{\bar{M}}_{4;k,k}} - \sqrt{\left(\widehat{\bar{M}}_{4;i,i} + \widehat{\bar{M}}_{4;k,k}\right)^2 + 4\widehat{\bar{M}}_{4;i,k}}\right]. \quad (38)$$

Once the noise power is estimated, the signal power over each antenna element, i.e. $P_i (i = 0, 1, \ldots N_a - 1)$, which should be always real and non-negative, can be estimated using (34), so that we have

$$\widehat{P}_{i;M_4} = \max\left(0, \sqrt{\widehat{\bar{M}}_{4;i,i}} - \widehat{N}_{M_4}\right). \quad (39)$$

Finally our new SNR estimates (one per antenna element i) are given by :

$$\widehat{\rho}_{i;M_4} = \frac{\widehat{P}_{i;M_4}}{\widehat{N}_{M_4}}. \quad (40)$$

Note that the same equations are used regardless of the modulation type or order. Our new method does, therefore, not require the a priori knowledge of the modulation type or order to estimate the SNR, contrarily to other NDA estimation methods.

V. Simulations

In this section, we will focus on the performance of our new M_4 SNR estimation method and compare it with the performance of the main existing methods, namely the M_2M_4 and Gao's methods. Monte Carlo simulations will be run with 10000 realizations. We choose also $N_a = 3$ antennas and an observation interval of $K = 2000$ symbols. The required lookup table for the Gao's method, used in our simulations, is very large (with SNR ranging from -100 dB to 100 dB

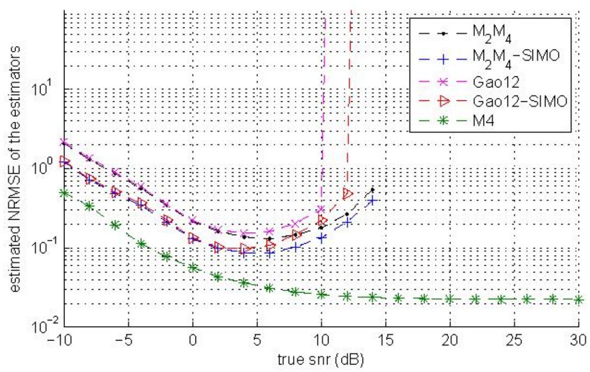

Fig. 1. SNR NRMSE on one of the 3 antennas with the same experienced SNR, 64-QAM

in step of 0.02 dB, for more accuracy). Linear interpolation is then used for the numerical inversion of Gao's function, to insure fairness in the comparison.

Fig.1 shows the estimated Normalized Root Mean Square Error (NRMSE) when we experience the same SNR on the 3 antennas and when the regular rectangular 64-QAM modulation is used. We can obviously see that the use of an array of antennas can lead to a remarkable increase in the estimation accuracy, over all the SNR values. But, we notice also that the SISO and SIMO versions of M_2M_4 and Gao's methods become unable to estimate the SNR when it exceeds about 15 dB. In contrast, the new M_4 method appears to be the best over all the SNR values and can estimate the SNR with accuracy even when it is high.

In wireless communications, SIMO channels would be fading and SNR is not always expected to be the same on all the antenna elements. The next considered scenario will suppose that one of the 3 antennas has 20 dB higher SNR than the SNR experienced on the other elements.

Fig. 2 illustrates the estimated NRMSE on the antenna element that experiences high SNR, as a function of the true SNR on the other antennas, when 16-QAM modulation is used. We observe that the SIMO versions of the M_2M_4 and Gao's methods are better than the SISO versions for all the SNR values. The two versions appear to be unable to estimate the SNR when it is higher than 12 dB. However, it is obvious that the M_4 method is the best one for all the SNR values.

Fig. 3 illustrates the NRMSE on one of the other antenna elements which are with equal SNR, for the same modulation order. We notice, in this case, that the SISO versions are better for SNR values in the medium range (from -4 dB to 12 dB). This is due to the contribution of the noise estimate associated with the high SNR antenna element which is very inaccurate. The M_4 method seems again to have the most satisfactory behavior over the entire range with clearly superior performance.

Note that similar performance improvements have been observed for other modulation orders like QPSK and 128-QAM, but were not included for the sake of briefness and clarity.

Finally, we can say that, in all the cases, the new M_4 estimator seems, clearly, to be the best method, and the only viable alternative for NDA SNR estimation over a wide SNR range.

VI. Conclusion

In this document, a new SIMO SNR estimation method, for arbitrary QAM modulation, the M_4 was presented. Our new method assumes that the noise components at all the antenna elements can be adequately modeled by complex Gaussian variables, of equal average power, which are temporally and spatially white (uncorrelated between antenna elements). Its performance was compared with those achieved by the best existing NDA moment-based SNR estimation methods, namely the M_2M_4 and Gao's methods, with SIMO extensions developed in this paper. The superiority of the new M_4 method ,against these two methods, was shown over both fading and non-fading channels. Moreover, although the new M_4 method was presented in this paper in the context of multi-antenna systems, it is worth nothing that the method could be directly applicable to other systems having other types of diversity, beside or in addition to spatial diversity. For instance, the method is directly applicable to SISO CDMA systems with path diversity, as managed, for example, by RAKE receivers. In that context, one would simply consider SNR per path instead of SNR per antenna element. It could also be applicable to a multi-carrier SIMO system over

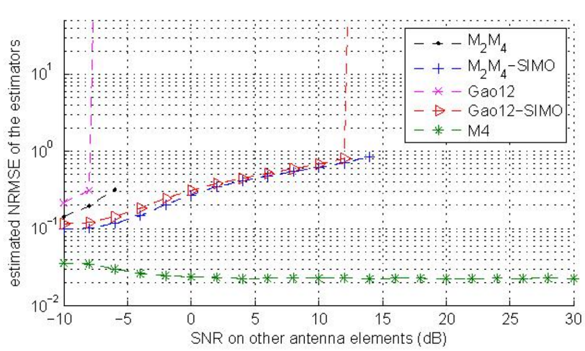

Fig. 2. SNR NRMSE on the high SNR antenna element, 16-QAM

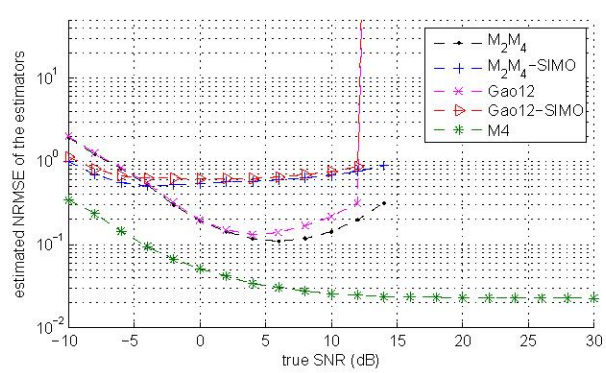

Fig. 3. SNR NRMSE on one of the low SNR antenna elements, 16-QAM

multiple adjacent subcarriers, with a per subcarrier SNR instead of a SNR per antenna element.

REFERENCES

[1] N. Nahi and R.Gagliardi, "On the estimation of signal-to-noise ratio and application to detection and tracking systems," *University of Southern California, Los Angeles, EE Report 114,* July 1964.

[2] T. Benedict and T. Soong, "The joint estimation of signal and noise from sum envelope," *IEEE Trans. Inform Theory,* vol. IT-13, pp. 447-457, July 1967.

[3] R. B. Kerr, "On signal and noise level estimation in a coherent PCM channel," *IEEE Trans. Aerosp. Electron. Syst.,* vol. AES-2, pp. 450-454, July 1966.

[4] R. M. Gagliardi and C. M. Thomas, "PCM data reliability monitoring through estimation of signal-to-noise ratio," *IEEE Trans. Commun.,* vol. COM-16, pp. 479-486, June 1968.

[5] R. Matzner, "An SNR estimation algorithm for complex baseband signals using higher order statistics," *Facta Universitatis (Nis),* no. 6, pp. 41-52, 1993.

[6] D.R. Pauluzzi and N.C. Beaulieu, "A comparison of SNR estimation techniques for the AWGN channel," *IEEE Transactions on Communications,* vol. 48, pp. 1681-1691, October 2000.

[7] P. Gao and C. Tepedelenlioglu, “SNR estimation for non-constant modulus constellations,“ *IEEE Trans. Signal Processing,* vol. 53, no. 3, March 2005, pp. 865-870.

[8] A.T. Arnholt and J.L. Hebert. “Optimal combinations of pairs of estimators,“ Interstat (2001), pp. 1-8.