Capacity of Log-normal Fading Channels

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ABSTRACT

This paper investigates the capacity of log-normal fading channels with receiver channel state information. We provide a closed-form expression for the ergodic capacity of the log-normal fading channel. Since the developed expression involves an infinite series, we show that the error that results from the truncation of this series is insignificant. Relying on the fact that the sum of log-normal Random Variables (RV) is well approximated by another log-normal RV, we further apply the obtained results to find the capacity of correlated log-normal fading channels with Maximum Ratio Combining and Equal Gain Combining. The analytical expressions obtained match perfectly the capacity given by simulations.

Categories and Subject Descriptors: J.2 [Physical Sciences and Engineering]: Mathematics and statistics; G.3 [Probability and Statistics]: Distribution functions.

General Terms: Performance, Theory.

Keywords: Channel capacity, log normal distributions, maximum ratio combining and equal gain combining.

1. INTRODUCTION

Capacity of fading channels has received (and is still receiving) an extensive interest. This concern is motivated by the need for a valuable tool to assess the achievable performance of communication links over fading channels. This interest mainly started at the beginning of the nineties with the seminal work of Lee [1], in which he derived the capacity of a Rayleigh fading channel. Since then, additional results are rapidly becoming available. In [2], the author extended the results presented in [1] by deriving the capacity of Rayleigh fading channels under MRC diversity. In [3] and in [4], the authors derived the capacity of Rayleigh fading channels under different diversity schemes and different rate adaptation and transmit power configurations. Other fading channels like Rician, Hoyt, Nakagami and Weibull fading channels were studied in [5], [6] and [7].

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Although a huge amount of research has addressed the capacity of different kind of fading channels, the results on the capacity of log-normal fading channels are rather scarce. This dearth does not imply that the study of the log-normal capacity is less interesting. Indeed, in different cases, wireless channels are modeled as log-normal. This holds particularly for slowly varying channels like indoor channels. Indeed, both the small and the large scale effects are compounded, and consequently the log-normal fading accurately describes the distribution of the channel path gain. In addition, if spatial diversity is used at the receiver, then the effects of the multipath will be mitigated and the performance of the communication system will be only affected by the log-normal shadowing. Last but not least, the log-normal distribution is found to be the best fit to characterize Ultra Wideband (UWB) channels [8]. Therefore a closed-form expression of the capacity of the log-normal channel is of great use in order to assess the ultimate performance that one can achieve in such environments. Upper and lower bounds for the capacity were developed in [9], but as we will see later, these bounds are loose for low SNR. Here, we will provide a closed-form expression for the ergodic capacity of the log-normal channel with channel state information at the receiver. Since the developed expression contains an infinite series, we also study the error that results from the truncation of this series. Relying on the fact that the sum of log-normal Random Variables (RV) is well approximated by another log-normal RV, we further apply the obtained results to find the capacity of correlated log-normal fading channels with Maximum Ratio Combining and Equal Gain Combining.

The remainder of the paper is organized as follows. In Section 2, we derive the capacity of the log-normal channel. Section 3 extends the obtained results to approximate the capacity of maximum ratio combining and equal gain combining in a correlated environment and section 4 concludes the paper.

2. THE CAPACITY OF THE LOG-NORMAL CHANNEL

The ergodic capacity of a fading channel is known to be given by [10]

$$E[C] = \int_0^{+\infty} \log_2(1+\gamma) f_\gamma(\gamma) d\gamma, \tag{1}$$

where $f_\gamma(\gamma)$ denotes the probability density function (pdf) of the fading process.

For a log-normal fading channel, the pdf is given by

$$f_\gamma(\gamma) = \frac{\xi}{\sigma\sqrt{2\pi}\gamma} e^{-\frac{(\xi \ln \gamma - \mu)^2}{2\sigma^2}}, \tag{2}$$

where $\xi = \frac{10}{\ln(10)} = 4.3429$, σ is the logarithmic standard deviation expressed in dB and $\mu = \Gamma_{\text{dB}} - \frac{\sigma^2}{2\xi}$ [1] is the logarithmic mean of the log-normal RV also expressed in dB. $\Gamma_{\text{dB}} = \xi \ln(\Gamma)$ denotes the average Signal to Noise Ratio (SNR) in dB.
The ergodic capacity of the log-normal channel is therefore given by:

$$E[C] = \frac{\xi}{\sigma\sqrt{2\pi}\ln(2)} \int_0^{+\infty} \frac{\ln(1+\gamma)}{\gamma} e^{-\frac{(\xi \ln \gamma - \mu)^2}{2\sigma^2}} d\gamma, \tag{3}$$

2.1 First result

The capacity can be written as

$$\begin{aligned} E[C] &= \frac{e^{-\frac{\mu^2}{2\sigma^2}}}{2\ln(2)} \left[\sum_{k=1}^{K} \frac{(-1)^{k+1}}{k} \operatorname{erfcx}(\frac{\sigma k}{\xi\sqrt{2}} + \frac{\mu}{\sqrt{2}\sigma}) \right. \\ &+ \left. \sum_{k=1}^{K} \frac{(-1)^{k+1}}{k} \operatorname{erfcx}(\frac{\sigma k}{\xi\sqrt{2}} - \frac{\mu}{\sqrt{2}\sigma}) \right] + R_K \\ &+ \frac{\mu}{2\xi\ln(2)} \operatorname{erfc}(-\frac{\mu}{\sqrt{2}\sigma}) + \frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\xi\sqrt{2\pi}\ln(2)}, \end{aligned} \tag{4}$$

where K is a sufficiently large integer, $\operatorname{erfcx}(x)$ is a built-in MATLAB function called the scaled complementary error function and is given by:

$$\begin{aligned} \operatorname{erfcx}(x) &= e^{x^2} \operatorname{erfc}(x) & (5) \\ &= \frac{2e^{x^2}}{\sqrt{\pi}} \int_x^{+\infty} e^{-t^2} dt, & (6) \end{aligned}$$

and R_K will be given by one of the two expressions bellow:

- If $\mu \neq 0$,

$$\begin{aligned} R_K &\cong \frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\mu\sqrt{2\pi}\ln(2)} (-1)^K \left[\beta(K+1-\frac{\xi\mu}{\sigma^2}) \right. \\ &- \left. \beta(K+1+\frac{\xi\mu}{\sigma^2}) \right], \end{aligned} \tag{7}$$

  where $\beta(\cdot)$ is given by (8.372) in [11] as:

$$\beta(x) = \sum_{k=0}^{+\infty} \frac{(-1)^k}{k+x} = \frac{1}{2}\left(\psi\left(\frac{x+1}{2}\right) - \psi\left(\frac{x}{2}\right) \right), \tag{8}$$

  where $\psi(\cdot)$ is the Digamma function.

- If $\mu = 0$,

$$R_K \cong \frac{\xi\sqrt{2}}{\sigma\sqrt{\pi}\ln(2)} \left[\frac{\pi^2}{12} - \sum_{k=1}^{K} \frac{(-1)^{k+1}}{k^2} \right]. \tag{9}$$

[1]Note that the fading process is normalized, i.e., $E[\gamma] = e^{\frac{\mu}{\xi} + \frac{\sigma^2}{2\xi^2}} = \Gamma$.

Proof

By letting $u = \ln(\gamma)$ in (3), the capacity can be rewritten as

$$E[C] = \frac{\xi}{\sigma\sqrt{2\pi}\ln(2)} \int_{-\infty}^{+\infty} \ln(1+e^u) e^{-\frac{(\xi u - \mu)^2}{2\sigma^2}} du. \tag{10}$$

With some manipulations we obtain that:

$$\begin{aligned} E[C] &= \frac{\xi}{\sigma\sqrt{2\pi}\ln(2)} \left[\int_0^{+\infty} \ln(1+e^{-u}) e^{-\frac{(\xi u + \mu)^2}{2\sigma^2}} du \right. \\ &+ \left. \int_0^{+\infty} \ln(1+e^{u}) e^{-\frac{(\xi u - \mu)^2}{2\sigma^2}} du \right]. \end{aligned} \tag{11}$$

Noticing that $\ln(1+e^u) = u + \ln(1+e^{-u})$ and making this change in the second integral, we obtain:

$$\begin{aligned} E[C] &= \frac{\xi}{\sigma\sqrt{2\pi}\ln(2)} \left[\int_0^{+\infty} \ln(1+e^{-u}) e^{-\frac{(\xi u + \mu)^2}{2\sigma^2}} du \right. \\ &+ \int_0^{+\infty} \ln(1+e^{-u}) e^{-\frac{(\xi u - \mu)^2}{2\sigma^2}} du \\ &+ \left. \int_0^{+\infty} u e^{-\frac{(\xi u - \mu)^2}{2\sigma^2}} du \right]. \end{aligned} \tag{12}$$

The last term in this equation can be easily expressed as follows:

$$\int_0^{+\infty} u e^{-\frac{(\xi u - \mu)^2}{2\sigma^2}} du = \frac{\sigma^2}{\xi^2} e^{\frac{-\mu^2}{2\sigma^2}} + \mu \frac{\sigma}{\xi^2} \sqrt{\frac{\pi}{2}} \operatorname{erfc}(-\frac{\mu}{\sqrt{2}\sigma}). \tag{13}$$

Let us now introduce the following series expansion, which holds for $0 \le x \le 1$ ((1.512) in [11]),

$$\ln(1+x) = \sum_{k=1}^{+\infty} (-1)^{k+1} \frac{x^k}{k}. \tag{14}$$

Since $0 \le e^{-u} \le 1$ for $u \ge 0$, then

$$\ln(1+e^{-u}) = \sum_{k=1}^{+\infty} (-1)^{k+1} \frac{e^{-ku}}{k}. \tag{15}$$

Consequently by injecting this identity in $E[C]$, the capacity becomes[2]:

$$\begin{aligned} E[C] = \frac{\xi}{\sigma\sqrt{2\pi}\ln(2)} & \left[\sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k} \int_0^{+\infty} e^{-k\gamma} e^{-\frac{(\xi\gamma + \mu)^2}{2\sigma^2}} d\gamma \right. \\ &+ \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k} \int_0^{+\infty} e^{-k\gamma} e^{-\frac{(\xi\gamma - \mu)^2}{2\sigma^2}} d\gamma \\ &+ \left. \frac{\sigma^2}{\xi^2} e^{\frac{-\mu^2}{2\sigma^2}} + \frac{\mu\sigma}{\xi^2} \sqrt{\frac{\pi}{2}} \operatorname{erfc}(-\frac{\mu}{\sqrt{2}\sigma}) \right]. \end{aligned} \tag{16}$$

We can prove that:

$$\int_0^{+\infty} e^{-k\gamma} e^{-\frac{(\xi\gamma \pm \mu)^2}{2\sigma^2}} d\gamma = e^{-\frac{\mu^2}{2\sigma^2}} \sqrt{\frac{\pi\sigma^2}{2\xi^2}} \operatorname{erfcx}(\frac{\sigma k}{\xi\sqrt{2}} \pm \frac{\mu}{\sqrt{2}\sigma}). \tag{17}$$

[2]It can be easily shown that the sum and the integral can be interchanged.

Consequently, the capacity of a log-normal fading channel will be given by[3]

$$E[C] = \frac{e^{-\frac{\mu^2}{2\sigma^2}}}{2\ln(2)}\left[\sum_{k=1}^{+\infty}\frac{(-1)^{k+1}}{k}\,\mathrm{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}+\frac{\mu}{\sqrt{2}\sigma}) + \sum_{k=1}^{+\infty}\frac{(-1)^{k+1}}{k}\,\mathrm{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}-\frac{\mu}{\sqrt{2}\sigma})\right] + \frac{\mu}{2\xi\ln(2)}\mathrm{erfc}(-\frac{\mu}{\sqrt{2}\sigma}) + \frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\xi\sqrt{2\pi}\ln(2)}. \quad (18)$$

This expression can be also rewritten as

$$E[C] = C_K + R_K, \quad (19)$$

where C_K is the truncated series plus the last two terms and R_K is the rest of the series and is given by:

$$R_K = \frac{e^{-\frac{\mu^2}{2\sigma^2}}}{2\ln(2)}\left[\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k}\,\mathrm{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}+\frac{\mu}{\sqrt{2}\sigma}) + \sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k}\,\mathrm{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}-\frac{\mu}{\sqrt{2}\sigma})\right]. \quad (20)$$

For K sufficiently large, we have the following approximation:

$$\mathrm{erfcx}(k) \cong \frac{1}{k\sqrt{\pi}}, \quad k \geq K+1. \quad (21)$$

Using this identity, we obtain that

$$R_K \cong \frac{e^{-\frac{\mu^2}{2\sigma^2}}\xi}{\sigma\sqrt{2\pi}\ln(2)}\left[\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k(k+\frac{\xi\mu}{\sigma^2})} + \sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k(k-\frac{\xi\mu}{\sigma^2})}\right]. \quad (22)$$

If $\mu \neq 0$, in order to obtain (7), we use the fact that

$$\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k(k\pm\frac{\xi\mu}{\sigma^2})} = \pm\frac{\sigma^2}{\xi\mu}\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k} \mp \frac{\sigma^2}{\xi\mu}(-1)^K\beta(K\pm\frac{\xi\mu}{\sigma^2}+1). \quad (23)$$

If $\mu = 0$, we obtain (9) using the fact that

$$\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k^2} = \frac{\pi^2}{12} - \sum_{k=1}^{K}\frac{(-1)^{k+1}}{k^2}. \quad (24)$$

2.2 Second result

We can prove that R_K can be bounded as follows

$$|R_K| < \frac{\xi\sqrt{2}e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{\pi}\sigma\ln(2)(K+1+\frac{\xi\mu}{\sigma^2})(K+1-\frac{\xi\mu}{\sigma^2})}. \quad (25)$$

This result suggests that for a relatively large value of K, R_K can be neglected in (4).
This observation is better illustrated by Fig. 1 showing the truncation error. This error is defined by the ratio of the right hand side of the inequality to the capacity $E[C]$. Clearly, it is possible to do a truncation in (18) without undermining the accuracy of the formula.

[3] We should note here that Schwartz and Yeh [12] obtained a similar expression in the context of approximating the distribution of the sum of log-normal random variables.

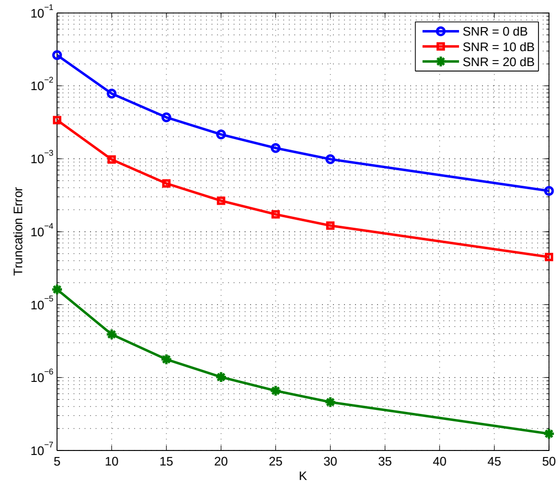

Figure 1: The truncation error as a function of K ($\sigma = 5$).

Proof

Starting from

$$|R_K| < \frac{e^{-\frac{\mu^2}{2\sigma^2}}}{2\ln(2)}\left[\left|\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k}\,\mathrm{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}+\frac{\mu}{\sqrt{2}\sigma})\right| + \left|\sum_{k=K+1}^{+\infty}\frac{(-1)^{k+1}}{k}\,\mathrm{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}-\frac{\mu}{\sqrt{2}\sigma})\right|\right], \quad (26)$$

we prove (25) by noting that the two series that intervene in (18) are alternating series. Indeed, $\left\{\frac{\mathrm{erfcx}(\frac{\sigma k}{\xi\sqrt{2}}\pm\frac{\mu}{\sqrt{2}\sigma})}{k}\right\}_k$ are sequences of positive decreasing terms that converge to 0 when $k \to \infty$. Hence, the two sums in the right hand side of the last inequality are the remainders of two alternating series. However, for an alternating series we have the following result

$$\left|\sum_{k=K+1}^{+\infty}(-1)^{k+1}a_k\right| < a_{K+1}. \quad (27)$$

Therefore applying this inequality gives

$$|R_K| < \frac{e^{-\frac{\mu^2}{2\sigma^2}}}{2(K+1)\ln(2)}\left[\mathrm{erfcx}(\frac{\sigma(K+1)}{\xi\sqrt{2}}+\frac{\mu}{\sqrt{2}\sigma}) + \mathrm{erfcx}(\frac{\sigma(K+1)}{\xi\sqrt{2}}-\frac{\mu}{\sqrt{2}\sigma})\right]. \quad (28)$$

And finally, by using the approximation given by (21), and after some simplification, we obtain (25).
Fig. 2 shows the capacity versus Γ_{dB} for a log-normal RV with $\sigma = 5$ dB and $K = 10$. This figure depicts clearly the adequacy between the results obtained by simulations and those generated by the developed formula. This figure depicts also the bounds obtained in [9], which are given by

$$\begin{cases} E[C]_{\rm up}=\frac{1}{\log(2)}\left(\frac{\mu}{\xi}+\exp(-\frac{\mu}{\xi}+\frac{\sigma^2}{2\xi^2})\right), \\ E[C]_{\rm low}=\frac{1}{\log(2)}\left(\frac{\mu}{\xi}+Q(\frac{\mu}{\sigma})-\exp(\frac{\mu}{\xi}+\frac{\sigma^2}{2\xi^2})Q(\frac{\mu}{\sigma}+\frac{\sigma}{\xi})\right), \end{cases}$$

where $Q(\cdot)$ is the gaussian integral function and can be related to the complementary error function as follows

$$Q(x)=\frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right). \qquad (29)$$

These bounds are loose for low SNR; the upper bound highly overestimates the capacity whereas the lower bound underestimates it. However, our formula accurately characterizes the capacity for all SNR values.

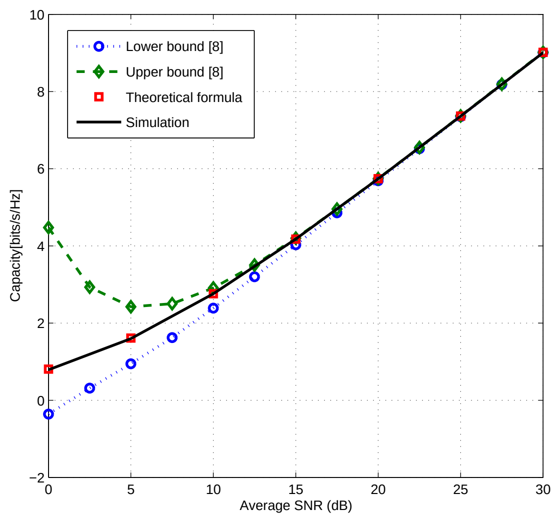

Figure 2: Capacity in log-normal fading ($\sigma=5$) for the SISO channel.

3. CAPACITY WITH MRC AND EGC

The instantaneous received SNR at the output of an M-branch maximum ratio combiner and equal gain combiner are, respectively, given by

$$\begin{cases} \gamma_{mrc}=\sum_{m=1}^{M}\gamma_m, \\ \gamma_{egc}=\frac{1}{M}(\sum_{m=1}^{M}\sqrt{\gamma_m})^2. \end{cases}$$

Exact expressions for the probability density functions of the RVs γ_{mrc} and γ_{egc} are unfortunately unknown. Notice however, that these RVs consist of a sum of log-normal RVs[4]. We can therefore hinge on the log-normal approximation which states that the sum of log-normal Random Variables (RV) can be well approximated by another log-normal RV. Consequently γ_{mrc} and γ_{egc} are viewed as log-normal RVs thereby allowing us to use the previously established results for the SISO channel.

The logarithmic mean and logarithmic variance of the log-normal approximations of γ_{mrc} and γ_{egc} can be estimated by various methods. Here, we use the well known Fenton-Wilkinson (F-W) [13] method because it provides a closed-form expression of the parameters of the log-normal RV and because of its simplicity. However, it should be noted that the F-W method performs badly for large standard deviations. In such cases, other methods should be used instead. Among these methods, we refer the interested reader to the method in [14] which seems to provide good results even for high standard deviations.

Without loss of generality, we assume in the following that the different diversity branches experience identical fading, i.e., each branch has an average SNR equal to Γ and standard deviation equal to σ. We assume also that the branches can be correlated and that the correlation factor ρ, is constant for each pair of branches. We have therefore the following expressions

$$\begin{cases} \mu_{mrc}=\xi\ln(M\Gamma)-\frac{\sigma^2_{mrc}}{2\xi}, \\ \sigma^2_{mrc}=\xi^2\ln\left(\frac{(M-1)e^{\rho\frac{\sigma^2}{\xi^2}}+e^{\frac{\sigma^2}{\xi^2}}}{M}\right), \end{cases}$$

and

$$\begin{cases} \mu_{egc}=\xi\ln\left(\Gamma+\Gamma(M-1)e^{(\rho-1)\frac{\sigma^2}{4\xi^2}}\right)-\frac{\sigma^2_{egc}}{2\xi}, \\ \sigma^2_{egc}=4\xi^2\ln\left(\frac{(M-1)e^{\rho\frac{\sigma^2}{4\xi^2}}+e^{\frac{\sigma^2}{4\xi^2}}}{M}\right). \end{cases}$$

The capacity for MRC and EGC is obtained therefore by substituting these values in (4).

Fig. 3 shows the capacity in a log-normal channel with maximum ratio combining and equal gain combining. It is observed that the capacity given by the analytical formula match perfectly the capacity given by the simulation. Also as expected the capacity increases as the number of antennas increases.

Fig. 4 and 5 show the impact of the correlation on the capacity of MRC and EGC. As one should expect the capacity decreases as the correlation factor increases.

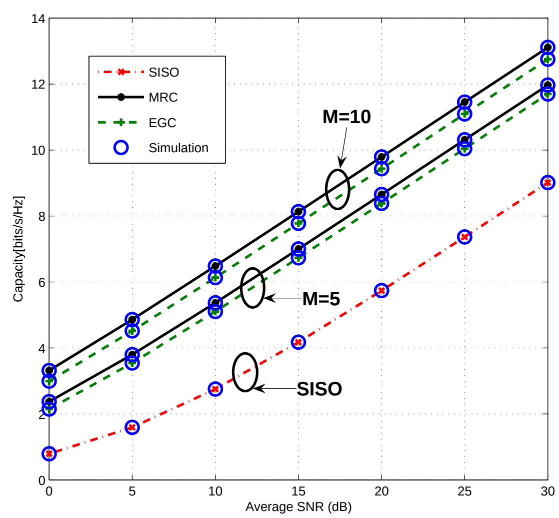

Figure 3: Capacity in log-normal fading ($\sigma=5$) for SISO, MRC and EGC ($\rho=0$ and $K=10$).

[4]Since the square, the square root as well as the multiplication by a constant of a log-normal RV is a log-normal RV, γ_{egc} can be also seen as a sum of log-normal RVs.

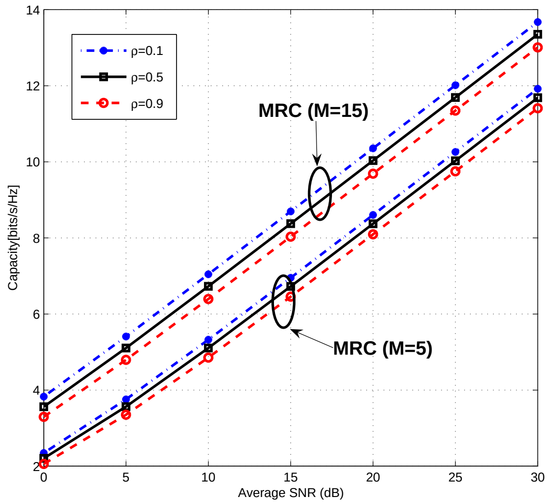

Figure 4: Capacity in log-normal fading ($\sigma = 5$) of MRC for correlated branches.

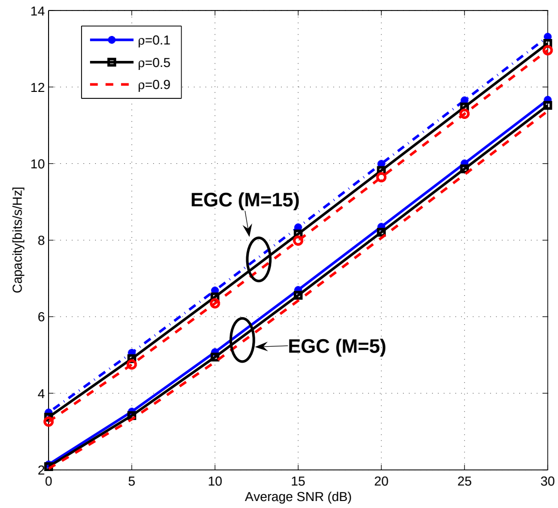

Figure 5: Capacity in log-normal fading ($\sigma = 5$) of EGC for correlated branches.

4. CONCLUSION

In this paper we have provided a closed-form expression of the ergodic capacity of log-normal fading channels with receiver channel state information. Since the sum of log-normal Random Variables (RV) is well approximated by another log-normal RV, the developed formula can be used as well to evaluate the capacity of uncorrelated/correlated log-normal channels with Maximum Ratio Combining and Equal Gain Combining. The analytical expressions obtained match perfectly the capacity given by simulations.

5. ACKNOWLEDGMENT

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