A TWO-STAGE APPROACH TO LOCALIZE SPATIALLY DISTRIBUTED SOURCES

Mehrez Souden, Sofiène Affes, and Jacob Benesty

INRS-ÉMT, 800, de la Gauchetière O., Suite 6900,
Montréal, H5A-1K6, Qc, Canada

ABSTRACT

We present a new technique to localize multiple scattered sources and estimate their angular spreads (ASs). We take into account the partial stationarity of the channel and propose a two-stage approach to estimate the ASs and the nominal angles of arrival (AoAs) of the sources. First, we blindly estimate the channels over several data blocks regularly spaced by intervals larger than the coherence time but each, short enough in length, to make time variations negligible within the block duration. Second, for each spatially scattered source, we separately process the corresponding sequence of quasi-independent channel realization estimates as a new single-scattered-source observation over which we apply Taylor series expansions to transform the estimation of the nominal AoAs and the ASs of the corresponding scattered source into a simple localization of two closely-spaced uncorrelated rays (i.e., point sources). Rays' locations allow accurate retrieval of the AoAs and ASs. Simulations confirm the efficiency of the proposed approach in the most adverse conditions.

1. INTRODUCTION

In mobile communication systems, the performance of source localization algorithms are largely affected by the multipath phenomenon. Indeed, for the uplink, the energy transmitted by a single source (mobile terminal) arrives at the receiver within a cluster of rays randomly distributed around the nominal AoA due to the local scatterers. This phenomenon has a negative impact on classical localization algorithms [1]. In this context, the nominal AoA and the AS (i.e., the standard deviation of the AoA of a locally scattered source) are two key parameters in the design of source localizers [2] and optimal detectors [3].

Recently, the case of narrow-band scattered sources has been investigated in several works as [4]. Therein, incoherent distribution of sources has been assumed. This hypothesis stems from the particular situation of a highly varying channel in wireless communication systems. Unfortunately, this assumption is quite hard to satisfy in several real-world cases. Indeed, the channel realizations are closely related to the motion speed of the mobile terminals or equivalently the scatterers within their vicinity. Fast fading channels can be encountered with fast moving sources. However, they appear as static for slowly moving ones [5]-[7]. In addition, the channel stationarity assumption (at least during the estimation process) has been long exploited to develop blind and pilot signals-based channel estimation algorithms [8]-[10], [5], etc. In Section 4, we empirically prove that this stationarity feature affects the technique proposed in [4]. This fact accounts for the relevance of the current work.

In this paper, we present a new two-stage algorithm for the estimation of the nominal AoAs and the ASs of locally scattered sources. First, we exploit the sources independence to estimate the channel realizations over several data blocks. The channel is assumed as stationary over each block. Second, we match the channel estimates and exploit Taylor series expansions proposed in [4] to transform the estimation of the AoA and the AS of every source using a uniform linear array (ULA) of sensors into a simpler task consisting in localizing two point sources symmetrically positioned around the nominal AoA. The resulting procedure takes advantage of the capabilities of the channel estimation preprocessing-stage to accurately estimate the required parameters even in adverse contexts.

2. PROBLEM STATEMENT AND ASSUMPTIONS

We suppose N narrow-band, stationary, ergodic, and *independent* sources. Each source is scattered by a large number of scatterers within its vicinity to generate L wavefronts. This scenario is practical in the radiocommunications context where every source models a mobile terminal surrounded by scatterers [7]. At instant t, the considered sources, represented by an N-dimensional vector $\mathbf{s}(t) = [s_1(t) \; ... \; s_N(t)]^T$, impinge on M sensors yielding an M-dimensional observations vector $\mathbf{x}(t) = [x_1(t) \; ... \; x_M(t)]^T$. The channel is then modelled as an $M \times N$ matrix $\mathbf{B}(t)$, and $\mathbf{x}(t)$ is expressed as:

$$\mathbf{x}(t) \;=\; \mathbf{B}(t)\,\mathbf{s}(t) + \mathbf{n}(t), \qquad (1)$$

where $\mathbf{n}(t) = [n_1(t) \; ... \; n_M(t)]^T$ is an unknown noise vector composed of M Gaussian i.i.d centered stationary signals with variance σ_n^2. The L wavefronts generated from the qth source are impinging from different directions $(\tilde{\theta}_{ql})_{1 \le l \le L}$, assumed to be symmetrically distributed around the nominal AoA, θ_q, on the sensors array. Hence, the qth channel matrix column is expressed as:

$$\mathbf{b}_q(t) = \sum_{l=1}^{L} \gamma_{ql}(t)\mathbf{a}\left[\theta_q + \tilde{\theta}_{ql}(t)\right], \qquad (2)$$

where $\mathbf{a}$ represents the nominal steering vector whose expression strongly depends on the geometry of the sensors array. In this work, we consider only the case of

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a ULA of sensors. Hence, $\mathbf{a}$ is expressed as $\mathbf{a}(\theta) = [1 \; e^{j2\pi\Delta\sin(\theta)} \; ... \; e^{j2(M-1)\pi\Delta\sin(\theta)}]^T$ where Δ is the sensors separation in wavelengths. The channel gains, $(\gamma_{ql})_{1\le l\le L}$, are commonly modelled as uncorrelated, zero-mean complex Gaussian random variables. This corresponds to Rayleigh fading for a large number of scatterers. Actually, γ_{ql} and $\tilde{\theta}_{ql}$ fully characterize the lth wavefront generated from the qth source, and are the realizations of the stochastic processes γ_q and $\tilde{\theta}_q$, respectively. We also assume as in [4] that $\tilde{\theta}_q$ is centered and symmetrically distributed with low standard deviation. This hypothesis is practical for macrocell environments in radiocommunication systems [7].

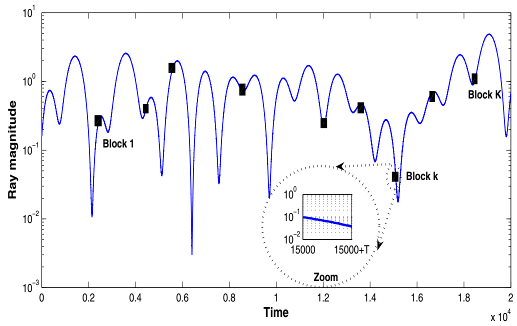

Fig. 1. An illustration of our processing strategy exploiting both fast and slow channel variations.

In practice, the channel may be slowly varying due to the low speed of the mobile terminal. Consequently, the random variables $\tilde{\theta}_q$ and γ_q are slowly varying. This fact directly affects the estimation of the parameters σ_{θ_q} and θ_q; $\forall\, q \in \{1, ..., N\}$. In Figure 1, we see that within a short data block, the ray's magnitude can be assumed as constant. Actually, the same slow variations behavior is observed with the the other parameters characterizing the ray. This special stationarity feature of the channel is commonly exploited in several works [5], [8]-[10], etc. However, the ray's magnitude (consequently the channel) remarkably changes between two distant-enough blocks. In this work, we take advantage of both aspects of slow/fast channel variations. Indeed, we first estimate the channel over K short data blocks. Then, we combine all the estimates to retrieve the ASs and the AoAs as explained below.

3. CHANNEL PARAMETERS ESTIMATION

3.1. Preprocessing: blind channel identification

Since the sources are independent, we can use the blind channel identification (through independent component analysis) as a preprocessing step to estimate the ASs and the AoAs. Hence, the performance of the proposed method strongly depends on this stage. Precisely, a fast convergent and accurate channel estimation algorithm is required to have less computational complexity and acceptable accuracy with a limited number of snapshots. This preprocessing has two main advantages: (i) It transforms the general multi-source problem in hand into the estimation of the AS and nominal AoA of every source separately. (ii) In case of *colored* Gaussian noise, the channel realizations can be estimated using fourth-order statistics [8], rendering the estimation of the ASs and the AoAs possible in the second stage even in colored noise, in contrast to previous techniques.

Without loss of generality, we will consider a spatially white Gaussian noise and use in our simulations in Section 4 the algorithm proposed in [9, 10]. We run the blind channel identification algorithm over each of the K blocks (cf. Figure 1). The K channel matrix realizations are blindly estimated up to some scale and permutation indeterminacies for the K data blocks. In other words, if we note $\mathbf{B}(k)$ as the kth channel matrix realization, its estimate is expressed as:

$$\hat{\mathbf{B}}(k) = \mathbf{B}(k)\mathbf{P}(k)\mathbf{D}(k) + \mathbf{E}(k), \tag{3}$$

where $\mathbf{D}(k) = \mathrm{diag}[\alpha_1(k) \; ... \; \alpha_N(k)]$ is a diagonal matrix composed of scalar indeterminacies, $\mathbf{P}(k)$ is a permutation matrix, and $\mathbf{E}(k)$ is an "error matrix" representing the estimation residue of the preprocessing step.

3.2. Covariance matrix and practical considerations

Here, we suppose that the permutation indeterminacies are solved. We will address this issue in the following subsection. The qth column of the channel matrix has K realizations $[\mathbf{b}_q(k)]_{1\le k\le K}$ whose estimates are:

$$\begin{aligned}\hat{\mathbf{b}}_q(k) &= \alpha_q(k)\mathbf{b}_q(k) + \mathbf{e}_q(k)\\ &= \alpha_q(k)\sum_{l=1}^{L}\gamma_{ql}(k)\mathbf{a}\left[\theta_q + \tilde{\theta}_{ql}(k)\right] + \mathbf{e}_q(k),\end{aligned} \tag{4}$$

where $\mathbf{e}_q(k)$ is the column vector of $\mathbf{E}(k)$ defined in (3). Notice here that (4) has the same form as the data model that has been long considered in the literature to estimate the AS and the nominal AoA [4]. In addition, the fundamental hypothesis of incoherently distributed sources with fast channel realizations is now satisfied if the K data blocks are enough spaced such that for $q \in \{1, ..., N\}$, $[\gamma_{ql}(k)]_{1\le k\le K}$ models a sequence of realizations of a random variable which is independent of $[\gamma_{ql'}(k)]_{1\le k\le K}$ $\forall\, l,\ l' \in \{1, ..., L\}$ such that $l \neq l'$. The scale indeterminacies $[\alpha_q(k)]_{1\le k\le K}$ have no effect as it will be demonstrated later. Therefore, we can successfully utilize the same procedure presented in [4] to estimate the AS and the nominal AoA of the qth source. It is also important to point out that the estimation error, $\mathbf{e}_q(k)$, is not necessarily a spatially white process and could be correlated with $\mathbf{b}_q(k)$. Thus, one must take a special care in the channel identification stage so that this estimation error is as low as possible.

Finally, we consider the following covariance matrix of the qth channel vector to estimate the qth AS and nominal AoA:

$$\mathbf{R}_q = \mathrm{E}\{\mathbf{b}_q\mathbf{b}_q^H\}. \tag{5}$$

However, recall that only an estimate of $\mathbf{b}_q$ is available in (4). Hence, we will approximate $\mathbf{R}_q$ using:

$$\hat{\mathbf{R}}_q = \frac{1}{K}\sum_{k=1}^{K}\hat{\mathbf{b}}_q(k)\hat{\mathbf{b}}_q^H(k) \tag{6}$$

which is a consistent estimator of $\mathbf{R}_q$ up to a scale factor, $\mathrm{E}\{|\alpha_q|^2\}$, induced by the scale indeterminacies.

3.3. Channels matching

The point here is how to classify the estimated sources (or equivalently the column vectors of the random channel realizations' estimates) over the K data blocks. Two main cases must be considered to perform this task.

Scenario 1: the sources are spatially very close such that the wavefronts generated from a couple of sources overlap. In this case, some properties of the sources can be exploited. First, suppose that the sources are correlated over time (at least between two consecutive T-length data blocks). In this case, the calculation of the estimated sources' correlations for two consecutive data blocks could be exploited to match the independent components and solve the permutations indeterminacies. This assumption can be further relaxed to the temporal dependence. Indeed, knowing that the sources are mutually independent, one can easily use some higher order-statistics-based criteria (e.g., maximizing the cross-cumulants). Other properties of the sources can also be utilized depending on the considered application. For instance, in the context of CDMA systems, one can take advantage of the spreading codes to classify the channels. For digital signals, a waveform matching of the estimated sources could be exploited if the signals have different waveforms.

Scenario 2: the sources' angular separations are much larger than the ASs such that the wavefronts generated from at least two sources overlap. One can maximize the normalized columns scalar product of every estimate of the channel realization with a reference one (chosen randomly) to identify the K-length realizations sequence for every channel column vector. The solutions provided for the previous case also apply here.

3.4. Approximative two-ray model

We briefly review the procedure that transforms the estimation of the nominal AoA and the AS of a single source (indexed by q) into the localization of two point sources as in [4]. For low angular deviation values, the first order Taylor series expansion can be used to express the qth spatial frequency as:

$$\begin{aligned} 2\pi\Delta\sin(\theta_q+\tilde{\theta}_q) &\approx 2\pi\Delta\sin(\theta_q)+2\pi\Delta\tilde{\theta}_q\cos(\theta_q) \\ &\triangleq \omega_q+\tilde{\omega}_q, \end{aligned} \tag{7}$$

where $\tilde{\omega}_q$ is the spatial frequency deviation resulting from the angular deviation. According to the previous representation, $\tilde{\omega}_q$ and $\tilde{\theta}_q$ have approximately the same probability density function. Furthermore, we can easily establish that the standard deviation of $\tilde{\omega}_q$ is expressed as:

$$\sigma_{\omega_q}=2\pi\Delta\cos(\theta_q)\sigma_{\theta_q}. \tag{8}$$

Hence, determining θ_q and σ_{θ_q} amounts to estimating ω_q and σ_{ω_q} $\forall\ q\in\{1,...,N\}$. Now, using this first order Taylor series, it can be established that $\mathbf{R}_q$ is expressed as [3, 4]:

$$\mathbf{R}_q=\mathbf{D}_{\mathbf{a}}(\omega_q)\Xi(\sigma_{\omega_q})\mathbf{D}_{\mathbf{a}}{}^*(\omega_q), \tag{9}$$

where:

$$\mathbf{D}_{\mathbf{a}}(\omega_q)=\mathrm{diag}[\mathbf{a}(\omega_q)] \tag{10}$$

and $\Xi(\sigma_{\omega_q})=\mathbf{R}_q$ when $\omega_q=0$. The (p,r)th entry of $\Xi(\sigma_{\omega_q})$ is:

$$\xi_{pr}\approx\Phi_{\tilde{\omega}_q}\left[(p-r)\sigma_{\omega_q}\right], \tag{11}$$

with $\Phi_{\tilde{\omega}_q}$ being the characteristic function of $\tilde{\omega}_q$. This representation was exploited in [3] to explicit the effect of the angular spread on the coherence of the received signal. Nevertheless, it still requires the knowledge of the distribution of the angular deviation. To circumvent this limitation, a second order Taylor series expansion for $\mathbf{a}$ was utilized in [4] to find that $\mathbf{R}_q$ can be approximated as:

$$\mathbf{R}_q\approx\frac{1}{2}\mathbf{A}(\omega_q+\sigma_{\omega_q},\omega_q-\sigma_{\omega_q})\mathbf{A}^H(\omega_q+\sigma_{\omega_q},\omega_q-\sigma_{\omega_q}), \tag{12}$$

where:

$$\mathbf{A}(\omega_q+\sigma_{\omega_q},\omega_q-\sigma_{\omega_q})=\left[\mathbf{a}(\omega_q+\sigma_{\omega_q})\quad\mathbf{a}(\omega_q-\sigma_{\omega_q})\right]. \tag{13}$$

The approximation in (12) has two main advantages. First, the resulting representation is independent of the angular distribution and depends explicitly on the nominal AoA and the AS only. Second, the originally complicated angular spread estimation problem is transformed into a simpler one consisting in recovering two AoAs. Thus, a classical localization algorithm could be used to solve this problem. In the sequel, we will use the algorithm root-MUSIC [11] resulting in the so-called "spread root-MUSIC" [4]. Notice also that the application of the localization algorithm to $\Xi(\sigma_{\omega_q})$ leads empirically to two symmetrical values $\{\lambda(\sigma_{\omega_q}),-\lambda(\sigma_{\omega_q})\}$ where λ is positive function which has no analytical expression but can be empirically determined. For low AS values, $\lambda(\sigma_{\omega_q})\approx\sigma_{\omega_q}$ [4]. This approximation will be used next.

4. SIMULATION RESULTS

In what follows, we consider $L=50$ wavefronts of every source such that for $q\in\{1,...,N\}$, $(\gamma_{ql})_{1\leq l\leq L}$ are centered, Gaussian, and i.i.d with $\mathrm{E}\{|\gamma_{ql}|^2\}=1/L$. The angular deviations of these wavefronts are uniformly distributed i.e., $\tilde{\theta}_q\in[-\sqrt{3}\sigma_{\theta_q},\sqrt{3}\sigma_{\theta_q}]$. The signals are received over a ULA of $M=6$ sensors with a half wavelength elements separation. As a performance index, we calculate the root mean squared error (RMSE) over $MC=5.10^2$ Monte-Carlo runs in all our simulations.

To prove the relevance of the proposed method, we start by checking the effect of the channel partial stationarity on the algorithm spread root-MUSIC. We consider the case of a single scattered BPSK source, and vary the stationarity window length of the channel. In other words, we suppose that the channel [characterized by $(\gamma_{1l})_{1\leq l\leq L}$ and $(\tilde{\theta}_{1l})_{1\leq l\leq L}$] is constant over T_s snapshots and estimate the AS and the nominal AoA. We suppose that $\sigma_{\theta_1}=3°$, $\theta_1=0°$, $SNR=10$ dB, and we estimate the statistics over $T=5.10^2$ snapshots. Figure 2 clearly shows that the accuracy of this algorithm deteriorates as T_s increases.

We consider a second scenario of two scattered sources located at $\theta_1=0°$ and $\theta_2=5°$. The ASs of both signals are $\sigma_{\theta_1}=\sigma_{\theta_2}=3°$ (i.e., adverse scenario of two scattered sources with spatial overlap). We chose

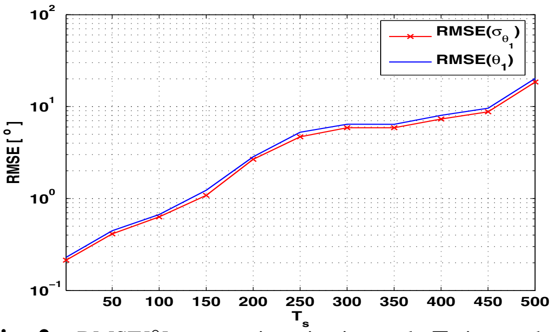

Fig. 2. RMSE[°] vs. stationarity interval, T_s in number of snapshots, at SNR= 10 dB and $\sigma_{\theta_1} = 3°$.

$T = K = 10^2$ so that we ensure an acceptable accuracy in blindly estimating the channel matrix realizations, the ASs, and the nominal AoAs. We suppose that the channel is constant over the T snapshots and take different Gaussian realizations of the coefficients $[\gamma_{ql}(k)]_{1 \le k \le K}$ for $l \in \{1, ..., L\}$ and $q \in \{1, ..., N\}$ as assumed in Section 2. To solve the permutation indeterminacies in estimating the channel, we suppose that the same signals are retransmitted over the K blocks. We compare the proposed approach to the direct one where we calculate the statistics over the KT snapshots[1] without taking into account the channel partial stationarity. In Figures 3 and 4, we plot the RMSE for the ASs and nominal AoAs achieved by both approaches with respect to the SNR. Clearly, the proposed two-stage approach outperforms the direct one. Indeed, since $|\theta_1 - \theta_2| < \sqrt{3}(\sigma_{\theta_1} + \sigma_{\theta_2})$, the resulting wavefronts from both sources overlap and transforming the original problem into the localization of four rays is confusing since the rays can not be identified and matched properly to estimate the ASs and the nominal AoAs. For the new method, we mitigate this problem since every single channel vector random realizations are first identified using the spatial independence and the temporal correlation of the sources. After that, we exploit the two-ray approximation for every channel column vector.

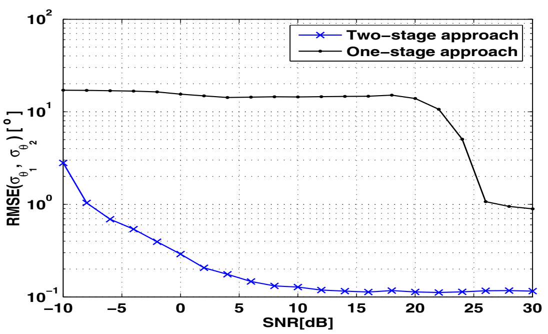

Fig. 3. RMSE($\sigma_{\theta_1}, \sigma_{\theta_2}$)[°] vs. SNR[dB] at $\sigma_{\theta_1} = \sigma_{\theta_2} = 3°$, $\theta_1 = 0°$, and $\theta_2 = 5°$.

[1] Other combinations of the observations from different data blocks were considered. The one used here seemed to be the best in terms of complexity and accuracy.

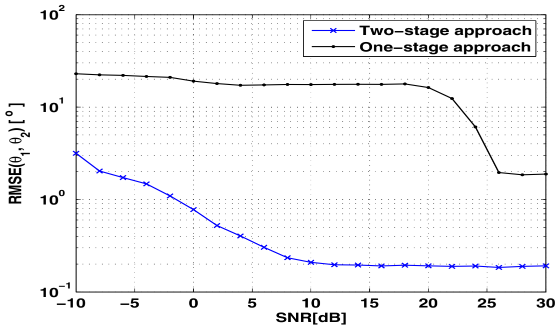

Fig. 4. RMSE(θ_1, θ_2)[°] vs. SNR[dB] at $\sigma_{\theta_1} = \sigma_{\theta_2} = 3°$, $\theta_1 = 0°$, and $\theta_2 = 5°$.

5. CONCLUSION

In this paper, a two-stage approach to estimate the ASs and the nominal AoAs of scattered sources was proposed. The first stage consists in channel identification while the second determines the required spatial parameters separately from each channel estimate. Simulation results demonstrate that this new method is suitable for accurate localization and estimation of the ASs of the scattered sources even in the most adverse conditions.

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