A NEW APPROACH TO BLIND SEPARATION OF TWO SOURCES WITH THREE SENSORS

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ABSTRACT

An exhaustive investigation of the overdetermined blind source separation (BSS) 3×2 problem is presented. We establish a new relationship between the column vectors of the channel matrix using second order statistics (SOS) only. This relationship is then exploited to factorize the separating matrix into a whitening term, whose expression depends on whether the observations are corrupted by noise or not, and a 2×2 orthogonal matrix that can be determined using high order statistics (HOS). Simulation results demonstrate that resorting to the new form of the whitening matrix, mainly in the case of noisy ill-conditioned BSS, results in improved performance compared to the common whitening-based techniques.

1. INTRODUCTION

Blind source separation has recently emerged as a wide field of research with several practical applications including radiocommunications, speech enhancement, biomedical signal processing, etc. This fact is due to the inherent goal of the BSS which consists in retrieving unknown sources by processing their mixtures only.

So far, numerous BSS algorithms have been proposed. Besides, a well known trend has been to consider linear mixtures of independent sources. Consequently, most of the BSS problems have been solved by independent component analysis (ICA) which exploits the independence of the source signals to separate them. Except some works as [1], the BSS-ICA schemes are generally based on the fourth order (FO) cost functions. In these schemes, the complexity of the BSS-ICA problem is greatly reduced by whitening the observations using SOS.

In this paper, we aim at separating two sources using three (or more sensors). This so called Two-Input Multiple-Output (TIMO) data model is practical in several situations [2] where it is convenient to have less antennas at the transmitting end due to cost or space constraints or when antenna selection is performed. Furthermore, it was shown in [3] that such a model is desirable since it leads to simple yet accurate solutions.

By focusing on the case of 3 x 2 BSS-ICA problem, we derive a new relationship between the column vectors of the channel matrix using SOS only. Interestingly, this relationship leads, in a generic form, to the factorization of the separating matrix into whitening and rotation terms. Based on this factorization, we further elaborate new forms for the whitening matrix which take into account the noise level. We show by simulations that the proposed approach enhances the BSS-ICA performance.

2. PROBLEM STATEMENT AND ASSUMPTIONS

We assume N stationary ergodic sources represented by an N-dimensional vector $\mathbf{s}(t) = [s_1(t) \ldots s_N(t)]^T$ and mixed by an $M \times N$ ($M \geq N$) unknown channel matrix $\mathbf{A}$ to yield an M-dimensional vector of observations $\mathbf{x}(t) = [x_1(t) \ldots x_M(t)]^T$ at time t:

$$\mathbf{x}(t) = \mathbf{A}\,\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{n}(t) = [n_1(t) \ldots n_M(t)]^T$ is an unknown noise vector composed of M Gaussian i.i.d centered stationary signals with variance σ_n^2. BSS consists in recovering the N sources, by solely processing the observations $\mathbf{x}(t)$. In other words, an unknown $M \times N$ separating matrix $\mathbf{G}$ is to be determined such that the estimate vector $\mathbf{y}(t)$ defined by:

$$\mathbf{y}(t) = \mathbf{G}\,\mathbf{x}(t) \tag{2}$$

is as close as possible to the original source vector $\mathbf{s}(t)$. In the sequel, we will omit the time index t for the sake of clarity. We also stress that only the case of $N = 2$ and $M = 3$ is considered in this work[1]. To have a tractable problem (described by equations (1) and (2)), several assumptions are commonly required: (**H1**) The source signals $(s_i)_{i\in\{1,\ldots,N\}}$ are mutually independent, with variances $(\sigma_i^2)_{i\in\{1,\ldots,N\}}$ respectively, and optionally centered. Sources independence

[1] We are currently investigating the general case $M \times N$.

is the core hypothesis for most of BSS-ICA literature. (**H2**) At most one of the sources is Gaussian. (**H3**) The channel matrix $\mathbf{A}$ is full column rank. (**H4**) The noise and source components are independent. (**H5**) The sources' kurtoses have the same sign.

3. CLASSICAL BSS-ICA WITH STANDARD WHITENING

Except some works as in [1], the classical BSS-ICA approach consists in two main steps: whitening (or standardization [3]) followed by a rotation of the observations such that the resulting components are as independent as possible. While the first step can be performed based only on the SOS, the identification of the independent components is achieved by use of the HOS.

In the prewhitening-based algorithms such as [3, 4, 5], a square root decomposition (such as Schur decomposition or EVD) of the observed covariance matrix is usually employed to estimate the number of sources and standardize the observations. To be specific, the covariance matrix of the observations is given by:

$$\mathbf{R}_x = \mathrm{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma_n^2\mathbf{I}. \tag{3}$$

Due to the scale indetermination [6], BSS-ICA problems are generally solved for unit variance sources (i.e., $\mathbf{R}_s = \mathbf{I}$). If the sources have different variances, the matrix:

$$\mathbf{B} = \mathbf{A}\mathbf{R}_s^{1/2}. \tag{4}$$

is to be considered instead of $\mathbf{A}$. In the sequel, we adopt this notation for the channel matrix. The SVD of $\mathbf{B}$ yields:

$$\mathbf{B} = \mathbf{V}\mathbf{\Lambda}^{1/2}\mathbf{V}_{\mathbf{r}}, \tag{5}$$

where $\mathbf{V}$ and $\mathbf{V}_{\mathbf{r}}$ are unitary matrices and $\mathbf{\Lambda}$ is diagonal. If we further note $\mathbf{R}_w = \mathbf{R}_x - \sigma_n^2\mathbf{I}$, equation (3) becomes:

$$\mathbf{R}_w = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H. \tag{6}$$

The whitening matrix is given by:

$$\mathbf{W} = \mathbf{\Lambda}^{-1/2}\mathbf{V}^H. \tag{7}$$

Actually, in the particular case 3×2, a projection on the signal subspace by taking the first two column vectors of the matrix $\mathbf{V}$ corresponding to the two non-null eigenvalues forming the diagonal of the 2×2 matrix $\mathbf{\Lambda}$ (the third line and column which are null are eliminated) is also performed.

Notice, here, that the third dimension corresponding to the noise subspace is employed to determine the noise power only. In contrast, we will show in the following section that this subspace can be exploited differently to yield an explicit relationship between the channel vectors. The 2-dimensional vector of the whitened observations:

$$\mathbf{x}_w = \mathbf{W}\mathbf{x} \tag{8}$$

represents a rotated version of the original sources. In other words, there exist a unitary matrix $\mathbf{Q}$ defined as:

$$\mathbf{Q} = \begin{pmatrix} \cos(\theta)e^{j\phi} & \sin(\theta)e^{-j\phi} \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \tag{9}$$

where θ and ϕ are real valued, such that:

$$\mathbf{x}_w = \mathbf{Q}\mathbf{s}. \tag{10}$$

Resorting to HOS of $\mathbf{x}_w$ is inevitable since SOS are not sufficient to determine the optimal rotation $\mathbf{Q}$. Besides, the particular case of two sources had been investigated in [3, 4]. There, the maximization of the FO contrast function led to analytical expressions for the optimal values of θ and ϕ based on FO statistics.

Once the optimal rotation, $\mathbf{Q}_{\mathrm{opt}}$, is determined, the channel matrix is completely identified up to some complex scale and permutation indeterminacies [6]:

$$\mathbf{B} = \mathbf{V}\mathbf{\Lambda}^{1/2}\mathbf{Q}_{\mathrm{opt}}, \tag{11}$$

In a noise-free situation, the separating matrix $\mathbf{G}$ is expressed as follows:

$$\mathbf{G} = \mathbf{Q}_{\mathrm{opt}}^H\mathbf{W}, \tag{12}$$

whereas for noisy mixtures, it is recommended to use the matrix corresponding to minimum mean square error (MMSE) to separate the sources once the channel matrix is identified:

$$\mathbf{G} = \mathbf{B}^H\mathbf{R}_x^{-1}. \tag{13}$$

4. NEW APPROACH

In contrast to the standard method described in the previous section, we further exploit the information contained in the noise subspace to establish an explicit relationship between the two column vectors of the channel matrix. This relationship will allow us to deduce two different expressions for the so-called whitening matrix depending on the noise level.

4.1. Explicit relationship between the channel vectors

Taking into account (**H3**), the covariance matrix, $\mathbf{R}_w = \mathbf{B}\mathbf{B}^H$, is of rank 2 . Hence, its kernel is one dimensional. We can verify that the vector $\mathbf{v}_0 = [v_{01}\ v_{02}\ v_{03}]^T$ whose entries satisfy the following system of equations:

$$\begin{cases} v_{01} & = b_{21}^*b_{32}^* - b_{31}^*b_{22}^* \\ v_{02} & = b_{31}^*b_{12}^* - b_{11}^*b_{32}^* \\ v_{03} & = b_{11}^*b_{22}^* - b_{21}^*b_{12}^* \end{cases}, \tag{14}$$

where $(b_{ij})_{1\leq i\leq 3,\ 1\leq j\leq 2}$ are the entries of the channel matrix $\mathbf{B}$, spans this subspace [7]. Interestingly, we can also check that the vector $\mathbf{v} = [v_1\ v_2\ v_3]^T$ having the following entries:

$$\left\{\begin{array}{ll} v_1 & = r_{12}r_{23} - r_{13}r_{22} \\ v_2 & = r_{21}r_{13} - r_{23}r_{11} \\ v_3 & = r_{11}r_{22} - r_{12}r_{21} \end{array}\right. , \qquad (15)$$

where $r_{ij} = \mathrm{E}\{x_i x_j^*\}$ ($\forall\ i, j \in \{1,2,3\}$), spans this subspace too. Hence, there exists a complex valued constant $\alpha \neq 0$ such that:

$$\mathbf{v}_0 = \alpha \mathbf{v}. \qquad (16)$$

Now combining equations (3), (14), and (16), we obtain the following non-linear system of equations:

$$\left\{\begin{array}{lll} b_{11}^* b_{22}^* - b_{21}^* b_{12}^* & = \alpha v_3 & (L1) \\ b_{11} b_{21}^* + b_{12} b_{22}^* & = r_{12} & (L2) \\ b_{31}^* b_{12}^* - b_{11}^* b_{32}^* & = \alpha v_2 & (L3) \\ b_{11} b_{31}^* + b_{12} b_{32}^* & = r_{13} & (L4) \\ b_{21}^* b_{32}^* - b_{31}^* b_{22}^* & = \alpha v_1 & (L5) \\ b_{21} b_{31}^* + b_{22} b_{32}^* & = r_{23} & (L6) \end{array}\right. . \qquad (17)$$

Performing the following operations on system (17) [7]:

$$\left\{\begin{array}{ll} L1 & \leftarrow b_{21}L1 - [b_{22}L2]^* \\ L2 & \leftarrow b_{22}L1 + [b_{21}L2]^* \\ L3 & \leftarrow b_{11}L3 - b_{12}^* L4 \\ L4 & \leftarrow b_{12}L3 + b_{11}^* L4 \\ L5 & \leftarrow [b_{31}L5]^* - b_{32}L6 \\ L6 & \leftarrow [b_{32}L5]^* + b_{31}L6 \end{array}\right. , \qquad (18)$$

we transform the system (17) into the following linear system of equations:

$$\mathbf{R}\,\mathbf{b} \ = \ \mathbf{0}, \qquad (19)$$

where

$$\mathbf{R} \ = \ \begin{pmatrix} -r_{31}^* & 0 & r_{11} & -\alpha v_2 & 0 & 0 \\ r_{22} & -r_{12}^* & 0 & 0 & -\alpha v_3 & 0 \\ 0 & r_{33} & -r_{32} & 0 & 0 & -\alpha v_1 \\ \alpha^* v_2^* & 0 & 0 & -r_{31} & 0 & r_{11} \\ 0 & \alpha^* v_3^* & 0 & r_{22} & -r_{12} & 0 \\ 0 & 0 & \alpha^* v_1^* & 0 & r_{33} & -r_{32}^* \end{pmatrix}$$

and

$$\mathbf{b} \ = \ [b_{11}^*\ b_{21}^*\ b_{31}^*\ b_{12}\ b_{22}\ b_{32}]^T.$$

In compact notations, equation (19) can be written as:

$$\mathbf{R} \ = \ \begin{pmatrix} \mathbf{C} & -\alpha \mathbf{D} \\ \alpha^* \mathbf{D}^* & \mathbf{C}^* \end{pmatrix} \qquad (20)$$

$$\mathbf{b} \ = \ [\mathbf{b}_1^H\ \mathbf{b}_2^T]^T. \qquad (21)$$

Vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ are, respectively, the first and the second column vectors of the channel matrix. On the other hand, $\alpha \neq 0$ and $\mathbf{B}$ is of rank 2. Therefore, $v_i \neq 0\ \forall\ i \in \{1,2,3\}$ and $\mathbf{D}$ is full rank. Hence, we conclude that the vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ are closely related by the following equations [7]:

$$\left\{\begin{array}{ll} \mathbf{b}_2 & = \frac{1}{\alpha}\mathbf{D}^{-1}\mathbf{C}\mathbf{b}_1^* \\ \mathbf{b}_1 & = -\frac{1}{\alpha}\mathbf{D}^{-1}\mathbf{C}\mathbf{b}_2^* \end{array}\right. . \qquad (22)$$

By an appropriate use of the noise subspace, we were able to establish an explicit relationship between the column vectors of the channel matrix. To the best of our knowledge, this new relationship is first identified in this work.

4.2. Parametrization

Using equation (22), it is easy to check that $\mathbf{b}_1$ and $\mathbf{b}_2$ are the eigenvectors of the matrix

$$\mathbf{F} \ = \ \mathbf{D}^{-1}\mathbf{C}\mathbf{D}^{-1*}\mathbf{C}^*,$$

associated with the eigenvalue $-|\alpha|^2$ (of multiplicity 2). The signal subspace, spanned by the two eigenvectors $\mathbf{u}_1$ and $\mathbf{u}_2$ of $\mathbf{F}$ associated with the double-multiplicity eigenvalue $-|\alpha|^2$, is now well defined using the noise subspace. In other words, $\mathbf{b}_1$ and $\mathbf{b}_2$ are linear combinations of $\mathbf{u}_1$ and $\mathbf{u}_2$:

$$\left\{\begin{array}{ll} \mathbf{b}_1 & = \eta_1^* \mathbf{u}_1 - \eta_2^* \mathbf{u}_2 \\ \mathbf{b}_2 & = \eta_1 \mathbf{u}_2 + \eta_2 \mathbf{u}_1 \end{array}\right. , \qquad (23)$$

where η_1 and η_2 are two unknown complex variables that will be determined in the sequel. Note that only $|\alpha|$ can be exactly computed by EVD of $\mathbf{F}$. Therefore, $\mathbf{b}_1$ and $\mathbf{b}_2$ are linked up to a phase indetermination in equation (22). To find the unknowns η_1 and η_2, recall that:

$$\|\mathbf{b}_1\|^2 + \|\mathbf{b}_2\|^2 = (|\eta_1|^2 + |\eta_2|^2)(\|\mathbf{u}_1\|^2 + \|\mathbf{u}_2\|^2) = \mathrm{tr}\{\mathbf{R}_x\}. \qquad (24)$$

Thus, if we normalize $\mathbf{u}_1$ and $\mathbf{u}_2$ by $\sqrt{\frac{\mathrm{tr}\{\mathbf{R}_x\}}{(\|\mathbf{u}_1\|^2 + \|\mathbf{u}_2\|^2)}}$, we obtain:

$$|\eta_1|^2 + |\eta_2|^2 \ = \ 1, \qquad (25)$$

and η_1 and η_2 can be expressed as:

$$\left\{\begin{array}{ll} \eta_1 & = \cos(\theta) e^{j\phi_1} \\ \eta_2 & = \sin(\theta) e^{j\phi_2} \end{array}\right. , \qquad (26)$$

where ϕ_1, ϕ_2, and θ are real valued. Now, taking into account the fact that $\mathbf{b}_1$ and $\mathbf{b}_2$ are determined up to a phase ambiguity, we eventually deduce the parametrized relationship between these two vectors:

$$\left\{\begin{array}{ll} \mathbf{b}_1 & = \cos(\theta) e^{j\phi} \mathbf{u}_1 - \sin(\theta) \mathbf{u}_2 \\ \mathbf{b}_2 & = \sin(\theta) e^{-j\phi} \mathbf{u}_1 + \cos(\theta) \mathbf{u}_2 \end{array}\right. , \qquad (27)$$

where θ and ϕ are parameters that can be determined using HOS only. In a more compact representation, the matrix $\mathbf{B}$ can be expressed as:

$$\mathbf{B} = \mathbf{U}\mathbf{Q}, \qquad (28)$$

where $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2]$ and $\mathbf{Q}$ is as defined in equation (9).

Based on the relationship established in equation (22) and the appropriate parametrization in equation (27), we find a generic factorization for the channel matrix similar, in essence, to that given in equation (11): a 3×2 matrix multiplied by a rotation. However, both first parts of $\mathbf{B}$ in equations (11) and (28) are different since they emanate from different methods.

4.3. Whitening matrix

Based on expression (28) for the channel matrix, we deduce the conventional factorization of the separating matrix $\mathbf{G}$ into a whitening term $\mathbf{W}$ and a rotation $\mathbf{Q}^H$. To be specific, $\mathbf{G}$ is expressed as:

$$\mathbf{G} = \mathbf{Q}^H \mathbf{W}. \tag{29}$$

Our goal now is to establish an optimal expression for $\mathbf{W}$ that takes into account whether the observed mixtures are corrupted by noise or not.

4.3.1. Noise-free mixtures

In the case of noise-free mixtures, the optimal separator $\mathbf{G}$ is obviously the pseudo-inverse of the channel matrix corresponding to the zero-forcing (ZF) criterion. Using equation (28), we obtain:

$$\mathbf{W} \ = \ \mathbf{U}^{\#}. \tag{30}$$

where $(.)^{\#}$ stands for the pseudo-inverse operator.

4.3.2. Noisy mixtures

As mentioned previously, when the observed mixtures are corrupted by noise, the matrix corresponding to the MMSE criterion and given in equation (13) is the optimal source separator. Hence, we obtain a new optimal expression for the whitening matrix:

$$\mathbf{W} \ = \ \mathbf{U}^H \mathbf{R}_x^{-1}. \tag{31}$$

This new form takes into account the noise effect in recovering the independent components from the whitened data in contrast to the conventional approach where the whitening matrix is set without considering the noise level. In other words, we exploit the MMSE criterion in both stages of the BSS-ICA problem: firstly in identifying the optimal rotation and secondly in separating the sources. We will prove by the simulations that this new method improves the BSS performance.

4.3.3. HOS-based resolution

The optimal rotation $\mathbf{Q}$ which is fully defined by the parameters θ and ϕ is found by resorting to HOS of the whitened data. The closed-form solutions provided in [3, 4] can be exploited to determine these parameters.

5. SIMULATION RESULTS

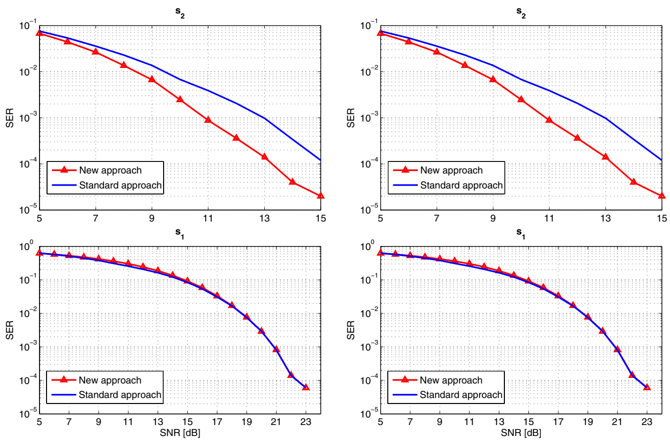
In order to prove the advantages of the proposed method, we compare it with the classical BSS-ICA with standard whitening described in section 3. We emphasize that our comparison is limited to the effect of the SOS only. Consequently, the second stage which consists in estimating the rotation matrix by HOS is common for both of the compared techniques. In the noise-free case, we found that the performance of the proposed technique is similar to the standard whitening-based one. Hence, we mainly focus our discussions on the noisy-mixtures case.

As mentioned in section 4.3.2, by using the MMSE criterion we obtained a new expression for the so-called whitening matrix which leads to an improved performance when employed to determine the optimal rotation matrix $\mathbf{Q}$. Once $\mathbf{Q}$ is determined, we use the MMSE criterion again to separate the sources. We compare our approach to the standard one where the conventional whitening matrix given in section 3 is used to whiten the observations and the MMSE criterion is employed to separate the sources only.

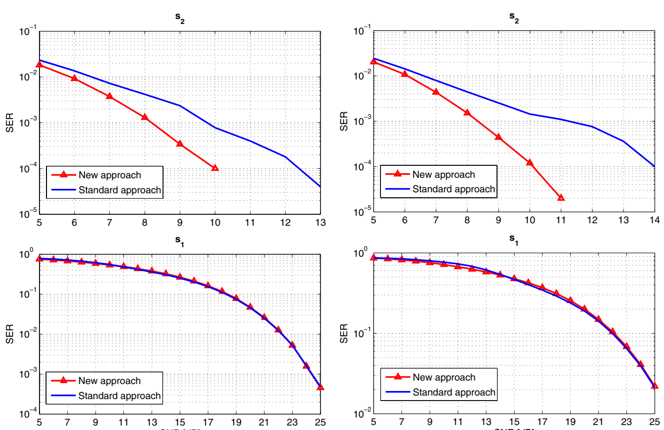
It goes without saying that the ultimate goal of BSS is to minimize the errors on the estimated sources. For instance, the symbol error rate (SER) in communication systems is a very reliable performance criterion for BSS-ICA techniques. We use 10^5 samples of two independent BPSK sources to evaluate the SER in our simulations. However, for all tested scenarios, we process the data in blocks of 500 samples each and estimate the required SOS and HOS to determine the separating matrices for every block. SER results are then averaged over all blocks. We particularly focus on the case where the BSS problem is ill conditioned. In fact, we consider that $\frac{\sigma_2^2}{\sigma_1^2} \neq 1$ (or equivalently the two channel vectors have different amplitudes) and assess the effect on the SER of the conditioning of the channel matrix $\mathbf{A}$ (with normalized columns), defined as the highest to the lowest (non-zero) singular values ratio. For better illustration of the results, we perform an averaging over 100 realizations of the channel matrix $\mathbf{A}$ for every conditioning.

In figure 1, we represent the variations of the SER versus SNR for both sources. Notice how the performance of our approach becomes increasingly remarkable compared to the standard method for the source s_2. We clearly see that increasing the conditioning of the channel matrix from 5 to 8 results in a loss of performance (in terms of the required SNR at a given SER) for the standard whitening-based technique especially when the ratio $\frac{\sigma_2^2}{\sigma_1^2} = 10$ dB, while the performance of our approach are almost unaffected. For the "weakest" source s_1 our approach exhibits the same level of performance as the conventional one.

It is also of interest to evaluate the variations of the required SNR with respect to the conditioning of the channel matrix to achieve a fixed SER value. Figure 2 represents the variations of the required SNR [dB] with respect to the channel matrix conditioning to achieve a $SER = 10^{-2}$ for $\frac{\sigma_2^2}{\sigma_1^2} = 6$ dB to extract s_2. At low conditioning both approaches exhibit the same level of performance. Nevertheless, our new method performs much better than the standard one when the BSS problem becomes ill-conditioned. The achieved gain in SNR can go up to 2 dB at 10^{-2} SER only.



(a) $\frac{\sigma_2^2}{\sigma_1^2} = 6$ dB; $\text{cond}(\mathbf{A}) = 5$ (b) $\frac{\sigma_2^2}{\sigma_1^2} = 6$ dB; $\text{cond}(\mathbf{A}) = 8$



(c) $\frac{\sigma_2^2}{\sigma_1^2} = 10$ dB; $\text{cond}(\mathbf{A}) = 5$ (d) $\frac{\sigma_2^2}{\sigma_1^2} = 10$ dB; $\text{cond}(\mathbf{A}) = 8$

Fig. 1. SER vs SNR for s_2 and s_1 for different values of $\frac{\sigma_2^2}{\sigma_1^2}$ and conditioning of $\mathbf{A}$ ($\text{cond}(\mathbf{A})$).

6. CONCLUSION

In this paper, a new relationship between the column vectors of the channel matrix was elaborated to solve the 3×2 BSS-ICA problem . To proceed, we fully exploited the information provided by the SOS. Then, we reduced the complexity of the problem in hands by an appropriate parametrization. We further exploited the optimal form of the separating matrix (ZF/MMSE of the channel matrix for noise-free/noisy mixtures, respectively) to elaborate appropriate expressions for the whitening matrix. Simulation results proved the robustness of the proposed approach to solve the ill-conditioned BSS problem with noisy mixtures. Although we focused on the particular case of three observations in this paper, the more general case ($M > 3$) can be easily investigated by selecting three antennas from M which transforms it into 3x2 subproblems.

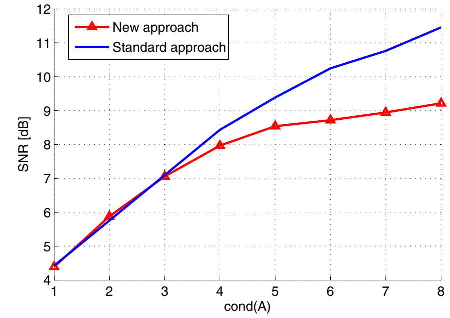


Fig. 2. Required SNR versus $\text{cond}(\mathbf{A})$ to achieve $SER = 10^{-2}$ at $\frac{\sigma_2^2}{\sigma_1^2} = 6$ dB.

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