ON THE BER PERFORMANCE OF PULSE-POSITION-MODULATION UWB RADIO IN MULTIPATH CHANNELS

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ABSTRACT

The impact of multipath propagation on ultra wide band (UWB) radio, which is carrier free, is quite different from that on conventional radio. This paper presents a theoretical analysis of bit error rate (BER) performance of binary pulse-position-modulation (PPM) UWB radio in multipath channels. The BER formula is derived. Particular attention is paid to the effect of the propagation time-delay on the BER performance. It is also noted that, for the PPM scheme, the multipath propagation time-delay may lead to unbalanced BERs over "0"-"1" in some particular multipath channels.

1. INTRODUCTION

Ultra wide band (UWB) radio, also called impulse radio, has recently been proposed for wireless communications systems [1]-[4]. It employs short duration pulses or impulses to convey information rather than modulating a sinusoidal carrier with information symbols. Therefore, multipaths could be resolvable if the duration of a pulse is much shorter than the multipath propagation time-delay. This makes UWB radio very attractive for wireless communications in dense multipath environments.

It is of interest to provide insights into the multipath propagation characteristics of this uncommon wireless technique. Extensive propagation measurements indicated that the UWB signal does not suffer serious fading in modern office buildings [5], [6]. Therefore, very little fading margin is required to ensure a reliable communication. The propagation measurements in rural terrain were reported in [7], where mean delay, delay spread, propagation loss and forestation losses were observed. The multipath performance of a perfect UWB rake-receiver based on energy capture was discussed in [9], [10]. A spatial beamformer was applied to UWB radio in [11] and spatio-temporal diversity, based on measured data, was discussed in [12].

This paper investigates theoretically the bit error rate (BER) performance of UWB radio in multipaths. In section 2, the PPM scheme is described. The BER performance is analyzed in section 3, where the impact of signal design parameters, particularly the modulation index, on the system performance is discussed in detail. Computer simulation results are presented in section 4 and conclusions are drawn in section 5.

2. SIGNAL MODEL

The PPM signal can be modeled as

$$S^{(k)}(t)=\sum_j W(t-jt_f-c_j^{(k)}t_c-\delta\, d_{[j/N_s]}^{(k)}) \qquad (1)$$

where k denotes the kth user in a multi-user system; $W(t)$ is a pulse with duration of w_b such that $\int_{-\infty}^{\infty} W(t)dt=0$; $t_f >> w_b$ is the pulse repetition time; $c_j^{(k)} = c_{j+mP_c}^{(k)}$, $m=0,1,2\cdots$, is the jth code of the kth user's PN sequence with period P_c, $0<c_j^{(k)}\le N_h$, N_h is an integer; t_c is a time-shift unit incurred by the PN sequence; δ is referred to as modulation index in this paper, which is a time-shift unit incurred by the binary symbol $d_{[j/N_s]}^{(k)}$, $d_{[j/N_s]}^{(k)}\in\{0,\ 1\}$, where "[ ]" denotes the integer operator. It is clear from (1) that N_s pulses represent one binary information symbol. The jth pulse is emitted at time $jt_f+c_j^{(k)}t_c+\delta\, d_{[j/N_s]}^{(k)}$. Therefore, the PPM scheme may be viewed as time modulation (TM).

The bandwidth of a pulse is $B_c\approx 1/w_b$. The duration of each symbol is constrained by $T_{bit}=N_s t_f$, and the symbol transmission rate is given by $R_b=1/T_{bit}$. The choice of the parameters should be satisfied with $(N_h t_c+\delta+w_b)\le t_f$. If $N_h t_c+\delta+w_b$ is too small, there may be a large collision probability of multiple users' signals. It would be better that $N_h t_c+\delta+w_b$ is designed to be close to t_f. In this case, with well-designed PN sequences, multiple access interference can be modeled as a Gaussian process [5]. Modulation index δ and pulse duration w_b are normally in the same order and δ should be designed such that the cross-correlation between $W(t)$ and $W(t-\delta)$ is minimum.

The normalized auto-correlation function of $W(t)$ is defined as

$$R(\tau) = \int_{-\infty}^{\infty} W(t)W(t-\tau)dt \,/\, \int_{-\infty}^{\infty} W^2(t)dt \quad (2)$$

Let E_p denote the energy of $W(t)$, then

$$E_p = \int_{-\infty}^{\infty} W^2(t)dt \quad (3)$$

Thus the energy of one bit symbol is $E_{bit} = N_s E_p$.

3. BER PERFORMANCE ANALYSIS

For simplicity, only one user is considered. It is assumed that the receiver has achieved correct timing and PN sequence synchronization from the signal component received from the direct path. It is further assumed that the binary symbols have equal transmission probabilities. The demodulation format considered here is based on correlation.

Let α_l and τ_l denotes, respectively, the propagation attenuation and time-delay over the lth path out of L discrete multipaths. Then, the received signal is

$$r(t) = \sum_{l=0}^{L-1}\sum_{j} \alpha_l W(t - jt_f - c_j t_c - \delta\, d_{[j/N_s]} - \tau_l) + n(t) \quad (4)$$

where $n(t)$ is the additive white Gaussian noise with double-sided power density of $N_0/2$ Watts/Hz.

For a binary correlation receiver, the statistic within one symbol duration can be defined as

$$g = \int_0^{T_{bit}} r(t)[W_{bit}(t-\tau_0) - W_{bit}(t-\tau_0-\delta)]dt \quad (5)$$

where τ_0 denotes the propagation time-delay of the direct path, and $W_{bit}(t)$ is a locally generated pulse train to be correlated with the received signals. To receive the ith symbol, we have

$$W_{bit}(t) = \sum_{j=(i-1)N_S}^{iN_S-1} W(t - jt_f - c_j t_c) \quad (6)$$

From (6), the detection decision is

$$g \ge 0 \;\Rightarrow\; \text{“0”} \quad (7)$$

$$g < 0 \;\Rightarrow\; \text{“1”} \quad (8)$$

Since the noise $n(t)$ is a Gaussian process and $W(t)$ is deterministic, the received signal $r(t)$ is also Gaussian. The statistic g is a linear function of $r(t)$. As a consequence, g is a Gaussian random process.

If "0" was transmitted, the conditional mean value of the statistic is

$$u_0 = E\{g/0\} = E_{bit}\sum_{l=0}^{L-1}\alpha_l\{R(\tau_l-\tau_0) - R(\tau_l-\tau_0-\delta)\} \quad (9)$$

The conditional variance is

$$\sigma_0^2 = Var\{g/0\} = N_0 E_{bit}(1-\rho) \quad (10)$$

where

$$\rho = \int_{-\infty}^{\infty} W_{bit}(t)W_{bit}(t-\delta)dt \Big/ \int_{-\infty}^{\infty} W_{bit}^2(t)dt \quad (11)$$

$$-1 < \rho \le 1 \quad (12)$$

If "1" was transmitted, the conditional mean value of the statistic is

$$u_1 = E\{g/1\} = E_{bit}\sum_{l=0}^{L-1}\alpha_l\{R(\tau_l-\tau_0+\delta) - R(\tau_l-\tau_0)\} \quad (13)$$

The conditional variance is

$$\sigma_1^2 = Var\{g/1\} = N_0 E_{bit}(1-\rho) \quad (14)$$

From (10) and (14) we may let $\sigma^2 = \sigma_0^2 = \sigma_1^2$. Thus the conditional probability density functions are simply

$$p(g/0) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\{-\frac{(g-u_0)^2}{2\sigma^2}\} \quad (15)$$

$$p(g/1) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\{-\frac{(g-u_1)^2}{2\sigma^2}\} \quad (16)$$

Therefore, given that "0" was transmitted, the BER is simply the probability that g<0, i.e.,

$$P_E(1/0) = \int_{-\infty}^{0} p(g/0)\,dg = \int_{u_0/\sigma}^{\infty}\frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})dx \quad (17)$$

Similarly, if we assume that "1" was transmitted, the BER is given by

$$P_E(0/1) = \int_{0}^{\infty} p(g/1)\,dg = \int_{-u_1/\sigma}^{\infty}\frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})dx \quad (18)$$

Since "0" and "1" are transmitted with identical *a priori* probability, the average BER is

$$P_E = \frac{1}{2}\{Q(\frac{u_0}{\sigma}) + Q(-\frac{u_1}{\sigma})\} \quad (19)$$

where

$$Q(z) = \int_z^{\infty}\frac{1}{\sqrt{2\pi}}\exp(-\frac{z^2}{2}).$$

We can see from (19) that the BER is determined by u_0, u_1 and σ. It can be further seen from (9) and (13) that u_0 and u_1 are dependent upon path attenuation α_l and propagation time-delay τ_l.

It should be noted that u_0 might not be equal to u_1. Let $\tau = \tau_l - \tau_0$ for lth path and define

$$f(\tau) = u_0(\tau) - (-u_1(\tau)) = E_{bit}\{R(\tau+\delta) - R(\tau-\delta)\} \quad (20)$$

where

$$u_0(\tau) = E_{bit}\{R(\tau) - R(\tau - \delta)\} \quad (21)$$

$$-u_1(\tau) = E_{bit}\{R(\tau) - R(\tau + \delta)\} \quad (22)$$

From (17) and (18), we have

$$f(\tau) > 0 \Rightarrow P_E(1/0) < P_E(0/1)$$

$$f(\tau) < 0 \Rightarrow P_E(1/0) > P_E(0/1)$$

$$f(\tau) = 0 \Rightarrow P_E(1/0) = P_E(0/1)$$

From (21), (22), (17) and (18), we can see that the condition upon which $P_E(1/0) = P_E(0/1)$ is

$$R(\tau - \delta) = R(\tau + \delta) \quad (23)$$

Once this condition is not satisfied, unbalanced BERs arise over "0"-"1", i.e., $P_E(1/0) \neq P_E(0/1)$. This is because PPM is actually a time modulation scheme and hence the modulation index δ may be confused with the propagation time-delay in multipath environments. For example, supposing $\tau_0 = 0$, the arrival time of the direct path component should be $jt_f + c_j^{(k)} t_c$ when "0" is emitted. If there exists a reflect path whose propagation time-delay τ_l leads to $jt_f + c_j^{(k)} t_c + \tau_l$ = $jt_f + c_j^{(k)} t_c + \delta$, which should be the arrival time of signals transmitting "1", the detection decision result would be "1" instead of "0". Consequently, incorrect decision happens.

Although the above conclusion is observed from the assumption of transmitting binary symbol "0" and "1", it could be extended to the scheme of transmitting "-1" and "+1". As a result, the modulation index δ must be carefully designed in order to achieve satisfactory BER performance in practical multipath environments.

4. COMPUTER SIMULATION RESULTS

The following pulse waveform is adopted in the computer simulations:

$$W(t) = 2\sqrt{e}\, A \pi\, t\, f_c e^{-2(\pi t f_c)^2} \quad (24)$$

The parameters used to produce UWB signals are taken as: $t_f = 1.25 \times 10^{-6} s$, $w_b = 1.25 \times 10^{-8} s$, $f_c = 80 MHz$, $N_s = 20$, $N_p = 100$, and $N_h = 100$. The information symbols to be transmitted are randomly generated. The average power of the modulated signal is 3.84mW. Modulation index is $\delta = 4.853 \times 10^{-9} s$. The noise present at the input of the correlator is the white Gaussian noise.

To evaluate the impact of different multipath channels on the BER performance, we considered four channel models. Four paths are assumed in each channel model. The multipath delays and power profiles are listed in Table I. The propagation attenuation α_l is a constant for Channel (b), (c) and (d). Channel (a) is one direct path only. Channel (b) consists of multipaths whose propagation distance-differences are about 1m. The maximum propagation time-delay is less than the modulation index δ. In channel (c), the propagation distance-differences among multipaths are about 3m. The minimum propagation time-delay is about twice larger than the modulation index δ. Channel (d) may be the worst channel as τ_3 is equal to δ. Ten thousand runs of simulation were performed for each given signal-to-noise rate (SNR) present at the correlator input.

TABLE I
PARAMETERS OF MULTIPATH CHANNELS

Ch.	Path 1		Path 2		Path 3		Path 4	
	τ_0	α_0	τ_1	α_1	τ_2	α_2	τ_3	α_3
a	0	1.0	0	0	0	0	0	0
b	0	0.9	3.33	0.23	4.58	0.3	1.67	0.2
c	0	0.9	10.	0.23	12.5	0.3	16.25	0.2
d	0	0.9	3.33	0.23	4.58	0.3	4.853	0.2

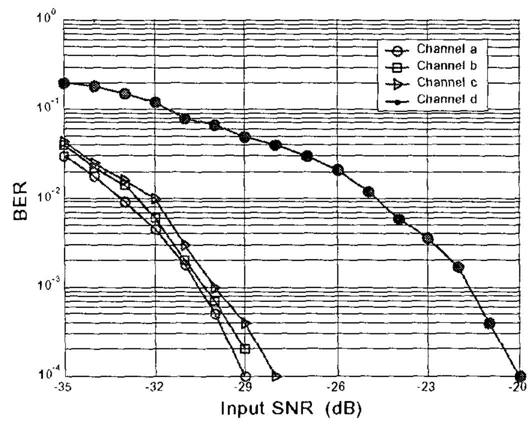

Fig. 1. PPM BER performance

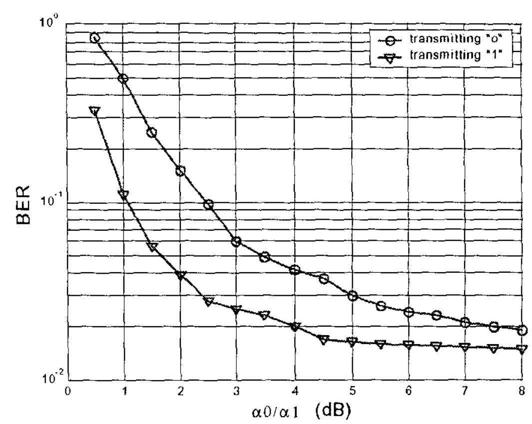

Fig. 2. PPM BERs over "0" and 1

Fig. 1 shows the BER performance results for the selected four multipath channels. The simulation results for channel (d) indicate that for a given modulation index δ, if there exists $\tau = \delta$, the BER performance is deteriorated seriously. Careful comparison between channels (b) and (d) suggests that the BER performance degrades dramatically simply if just one of the propagation time-delays is set equal to δ.

Fig. 2 illustrates unblanced BERs over "0"-"1". To see clearly the impact of propagation time-delay on the BER performance when transmitting "0" and "1", respectively, two propagation paths were considered. The attenuation ratio of path 1 to path 2 is denoted as α_0 / α_1. The corresponding propagation time-delays are $\tau_0 = 0$ and $\tau_1 = \delta$. The SNR present at the correlator input is set to be –32dB. For each α_0 / α_1, ten thousand runs of simulation were performed. Fig. 2 shows clearly that the BER of transmitting "0" is larger than that of transmitting "1". This result confirms our observation of unbalanced BERs over "0"-"1" in section 3. Indeed, for the given simulation parameters, we have

$$f(\tau_1) = E_{bit}\{R(\tau_1 + \delta) - R(\tau_1 - \delta)\} \approx -E_{bit} < 0.$$

5. CONCLUSION

The BER performance of binary PPM in UWB radio is analyzed in multipath channels. Computer simulation results show that the BER performance of binary PPM is close to that of an ideal channel with one direct path only. On the other hand, if one propagation time-delay is equal to the modulation index δ, unbalanced BER over "0"-"1" arises, thereby degrading the BER performance dramatically. Accordingly, the modulation index δ of PPM should be carefully designed so that the effect of the multipath propagation time-delay on the BER performance can be reduced.

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