Partial Interference Subspace Rejection in CDMA Systems

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Abstract

Previously presented Interference Subspace Rejection (ISR) [3, 8, 4] proposed a family of new efficient multi-user detectors for CDMA. We reconsider in this paper, the modes of ISR using Decision Feedback (DF). DF modes share similarities with the PIC but attempt to cancel interference by nulling rather than subtraction. However, like the PIC they are prone to wrong tentative decisions. We propose a modification to DF modes that performs partial ISR instead of complete interference cancellation. When tentative decisions are correct, interference is therefore not perfectly rejected anymore. This drawback is compensated by improved robustness to wrong tentative decisions. We show that in hard handoff systems, partial ISR can only provide negligible performance improvements in high loaded systems outside the region of interest due to out-sector[1] interference. In situations where both in-sector and out-sector interferences are cancelled, as may occur in soft handoff situations, BER performance may improve by more than 1 dB.

1 Introduction

Subtractive multi-user receivers for CDMA mitigate interference by reconstructing and subtracting interference from the total received signal (before or after despreading), using estimates of the channel response along with tentative decisions of interfering symbols. Usually tentative decisions are based on Hard-Decision (HD) Maximal Ratio combined (MRC) signal estimates, which exploits prior knowledge of the alphabet of transmitted symbols (for instance {-1,1} with BPSK). HD may be replaced by SD in which the decision device is generalized, to allow for an alphabet different from the actual alphabet used for the link [6, 13]. For instance the linear PIC [7] uses a linear SD mapping function (*i.e.*, $y = x$) and scale, by the instantaneous signal estimate before subtraction, rather than scaling by the sign of the bit (*i.e.*, $y = \text{sign}(x)$). One can understand the SD method as one which "weights" the amount of interference before subtracting it. In the low SNR region, instantaneous signal estimates are near random (at least in the limit). For that reason, HD has proven to be better in the low SNR region, whereas SD is better at high SNR [6]. On the contrary, SD performs better than HD if the channel is estimated reliably [10]. However, when used in a multi-stage scheme, SD is better and it was even shown to approach the MMSE detector in [7, 12].

Like subtractive receivers, DF modes of ISR are sensitive to wrong decisions [3, 2, 8]. We present a modification to ISR, in which only a fraction of interference is removed. Unlike in traditional SD schemes, this fraction is not determined by the instantaneous signal estimate (*i.e.*, using a mapping function). Instead the weights are optimized to achieve minimal residual variance subject to the BER of tentative decisions. [1] exploited same idea but with application to the PIC.

[1]Out-sector interference refers to interference from other sectors of the same site and interference from other sites.

2 Signal Model and Overview of ISR

2.1 Signal Model

We provide in this section a very brief summary of the signal model and the concept of ISR. For details, the reader is referred to [3, 2]. We consider the uplink of an asynchronous CDMA communication system. The system consist of N_u users; one user ($\underline{Y}_n^d$), which we denote the desired user, and $NI = N_u - 1$ interfering users ($\underline{I}_n^i, i = 1, \cdots, NI$), which we denote the interfering users. The signal received at the antenna array of M sensors after down conversion, pulse matched filtering, sampling at the chip rate, and framing into vectors of duration MN_T, can then be formulated as

$$\underline{Y}_n = \underbrace{\underline{Y}_n^d}_{\text{desired}} + \underbrace{\sum_{i=1}^{NI} \underline{I}_n^i}_{\text{interferers}} + \underbrace{\underline{N}_n}_{\text{AWGN}}, \quad (1)$$

where the noise term, $\underline{N}_n$, which incorporates thermal noise and other unknown sources of interference, is assumed to be AWGN. The signals ($\underline{Y}_n^d$ and $\underline{I}_n^i$) hold contributions from a number[2] of consecutive BPSK bits spread by PN codes with processing gain L. $\underline{Y}_n^d$ decomposes into $\underline{Y}_n^d = \underline{Y}_{0,n}^d s_n^d + \underline{Y}_{ISI,n}^d$, where $\underline{Y}_{0,n}^d$ is the spread-channel response of the desired bit n, $s_n^d = \psi_n^d b_n^d$ is the signal component, where b_n is the bit and $(\psi_n^d)^2$ is the total received power; and $\underline{Y}_{ISI,n}^d$ is the contribution of other bits of the desired user in the observation frame which causes ISI.

2.2 ISR

In ISR, we define the beamformer which satisfies

$$\begin{cases} \underline{W}_n^{d\,H} \underline{Y}_{0,n}^d = 1 & \text{distortionless response} \\ \underline{W}_n^{d\,H} \underline{I}_{0,n}^i = 0 & \text{null interference response} \end{cases}; \quad (2)$$

where $\underline{Y}_{0,n}^d$ represents the contribution of the n^{th} bit in $\underline{Y}_n^d$, the desired signal term[3]. In practice the ISR beamformer in Eq. 2 is implemented by

$$\underline{W}_n^d = \left(\underline{Y}_{0,n}^{d\;H} \Pi_n^d \underline{Y}_{0,n}^d\right)^{-1} \Pi_n^d \underline{Y}_{0,n}^d = K_d \Pi_n^d \underline{Y}_{0,n}^d, \quad (3)$$

where Π_n^d is an estimate of the projector which projects the desired response onto the subspace orthogonal to the interference. The projector is computed from

$$\Pi_n^d = \mathbf{I}_{MN_T} - \hat{\mathbf{C}}_n \left(\hat{\mathbf{C}}_n^H \hat{\mathbf{C}}_n\right)^{-1} \hat{\mathbf{C}}_n^H, \quad (4)$$

where the constraint matrix $\hat{\mathbf{C}}_n$ is the essential part of ISR. Its columns span the interference subspace. In ISR different philosophies to define the constraints matrix are defined; these are termed modes. For instance, in the mode ISR by realizations (ISR-R), the columns of $\hat{\mathbf{C}}_n$ are the estimated interfering users, $\hat{\underline{I}}_n^i$, reconstructed with the aid of tentative symbol decisions and estimates of the channel response. For a description of other modes please refer to [3, 2, 8]; here we shall only consider ISR-R for simplicity.

Tentative decisions are mostly MRC estimates[4] or past processed ISR estimates which fall inside the observation. The channel is estimated using the Spatio-Temporal Array-Receiver [5].

[2] Determined by the chosen time duration of the N_T observation chips.

[3] Therefore, $\underline{Y}_n^d - \underline{Y}_{0,n}^d$ is ISI.

[4] The MRC beamformer arrives from Eq. 3 by replacing the projector by the identity matrix.

3 Partial ISR

When the ISR beamformer of Eq. 3 is computed, the ISR signal estimate is $\hat{s}_n^d = \underline{W}_n^d \underline{Y}_n$. This can also be written as (Eq. 4 into Eq. 3)

$$\hat{s}_n^d = K_d \hat{\underline{Y}}_{0,n}^{d^H} \left[\underline{Y}_n - \hat{\mathbf{C}}_n \left(\hat{\mathbf{C}}_n^H \hat{\mathbf{C}}_n\right)^{-1} \hat{\mathbf{C}}_n^H \underline{Y}_n\right]. \quad (5)$$

This reformulation is useful because it can be understood as the observation (first term) from which we subtract reconstructed interference (last term). Omitting the noise vector for convenience of presentation, we can therefore write

$$\hat{s}_n^d = \hat{\underline{Y}}_{0,n}^{d^H} \left(\underline{Y}_{0,n}^d s_n^d + \sum_{i=1}^{NI} \underline{I}_n - \sum_{i=1}^{NI} \check{\underline{I}}_n\right), \quad (6)$$

where $\underline{I}_n^i, i = 1, \cdots, NI$ are the actual interfering signals and $\check{\underline{I}}_n^i, i = 1, \cdots, NI$ stand as the ISR estimates of the interference not to be confused with the estimates first used to form the constraints, $\hat{\underline{I}}_n^i, i = 1, \cdots, NI$.

Assuming that channel identification is accurate, imperfect cancellation of residuals (*i.e.*, $\Delta\underline{I}_n^i = \underline{I}_n^i - \check{\underline{I}}_n^i$) are primarily due to wrongful tentative decisions. Introducing the variance of the residuals after beamforming, $\xi_{i,n}^2$, we can therefore state the following:

$$\text{ISR}: \begin{cases} \text{No errors} & \Rightarrow \xi_{i,n}^2 = 0 \\ \text{Some errors} & \Rightarrow \xi_{i,n}^2 = 4E \neq 0 \end{cases} \quad (7)$$

where $E > 0$ symbolizes a generalized stochastic variable used to describe the variance of the residuals due to generalized error events. In other words, DF ISR attempts to cancel interference completely, but when unsuccessful, the penalty is bursty residual interference. New idea: We introduce a weight, to be applied to interfering signals. Instead of subtracting the total reconstructed interference, we subtract only a fraction; that is,

$$\hat{s}_n^d = \hat{\underline{Y}}_{0,n}^{d^H} \left(\underline{Y}_{0,n}^d s_n^d + \sum_{i=1}^{NI} \underline{I}_n - \sum_{i=1}^{NI} w_n(i) \check{\underline{I}}_n\right), \quad (8)$$

where $0 \leqslant w(i) \leqslant 1$ is the weight associated with interferer i. In practice, Eq. 8 is implemented by introducing a diagonal matrix which holds the weight into Eq. 5; that is

$$\hat{s}_n^d = \hat{\underline{Y}}_{0,n}^{d^H} \left[\underline{Y}_n - \hat{\mathbf{C}}_n^d \mathbf{W}_n \left(\hat{\mathbf{C}}_n^H \hat{\mathbf{C}}_n\right)^{-1} \hat{\mathbf{C}}_n^H \underline{Y}_n\right] \quad (9)$$

where for ISR-R

$$\mathbf{W}_n = \text{diag}\left[w_n(1), w_n(2), \cdots, w_n(NI)\right]. \quad (10)$$

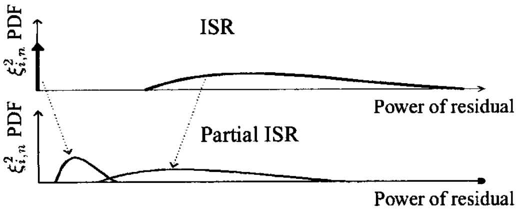

Figure 1. PDF of residual interference variance without (above) and with (below) weights.

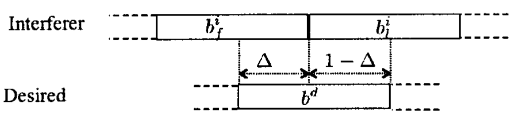

Figure 2. Definition of fractional overlap of bits between signal and interferer.

When $w_n(i) < 1$ interference is never completely cancelled, although decisions are correct. However, when tentative decisions are wrong, the penalty is reduced as well. With reference to Eq. 7 we can now modify the statements to

$$\text{Partial ISR:} \begin{cases} \text{No errors} & \Rightarrow \xi_{i,n}^2 = (1-w(i))^2 E_1 \\ \text{Some errors} & \Rightarrow \xi_{i,n}^2 = (1+w(i))^2 E_2 \end{cases} \quad (11)$$

where E_1 and E_2 are stochastic variables obeying (with reference to Eq.7): $\mathrm{E}\{E\} = \mathrm{E}\{E_1\} = \mathrm{E}\{E_2\}$. Note though that the total expected variance of the residuals have changed.

Fig. 1 is a simplified illustration of the PDF of the residuals with and without weights. When no weights are used, residual interference noise is bursty and appears only when tentative decisions are wrong. When a weight is used, interference is never perfectly cancelled, but it has become less bursty, and has lower overall variance.

3.1 Choosing the optimal weights

We consider here the optimization of weights in the MSE sense, and we therefore wish to derive the weights which result in the lowest expected variance of the residuals. To proceed we refer to the result in [8], where it was claimed that errors of interfering bits, which temporarily overlap the current bit of the objective user, are by far dominant. We therefore limit ourselves to consider error events due to interfering bits temporally overlapping the current bit of the desired user. We further limit our attention to non-selective fading (one path propagation). Any bit of a desired user will then be overlapped by no more than two bits for any interferer. Fig. 2 defines the fractional overlaps of the first interfering bit (b_f^i) and the last interfering bit (b_l^i), Δ and $1-\Delta$, respectively. Let p_e^i denote the probability that b_f^i and b_l^i are in error; then the variance of the error due to no errors, b_f^i in error, b_l^i in error and both b_f^i and b_l^i in error, and their respective probabilities are[5]

$$\begin{cases} (1-w(i))^2\sigma_i^2 & 1-2p_e^i+p_e^{i\,2} \\ \left[(1+w(i))^2\Delta+(1-w(i))^2(1-\Delta)\right]\sigma_i^2 & p_e^i(1-p_e^i) \\ \left[(1+w(i))^2(1-\Delta)+(1-w(i))^2\Delta\right]\sigma_i^2 & p_e^i(1-p_e^i) \\ (1+w(i))^2\sigma_i^2 & p_e^{i\,2} \end{cases}$$

From [8] it can be verified that normally $\sigma_i^2 < (\psi^i)^2/(ML)$ where $(\psi^i)^2$ is the total received power of interferer i, M is the number of receiving antennas, and L is the spreading factor. From the results above we compute the average residual error and arrive after a few steps at

$$\begin{aligned} \mathrm{E}\{\xi_{i,n}^2\} &= \left[(1-w(i))^2(1-2p_e^i+(p_e^i)^2)\right] \\ &\quad + \left((1+w(i))^2+(1-w(i))^2\right)p_e^i(1-p_e^i) \\ &\quad + (1+w(i))^2(p_e^i)^2\big]\,\sigma_i^2. \end{aligned} \quad (12)$$

To find the minimum, the derivative of Eq. 12 with respect to $w(i)$ is set equal to null. After tedious labor, we arrive at

$$w(i) = 1 - 2p_e^i. \quad (13)$$

Therefore, choosing the weights from Eq. 13 minimizes the variance of the residual error. [1] arrived at the same result investigating the PIC. It is surmised that the result readily applies to the non-selective case since the results are independent of the fractional overlaps, which was seen already in Eq. 12.

Weighting also mitigates white noise enhancement. This suggests a similarity with the MMSE [15, 9]. [7, 12] presented a family of multistage PIC and SIC versions which proved to approach the MMSE.

3.2 Estimation of optimal weights

In DF ISR, tentative decisions are mostly MRC estimates[6]. The powers of interfering users $\hat{\psi}^i$, can be estimated as described in [3, 2]. If interference is assumed to be complex Gaussian noise, the instantaneous p_e^i may be estimated from

$$I_0^i = \sum_{j=1,j\neq i}^{NI} (\hat{\psi}^i)^2 + (\hat{\psi}^d)^2 \quad (14)$$

$$p_e^i \simeq \mathrm{Q}\left(-\sqrt{2\frac{(\hat{\psi}^i)^2}{N_0 T + I_0^i}}\right) \quad (15)$$

[5] These results are also valid for the PIC; however, for the PIC $\sigma_i^2 = (\psi^i)^2/(ML)$, which indicates that ISR is superior to the PIC. See [8] for further details.

[6] Although some decisions may arrive from past estimated ISR bits.

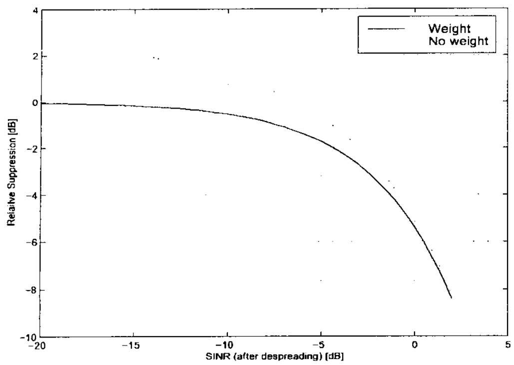

Figure 3. Relative suppression of interference with and without weights as a function of the total SINR, which is determinative for the error rate of tentative decisions.

where N_0 is the two-sided spectral density of AWGN, and T is the duration of one bit. Normally N_0 is insignificant, and can be disregarded. In reality, interference is basically a real valued Gaussian source with random complex phase, which is why Eq. 15 is an approximation even if I_0 and N_0 are perfectly known. For more insight into this problem, we refer to [11].

It is obvious that partial rejection may be particularly useful when the BER of the interferers is high. Otherwise, the optimal weights are close to unity and we approach the classical solution with no weights. To illustrate this, we consider a simplified situation, where we wish to reject N_u users all with power $(\psi_u)^2$, subject to the constraint that the total residual interference power after despreading divided by the power of the desired user is constant, *i.e.*, $\frac{N_u(\psi_u)^2}{(\psi_d)^2 L}$ = constant. If interference is assumed Gaussian, the bit error probability of tentative decisions, when MRC is used, is computed from BER=$Q\left(-\sqrt{2 \cdot SINR}\right)$ where SINR is the signal to interference noise ratio after despreading.

In Fig. 3 we show the relative residual suppression error with and without weights. When there are a few interferers, the SINR is high[7] and the weights offer no gain of significance. When there are many interferers, the SINR is low and partial interference subspace rejection can be useful. In other words, we can expect improvements in situations where the desired user experience interference from many sources. It is noted that in the limit as SINR$\rightarrow -\infty$, partial ISR performs as if no rejection is attempted (0 dB relative suppression), whereas ISR approaches 3 dB and therefore effectively amplifies interference. Note that although the limit of 3 dB is true with the PIC, for instance, the interference level is still lower with ISR as was shown in [8].

[7] Due to little mutual interference.

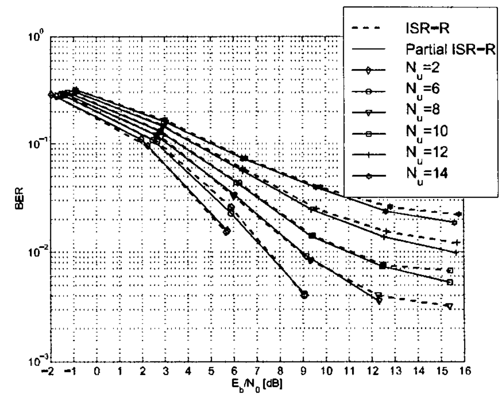

Figure 4. ISR and partial ISR compared.

4 Simulation Results

We consider the uplink of a cellular CDMA system employing differential BPSK modulation at a 2.05 GHz carrier. The chip rate is 3.84 Mcps, and the processing gain is $L = 16$ chips (*i.e.*, data rate of 240 Kbps). The number of receiving antennas is assumed to be $M = 1$ although results presented herein apply to multiple antenna configurations. The users' signals experience selective Rayleigh fading[8] with three propagation paths having relative average strengths of 0, -6, and -10 dB. The frequency error between Tx and Rx oscillators is 300 Hz. The delays are chosen randomly with a maximum spread of 10 chips. The observation frame has the dimension $N_T = 128$ chips. To be even more realistic in the simulations, we use STAR [5] to estimate the channels.

In Fig 4 we verify the BER performance of partial ISR with various SNRs in a conventional system with N_u = 2, 6, 8, 10, and 12 users. Partial ISR-R provides no gain at low SNR and only little gain at high SNR. Normally it is not possible to work at the SNRs where partial ISR can improve performance due to out-sector interference. If the amount of out-sector interference to in-sector interference is $f_{oi} = 0.6$ [14], and the load is $N_u = 8$ users this means that the working point is around $10\log(L/(N_u f_{io})) = 5.2$ dB where no gain of partial ISR over conventional ISR can be measured. The potential power of partial ISR is hence reduced to quasi-isolated cell systems ($f_{oi} \ll 1$).

If soft handoff is an option, the BS may monitor out-sector interference and cancel it as well. Out-sector interference is, however, power-controlled by other BSs, resulting in greater power fluctuations but less average power due to higher path loss. As an approximation, we let out-sector interference be power controlled by the objective BS but

[8] At 8.9 Hz corresponding to a speed of 5 Kmph.

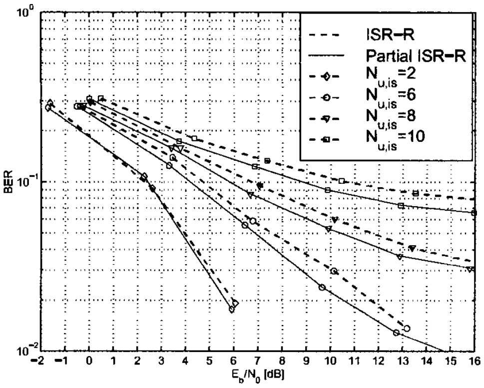

Figure 5. ISR-R and partial ISR-R with out-sector interference rejection. $N_{u,ic}$ is the number of in-sector users. For each in-sector users there are $2N_{u,oc}$ users transmitting 4.8 dB lower average power.

aim at an average received power 4.8 dB lower. The total number of users, N_u, communicating with the BS, is distributed as $N_{u,is}$ in-sector users, and $N_{u,os}$ out-sector users where the number of out-sector users is twice the number of in-sector users; therefore, $N_{u,os} = 2N_{u,is}$. The out-sector users which communicate with the BS, therefore cause a relative interference of $f_{oi} = 0.6$ compared to in-sector users.

The E_b/N_0-BER plot is shown in Fig. 5 for $N_{u,is} = 2$, 6, 10, and 12. At low system loads, partial ISR-R provides no significant improvement. However, at higher loads, differences become evident and in the limit, weights allow for support of two more users.

5 Conclusions

We presented partial ISR as a method to mitigate errors in the tentative decisions used to reconstruct interference. In situations where only in-sector interference is rejected at the BS, partial ISR provide improvements only with high loads. Due to out-sector interference, however, it will normally not be possible to work with these loads. However, partial ISR may find application if the BS attempts to cancel out-sector interference, which may for instance occur in soft handoff situations, because it mitigates the high BER of tentative decisions, which are used for the reconstruction of out-sector interference.

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