AN ALGORITHM FOR MULTI-SOURCE BEAMFORMING AND MULTI-TARGET TRACKING: FURTHER RESULTS

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ABSTRACT

We herein propose an optimal beamformer for the extraction and the tracking of partially- or fully-coherent sources in colored noise. We adaptively implement it in a simple structure and combine it with a "source-subspace" tracking procedure. We finally show its effectiveness and its fast tracking capacity by simulations.

1 INTRODUCTION AND FORMULATION

Array processing techniques (*e.g.* beamforming, localization, tracking, etc$\cdots$) are very sensitive to source correlation with other desired interferers or noise. To particularly prevent signal cancellation in adaptive beamforming [1], the spatial averaging technique proposed and analyzed in [2,3] implements a "partial" decorrelation feature for a suboptimal extraction of partially-coherent sources in uncorrelated white noise. Another averaging technique using a structured correlation matrix [4] optimally reduces uncorrelated white noise, but its performance still improves at the cost of an increasing complexity and a larger number of linear and equidistant sensors. In [5], array filters are designed for attenuating coherent interference with a suboptimal reduction of uncorrelated white noise, while a search strategy is proposed in [6] to partially activate the beamforming sub-structure with the minimum cancellation effect.

In this paper, we simply propose a "full" decorrelation feature in beamforming by spatial filtering. We adapt from previous versions [7,8] a multi-source Linearly Constrained Minimum Variance (LCMV) beamformer and adaptively implement it in a new multi-dimensional Generalized Sidelobe Canceller (GSC) structure [9]. This new beamformer is combined with a "source-subspace" tracking procedure of steering vectors to achieve optimal extraction and tracking of partially- or fully-coherent sources in colored noise.

At time t, we consider the following model for the observation vector X_t received by an array of m sensors:

$$\begin{aligned} X_t &= G_t S_t + N_t \ , \qquad (1) \\ G_t &= [F(\kappa_{1,t}), F(\kappa_{2,t}), \cdots, F(\kappa_{p,t})] \ , \\ F(\kappa) &= \left[e^{-j\xi_1\kappa}, e^{-j\xi_2\kappa}, \cdots, e^{-j\xi_m\kappa}\right]^T \ , \end{aligned}$$

where $S_t = [s_{1,t}, s_{2,t}, \cdots, s_{p,t}]^T$ is the signal vector of $p \leq m$ desired plane-wave sources, N_t is the noise vector, and G_t is a $m \times p$ steering matrix whose columns belong to an array manifold defined by the propagation modeling function F in the far-field. $\xi_1, \cdots, \xi_m$ are linear sensors with arbitrary positions whose centroid is at the origin, whereas $\kappa_1, \cdots, \kappa_p$ are source wave-numbers. We assume that erroneous estimates are initially provided by $\hat{G}_0 = [F(\hat{\kappa}_{1,0}), F(\hat{\kappa}_{2,0}), \cdots, F(\hat{\kappa}_{p,0})]$. We make no particular assumptions on the statistics of the signals.

In [7,8], we implemented a multi-source beamforming and tracking algorithm in 3 steps. Given an estimate of G_t at time t say $\hat{G}_t$, we first estimate S_t by any $m \times p$ beamforming matrix W_t under the distortionless and "non-mixture" constraint $W_t^H \hat{G}_t = I_p$ by:

$$\hat{S}_t = W_t^H X_t \ . \qquad (2)$$

Proposed structures of W_t and ways to select it will be discussed soon in the next section. We secondly update the steering matrix G_t and track its variations in a LMS-like (Least Mean Square) equation by:

$$\tilde{G}_{t+1} = \hat{G}_t + \mu \ (X_t - \hat{G}_t \hat{S}_t) \ \hat{S}_t^H \ , \qquad (3)$$

where μ is an adaptation step-size possibly normalized (*i.e.* NLMS). We finally fit the column vectors of $\tilde{G}_{t+1}$ to lie in the array manifold by:

$$\begin{aligned} \tilde{\kappa}_{i,t+1} &= \hat{\kappa}_{i,t} - \frac{\sum_{k=1}^{m} \xi_k \mathrm{Im}\left\{\log(\tilde{G}_{k,i,t+1} e^{j\xi_k\hat{\kappa}_{i,t}})\right\}}{\sum_{k=1}^{m} \xi_k^2} \ , \\ \hat{\dot{\kappa}}_{i,t+1} &= (1-\alpha)\hat{\dot{\kappa}}_{i,t} + \alpha(\tilde{\kappa}_{i,t+1} - \tilde{\kappa}_{i,t}) \ , \\ \hat{\kappa}_{i,t+1} &= \tilde{\kappa}_{i,t+1} + \hat{\dot{\kappa}}_{i,t+1} \ , \quad \text{for } i = 1, \cdots, p \ ; \\ \hat{G}_{t+1} &= [F(\hat{\kappa}_{1,t+1}), F(\hat{\kappa}_{2,t+1}), \cdots, F(\hat{\kappa}_{p,t+1})] \ . \qquad (4) \end{aligned}$$

α is a smoothing factor used for the speed estimation such that $\hat{\dot{\kappa}}_{i,0} = 0$ for $i = 1, \cdots, p$. We introduced kinematics in (4) to improve the tracking behavior of the algorithm especially for crossing targets as explained in [7,8]. We also proposed in [8] a simple procedure for the source number tracking. Details are not given for the sake of simplicity, but they can be found in [7,8].

At this stage, equation (3) can be viewed as a gradient-based subspace tracking method which minimizes the cost function of G:

$$E\left[\left\|\left(I_m - GW^H\right)X_t\right\|^2\right] , \qquad (5)$$

under the constraint $W^H G = I_p$ in (2) such that the column vectors of G be in the array manifold in (4). In (3), the orthogonal projection to $\hat{G}_t$ of X_t by $I_m - \hat{G}_t W_t^H$ reaches the criterion of (5) when $\hat{G}_t$ directly converges to G_t in the array manifold. It particularly implements a direct spatial decorrelation feature of the sources.

Related subspace tracking methods can be found in [8]. However, contrarily to these techniques, we analyze the spatial structure of the observation signals instead of their eigen-structure regardless of the signal statistics. As we directly track the "source-subspace" defined by G_t instead of the signal or noise eigen-subspaces, we no longer need uncorrelated white noise and a known number of mutually uncorrelated sources. These assumptions are usually necessary to recover the spatial information from an intermediate eigen-decomposition whereas the proposed algorithm avoids them.

Notice that W_t implements the spatial decomposition and decorrelation of the sources by a joint-orthogonality between the beamforming and steering matrices (*i.e.* $W^H \hat{G}_t = I_p$). It directly rules the behavior of the learning curve until convergence to the "source-subspace" criterion in (5). It finally controls the quality of source extraction and noise reduction. We next propose an optimal structure of the beamformer W_t which outperforms former versions we proposed in [7,8].

2 NEW BEAMFORMING STRUCTURE

Our main contribution in this work is about the selection of the beamformer W_t in (2).

We previously proposed the multi-DS (Delay-Sum) beamformer [7,8]:

$$W_t = \hat{G}_t\left(\hat{G}_t^H \hat{G}_t\right)^{-1} . \qquad (6)$$

Under the constraint $W_t^H \hat{G}_t = I_p$, the multi-DS is optimal for the extraction of coherent sources in uncorrelated white noise [7,8]. We also proposed in [8] adaptive GSC implementations of p parallel Minimum Variance Distortionless Response (MVDR) beamformers (*i.e.* $\mathrm{diag}[W_t^H \hat{G}_t] = \mathrm{diag}[I_p]$) [9]. When uncorrelated, the sources are optimally extracted in colored noise. Each column GSC beamformer of W_t particularly views other sources as jammers and tries to put deep nulls in their directions (*i.e.* $W_t^H \hat{G}_t \simeq I_p$). Although the desired source positions are simultaneously tracked, $W_t^H \hat{G}_t = I_p$ is not explicitly implemented to force the spatial decorrelation feature. This excludes the case of coherent sources due to risks of source cancellation in adaptive beamforming [1].

We herein propose the multi-LCMV beamformer W_t which minimizes the output distortion $E\left[\|S_t - \hat{S}_t\|^2\right]$ under the constraint $W_t^H G_t = I_p$ by:

$$\begin{aligned} W_t &= R_X^{-1} G_t (G_t^H R_X^{-1} G_t)^{-1} \\ &= R_N^{-1} G_t (G_t^H R_N^{-1} G_t)^{-1} , \end{aligned} \qquad (7)$$

where R_X and R_N respectively denote the correlation matrices of X_t and N_t in (1). The second expression of W_t is easily derived with the inversion lemma under the distortionless and "non-mixture" constraint. Under this constraint, the beamformer is optimal for the colored noise reduction and the extraction of coherent sources (other linear constraints are not excluded if necessary).

The multi-LCMV can be iteratively implemented from (7) by:

$$W_t = \hat{R}_X^{-1} \hat{G}_t (\hat{G}_t^H \hat{R}_X^{-1} \hat{G}_t)^{-1} , \qquad (8)$$

where $\hat{R}_X$ is a sample correlation matrix of X_t. This iterative structure is even robust to a partial correlation between sources and noise. However, to avoid matrix computations in (8) involving inversions and a slow tracking capacity of time-variations, we adaptively implement the beamformer in the new multi-dimensional adaptive GSC structure denoted by multi-LCMV-GSC as follows:

$$\begin{aligned} Y_t &= W_t^{cH} X_t = \left(\hat{G}_t^H \hat{G}_t\right)^{-1} \hat{G}_t^H X_t , \\ X_t^n &= P_t^H X_t , \\ \hat{S}_t &= Y_t - W_t^{nH}\ X_t^n , \\ W_{t+1}^n &= W_t^n - \eta X_t^n \hat{S}_t^H , \end{aligned} \qquad (9)$$

where Y_t is the output of the multi-DS beamformer W_t^c subject to the constraint $W_t^{cH} \hat{G}_t = I_p$ (see (6)). P_t is the $m \times (m-p)$ blocking matrix of the noise subspace (*i.e.* $P_t^H \hat{G}_t = 0_{(m-p)\times p}$), X_t^n is the noise reference vector and the $(m-p)\times p$ matrix W_t^n is the noise filter of the multi-LCMV-GSC. The step-size η is possibly normalized (*i.e.* NLMS-GSC, see [8]). The total beamformer is $W_t = W_t^c - P_t W_t^n$ and it satisfies $W_t^H \hat{G}_t = I_p$.

Notice that the multi-DS branch in (9) given by the constrained beamformer W_t^c requires an order of $O(mp^2 + mp + p^2 + p^3)$ operations. When $p \gg 1$, we propose an adaptive estimation of Y_t to reduce its computational complexity to $O(mp)$ by:

$$Y_t^{i+1} = Y_t^i + \zeta_{Y}\ (\hat{G}_t^H \hat{G}_t)\ \left(\hat{G}_t^H X_t - (\hat{G}_t^H \hat{G}_t) Y_t^i\right) , \qquad (10)$$

where $Y_t^0 = 0_p$ and where ζ_Y is a step-size. Just few iterations are sufficient for convergence (*e.g.* $i \simeq 5$).

Notice also that the calculation of the blocking matrix P_t requires an orthogonal completion of $\hat{G}_t$ such that $P_t^H P_t = I_{m-p}$ and $P_t^H \hat{G}_t = 0_{(m-p)\times p}$. Orthogonality of P_t is needed here to achieve the best convergence rate of W_t [9]. In the same way, we adaptively estimate it to reduce the complexity of its calculation from $O(m^3)$ to $O(m(m-p)^2)$ within few iterations by:

$$P_t^{i+1} = P_t^i + \zeta_P \left[I_m - \left(\frac{1}{2m} \hat{G}_t \hat{G}_t^H + P_t^i {P_t^i}^H \right) \right] P_t^i \ , (11)$$

where $P_t^0 = P_{t-1}$ and where ζ_P is a step-size.

The multi-LCMV-GSC finally requires a total order of complexity of $O(m(m-p)^2 + mp)$. It is optimal for the extraction of correlated sources in incoherent colored noise [1]. When the noise is uncorrelated and white, the adaptive beamformer W_t can be reduced to its optimal multi-DS branch in (10) with a total order of complexity of $O(mp)$. The multi-DS structure is even robust to a partial coherence between the desired sources and the noise, but is suboptimal for the colored noise reduction.

3 EFFICIENCY IN COHERENT FIELDS

Here, we assess by simulations to what extent the proposed algorithm is efficient in coherent and noisy environments.

Without loss of generality, we consider for the sake of simplicity a uniform linear array of $m = 16$ sensors. This antenna is receiving $p = 2$ desired sources with equal unit powers and uncorrelated white noise at a Signal to Noise Ratio (SNR) of 10 dB. We will always assume this configuration in the following, except in figure 2 where we will just add a jammer to simulate colored noise.

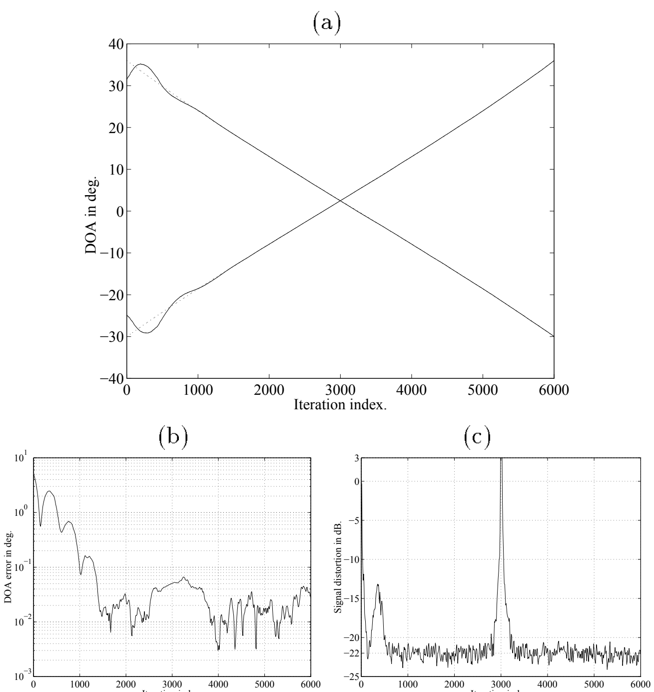

Figure 1: Performance of multi-DS in the presence of 2 fully-correlated sources (*i.e.* identical).

In figure 1-a, we successfully track 2 fully-coherent and linearly crossing sources with initial DOA (Direction of Arrival) errors of 5 deg. The mean of absolute DOA errors in figure 1-b is reduced to the range of 10^{-2} deg, while the mean of output distortion in figure 1-c is minimized to its optimal value (*i.e.* almost $-10\log_{10}(m) - \mathrm{SNR} = -22$ dB). Although the signal-subspace is degenerate, the spatial filtering by the distortionless and "non-mixture" constraint simply implements a "full" decorrelation feature and enables the spatial decomposition and the tracking of the source-subspace by the cost function in (5). Previous methods propose a "partial" decorrelation which improves only at the cost of an increasing complexity and/or a higher number of sensors [2-6]. Besides, they do not process mobile sources.

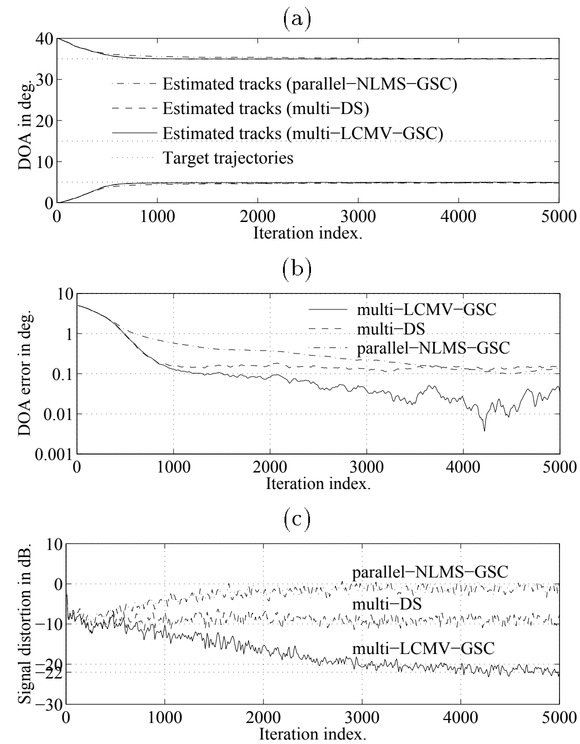

Figure 2: Performance of parallel-NLMS-GSC, multi-DS, and multi-LCMV-GSC in the presence of 2 correlated sources with a correlation factor of 0.9 and a jammer with equal power all at a SNR of 10 dB.

In figure 2-a, we now consider 2 partially-coherent and immobile sources with a correlation factor of 0.9. We also add a jammer with an equal unit power and a high interference ratio to simulate spatially colored noise. We simultaneously run the multi-DS [7,8], the parallel-NLMS-GSC [8] and the new multi-LCMV-GSC of equation (9) for comparisons. All versions correct the initial DOA errors of 5 deg, but the multi-LCMV-GSC offers the minimum mean of absolute DOA errors in figure 2-b. In this figure, the new beamformer shows the best tracking behavior until convergence to the source-subspace criterion in (5) because it optimally reduces colored noise. In figure 2-c, the multi-LCMV-GSC also shows the best learning curve in the mean of output distortion. It perfectly cancels the jammer and reaches its optimal performance at the steady state. On the other hand, the multi-DS is not able to cancell the interference of the jammer whereas the parallel-NLMS-GSC cancels the desired sources due to their correlation [1]. Actually, we also carried out other experiments with spatially correlated white noise. The results are not given for the lack of space, but the new structure is simple and easy to test. Previous methods do not process colored noise and usually assume uncorrelated white noise [2-6].

4 FAST TRACKING CAPACITY

We assess in this section the tracking behavior of the proposed algorithm in highly non-stationary and time-varying environments.

In figure 3-a, we successfully correct initial DOA errors of 2 deg and track 2 high speed sources crossing linearly with an absolute speed as high as 1.32 deg/iteration. Other tracking algorithms can be found in [8], but the source speeds are usually far below this range. This experiment shows that the projection of the steering vectors in the array manifold with a kinematic model for time-variations tremendously improves the tracking capacity. It proves that a filter regularization in a given set with time-variations modeling speeds up the convergence and significantly enhances the tracking in adaptive filtering.

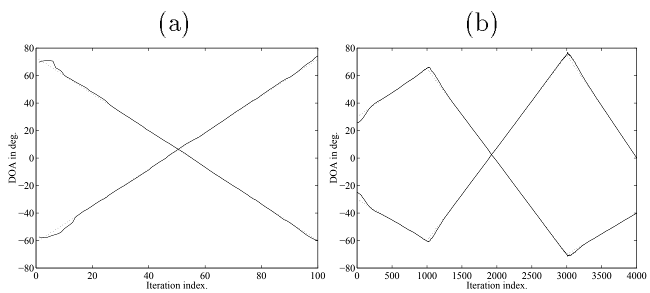

Figure 3: (a): 2 sources moving linearly with speeds of absolute value 1.32 deg/iteration. (b): 2 sources moving and crossing linearly with 2 sharp shifts in direction.

In figure 3-b, we successfully correct DOA errors of 5 deg and track 2 sources moving and crossing linearly with 2 sharp shifts in direction. Realistic maneuvering targets cannot follow such trajectories, but the experiment illustrates well the high adaptation capacity of the algorithm to sudden and discontinuous changes in the kinematic model such as the speed variations.

At the steady state, all figures show that the algorithm is unbiased and that it perfectly handles crossovers at various speeds. In the performance and convergence analyses made in [8], we particularly show that an adequate selection of the adaptation step-sizes μ and α is necessary in equations (3) and (4) respectively. We also prove that the proposed algorithm and MUSIC (see reference in [8]) have the same asymptotic DOA misadjustment for the immobile sources whereas our method has a better tracking behavior for the mobile case [8].

5 CONCLUSION

We proposed an optimal and simple algorithm for the extraction and the tracking of partially- or fully-coherent sources in colored noise. Other issues related to 2D array-calibration and partially blind or wideband beamforming are described or referenced in [10].

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