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Robust Multisource Beamforming Via LMS-Like Target Tracking

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Abstract. A new algorithm for simultaneous robust multisource beamforming and multitarget tracking is proposed. Using a set of optimal conventional beamformers in the presence of uncorrelated white noises, the source signals are first extracted at each iteration. The steering vectors which are bound to be in the array manifold (i.e. a set of parametrized vectors), are time-updated by a correcting/tracking procedure generalized from a former work we have derived in the single source case. A separate projection of each steering vector on the array manifold enables us to extract the corresponding location parameter (i.e. the parametrizing variable), and to resume the iterations of the adaptive algorithm. The beamforming and tracking performances are then shown to be identical to the single source case. However, it is shown that the additional use of some kinematic parameters (i.e. speed, acceleration, etc$\cdots$) inferred from the reconstructed trajectories improves the tracking performance, and overcomes some of the problems of crossing targets. The efficiency of multitarget tracking and the robustness of multisource beamforming are confirmed by simulation.

1. Introduction

Recently, we proposed in [1,2] a robust adaptive beamforming algorithm based on a LMS-like target tracking for a single source, by allowing the steering vector of a classical beamformer to be time-adapted in the array manifold. This algorithm proved to be robust to strong localization errors without introducing any SNR loss, and to have an efficient tracking behavior when in the presence of mobile sources (target and jammers).

In this paper, our purpose is to generalize this algorithm to the simultaneous extraction and tracking of multiple sources. To do so, we assume that the number and the location parameters of all point sources are initialized by an approximate localization technique. Those sources which are not localized will have relatively small power, and will be confused with spatially diffuse noise.

Given the propagation model, the idea is to simultaneously time-adapt all the steering vectors in the same way as in [1,2], with a separate projection of each vector on the array manifold. The source signals vector is estimated using a set of beamformers. For one selected desired source, classical adaptive beamformers such as Frost or GSC could be used successfully to adaptively cancel all the present jammers (possibly including those unlocalized sources). Instead, we simply propose the use of a set of conventional beamformers which are shown to have an optimal performance in the presence of uncorrelated white noise. Their advantage of being non adaptive is that they avoid the source signal cancellation, even in the presence of partially coherent interference. Though the unlocalized sources are not specifically canceled, the performance remains barely unchanged since we assumed these sources to be confused with the white noise. However, adaptive beamformers could use in parallel the updated location parameters given by the proposed algorithm for a final and separate source extraction, when in the presence of no coherent source interference.

The simulations performed with conventional beamformers confirm the results we expected from the generalization of the convergence and performance analyses made in [1,3]. Indeed, the algorithm has the same robustness to location errors, and presents a comparable tracking behavior for each source.

However, two problems are encountered as expected theoretically. First, the location estimator is biased whenever the location parameter increment is not zero mean (it can no longer be considered as a reasonable assumption, when in the presence of maneuvering targets). Second, the algorithm cannot deal with crossing targets. The presence of a unique source in the spread of a locking range is a major hypothesis we had to assume for the convergence proof made in [1,3]. We thus introduce an additional procedure based on the use of some kinematic parameters such as the speed. It is shown to remove the bias, and to overcome some of the problems of crossing targets by LMS-like iteration blocking and trajectory prediction.

2. Mathematical Formulation

We consider the following model of plane wave propagating signals received by a linear array (see Figure 1):

$$X_t = G_t S_t + N_t, \tag{1}$$
$$G_t = [F(\theta_{1,t}), F(\theta_{2,t}), \cdots, F(\theta_{m,t})], \tag{2}$$

where X_t is the m-dimensional observation vector, $S_t = [s_{1,t}, s_{2,t}, \cdots, s_{p,t}]^T$ is the vector of p desired narrowband signals to be extracted ($p \leq m$), N_t is an additive noise vector, and G_t is the transfer function (i.e. the $m \times p$ steering matrix) between the emitted sources S_t and the m-sensor antenna array. All the quantities considered herein are complex. F is a parametric modeling function determining the propagation law and the configuration geometry, and $\Theta_t = [\theta_{1,t}, \theta_{2,t}, \cdots, \theta_{p,t}]^T$ is the location parameter of interest. $\theta_{i,t}$ represents the DOA or the spatial coordinates of the source $s_{i,t}$. The subscript t stands for time index.

We consider here the simple case of a linear array and a plane wave propagation model. So, the parameterizing function F is given by:

$$F(\theta) = e^{-j\tau}\left[e^{-j\kappa x_1}, e^{-j\kappa x_2}, \cdots, e^{-j\kappa x_m}\right]^T, \tag{3}$$

where $\theta \triangleq [\kappa\ \tau]^T$ represents a single source location parameter. The wavenumber $\kappa \triangleq \frac{2\pi \sin(\phi)}{\lambda}$ where $\phi \in [-\pi/2, \pi/2[$ is the DOA, and λ is the wave length. $[x_1, x_2, \cdots, x_m]^T$ is the sensor positions vector. τ represents the phase delay from the origin to within about an integer multiple of 2π, and is obviously restricted to $[0, 2\pi[$.

We further make the following assumptions:

A1: G, N and S are mutually independent.

A2: Channel G is slowly time varying in comparison to the variations of N and s. Hence we are able to estimate G properly, then update it.

A3: A possibly erroneous approximation of Θ_0, say $\hat{\Theta}_0$, is initially provided either by an approximate *a priori* guess, or by a given localization technique.

A4: N is a white noise with zero mean Gaussian distribution and autocorrelation matrix $\sigma_N^2 I$.

A5: The DOA's are far apart enough to make the source separation possible. The validity of this assumption is tested continuously. Whenever it is invalid, we will use an alternative procedure presented in section 4.

Given these assumptions, our main purpose in this paper is to reduce the error between G and its estimate $\hat{G}$. The idea is to constantly correct the steering matrix $\hat{G}$ and time-adapt the classical beamformers say W to the new look-directions. The beamformers should thus provide a robust multisource signal extraction, and an efficient method for multitarget tracking.

3. Multisource beamforming

The algorithm for robust multisource beamforming via LMS-like target tracking can be summed up by the following steps (see Figure 1):

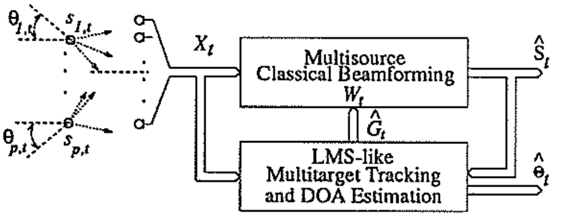

Figure 1. Block diagram of the algorithm.

Step I Beamforming

At iteration t, we suppose that an estimation of G_{t-1}, say $\hat{G}_{t-1}$ is available. As assumption A2 states that G is slowly time varying, it is possible to estimate S_t with the steering matrix G_t approximated by $\hat{G}_{t-1}$ at time t. Hence, it is straightforward to show that conventional beamformers are optimal for the minimization of the output distortion under the condition that $W_t^H G_t = I_p$:

$$\hat{S}_t = W_t^H X_t \triangleq \left(\hat{G}_{t-1}^H \hat{G}_{t-1}\right)^{-1} \hat{G}_{t-1}^H X_t. \tag{4}$$

A^H denotes the conjugate transpose of A and W_t is the $m \times p$ beamforming matrix. It must be noted by the virtue of assumption A5, that the matrix G_t is full rank and that $G_t^H \hat{G}_t$ is inversible. Moreover, the condition $W_t^H \hat{G}_{t-1} = I_p$ is fulfilled. Hence, for each one of the p sources say s_i, the corresponding beamformer W_i considers the remaining sources s_j $(j \neq i)$ as jammers and rejects them.

Step II LMS-like

The resulting estimate of S_t, say $\hat{S}_t$, can be used in a LMS-like procedure to track or correct the steering matrix variations:

$$\tilde{G}_t \quad = \quad \hat{G}_{t-1} + \mu\,(X_t - \hat{G}_{t-1}\hat{S}_t)\hat{S}_t^H. \tag{5}$$

It should be noted at this stage that the column vectors $\tilde{G}_{i,t}$ of the LMS-like updated matrix $\tilde{G}_t$ obtained in (5), do not necessarily belong to the array manifold. For this reason, we denote them at the present by $\tilde{G}_t$ in (5). The convergence analysis in [1,3] shows that equation (5) does not track correctly the steering vectors when assumption A5 is not verified. This is actually due to the fact that the noise covariance matrix defined for one selected source can no longer be approximated by the "nice" form $\sigma^2 I$, when one or more sources are present in its locking range. To overcome this problem, we will introduce a new procedure presented later in section 4.

Step III DOA estimation

At present, we consider that assumption A5 is valid. In this case, the estimator $\tilde{G}_{i,t}$ of $G_{i,t}$ (i^{th} column of G_t) can be improved by a DOA adjustment with respect to a projection over the array manifold, performed separately for each source as in [1,2]. The i^{th} location parameter $\hat{\theta}_{i,t} \triangleq [\hat{\kappa}_{i,t}, \hat{\tau}_{i,t}]^T$ is then updated as follows for $1 \leq i \leq p$:

$$\hat{\theta}_{i,t} = \hat{\theta}_{i,t-1} - K\left[\begin{array}{c} \sum_{q=1}^m x_q \mathrm{Im}\left(\tilde{G}_{q,i,t} e^{jx_q\hat{\kappa}_{i,t-1}}\right) \\ \sum_{q=1}^m \mathrm{Im}\left(\tilde{G}_{q,i,t} e^{jx_q\hat{\kappa}_{i,t-1}}\right) \end{array}\right], \tag{6}$$

where $\tilde{G}_{q,i,t}$ is the q^{th} component of the i^{th} column vector $\tilde{G}_{i,t}$ of the matrix $\tilde{G}_t$ at time t, and where Im(.) denotes the imaginary part. The matrix K is given by:

$$K \triangleq \left[\begin{array}{cc} \sum_{q=1}^m x_q^2 & \sum_{q=1}^m x_q \\ \sum_{q=1}^m x_q & \sum_{q=1}^m 1 \end{array}\right]^{-1}. \tag{7}$$

It must be noted that another alternative is also presented in [1,2] instead of (6).

The delays $\hat{\tau}_{i,t}$ are actually estimated to make the projection consistent. They are now set to zero to make the origin at the array center (see [1,2] for more details). Hence, we finally reconstruct the steering matrix:

$$\begin{aligned} \hat{G}_{i,t} \quad &= \quad F\left([\hat{\kappa}_{i,t}, 0]^T\right) \quad , \quad 1 \leq i \leq p \ ; \\ \hat{G}_t \quad &\triangleq \quad \left[\hat{G}_{1,t}, \hat{G}_{2,t}, \cdots, \hat{G}_{p,t}\right]. \end{aligned} \tag{8}$$

Except the matrix inversion in (4), all the above steps involve a number of operations proportional to the number of sensors m by the number of sources p. The computational complexity of this algorithm is then of order $O(mp + p^3)$.

The performance analysis of the proposed algorithm is the same as the single source case presented in [1,3], and shows for the i^{th} source $(1 \leq i \leq p)$ that:

$$E[\hat{\kappa}_{i,t}] = E[\hat{\kappa}_{i,t-1}](1 - \mu\sigma_{s_i}^2) + \mu\sigma_{s_i}^2 E[\kappa_{i,t}], \tag{9}$$

where $\sigma_{s_i}^2$ is the variance of $s_{i,t}$. This equation proves that the algorithm is able to track each target in the temporal mean, with a time constant given by $\tau_{E[\hat{\kappa}_i]} = 1/\left(\mu\sigma_{s_i}^2\right)$. In the case of a random walk process, let us define for the i^{th}

source the target motion increment or "speed" by $\dot{\kappa}_{i,t} \triangleq \kappa_{i,t+1} - \kappa_{i,t}$. Then we notice that the above algorithm gives unbiased location estimators, unless the increments are not zero mean. This point is addressed in the following section, where an efficient solution is given.

4. Speed estimation and tracking of crossing targets

We present in this section an additional step in the algorithm, to improve its performance regarding the bias of the location parameters, and to avoid some of the problems of crossing targets. This step is based on the prediction of the target trajectories, thanks to the extra information obtained on their kinematics. As known in the literature of motion target analysis [5-7], we use the estimated kinematic parameters to rule the behavior of the targets during the crossover interval. Whereas outside it, we propose the use of these parameters to remove the bias of the location estimators (see Figure 2) .

Let us assume the source speeds $\dot{\Theta}_t \triangleq (\Theta_t - \Theta_{t-1})$ to be slowly time-varying, as it can be stated by assumption A2. We are then able to estimate them by replacing equation (6) by the following procedure (for $1 \leq i \leq p$):

$$\begin{bmatrix} \tilde{\kappa}_{i,t} \\ \tilde{\tau}_{i,t} \end{bmatrix} = \begin{bmatrix} \hat{\kappa}_{i,t-1} \\ 0 \end{bmatrix} - K \begin{bmatrix} \sum x_q \mathrm{Im}\left(\tilde{G}_{q,i,t} e^{jx_q \hat{\kappa}_{i,t-1}}\right) \\ \sum \mathrm{Im}\left(\tilde{G}_{q,i,t} e^{jx_q \hat{\kappa}_{i,t-1}}\right) \end{bmatrix}, \quad (10)$$

$$\hat{\dot{\kappa}}_{i,t} = \alpha \hat{\dot{\kappa}}_{i,t-1} + (1-\alpha)\left(\tilde{\kappa}_{i,t} - \tilde{\kappa}_{i,t-1}\right), \quad (11)$$

$$\hat{\kappa}_{i,t} = \tilde{\kappa}_{i,t} + \hat{\dot{\kappa}}_{i,t}. \quad (12)$$

The speed $\hat{\dot{\kappa}}_{i,t}$ is estimated in (11) via the lowpass AR filtering of an intermediate location estimator $\tilde{\kappa}_{i,t}$ with a smoothing factor α. This factor must be chosen respectively to the stationarity of $\dot{\Theta}_t$ and the time constants of the algorithm. For trajectories with increments slowly time-varying in average, it can be seen easily that the intermediate location estimator in (10) is biased with a delay equal to the product of the speed by the corresponding time constant. Hence, it can be shown that the final location estimator in (12) catchs up with the bias at convergence, under some additional stability conditions on α and μ.

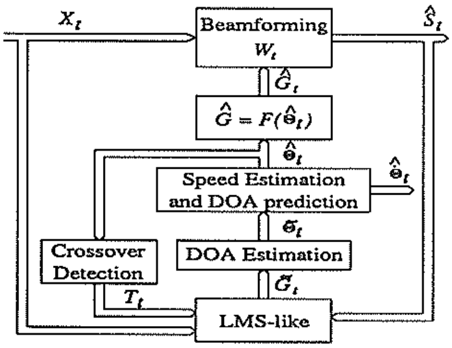

Figure 2. The block diagram with speed estimation.

However, when the targets are not distant enough, and when they are in the same locking range, the algorithm fails in tracking them properly. For each source, we therefore define the following test of validity of assumption A5:

$$T_{i,t} \triangleq \begin{cases} 0 & \text{if } \exists\, j \neq i \mid |\hat{\theta}_{i,t-1} - \hat{\theta}_{j,t-1}| \leq \epsilon \\ 1 & \text{otherwise} \end{cases}$$

$$T_t \triangleq [T_{1,t}, T_{2,t}, \cdots, T_{p,t}]^T. \quad (13)$$

If this assumption is not valid, the adaptation of the LMS-like equation (5) should be blocked for the corresponding crossing targets. Hence we modify it as follows:

$$\tilde{G}_t = \hat{G}_{t-1} + \mu \,\mathrm{diag}[T_t]\, \left(X_t - \hat{G}_{t-1}\hat{S}_t\right)\, \hat{S}_t^H. \quad (14)$$

We are now able to hold on the tracking of the crossing trajectories, even during the crossover intervals, where we can see that the speed estimates remain constant. This is actually a good and reasonable approximation for targets locally crossing with uniform speed.

In the case where the motion of the targets is more complex (e.g. uniformly accelerated trajectories, etc...), we may estimate higher order increments in an alternative procedure (i.e. acceleration, 3^{rd} derivative, etc...). It should be noted that the estimation and the use of higher order increments do not improve significantly the precision of the source localization, but enable however a better prediction of the target trajectory during the crossover interval (see Figure 3). This problem is addressed in a more general work, presently under preparation.

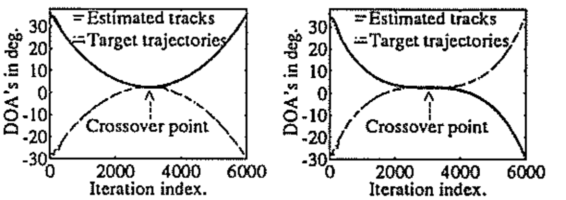

Figure 3. Target tracking with the estimation of higher order increments for complex crossovers.

This additional step finally improves the target tracking performance in real situations, and does not introduce a higher order computational complexity.

5. Simulation Results

To illustrate the efficiency of the algorithm proposed in section 3, we considered the case of an equidistant linear array of 16 sensors. 4 planewave narrowband and uncorrelated moving sources are emitting with a unit power from separate initial angles. Spatially diffuse white noise is added at a mean SNR of $10dB$.

The simulation results show that the algorithm generalized to the multisource case has the same properties as [1,2] regarding robustness to errors, tracking behavior (see Figure 4a), and source signal extraction (see Figure 4b: $22dB \simeq 10\ log_{10}(m) + SNR$ improvement). However, we can see in figure 4 that the tracking and the source signal extraction are both disturbed when two of the targets get into the same locking range or cross.

To illustrate the performance of the speed estimation procedure, we considered the case of two crossing targets with uniform speeds as shown in Figure 5.

Without the speed estimation procedure, both trackers produce an estimation delay and turn back at the crossover point. Although both sources remain tracked, the result is not satisfactory for source signal extraction and classification (see Figures 6a and 6b).

On the other hand, the speed estimation procedure removes the bias, and maintains the tracking during and after the crossover. Figures 6a and 6b show respectively $34dB$ improvement in source localization, and $3dB$ enhancement in beamforming. Of course, when the targets are too close around the crossover point, the sources are confused and cannot be extracted separately.

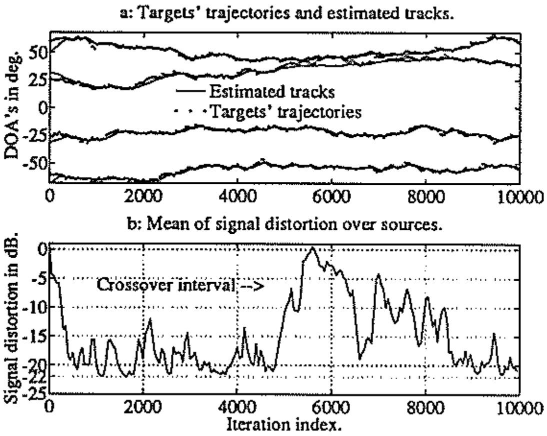

Figure 4. a) DOA trajectories and estimated tracks of 4 sources. b) Mean of signal distortion $\sum_{i=1}^{p} |\hat{s}_{i,t} - s_{i,t}|^2/p$.

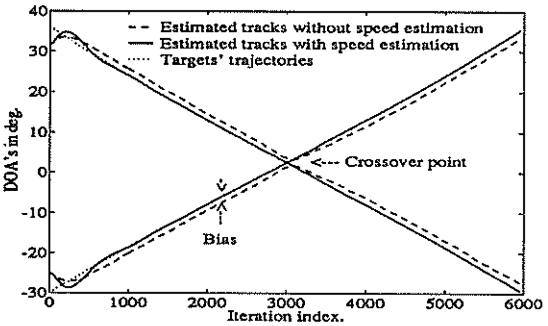

Figure 5. DOA trajectories and estimated tracks of two immobile crossing sources with and without speed estimation.

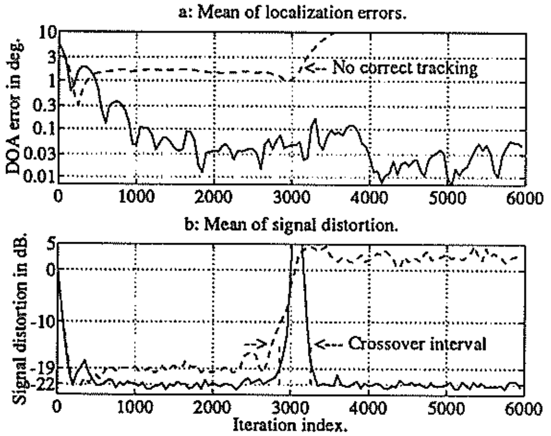

Figure 6. Performance of target tracking and source separation. a) Mean of localization errors $\sum_{i=1}^{p} |\hat{\phi}_{i,t} - \phi_{i,t}|/p$. b) Mean of total distortion $\sum_{i=1}^{p} E[|\hat{s}_{i,t} - s_{i,t}|^2]/p$.

6. Conclusions

We presented in this paper a new algorithm for robust multisource beamforming and multitarget tracking. We first described the generalization we have made of the work recently presented in the single source case. Given the assumption that the number and the approximate initial locations of the most significant sources are known, we proved the use of the conventional beamformers to be optimal in the presence of uncorrelated white noise. Using the estimated location parameters, it should be noted here that adaptive beamformers such as the GSC could be used in a master-slave structure to only extract the source signals, when no coherent interference is present. We also showed the resulting algorithm to offer the same robustness to location errors, and to have the same beamforming and tracking performances. Particularly in the case where the time increment of a location parameter is not zero mean, the location estimate is biased. Hence we proposed a new procedure based on the use of some estimated kinematic parameters (e.g. the speed), which is proved to yield unbiased estimates. Simultaneously, the estimated kinematic parameters are shown to solve some of the problems of crossing targets via the prediction of their trajectories.

In addition, the algorithm has a complexity of order $O(p^3 + pm)$ where p and m are respectively the number of sources and array sensors. Hence it can be implemented in a very easy way.

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