ROBUST ADAPTIVE BEAMFORMING VIA LMS-LIKE TARGET TRACKING

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ABSTRACT

This paper presents an algorithm implementing robustness in beamforming, by directly reducing localization errors in the presence of pointing errors or a single moving target. Given an initial position, the desired source signal is first estimated using a classical beamforming unit. It is in a second step, processed by an "LMS-like" gradient stochastic estimation procedure of the steering vector, to adaptively track the correct source position. The newly identified source position is projected over the array manifold, then finally transmitted in a feedback loop to the beamforming unit, closing in this way the global algorithm iteration. The simulation results show that robustness is effectively realized without any compromising output SNR loss. Moreover, they prove an efficient tracking behavior in the presence of mobile sources.

1 INTRODUCTION

The potential for using adaptive beamforming to improve the performance of source signal estimation was recognized in the early 1960's in the fields of signal processing [1]. Unfortunately, classical beamforming algorithms are very sensitive to localization errors, and can not be used for a reliable signal extraction unless a robustness feature is appended.

Different versions were developed to make the beamformer robust to such errors [2-4]. Robustness is usually introduced via a certain compromise fixing some constraints representing a tradeoff between signal distortion and output noise reduction . This solution avoids signal cancellation, with however an allowable threshold of distortion. Moreover, sources are assumed either immobile with relatively small pointing errors, or mobile with strictly limited motion range around a fixed position. Whenever this assumption is not valid by the presence of strong pointing errors or free moving targets, beamforming is no longer convenient to a reliable signal extraction.

This paper alternatively proposes to gain robustness through an efficient reduction of localization errors, using classical beamforming with time-corrected and adapted steering vector.

Regarding the reduction of localization errors, localization methods such as the minimum variance, the maximum likelihood, MUSIC, and the related minimum norm, could be used in the case of a stationary environment, to reliably estimate the correct source position and the corresponding steering vector. Such techniques see however their performances drop drastically in the presence of spatially nonstationary moving targets. They can be used in this case to estimate an initial source position allowing the initialization of some tracking algorithms, which look conceptually more suitable to adapt to nonstationary sources.

Several estimation techniques such as Kalman filtering have been applied in target motion analysis to estimate the trajectory of an object, using maximum *a priori* (*MAP*) or maximum likelihood (*ML*) estimators, etc$\cdots$ [5]. These methods are expensive in terms of complexity, and we found that a more efficient algorithm can be used instead.

Our approach is to time-adapt the steering vector of a classical beamformer [6]. To do so, we use a simple LMS-like tracking procedure correcting the steering vector by a gradient stochastic term depending on the classical beamforming output. The LMS-like iterative equation can be interpreted as the result of an identification problem. Given a parametric model corresponding to a propagation law, the parameterizing variables such as the DOA or the target coordinates are then extracted by a projection of the LMS-adapted steering vector over the array manifold, say Γ. The estimated parameters are finally used to reconstruct the steering vector of the classical beamformer for enhanced signal extraction.

2 MATHEMATICAL FORMULATION

We consider the following model of a plane wave propagating signal received by a linear array (see Figure 1):

$$X_t = G_t s_t + N_t, \qquad (1)$$
$$G_t = F(\theta_t), \qquad (2)$$

where X_t is the m-dimensional observation vector, s_t is the desired narrowband signal to be extracted, N_t is an additive noise vector, and G_t is the transfer function (i.e. steering vector) between the emitted source s_t and the m-sensor antenna array. All the quantities considered herein are complex, and the subscript t stands for time index.

The parameterizing function F is given by:

$$F(\theta) = e^{-j\tau}\left[e^{-j\kappa x_1}, e^{-j\kappa x_2}, \cdots, e^{-j\kappa x_m}\right]^T, \qquad (3)$$

where $\theta \triangleq [\kappa \;\; \tau]^T$. The wavenumber $\kappa \triangleq \frac{2\pi \sin(\phi)}{\lambda}$ where $\phi \in [-\pi/2, \pi/2[$ is the DOA, and λ is the wave length. $\Xi \triangleq [x_1, x_2, \cdots, x_m]^T$ is the sensor positions vector. τ represents

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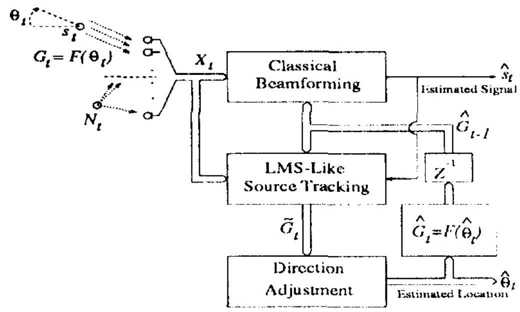

Figure 1: The block diagram of the algorithm.

the phase delay from the origin to within about an integer multiple of 2π, and is obviously restricted to $[0, 2\pi[$.

We further make the following assumptions:

A1: G, N and s are mutually independent.

A2: On each relatively short time interval, G has a Gaussian distribution with mean and autocorrelation matrix both slowly time varying in comparison to s and N variations.

A3: An approximation of θ_0 possibly erroneous, say $\hat{\theta}_0$, is provided initially either by an approximate *a priori* guess, or by a given localization technique.

A4: N is a white noise with zero mean Gaussian distribution and autocorrelation matrix $R_N = \sigma_n^2 I$.

We finally assume X to be the unique observation available, and the modeling function F to be given.

3 PROPOSED ALGORITHM

In this section, we present the algorithm in the simple case of a plane wave propagation model and a linear array.

Given the modeling equations and the assumptions made in section 2, robust adaptive beamforming via LMS-like target tracking can be summed up to the following steps (see Figure 1):

- At iteration t, we suppose that an estimation of G_{t-1}, say $\hat{G}_{t-1}$ is available. Thanks to assumption A2 stating that G is slowly time varying, it is possible to estimate s_t using one of the adapted classical beamforming techniques with $\hat{G}_{t-1}$ as the adapted steering vector. For illustration and simulations, we consider the case of the GSC algorithm without any loss of generality:

$$
\begin{aligned}
W_t^s &= \frac{\hat{G}_{t-1}}{\hat{G}_{t-1}^H \hat{G}_{t-1}} = \frac{\hat{G}_{t-1}}{m}, \qquad (4)\\
y_t &= W_t^{s\,H} X_t,\\
P &\triangleq \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix},
\end{aligned}
$$

$$
\begin{aligned}
X_t^n &= P\ \mathrm{diag}[\hat{G}_{t-1}^H]\ X_t,\\
\epsilon_t &= W_t^n\ X_t^n,\\
\hat{s}_t &= y_t - \epsilon_t,\\
W_{t+1}^n &= W_t^n - \eta\ \hat{s}_t^H X_t^n.
\end{aligned}
$$

It should be noted here that P is a $(m-1)\times m$ matrix (remark: $P\mathrm{diag}[\hat{G}_{t-1}^H]\hat{G}_{t-1} = 0$), and that W_t^n is a $(m-1)$-dimensional vector initialized at 0.

- The resulting estimate of s_t, say $\hat{s}_t$, can be used in a LMS-like procedure to track or correct the steering vector variations:

$$
\tilde{G}_t = \hat{G}_{t-1} + \mu\,(X_t - \hat{G}_{t-1}\hat{s}_t)\hat{s}_t^H. \qquad (5)
$$

It is actually the approximation result of a *MAP* estimation method, made possible thanks to the assumptions introduced in part 2. At this stage, we notice that the LMS-like updated vector $\tilde{G}_t$ obtained in (5) does not necessarily belong to the array manifold Γ. This is why we denote it at the present by $\tilde{G}_t$ in (5).

- This estimator of G_t can be improved by DOA adjustment respectively to a projection over the array manifold as follows:

$$
\hat{G}_t = \arg\min_{G\in\Gamma} d(\tilde{G}_t, G), \qquad (6)
$$

where d is a metric distance; or equivalently:

$$
\hat{\theta}_t = \arg\min_{\theta\in\Theta=F(\Gamma)} d(\tilde{G}_t, F(\theta)). \qquad (7)
$$

Using the log-distance $d_{\log}(Y,Z) \triangleq \|\log Y - \log Z\|$, we have:

$$
\hat{G}_t = F\left([\hat{\kappa}_t, 0]^T\right), \qquad (8)
$$

with:

$$
\hat{\theta}_t \triangleq \hat{\theta}_{t-1} - K \begin{bmatrix} \sum_{i=1}^m x_i \mathrm{Im}\left(\log(\tilde{G}_{t,i} e^{j x_i \kappa_{t-1}})\right) \\ \sum_{i=1}^m \mathrm{Im}\left(\log(\tilde{G}_{t,i} e^{j x_i \kappa_{t-1}})\right) \end{bmatrix}, \qquad (9)
$$

where Im(.) denotes the complex imaginary part, and:

$$
K \triangleq \begin{bmatrix} \sum_{i=1}^m x_i^2 & \sum_{i=1}^m x_i \\ \sum_{i=1}^m x_i & \sum_{i=1}^m 1 \end{bmatrix}^{-1}. \qquad (10)
$$

The Euclidean distance can be used successfully with a linearization technique of F around θ_t. Equation (9) can be interpreted as a linear regression of the arguments of $\tilde{G}_t$ components over the sensor positions. Hence, $\hat{\tau}_t$ is first estimated to make the linear regression consistent. It is finally set to zero (8), giving in this way the classical beamforming modeling where the delays are computed respectively to the array origin.

It should be noted that all the steps above involve a number of operations proportional to the number of sensors m. The computational complexity of this algorithm is then of order $O(m)$.

The performance analysis of the algorithm [6,7] proves that the initial localization error must be kept smaller than

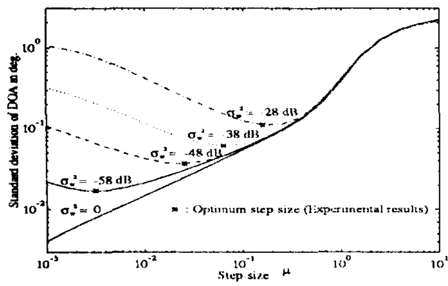

Figure 2: Steady state covariance of ϕ against step-size μ, for different values of $\sigma_w^2 \triangleq E[(\kappa_{t+1} - \kappa_t)^2]$.

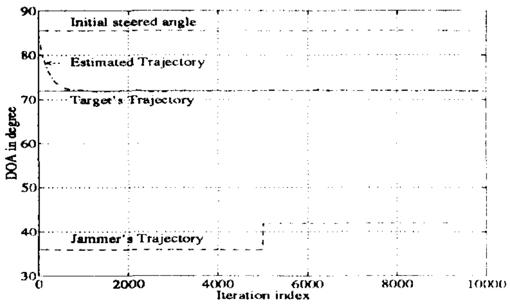

Figure 3: DOA trajectories in the case of an immobile source and a step moving jammer with $\sigma_j^2 = 1$.

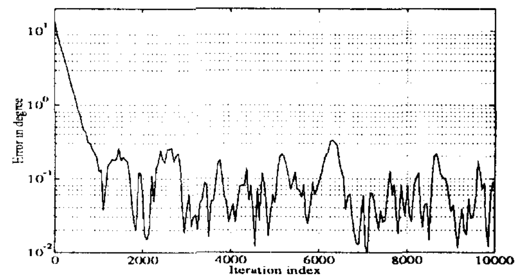

Figure 4: Absolute DOA error in the case of an immobile source and a step moving jammer.

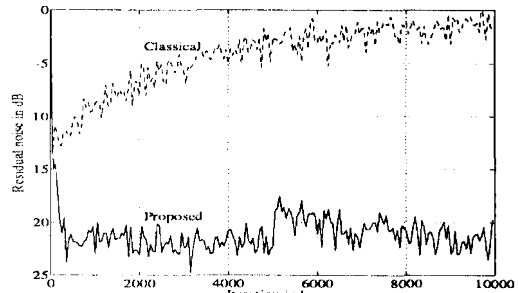

Figure 5: Total distortion $E[|s_t - \hat{s}_t|^2]$. Notice the proposed algorithm's adaptation to the jammer's step move.

the main lobe width, or say a locking range. Under such condition, the proof of convergence in mean and covariance is given, with some results regarding the stability condition over the step-size μ, and the steady state covariance of localization error. It is shown that the covariance decreases with higher SNR's and slower DOA variations; and that it depends on the sensors positioning, or equivalently on the array geometry. It is also proved that an optimal step-size can be selected to minimize the covariance (see Figure 2).

These theoretical results of the performance analysis are confirmed by simulations as expected intuitively [6,7].

4 SIMULATION RESULTS

In this section, we consider an equidistant linear array where the number of sensors is $m = 16$, $x_{i+1} - x_i = 1$, and the origin is at the array center. We take the values $\lambda = 2$, the source signal variance $\sigma_s^2 = 1$, and $\sigma_n^2 = 0.1$.

We study at the present the case of an immobile source with fixed DOA $\phi = 2\pi/5$.

To illustrate the efficiency of the presented algorithm, we run a simulation with a fixed step-size $\mu = 0.005$ in the presence of a step moving jammer initially at $\phi_j = \pi/5$, with a variance $\sigma_j^2 = 1$ (see Figure 3). Started with an initial DOA error $\delta\phi_0$ as high as $3\pi/40$, the presented algorithm is able to converge within a 200 sample time lag to the correct position. The simulations show that DOA errors are reduced to a very small range of about 10^{-2} degree (see Figure 4).

Figure 5 shows that the output SNR of the classical beamformer is quite equal to 0dB, while the presented algorithm allows an output SNR as high as the optimal performance 22dB$\simeq$ $(S/N) + 10\log_{10} m$. The resulting high resolution of localization actually enables the adapted classical beamformer to be brought back to its optimal performances. This is basically due to a very small desired source signal distortion of approximately -75dB (see Figure 6), combined with an optimum white noise reduction (12dB$\simeq$ $10\log_{10} m$ reduction) and an efficient jammer cancellation of approximatively 40dB by the GSC structure (see Figure 7). On the other hand, the classical beamformer considers the desired source as a jammer, and cancels it as expected while maintaining a unit gain in the initial corrupted direction (see Figures 5 and 7).

We run another simulation with a moving target in the same environment. Figure 8 shows that the algorithm is able to correct rapidly the steering error, and to track efficiently the DOA variations. DOA errors are reduced to about 10^{-1} degree and remain acceptable. The resolution of localization slightly degrades, but still enables the algorithm to properly extract the desired source signal and efficiently reduce the noise and the jammer as shown in Figure 9 (i.e. the total signal distortion is experimentally close to the optimal 22dB).

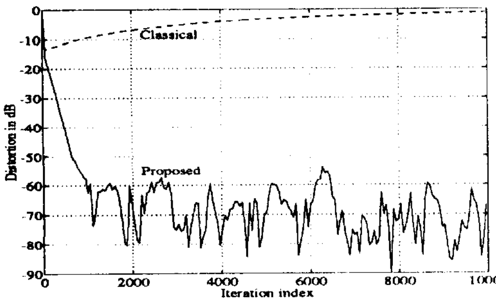

Figure 6: Source signal distortion $\left|1 - W_t^H F(\theta_t)\right|^2$ in the case of an immobile source and a step moving jammer.

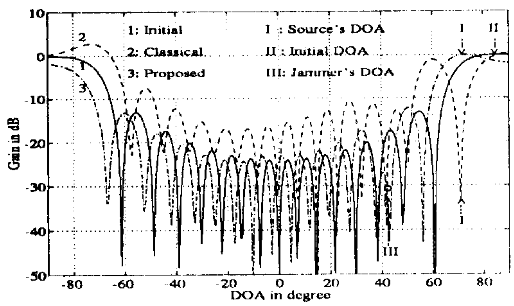

Figure 7: Array patterns at iterations $t = 0$ (1: array patterns are initially the same), and $t = 10000$ (2: array pattern of classical GSC, 3: array pattern of the proposed algorithm) in the case of immobile source and a step moving jammer, where $\sigma_J^2 = 1$.

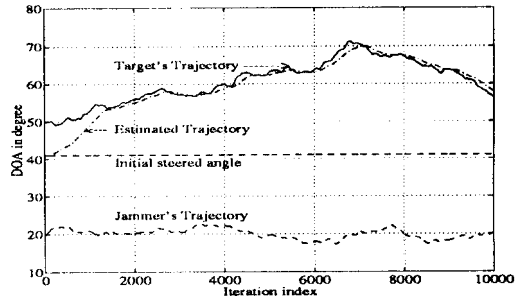

Figure 8: DOA trajectories in the case of a mobile source and a mobile jammer.

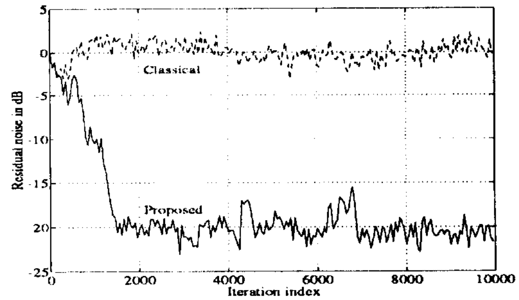

Figure 9: Total distortion for classical GSC and the proposed algorithm in the case of mobile sources. With the GSC algorithm, the desired signal is canceled after 2000 terations while the proposed algorithm gives an optimal performance, $10\log_{10} m + (S/N) = 22$dB.

5 CONCLUSION

We presented in this paper an algorithm for robust adaptive beamforming, based on a LMS-like correcting/tracking procedure. This algorithm avoids source signal cancellation by correcting the steering errors. It brings back an adapted classical beamformer to its optimal performance without any compromising loss in SNR, or any relative increase in source signal distortion. In addition, it proves to have high resolution capacity for localization, and an efficient behavior in moving target tracking. This algorithm has a complexity of order $O(m)$ where m is the number of sensors, and can be implemented in a very easy way.

At the present, the generalization of the presented algorithm to the multi-target tracking [8] and the wideband cases even for the near field propagation model is under study. Simultaneously, we are investigating the capacities of this algorithm to be adapted to calibration applications.

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