Closed-Form Error Analysis of Variable-Gain Multihop Systems in Nakagami-m Fading Channels

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***Abstract*—In this paper, infinite integrals involving the product of Bessel functions of different arguments are solved in closed-form. The obtained solutions form a framework for the error probability analysis of wireless amplify and forward (AF) systems with an arbitrary number of variable-gain relays operating over independent but not necessarily identical Nakagami-m fading channels. Here we show that the error probability can be described by generalized hypergeometric functions, namely, Gauss's and Lauricella's multivariate hypergeometric functions. This work represents a significant improvement over previous contributions and extends previous formulas pertaining to dual-hop transmissions over identical Nakagami-m fading channels. Numerical examples show an excellent match between simulation and theoretical results.**

***Index Terms*—Amplify and forward relaying, error probability, multi-hop wireless transmission, Nakagami fading, hypergeometric Lauricella function.**

I. INTRODUCTION

THE performance analysis of digital communication systems over fading environments has attracted a lot of research endeavor over the recent past. The derivation of closed-form expressions for key performance measures, namely the average error probability, is central to such research. Such closed-form results alleviate the need for Monte-Carlo simulations thereby enabling easy optimization of the overall system performance. In particular, numerous studies have been devoted to the performance analysis of multi-hop wireless systems over fading channels. Recently, the multi-hop concept has gained momentum in the context of cooperative wireless systems where relaying is used as a form of spatial diversity to overcome highly shadowed or deeply faded links [1]. The main idea is that communication is achieved by relaying the signal from the source to the destination via many intermittent terminals in between called relays. With relays that merely amplify and forward the incoming signal prior to relaying, AF transmission is the simplest and the cheapest to implement. Performance of such a system can be analyzed through the theoretical evaluation of certain performance metrics, namely, the average error probability. So far, despite many valuable contributions [2]-[12], the error analysis of the dual-hop case is still incomplete and there are no closed-form expressions for AF multihop systems with an arbitrary number of relays. In [2], [3], Hasna and Alouini presented an error probability analysis for dual-hop relaying system over identical Nakagami-m fading. Only recently have the authors in [4]-[6] considered the non-identical case for dual-hop transmission, but merely for integer values of the Nakagami-m fading parameter. Nevertheless, in practical scenarios, the m parameters often adopt non-integer values [13], which exclude the generality of [4], [5] and [6]. Other error analysis approaches bind the output SNR of the multi-hop relay link. For instance, it was upper bounded by the minimum value and the geometric mean of the SNRs at the hops, in [7] and [8], respectively. So far, there is no closed-form error probability analysis reported in the literature for multihop relaying systems with an arbitrary number of variable-gain relays over Nakagami-m fading. The most valuable contributions in this context can be found in [9], [10] and [12]. In reference [9], [10], the error probabilities of multi-hop multi-branch wireless communication systems are expressed as a double infinite integrals of the moment generating function (MGF) of the reciprocal of the instantaneous received SNR per branch. Therefore, in principle, exact evaluation of the error probability using the method in [9], [10] requires numerical computation of double integrals, which has been achieved by relying on the Gauss Quadrature Rule in [11]. In the theoretical approach presented in [12], the error probability performance of an AF multihop system is evaluated using single-integral expressions obtained in terms of the MGF of the reciprocal of the instantaneous received SNR. The obtained single integral formula is unfortunately not applicable to the multi-branch scenario, but it offers a more tractable solution than [9], [10] for the evaluation of the error probability of multihop transmissions. Motivated by the above considerations, in the present contribution, we derive for the first time this error probability in closed form. The obtained framework applies to the multihop scenario and can also be used to compute the exact formulas of the MGF of the end-to-end SNR in the multibranch multihop context. Our approach is inspired by [12] and generalizes both [2] and [4]. It turns out that the average error probability belongs to a special class of generalized hypergeometric series. These are the Lauricella's multivariate hypergeometric functions [14] of N variables $F_C^{(N)}$ for which some quite substantial mathematical apparatus is already known, like convergence properties and some analytical continuation formulas. Although the results are not expressible in common simple functions, they are at least expressible in this known type of functions, a significant improvement over previous results. In particular, new simple

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expressions for the error probability are derived in the dual-hop case, which is to date the most investigated one in the literature for its practical applications. The obtained formulas involve Appell's hypergeometric [14], Gauss' hypergeometric and Meijer's-G [15] functions.

The remainder of this paper is organized as follows. First, in section II, we derive closed-form solutions to the infinite integral containing the product of Bessel functions. Based on the obtained solutions, the error-rate performance for a variety of modulation schemes of AF multihop relaying systems with variable-gain relays is evaluated in section III. Section IV, derives new error probability results for the dual-hop case and shows how these results specialize for some less general fading scenarios of interest. Some numerical results are provided in section V. Finally, we conclude the paper while summarizing the main results in section VI.

II. Solution to the infinite Integral

This paper first addressees the calculation of the integrals

$$I(\nu,\mu,a,\Lambda,\beta)=\int_0^\infty s^\nu J_\mu\left(a\sqrt{s}\right)\prod_{i=1}^{N}K_{\lambda_i}\left(b_i\sqrt{s}\right)ds \quad (1)$$

where

$$\begin{aligned}\Lambda &= \{\lambda_1,...,\lambda_N\},\\ \beta &= \{b_1,...,b_N\},\\ &\Re(a),\mu>0,\\ &N>1.\end{aligned} \quad (2)$$

In (1), $J_\mu(\cdot)$ is the bessel function of the first kind and order μ [15, Eq. (8.440)], and $K_\lambda(\cdot)$ is the modified bessel function of the second kind and order λ [15, Eq. (8.485)]. The integral in (1) occurs in a number of wireless applications including the evaluation of the error probabilities and the ergodic capacity of wireless multihop systems. Yet, to the best of the author's knowledge, a closed-form solution for this integral is not known. Furthermore, a closed-form solution to the special case of (1) obtained when $N=2$, that is

$$\begin{aligned}&I_s\left(\nu,\mu,a,\lambda_1,\lambda_2,b_1,b_2\right)\\ &=\int_0^\infty s^\nu J_\mu\left(a\sqrt{s}\right)K_{\lambda_1}\left(b_1\sqrt{s}\right)K_{\lambda_2}\left(b_2\sqrt{s}\right)ds\end{aligned} \quad (3)$$

is not widely known and seems to have been found only when $b_1=b_2$ and $\lambda_1=\lambda_2$ (see [16]). None of references [16] or [15] gives a closed-form solution for either I or I_s . Nor does Mathematica give a closed-form solution for I or I_s. In this paper, we derive an explicit and general solution to (1) for any number $N>1$. Our analysis is only valid for real-valued non-integer λ_i. Nevertheless, practically, very similar results can be obtained at λ_i and $\lambda_i+\epsilon$ for sufficiently small ϵ values. By expressing the Bessel functions in terms of hypergeometric functions, namely, using

$$\begin{aligned}K_\lambda(z)&=2^{-\lambda-1}\Gamma(-\lambda)z^\lambda{}_0F_1(;1+\lambda,\frac{z^2}{4})+\\ &\quad 2^{\lambda-1}\Gamma(\lambda)z^{-\lambda}{}_0F_1(;1-\lambda,\frac{z^2}{4}),\end{aligned} \quad (4)$$

and

$$J_\mu(z)=\frac{1}{\Gamma(\mu+1)}\left(\frac{z}{2}\right)^\mu{}_0F_1\left(;1+\mu,-\frac{z^2}{4}\right), \quad (5)$$

where ${}_0F_1(a,b,z)$ denotes the confluent hypergeometric function [15], an alternative expression for I is shown to be given by

$$\begin{aligned}&I\left(\nu,\mu,a,\Lambda,\beta\right)=\frac{a^\mu}{2^\mu\Gamma(\mu+1)}\times\\ &\int_0^\infty s^{\nu+\frac{\mu}{2}}K_{\lambda_N}(b_N\sqrt{s}){}_0F_1\Big(;1+\mu,-\frac{a^2s}{4}\Big)\prod_{i=1}^{N-1}\left[V_i+W_i\right]ds,\end{aligned} \quad (6)$$

where

$$V_i=2^{\lambda_i-1}\Gamma\left(\lambda_i\right)\left(b_i\sqrt{s}\right)^{-\lambda_i}{}_0F_1\left(;1-\lambda_i,\frac{b_i^2s}{4}\right), \quad (7)$$

and

$$W_i=2^{-\lambda_i-1}\Gamma\left(-\lambda_i\right)\left(b_i\sqrt{s}\right)^{\lambda_i}{}_0F_1\left(;1+\lambda_i,\frac{b_i^2s}{4}\right). \quad (8)$$

In subsequent derivations, a more convenient expression for the product involved in (6) will be given using the following lemma. Let $V_1,...,V_N$ and $W_1,...,W_N$ denote two sets of N variables. Then, the following equality holds

Lemma 1:

$$\prod_{i=1}^{N-1}\left(V_i+W_i\right)=\sum_{i=0}^{N-1}\sum_{\tau(i,N-1)}\prod_{k=1}^{N-1}V_k^{i_k}W_k^{1-i_k}, \quad (9)$$

where $\tau(i,N-1)$ is the set of $N-1$-tuples such that $\tau(i,N-1)=\{(i_1,...,i_{N-1}):i_k\in\{0,1\},\sum_{k=1}^{N-1}i_k=i\}$. Indeed, by expanding the left side of (9), we can clearly notice that the i-th term can be viewed as $\binom{N-1}{i}$ combinations of the product of i_k variables V_k and $1-i_k$ variables W_k. Note that, when V_k and W_k are equal, (9) reduces to the Newton's binomial. Using the above equality, (6) will be given by

$$\begin{aligned}&I\left(\nu,\mu,a,\Lambda,\beta\right)=\frac{a^\mu}{2^{N-1+\mu}\Gamma(\mu+1)}\prod_{k=1}^{N-1}\left(\frac{b_k}{2}\right)^{\lambda_k}\\ &\sum_{i=0}^{N-1}\sum_{\tau(i,N-1)}\left\{\prod_{k=1}^{N-1}\left(\frac{2}{b_k}\right)^{2\lambda_ki_k}\Gamma\left(\lambda_k\right)^{i_k}\Gamma\left(-\lambda_k\right)^{1-i_k}\right\}M_{ij},\end{aligned} \quad (10)$$

where

$$\begin{aligned}M_{ij}=&\int_0^\infty s^\delta K_{\lambda_N}\left(b_N\sqrt{s}\right){}_0F_1\left(;1+\mu,-\frac{a^2s}{4}\right)\\ &\prod_{k=1}^{N-1}\left[{}_0F_1\left(;1-\lambda_k,\frac{b_k^2s}{4}\right)\right]^{i_k}\left[{}_0F_1\left(;1+\lambda_k,\frac{b_k^2s}{4}\right)\right]^{1-i_k}ds,\end{aligned} \quad (11)$$

whereby j stands for the j-th tuple of the set $\tau(i,N-1)$, and $\delta=\nu+\mu/2+\left(\sum_{k=1}^{N-1}\lambda_k\right)/2-\sum_{k=1}^{N-1}\lambda_ki_k$. The integral M_{ij} can be solved by expressing the Bessel K integrand in terms of Meijer's G-functions [15, Eq. (9.301)], namely, using $K_{\lambda_N}\left(b_N\sqrt{s}\right)=G_{0,2}^{2,0}\left(b_N^2s/4\,\middle|\,\begin{matrix}-\\ \lambda_N/2,-\lambda_N/2\end{matrix}\right)/2$. Then, considering the change of variable $z=b_N^2s/4$ yields

$$\begin{aligned}M_{ij}=&\left(\frac{4}{b_N^2}\right)^{\delta+1}\int_0^\infty\frac{1}{2z}G_{0,2}^{2,0}\left(z\,\middle|\,\begin{matrix}-\\ \xi,\xi-\lambda_N\end{matrix}\right){}_0F_1\left(;1+\mu,-\frac{a^2z}{b_N^2}\right)\\ &\prod_{k=1}^{N-1}\left[{}_0F_1\left(;1-\lambda_k,\frac{b_k^2z}{b_N^2}\right)\right]^{i_k}\left[{}_0F_1\left(;1+\lambda_k,\frac{b_k^2z}{b_N^2}\right)\right]^{1-i_k}dz,\end{aligned} \quad (12)$$

where $\xi = \delta + \lambda_N/2 + 1$. By further noticing that a single integral representation for the multivariate Lauricella hypergeometric function $F_C^{(n)}(a, b; c_1, ..., c_n; x_1, ..., x_n)$ is given by [14]

$$F_C^{(n)}(a, b; c_1, ..., c_n; x_1, ..., x_n) = \frac{1}{\Gamma(a)\Gamma(b)} \int_0^\infty G_{0,2}^{2,0}(t\,|a, b) \left(\prod_{k=1}^n {}_0F_1(; c_k, x_k t) \right) \frac{dt}{t}, \quad (13)$$

one can easily recognize that $I(\nu, \mu, a, \Lambda, \beta)$ can be expressed in terms of (13) as shown in (14) on the top of the next page. Using the multiples series representation of the Lauricella's hypergeometric function [14, Eq. (A.1.4)]

$$F_C^{(n)}(a, b; c_1, ..., c_n; x_1, ..., x_n) = \sum_{m_1=0}^{\infty} ... \sum_{m_n=0}^{\infty} \frac{(a)_{m_1+...m_n}(b_1)_{m_1+...m_n}}{(c_1)_{m_1}...(c_n)_{m_n}} \frac{x_1^{m_1}...x_n^{m_n}}{m_1!...m_n}; \quad \sqrt{|x_1|} + ... + \sqrt{|x_n|} < 1, \quad (15)$$

where $(a)_k = \Gamma(a+k)/\Gamma(k)$ denotes the Pochhammer symbol, it can be noted that the convergence of the total sum involved in (14) is governed by the threshold condition

$$|a^2| < \left(\sqrt{|b_1^2|} + \sqrt{|b_2^2|} + ... + \sqrt{|b_N^2|} \right)^2. \quad (16)$$

Hopefully, (14) can be extended to the complementary region of (16) by applying the analytical continuation formula of the Lauricella's hypergeometric function F_C [14]. Indeed, a Lauricella function in the argument z_i can be analytically continued to a sum of two Lauricella functions in the argument $z_i = x_i/x_n$ for $i = 1, ..., n-1$ and $z_n = 1/x_n$ according to (17). Although the derived analytic continuation of (14) is not shown here for lack of space, the former ensures that for all $a > 0$ and all admissible values of $\{b_i\}_{i=1}^N$, the absolute convergence of the series (14) is always guaranteed.

We now simplify (14) in an important case corresponding to $N = 2$. In such a setting, I in (14) reduces to I_s obtained as in (18), where $\xi_0 = \nu + \mu/2 + \lambda_1 + \lambda_2/2 + 1$, $\xi_1 = \nu + \mu/2 + \lambda_2 - \lambda_1/2 + 1$ and $F_4 = F_C^{(2)}$ is the fourth Appel hypergeometric function which is defined as

$$F_4[\alpha, \beta; \gamma, \gamma', x, y] = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(\alpha)_{j+k}(\beta)_{j+k}}{(\gamma)_j(\gamma')_k} \frac{x^j}{j!} \frac{y^k}{k!}, \quad |x|^{1/2} + |y|^{1/2} < 1. \quad (19)$$

The Appell functions [14] are well known, and numerical routines for their exact computation are available in packages such as Mathematica.

A. A Simplified Special Case of I

In the following, a simplified version of I is obtained when $\lambda_i, i = 1, ..., N$ are constrained to take integers plus one-half values, i.e., $\lambda_i = n_i + 1/2$, where n_i is an integer. In such a setting, we have

$$K_{\lambda_i}(b_i\sqrt{s}) = \sqrt{\frac{\pi}{2b_i}} e^{-b_i\sqrt{s}} \sum_{p=0}^{n_i} \frac{\Gamma(n_i+1+p)}{\Gamma(n_i+1-p)\Gamma(p+1)} (2b_i)^{-p} (\sqrt{s})^{-p-\frac{1}{2}}. \quad (20)$$

By inserting (20) into (1), I can be written as

$$I(\nu, \mu, a, \Lambda, \beta) = \frac{\frac{\pi}{2}^{N/2}}{\prod_{i=1}^N b_i} \int_0^\infty s^{\nu-\frac{N}{4}} J_\mu(a\sqrt{s}) e^{-\sum_{i=1}^N b_i\sqrt{s}} \prod_{i=1}^N \sum_{p=0}^{n_i} \Upsilon_{i,p} (\sqrt{s})^{-p} ds, \quad (21)$$

where

$$\Upsilon_{i,p} = \frac{\Gamma(n_i+1+p)(2b_i)^{-p}}{\Gamma(n_i+1-p)\Gamma(p+1)}. \quad (22)$$

The expression above encompasses the product of a set of polynomials of $x = \sqrt{s}$. It is well known that the product of a set of polynomials is another polynomial whose degree is the sum of the degrees of the polynomials in the set and the coefficient of x^p in the resulting polynomial is the sum of terms of the form $\prod_{k=1}^N b_{k,p_k}$ such that $\sum_{k=1}^N p_k = p$. Consequently, we obtain

Lemma 2:

$$\prod_{i=1}^N \left(\sum_{p=0}^{n_i} \Upsilon_{i,p} \xi^{-p} \right) = \sum_{p=0}^{n_\Sigma} \left(\sum_{w(p,N)} \prod_{i=1}^N \Upsilon_{i,p_i} \right) \xi^{-p}, \quad (23)$$

where $n_\Sigma = \sum_{i=1}^N n_i$ and $w(p, N)$ is the set of N-tuples such that $w(p, N) = \left\{ (p_1, ..., p_N) : p_k \in \{0, 1, ..., n_k\}, \sum_{k=1}^N p_k = p \right\}$. Using (23) and performing some algebraic manipulations, it follows that (21) reduces to

$$I(\nu, \mu, a, \Lambda, \beta) = \frac{\frac{\pi}{2}^{N/2}}{\prod_{i=1}^N b_i} \sum_{p=0}^{n_\Sigma} \left(\sum_{w(p,N)} \prod_{i=1}^N \Upsilon_{i,p_i} \right) \int_0^\infty s^{\nu-\frac{N}{4}-\frac{p}{2}} e^{-\sum_{i=1}^N b_i\sqrt{s}} J_\mu(a\sqrt{s}) ds. \quad (24)$$

Then, with the help of [15, Eq. (6.621)], the integral in (24) can be derived in closed form according to (25), where $F(a, b; c; x)$ is the Gauss hypergeometric function [15, Eq. (9.10)].

B. A Simplified Special Case of I_s

A special case of I_s corresponds to $b_1 = b_2 = b$ and $\lambda_1 \neq \lambda_2$. In this case, making use of [15, Eq. (7.821.1)] along with the identity

$$K_{\lambda_1}(b\sqrt{s}) K_{\lambda_2}(b\sqrt{s}) = \frac{\sqrt{\pi}}{2} G_{2,4}^{4,0}\left(b^2 s \,\middle|\, 0, \tfrac{\lambda_1+\lambda_2}{4}, \tfrac{\lambda_1-\lambda_2}{2}, \tfrac{\lambda_2-\lambda_1}{2}, -\tfrac{\lambda_1+\lambda_2}{2} \right), \quad (26)$$

a closed form of I_s is shown to be given by

$$I_s(\nu, \mu, a, \lambda_1, \lambda_2, b, b) = \frac{\sqrt{\pi}}{2} \left(\frac{4}{a^2} \right)^{\nu+1} G_{4,4}^{4,1}\left(\frac{4b^2}{a^2} \,\middle|\, \begin{matrix} -\nu - \frac{\mu}{2}, 0, \frac{1}{2}, -\nu + \frac{\mu}{2} \\ \frac{\lambda_1+\lambda_2}{2}, \frac{\lambda_1-\lambda_2}{2}, \frac{\lambda_2-\lambda_1}{2}, -\frac{\lambda_1+\lambda_2}{2} \end{matrix} \right). \quad (27)$$

$$I(\nu,\mu,a,\Lambda,\beta) = \frac{\left(\frac{a}{b_N}\right)^{\mu}\prod_{k=1}^{N-1}\left(\frac{b_k}{b_N}\right)^{\lambda_k}}{2^N\Gamma(\mu+1)}\left(\frac{4}{b_N^2}\right)^{\nu+1}\sum_{i=0}^{N-1}\sum_{\tau(i,N-1)}\Gamma(\xi)\Gamma(\xi-\lambda_N)\left\{\prod_{k=1}^{N-1}\left(\frac{b_N}{b_k}\right)^{2\lambda_k i_k}\Gamma(\lambda_k)^{i_k}\Gamma(-\lambda_k)^{1-i_k}\right\}$$
$$F_C^{(N)}\left(\xi,\xi-\lambda_N,1+\mu,\underbrace{1-\lambda_1}_{i_1},\underbrace{1+\lambda_1}_{1-i_1},...,\underbrace{1-\lambda_{N-1}}_{i_{N-1}},\underbrace{1+\lambda_{N-1}}_{1-i_{N-1}};-\frac{a^2}{b_N^2},\frac{b_1^2}{b_N^2},...,\frac{b_{N-1}^2}{b_N^2}\right). \tag{14}$$

$$F_C^{(n)}(a,b;c_1,...,c_n;x_1,...,x_n) = \frac{\Gamma(c_n)\Gamma(b-a)}{\Gamma(b)\Gamma(c_n-a)}(-x_n)^{-a}F_C^{(n)}(a,1+a-c_n;c_1,...,c_{n-1},1-b+a;z_1,...,z_n)+$$
$$\frac{\Gamma(c_n)\Gamma(a-b)}{\Gamma(a)\Gamma(c_n-b)}(-x_n)^{-b}F_C^{(n)}(b,1+b-c_n;c_1,...,c_{n-1},1-a+b;z_1,...,z_n). \tag{17}$$

$$I_s(\nu,\mu,a,\lambda_1,\lambda_2,b_1,b_2) = \frac{\left(\frac{a}{b_2}\right)^{\mu}\left(\frac{b_1}{b_2}\right)^{\lambda_1}\left(\frac{4}{b_2^2}\right)^{\nu+1}}{4\Gamma(\mu+1)}\left\{\Gamma(\xi_0)\Gamma(\xi_0-\lambda_2)\Gamma(-\lambda_1)F_4\left(\xi_0,\xi_0-\lambda_2,1+\mu,1+\lambda_1,-\frac{a^2}{b_2^2},\frac{b_1^2}{b_2^2}\right)+\right.$$
$$\left.\left(\frac{b_2}{b_1}\right)^{2\lambda_1}\Gamma(\lambda_1)\Gamma(\xi_1)\Gamma(\xi_1-\lambda_2)F_4\left(\xi_1,\xi_1-\lambda_2,1+\mu,1-\lambda_1,-\frac{a^2}{b_2^2},\frac{b_1^2}{b_2^2}\right)\right\}, \tag{18}$$

$$I(\nu,\mu,a,\Lambda,\beta) = \frac{\frac{\pi}{2}^{N/2}}{\prod_{i=1}^N b_i}\sum_{p=0}^{n_\Sigma}\left(\sum_{w(p,N)}\prod_{i=1}^N\Upsilon_{i,p_i}\right)\frac{\left(\frac{a}{2\sum_{i=1}^N b_i}\right)^{\mu}\Gamma\left(\mu+2\nu-\frac{N}{2}-p+2\right)}{\left(\sum_{i=1}^N b_i\right)^{2\nu-\frac{N}{2}-p+2}\Gamma(\mu+1)}$$
$$F\left(\nu+\frac{\mu-\frac{N}{2}-p}{2}+1,\nu+\frac{\mu-\frac{N}{2}-p+3}{2},\mu+1,-\frac{a^2}{(\sum_{i=1}^N b_i)^2}\right), \tag{25}$$

III. Application: Error Probabilities for Amplify and Forward Multi-hop Relaying Systems

Let us consider an N-hop wireless communication system where a source S communicates with a destination D trough $N-1$ intermediate terminals called relays. In the k-th time slot, the k-th relay R_k receives the signal from the immediately preceding relay and processes it by amplifying and forwarding it to the next hop R_{k+1}. Denoting by y_k the signal received by R_k, we have

$$y_k = v_k x_{k-1} + n_k, \quad k=1,...,N, \tag{28}$$

where v_k is the fading gain of the channel between terminals R_{k-1} and R_k, n_k denotes the additive white Gaussian noise received at the k-th terminal with power N_{0_k}, and x_k denotes the transmitted signal from the $(k-1)$-th relay given by

$$x_k = A_k y_k, \quad k=1,...,N-1, \tag{29}$$

where A_k is the amplification gain of the k-th terminal. The end-to-end instantaneous received SNR is given by [8] as

$$\gamma = \frac{\prod_{k=1}^N A_k^2\gamma_k}{\sum_{k=1}^N\prod_{j=k+1}^N A_j^2\gamma_j}, \tag{30}$$

where $\gamma_k = P_k|v_k|^2/N_{0_k}$ denotes the instantaneous received SNR over the channel between terminals R_{k-1} and R_k in which P_k is the transmitter power from terminal $R_{k-1}, k = 1,...,N$. As seen in (30), the instantaneous received SNR in an AF multihop transmission system depends on the relay amplification gains and the fading channel gains. AF relays can be classified into two categories, namely, variable-gain relays and fixed-gain relays. In the first case, the relay uses the channel information of the preceding hop to control the relay gain. In contrast, systems with blind relays use amplifiers with fixed gains resulting in a signal with variable power at the relay output. In this paper, we consider the first type of amplification gain which is generally chosen as

$$A_k = \sqrt{\frac{P_k}{P_{k-1}|v_k|^2}}, \quad k=1,...,N-1. \tag{31}$$

For this amplification gain, the relay amplifies its received signal, regardless of the received noise power[1]. Plugging this gain expression into (30), the end-to-end SNR is then given by

$$\gamma = \left[\sum_{l=1}^N\frac{1}{\gamma_l}\right]^{-1}. \tag{32}$$

Since the reciprocal of the end-to-end instantaneous received SNR γ is the sum of the inverse of the individual per-hop

[1]Since the amplification gain in systems with variable-gain relays is a function of the channel state information (CSI), variable-gain relays are also referred to as CSI-assisted relays [8], [9]

SNRs [8], then, the MGF of $\gamma^V = \frac{1}{\gamma}$ is the product of the individual MGFs pertaining to the different hops, thus implying

$$M_{\gamma^V}(s) = \prod_{l=1}^{N} M_{\frac{1}{\gamma_l}}(s), \tag{33}$$

where $M_{\frac{1}{\gamma_l}}(s)$ is the MGF of the SNR on the l-th hop. In Nakagami-m fading, $M_{\gamma^V}(s)$ is given by [2]

$$M_{\gamma^V}(s) = 2^N \left(\prod_{l=1}^{N} \frac{\left(\frac{m_l}{\bar{\gamma}_l}\right)^{m_l/2}}{\Gamma(m_l)} \right) s^{\frac{m_\Sigma}{2}} \prod_{l=1}^{N} K_{m_l}\left(2\sqrt{\frac{s m_l}{\bar{\gamma}_l}}\right), \tag{34}$$

whereby $\bar{\gamma}_i = \mathrm{E}(\gamma_i)$, with $\mathrm{E}(\cdot)$ denoting expectation, $m_i \geq 1/2$ is the Nakagami-m factor of the i-th hop and $m_\Sigma = \sum_{l=1}^{N} m_l$ is defined for the sake of notational convenience. In the sequel, considering AF multihop variable-gain relaying systems with operation over Nakagami-m fading channels, simple closed-form expressions for the error probabilities of AF multi-hop systems with variable gain relays are derived. Different modulation schemes are therefore considered, including binary and arbitrary rectangular M-QAM modulations.

A. Binary Modulations

For different binary modulation schemes, the bit error probability was recently derived in [12, Eq. (4b)] as a single integral form given by

$$P_e = \frac{1}{2} - \frac{\tau^{\eta/2}}{2\Gamma(\eta)} \int_0^\infty s^{\frac{\eta}{2}-1} J_\eta(2\sqrt{\tau s}) M_{\gamma^V}(s) ds, \tag{35}$$

where the parameters τ and η depend on the type of modulation detection scheme given in [17, Tab. 8.1] and $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [15, Eq. (8.350.2)]. By substituting appropriately (34) in (35) and using (1), P_e can be obtained as follows

$$\begin{aligned} P_e &= \frac{1}{2} - \frac{2^{N-1}\tau^{\eta/2}}{\Gamma(\eta)} \left(\prod_{l=1}^{N} \frac{(\frac{m_l}{\bar{\gamma}_l})^{m_l/2}}{\Gamma(m_l)} \right) \times \\ &\quad I\left(\frac{m_\Sigma + \eta}{2} - 1, \eta, 2\sqrt{\tau}, \Lambda, \beta\right), \end{aligned} \tag{36}$$

where

$$\begin{aligned} \Lambda &= \{m_1, m_2, ..., m_N\}, \\ \beta &= \left\{2\sqrt{\frac{m_1}{\bar{\gamma}_1}}, ..., 2\sqrt{\frac{m_N}{\bar{\gamma}_N}}\right\}. \end{aligned} \tag{37}$$

By properly substituting $I(\cdot,\cdot,\cdot,\cdot,\cdot)$ by its expression in (14), then after further manipulations, a closed-form expression of the error probability is obtained according to (38), where $\xi_\eta = m_\Sigma + \eta - \sum_{k=1}^{N-1} m_k i_k$. Note that equation (38) is a new closed-form expression for the bit error probability of binary modulations in AF relaying systems with variable gain relays under non-identical Nakagami-m fading. The values of the parameters η and τ are, for example, $(\eta, \tau) = (0.5, 1)$ for BPSK and $(\eta, \tau) = (0.5, 0.5)$ for BFSK modulation. Moreover, using the alternative expression of I when $m_i, i = 1, .., N$ are multiples of an integer plus one half, a simpler expression of P_e is obtained from (25) according to the formulas shown in (39), where $\Upsilon'_{i,p_i} = (\Gamma(n_i + 1 + p_i)/\Gamma(n_i + 1 - p_i)\Gamma(p + 1)) \left(2\sqrt{\frac{m_i}{\bar{\gamma}_i}}\right)^{-p_i}$. Notice that (39) is expressed in terms of the Gauss hypergeometric function $F(a; b, c; z)$, which is widely available.

B. M-Ary Modulations

An arbitrary rectangular $M_I \times M_J$ QAM signal constellation is assumed to be formed by drawing the in-phase and quadrature components from two independent M-ary pulse amplitude modulation (PAM) schemes, M_I-ary PAM and M_J-ary PAM, respectively. The symbol error probability of the ensuing M-ary rectangular QAM ($M = M_I M_J$) is [17]

$$\begin{aligned} P_e =& 2\left(1 - \frac{1}{M_I}\right)\mathrm{E}\left(Q\left(A\sqrt{\gamma}\right)\right) + 2\left(1 - \frac{1}{M_J}\right)\mathrm{E}\left(Q\left(B\sqrt{\gamma}\right)\right) \\ &- 4\left(1 - \frac{1}{M_I}\right)\left(1 - \frac{1}{M_J}\right)\mathrm{E}\left(Q\left(A\sqrt{\gamma}\right)Q\left(B\sqrt{\gamma}\right)\right), \end{aligned} \tag{40}$$

where $A = \sqrt{6/\left((M_I^2 - 1) + (M_J^2 - 1)\zeta\right)}$ and $B = \sqrt{\zeta}A$ where where ζ denotes the squared quadrature to in-phase distance ratio. It is seen that the evaluation of (40) involves the evaluations of two expectation forms, namely, the expectation of the Gaussian-Q function and the expectation of the product of two Gaussian-Q functions with different arguments. On the basis of the prominent results presented in [12, Eq. (6c)][2] the expectation of the Gaussian-Q function is obtained according to

$$\mathrm{E}\left(Q\left(A\sqrt{\gamma}\right)\right) = \frac{1}{2} - \frac{1}{2\pi} \int_0^\infty \frac{\sin\left(A\sqrt{2s}\right)}{s} M_{\gamma^V}(s) ds. \tag{41}$$

After substituting $M_{\gamma^V}(s)$ by its expression in (34) and making use of the identity

$$\sin\left(A\sqrt{2s}\right) = \sqrt{\frac{\pi A \sqrt{s}}{\sqrt{2}}} J_{\frac{1}{2}}\left(A\sqrt{2s}\right), \tag{42}$$

the expectation in (41) can be evaluated in closed form using (1) according to

$$\begin{aligned} \mathrm{E}\left(Q\left(A\sqrt{\gamma}\right)\right) &= \frac{1}{2} - \frac{2^{N-1}\sqrt{A}}{\sqrt{\pi\sqrt{2}}} \left(\prod_{l=1}^{N} \frac{\left(\frac{m_l}{\bar{\gamma}_l}\right)^{m_l/2}}{\Gamma(m_l)} \right) \\ &\quad I\left(\frac{m_\Sigma}{2} - \frac{3}{4}, \frac{1}{2}, A\sqrt{2}, \Lambda, \beta\right), \end{aligned} \tag{43}$$

where $I(\cdot,\cdot,\cdot,\cdot,\cdot)$ is obtained from (14) where Λ, β are given in (37). By properly substituting I by its expression, we obtain (44) shown on the top of the next page, where $\xi_q = m_\Sigma + \frac{1}{2} - \sum_{k=1}^{N-1} m_k i_k$.

By substituting A with B in (44), we obtain the closed-from solution for the second expectation with argument B in (40). Nevertheless, there are some challenges in the evaluation of the expectation of the product of two Gaussian Q-functions with different arguments, a process which involves the integration of the product of two Gaussian Q-functions with different arguments. In [18], the authors sidestepped this hurdle by introducing a simple and accurate exponential approximation

[2]It should be stressed that [12, Eq. (6c)] has a typo. It should read as in (41).

$$P_e = \frac{1}{2} - \frac{\tau^\eta \left(\prod_{l=1}^{N} \frac{(\frac{m_l \bar{\gamma_N}}{m_N \bar{\gamma_l}})^{m_l}}{\Gamma(m_l)}\right)}{2\Gamma(\eta)\Gamma(\eta+1)} \sum_{i=0}^{N-1} \sum_{\tau(i,N-1)} \Gamma(\xi_\eta)\Gamma(\xi_\eta - m_N) \left\{ \prod_{k=1}^{N-1} \left(\frac{m_N \bar{\gamma_k}}{m_k \bar{\gamma_N}}\right)^{m_k i_k} \Gamma(m_k)^{i_k} \Gamma(-m_k)^{1-i_k} \right\}$$
$$F_C^{(N)} \left(\xi_\eta, \xi_\eta - m_N, 1+\eta, \underbrace{1-m_1}_{i_1}, \underbrace{1+m_1}_{1-i_1}, ..., \underbrace{1-m_{N-1}}_{i_{N-1}}, \underbrace{1+m_{N-1}}_{1-i_{N-1}}; -\frac{\tau \bar{\gamma_N}}{m_N}, \frac{m_1 \bar{\gamma_N}}{m_N \bar{\gamma_1}}, ..., \frac{m_{N-1}\bar{\gamma_N}}{m_N \bar{\gamma_{N-1}}} \right). \quad (38)$$

$$P_e = \frac{1}{2} - \frac{\tau^\eta \pi^{N/2}}{2^{m_\Sigma + 2\eta + 1}\Gamma(\eta)\Gamma(\eta+1)} \left(\prod_{i=1}^{N} \frac{\left(\frac{m_i}{\bar{\gamma_i}}\right)^{\frac{m_i - 1}{2}}}{\Gamma(m_i)} \right) \sum_{p=0}^{n_\Sigma} \left(\sum_{w(p,N)} \prod_{i=1}^{N} \Upsilon'_{i,p_i} \right) \frac{\Gamma(2\eta + n_\Sigma - p)}{\left(\sum_{i=1}^{N} \sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^{2\eta + n_\Sigma - p}}$$
$$F\left(\eta + \frac{n_\Sigma - p}{2}, \eta + \frac{n_\Sigma - p + 1}{2}, \eta + 1, -\frac{\tau}{\left(\sum_{i=1}^{N} \sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^2} \right), \quad (39)$$

$$\mathrm{E}(Q(A\sqrt{\gamma})) = \frac{1}{2} - \frac{A \left(\prod_{k=1}^{N} \frac{\left(\frac{m_k \bar{\gamma_N}}{m_N \bar{\gamma_k}}\right)^{m_k}}{\Gamma(m_k)}\right)}{\pi\sqrt{2}} \sum_{i=0}^{N-1} \sum_{\tau(i,N-1)} \Gamma(\xi_q)\Gamma(\xi_q - m_N) \left\{ \prod_{k=1}^{N-1} \left(\frac{m_N \bar{\gamma_k}}{m_k \bar{\gamma_N}}\right)^{m_k i_k} \Gamma(m_k)^{i_k} \Gamma(-m_k)^{1-i_k} \right\}$$
$$F_C^{(N)} \left(\xi_q, \xi_q - m_N, \frac{3}{2}, \underbrace{1-m_1}_{i_1}, \underbrace{1+m_1}_{1-i_1}, ..., \underbrace{1-m_{N-1}}_{i_{N-1}}, \underbrace{1+m_{N-1}}_{1-i_{N-1}}; -\frac{A^2 \bar{\gamma_N}}{2m_N}, \frac{m_1 \bar{\gamma_N}}{m_N \bar{\gamma_1}}, ..., \frac{m_{N-1}\bar{\gamma_N}}{m_N \bar{\gamma_{N-1}}} \right), \quad (44)$$

of the product of two Gaussian Q-functions with different arguments given by

$$Q(A\sqrt{y})Q(B\sqrt{y}) \simeq \sum_{i,j=1}^{2} c_i c_j e^{-\left(A^2 b_i + B^2 b_j\right)y}, \quad (45)$$

where $\{c_i\} = \{\frac{1}{12}, \frac{1}{4}\}$ and $\{b_i\} = \{\frac{1}{2}, \frac{2}{3}\}$. The accuracy of the above tight upper bound was also discussed in [18]. Based on the above approximation and proceeding by using the McLauren series of $e^{(\cdot)}$ given in [15, Eq. (1.211.1)], it can be easily shown, using [15, Eq. (8.402)], that the expectation of the product of two Gaussian Q-functions with different arguments can be expressed as

$$\mathrm{E}\left(Q(A\sqrt{y})Q(B\sqrt{y})\right) = -\sum_{i,j=1}^{2} c_i c_j \sqrt{d_{ij}} \int_0^\infty M_{\gamma^V}(s) \frac{J_1\left(2\sqrt{d_{ij}s}\right)}{\sqrt{s}} ds, \quad (46)$$

where $d_{ij} = A^2 b_i + B^2 b_j$. Hence, a closed-form expression of (46) is obtained, using (1), as

$$\mathrm{E}\left(Q(A\sqrt{y})Q(B\sqrt{y})\right) \simeq -2^N \left(\prod_{l=1}^{N} \frac{\left(\frac{m_l}{\bar{\gamma_l}}\right)^{m_l/2}}{\Gamma(m_l)} \right) \sum_{i,j=1}^{2} c_i c_j \sqrt{d_{ij}} \, I\left(\frac{m_\Sigma - 1}{2}, 1, 2\sqrt{d_{ij}}, \Lambda, \beta\right). \quad (47)$$

By properly substituting I by its expression in (14), (47) can be evaluated as in (48) shown on the top of the next page. It is observed that employing (45) allows to evaluate (46) in closed form. Moreover, owing to the structure of (45), the obtained formulas, i.e., (44) and (48), have similar structures. This fact facilitates further numerical calculations. In the special fading condition corresponding to the case when the fading parameters of the different hops $m_i, i = 1, ..., N$ are odd multiples of one half, alternative simpler expressions of (44) and (48) could be obtained. In such a setting, applying (25) to (41) and (46), respectively, yields (49) and (50) shown on the next page.

Then, properly substituting (49) and (50) into (40) yields the error probability expression for multi-hop AF transmissions using an arbitrary M-QAM modulation over Nakagami-m fading with an odd multiple of one half fading parameter as given in (51). For an arbitrary $M_I \times M_J$ rectangular QAM constellation with Gray encoding, the error probability evaluation only involves taking the expectation of the Gaussian-Q function [19, Eq. (22)] according to

$$P_e = \frac{2}{\log_2(M_I \cdot M_J)} \left(\frac{1}{M_I} \sum_{k=1}^{\log_2(M_I)} \sum_{i=0}^{(1-2^{-k})M_I - 1} P(i,k)\mathrm{E}\left(Q\left(\sqrt{2w_i\gamma}\right)\right) + \frac{1}{M_J} \sum_{p=1}^{\log_2(M_J)} \sum_{j=0}^{(1-2^{-p})M_J - 1} P(j,p)\mathrm{E}\left(Q\left(\sqrt{2w_j\gamma}\right)\right) \right), \quad (52)$$

where $w_k = (2k+1)^2 \left(3\log_2(M_I \cdot M_J) / \left(M_I^2 + M_J^2 - 2\right)\right)$ and $P(i,j)$ are expressions of M_I and M_J given in [19]. In this case, using (43), we immediately obtain a closed-form expression for the error probability as given in (53). An

$$\mathrm{E}\left(Q(A\sqrt{y})Q(B\sqrt{y})\right)\simeq-\left(\prod_{k=1}^{N}\frac{\left(\frac{m_k\bar{\gamma_N}}{m_N\bar{\gamma_k}}\right)^{m_k}}{\Gamma(m_k)}\right)\sum_{i=0}^{N-1}\sum_{\tau(i,N-1)}\Gamma\left(\xi_q+\frac{1}{2}\right)\Gamma\left(\xi_q-m_N+\frac{1}{2}\right)\left\{\prod_{k=1}^{N-1}\left(\frac{m_N\bar{\gamma_k}}{m_k\bar{\gamma_N}}\right)^{m_k i_k}\Gamma(m_k)^{i_k}\Gamma(-m_k)^{1-i_k}\right\}$$
$$\sum_{i,j=1}^{2}c_ic_jd_{ij}F_C^{(N)}\left(\xi_q+\frac{1}{2},\xi_q-m_N+\frac{1}{2},2,\underbrace{1-m_1}_{i_1},\underbrace{1+m_1}_{1-i_1},...,\underbrace{1-m_{N-1}}_{i_{N-1}},\underbrace{1+m_{N-1}}_{1-i_{N-1}};-\frac{d_{ij}\bar{\gamma_N}}{m_N},\frac{m_1\bar{\gamma_N}}{m_N\bar{\gamma_1}},...,\frac{m_{N-1}\bar{\gamma_N}}{m_N\bar{\gamma_{N-1}}}\right). \quad (48)$$

$$\begin{aligned}\mathrm{E}(Q(A\sqrt{\gamma})) &= \frac{1}{2}-\frac{A\pi^{\frac{N}{2}-1}}{2^{m_\Sigma+\frac{1}{2}}}\left(\prod_{i=1}^{N}\frac{\left(\frac{m_i}{\bar{\gamma_i}}\right)^{\frac{m_i-1}{2}}}{\Gamma(m_i)}\right)\sum_{p=0}^{n_\Sigma}\left(\sum_{w(p,N)}\prod_{i=1}^{N}\Upsilon'_{i,p_i}\right)\frac{\Gamma(n_\Sigma-p+\frac{3}{2})}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^{n_\Sigma-p+\frac{3}{2}}}\\ &\quad F\left(\frac{n_\Sigma-p+\frac{3}{2}}{2},\frac{n_\Sigma-p+\frac{5}{2}}{2},\frac{3}{2},-\frac{A^2}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^2}\right). \end{aligned} \quad (49)$$

$$\begin{aligned}\mathrm{E}\left(Q(A\sqrt{y})Q(B\sqrt{y})\right) &\simeq -\frac{\pi^{\frac{N}{2}}}{2^{m_\Sigma+2}}\left(\prod_{i=1}^{N}\frac{\left(\frac{m_i}{\bar{\gamma_i}}\right)^{\frac{m_i-1}{2}}}{\Gamma(m_i)}\right)\sum_{i,j=1}^{2}c_ic_jd_{ij}\sum_{p=0}^{n_\Sigma}\left(\sum_{w(p,N)}\prod_{i=1}^{N}\Upsilon'_{i,p_i}\right)\\ &\quad \frac{\Gamma\left(n_\Sigma-p+2\right)}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^{n_\Sigma-p+2}}F\left(\frac{n_\Sigma-p}{2}+1,\frac{n_\Sigma-p+1}{2}+1,2,-\frac{d_{ij}}{(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}})^2}\right). \end{aligned} \quad (50)$$

$$\begin{aligned}P_e &= \left(1-\frac{1}{M_I}\right)+\left(1-\frac{1}{M_J}\right)-\frac{\pi^{\frac{N}{2}-1}}{2^{m_\Sigma+\frac{1}{2}}}\left(\prod_{i=1}^{N}\frac{\left(\frac{m_i}{\bar{\gamma_i}}\right)^{\frac{m_i-1}{2}}}{\Gamma(m_i)}\right)\sum_{p=0}^{n_\Sigma}\left(\sum_{w(p,N)}\prod_{i=1}^{N}\Upsilon'_{i,p_i}\right)\left\{\left(1-\frac{1}{M_I}\right)\frac{A\Gamma\left(n_\Sigma-p+\frac{3}{2}\right)}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^{n_\Sigma-p+\frac{3}{2}}}\right.\\ &\quad F\left(\frac{n_\Sigma-p+\frac{3}{2}}{2},\frac{n_\Sigma-p+\frac{5}{2}}{2},\frac{3}{2},-\frac{A^2}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^2}\right)+\left(1-\frac{1}{M_J}\right)\frac{B\Gamma\left(n_\Sigma-p+\frac{3}{2}\right)}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^{n_\Sigma-p+\frac{3}{2}}}\\ &\quad F\left(\frac{n_\Sigma-p+\frac{3}{2}}{2},\frac{n_\Sigma-p+\frac{5}{2}}{2},\frac{3}{2},-\frac{B^2}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^2}\right)-\sqrt{2}\pi\left(1-\frac{1}{M_I}\right)\left(1-\frac{1}{M_J}\right)\\ &\quad \left.\frac{\Gamma\left(n_\Sigma-p+2\right)}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^{n_\Sigma-p+2}}\sum_{i,j=1}^{2}c_ic_jd_{ij}F\left(\frac{n_\Sigma-p}{2}+1,\frac{n_\Sigma-p+1}{2}+1,2,-\frac{d_{ij}}{\left(\sum_{i=1}^{N}\sqrt{\frac{m_i}{\bar{\gamma_i}}}\right)^2}\right)\right\}. \end{aligned} \quad (51)$$

$$\begin{aligned}P_e &= 1-\frac{2^N}{\sqrt{\pi}\log_2(M_I\cdot M_J)}\left(\prod_{l=1}^{N}\frac{\left(\frac{m_l}{\bar{\gamma_l}}\right)^{m_l/2}}{\Gamma(m_l)}\right)\left(\frac{1}{M_I}\sum_{k=1}^{\log_2(I)}\sum_{i=0}^{(1-2^{-k})I-1}P(i,k)\sqrt{w_i}I\left(\frac{m_\Sigma-3/2}{2},\frac{1}{2},2\sqrt{w_i},\Lambda,\beta\right)+\right.\\ &\quad \left.\frac{1}{M_J}\sum_{p=1}^{\log_2(M_J)}\sum_{j=0}^{(1-2^{-p})M_J-1}P(j,p)\sqrt{w_j}I\left(\frac{m_\Sigma-3/2}{2},\frac{1}{2},2\sqrt{w_j},\Lambda,\beta\right)\right). \end{aligned} \quad (53)$$

---

alternative simpler expression of (53) is also obtained using (49).

IV. Dual-Hop AF Transmission Over Non-Identical Nakagami-m Fading

In some practical applications, a dual-hop transmission, i.e., $N = 2$, may be sufficient [2]. In this context, many authors

$$
\begin{aligned}
P_e \;=\; & \frac{1}{2} - \frac{\left(\frac{\tau\bar{\gamma}_2}{m_2}\right)^{\eta}\left(\frac{m_1\bar{\gamma 2}}{m_2\bar{\gamma}_1}\right)^{m_1}}{\eta B(m_2,\eta)}\left\{\frac{B\left(m_1+\eta,-m_1\right)}{B\left(m_2+\eta,m_1\right)}F_4\left(m_1+m_2+\eta, m_1+\eta, 1+\eta, 1+m_1, -\frac{\tau\bar{\gamma}_2}{m_2}, \frac{m_1\bar{\gamma 2}}{m_2\bar{\gamma}_1}\right)+\right. \\
& \left.\left(\frac{m_2\bar{\gamma}_1}{m_1\bar{\gamma}_2}\right)^{2m_1} F_4\left(m_2+\eta, \eta, 1+\eta, 1-m_1, -\frac{\tau\bar{\gamma}_2}{m_2}, \frac{m_1\bar{\gamma 2}}{m_2\bar{\gamma}_1}\right)\right\}. \qquad (54)
\end{aligned}
$$

have considered the error probability evaluation over non-identical Nakagami-m fading. So far, closed-form expressions are only available for integer values of the fading parameter, i.e. $m_i, i = 1,2 \in N$ [4], [5]. Nevertheless, in practical scenarios, the m parameters often take non-integer values [13]. In this work, we derive the error probability expressions in the m-complementary region of [4], i.e. $R\backslash N$. This allows applications of our analytical results over a larger range of the fading parameter values, thereby highlighting again the significance of this work. Using (18), the bit error probability for binary modulation of an AF dual-hop transmission is obtained as in (54) shown on the top of this page. where $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ is the beta function [15, Eq. (8.384.1)]. The above new expression of the bit error probability is valid for any non-integer value of $m_i, i = 1,2$. Another case, also not handled before, corresponds to identical ratios $\{m_l/\bar{\gamma}_l\}_{l=1}^{2}$ across the different hops, a scenario which includes as well the identically distributed fading [2], [3] as a special case. Defining $m/\bar{\gamma} = \{m_l/\bar{\gamma}_l\}_{l=1}^{2}$ and applying (27) yield

$$
\begin{aligned}
P_e = & \frac{1}{2} - \frac{\left(\frac{m}{\tau\bar{\gamma}}\right)^{\frac{m_1+m_2}{2}}\sqrt{\pi}}{\Gamma(\eta)\Gamma(m_1)\Gamma(m_2)} \times \\
& G_{4,4}^{4,1}\left(\frac{4m}{\tau\bar{\gamma}}\,\middle|\, \begin{matrix} 1-\eta-\frac{m_1+m_2}{2}, 0, \frac{1}{2}, 1-\frac{m_1+m_2}{2} \\ \frac{m_1+m_2}{2}, \frac{m_1-m_2}{2}, \frac{m_2-m_1}{2}, -\frac{m_1+m_2}{2} \end{matrix}\right). \qquad (55)
\end{aligned}
$$

By setting $m = m_1 = m_2$ and $\eta = \tau = 1$, we obtain an equivalent alternative representation for Hasna and Alouini's main result [2, Eq. (12)]. To prove the concordance of the two formulas, we use the Meijer's-G function property in [15, Eq. (9.31.1)] along with the identity

$$
\begin{aligned}
G_{3,3}^{3,1}\left(z\,\middle|\, \begin{matrix} a, c, a+1 \\ b, d, a \end{matrix}\right) = & \frac{\Gamma(b-1)\Gamma(d-1)}{\Gamma(c-a)} z^{a} \\
& \left(1 - {}_2F_1\left(b-a, d-a, c-a, -\frac{1}{z}\right)\right). \qquad (56)
\end{aligned}
$$

For completeness, it is worthwhile to mention that [2, Eq. (12)] can also be deduced from (54) by applying the analytical continuation formula of the Lauricella function followed by some algebraic manipulations using the Burchnall formulas [20, Eq. (37)],

$$
\begin{aligned}
& F_4(\alpha,\beta;\gamma,\gamma';x,x) \\
& = {}_4F_3\left(\alpha,\beta,\frac{1}{2}(\gamma+\gamma'),\frac{1}{2}(\gamma+\gamma'-1);\gamma,\gamma,\gamma+\gamma'-1;4x\right), \qquad (57)
\end{aligned}
$$

where ${}_pF_q(\cdot)$ is the generalized hypergeometric function [15, Eq. (9.14.1)]. In turn, the generalized hypergeometric function ${}_4F_3$ reduces to a simpler one when its parameters are constrained properly as

$$
{}_4F_3\left(a_1,a_2,a_3,a_4;b_1,a_3,a_4,z\right) = {}_2F_1\left(a_1,a_2,b_1,z\right), \qquad (58)
$$

where $F(a,b,c,z)$ is the Gauss hypergeometric function [15, Eq. (9.14.2)]. Hence, applying (17), (57) and (58) to (54), when $m = m_1 = m_2, \bar{\gamma} = \bar{\gamma}_1 = \bar{\gamma}_2$ and $\eta = \tau = 1$, yields [2, Eq. (12)].

For M-ary modulations and still using I_s in (18), the expectation of the Gaussian-Q function in (43), reduces when $N = 2$ to equation (59) shown on the next page. Moreover, when identical ratios $m/\bar{\gamma} = \{m_l/\bar{\gamma}_l\}_{l=1}^{2}$ are observed across the two hops, the expectation of the Gaussian-Q functions can be rewritten according to (27)

$$
\begin{aligned}
& \mathrm{E}(Q(A\sqrt{\gamma})) = \frac{1}{2} - \frac{1}{\Gamma(m_1)\Gamma(m_2)} \times \\
& \left(\frac{2m}{A^2\bar{\gamma}}\right)^{\frac{m_1+m_2}{2}} G_{4,4}^{4,1}\left(\frac{8m}{A^2\bar{\gamma}}\,\middle|\, \begin{matrix} \frac{1}{2}-\frac{m_1+m_2}{2}, 0, \frac{1}{2}, 1-\frac{m_1+m_2}{2} \\ \frac{m_1+m_2}{2}, \frac{m_1-m_2}{2}, \frac{m_2-m_1}{2}, -\frac{m_1+m_2}{2} \end{matrix}\right). \qquad (60)
\end{aligned}
$$

Moreover, the expectation of the product of two Gaussian-Q functions with different arguments can be derived as in (61). The derivation of the error probability expression of M-QAM modulations is then straightforward using (40).

V. Computational Methods and Numerical Examples

The aim of this section is to analyze the utility, accuracy, and numerical stability of the frameworks developed in the previous sections. All the results shown here have been analytically obtained by the direct evaluation of the expressions developed in this paper: either (38), (44), (48), (54), (60) and (61) for non-integer values of m, or (39), (49), (50) for integer plus one half values. The evaluation of these formulas involves the computation of some special hypergeometric functions, namely, the hypergeometric Lauricella functions.

A. Computational Methods

The third Lauricella function $F_C^{(n)}$ is typically computed with a finite summation approximating the infinite summation given in (15) as

$$
\begin{aligned}
& F_C^{(n)}\left(a,b;c_1,...,c_n;x_1,...,x_n\right) \\
& \quad = \sum_{q=0}^{q_{max}} (a)_q (b)_q \sum_{\Omega(q,n)} \prod_{k=1}^{n} \frac{x_k^{q_k}}{(c_k)_{q_k} q_k!}, \qquad (62)
\end{aligned}
$$

where $\Omega(q,n)$ is the set of n-tuples such that $\Omega(q,n) = \{(q_1,...,q_n) : q_k \in \{0,1,...,q\}, \sum_{k=1}^{n} q_k = q\}$. If a large q_{max} is required to obtain the desired accuracy, then (62) may have a high computational complexity. However, by

$$\mathrm{E}(Q(A\sqrt{\gamma}))=\frac{1}{2}-\frac{\sqrt{A\bar{\gamma}_1}}{\sqrt{m_1}B(\frac{1}{2},m_2)}\left[\left(\frac{m_2\bar{\gamma}_1}{m_1\bar{\gamma}_2}\right)^{m_2}\frac{B(m_2+\frac{1}{2},-m_2)}{B(m_2+\frac{1}{2},m_1)}F_4\left(m_1+m_2+\frac{1}{2},m_2+\frac{1}{2},\frac{3}{2},1+m_2,-A\frac{\bar{\gamma}_1}{m_1},\frac{m_2\bar{\gamma}_1}{m_1\bar{\gamma}_2}\right)+F_4\left(m_1+\frac{1}{2},\frac{1}{2},\frac{3}{2},1-m_2,-A\frac{\bar{\gamma}_1}{m_1},\frac{m_2\bar{\gamma}_1}{m_1\bar{\gamma}_2}\right)\right]. \quad (59)$$

$$\mathrm{E}(Q(A\sqrt{\gamma})Q(B\sqrt{\gamma}))\approx-2\sqrt{\pi}\sum_{i,j=1}^{2}c_ic_j\frac{\left(\frac{m}{d_{ij}\bar{\gamma}}\right)^{\frac{m_1+m_2}{2}}}{\Gamma(m_1)\Gamma(m_2)}G_{4,4}^{4,1}\left(\frac{4m}{d_{ij}\bar{\gamma}}\,\middle|\,{-\frac{m_1+m_2}{2},0,\frac{1}{2},1-\frac{m_1+m_2}{2}\atop \frac{m_1+m_2}{2},\frac{m_1-m_2}{2},\frac{m_2-m_1}{2},-\frac{m_1+m_2}{2}}\right). \quad (61)$$

reformulating the integral form of the Lauricella function in (13) such that it encompasses a term of $\exp(-t)$ [17, Ch. 2], the integral in (13) can be evaluated using the numerical integration method given in [21, Eq. (25.4.45)] for a variety of fading channel models. Then, one obtains

$$F_C^{(n)}(a,b;c_1,...,c_n;x_1,...,x_n)\approx\frac{\cos(\pi(a-b))}{2^{a+b-2}\Gamma(a)\Gamma(b)}\sum_{k=1}^{N_p}w_kt_k^{a+b-1} G_{1,2}^{2,1}\left(2t_k\,\middle|\,{\frac{1}{2}\atop b-a,a-b}\right)\left(\prod_{k=1}^{n}{}_0F_1\left(;c_k,x_k\frac{t_k^2}{4}\right)\right), \quad (63)$$

where t_k and w_k are, respectively, the k-th zero and weight of the Laguerre polynomial of order N_p [21, Eq. (25.4.45)]. Numerical results show that for $N_p=30$, this approximation provides a relative error for the average error rate below 0.3%. Notice that the other hypergeometric functions involved in this paper, namely, the Meijer's-G, Gauss hypergeometric and Appel functions are implemented in most popular computing softwares such as Matlab and Mathematica.

B. Numerical Results

In Fig. 1 we have reported the bit error probability induced by 16-QAM and QPSK modulations in a multihop variable gain relaying system with $N=2,4$. The identical and non-identical distributed fading cases are analyzed with an overall system fading severity $m_\Sigma=\sum_{i=1}^{N}m_i$ fixed to N. For the identical Nakagami case, all the hops undergo Rayleigh fading with $m_i=1, i=1,...,N$. For the non-identical case, the different hops may be subject to worse or better fading conditions than the identical case, such as $m_i\in\{0.5,1.5\}_{i=1,...,N}$. Since our analysis is only valid for non-integer fading parameters, then all the Rayleigh-case results were obtained using [22, Eq. (14)]. The error performance of the non-identical case are computed using (49) and (53) as well as the comparison with simulations. We can see a very good match over the range of SNRs of interest. As expected, we can observe that the error probability increases with the number of hops. It turns out that, although the overall system fading severity is the same, better performances are achieved when the different hops undergo identical fading. This is due to the fact that the AF multihop channel is a cascaded one and can be modeled as the product of N statistically independent Nakagami-m fading amplitudes. Therefore, the effect of a bad fading condition in one hop deteriorates the overall system performance even in the presence of a better fading condition in the following or preceding hops. Hence, the relay link is dominated by the more severely faded hop. This comment further corroborates the generality and usefulness of the analytical frameworks introduced in this paper, since real multihop channels are most likely non-identically distributed.

For multihop systems with variable gain relays under identical Nakagami fading with non-integer m, the error probability results are plotted in Fig. 2 for BFSK modulation. The BEP computations were performed using (38) and the computational methods in (63). As expected, the error performance improves as the fading severity decreases. In Fig. 3 we investigate the effect of power imbalance between the hops in a dual-hop BPSK transmission over non-identical Nakagami-m fading. The error probability is plotted against the first hop average SNR for $m_1=m_2=1.5$ and $m_1=0.7$and $m_2=1.5$. The higher average SNR resulting from one of the relays may be due to a Line Of Sight (LOS) signal component between the source terminal and the relay, or equivalently between the relay and the destination. Such a situation may occur in a cell-site scheme. Here, good accuracy is retained by the analytical models (54), which turn out to be not only very accurate but also numerically stable. Moreover, we can easily figure out that, such imbalance may be either beneficial or harmful for the overall system performance. Indeed, for $\bar{\gamma}_2>\bar{\gamma}_1$, it is advantageous compared to the balanced case, otherwise, it is detrimental.

Fig. 4 compares the exact bit error probabilities of BPSK multihop systems with variable-gain relays with the lower bound given in [8, Eq.(25)]. It can be seen that the proposed bounds lose their tightness with increasing SNR and, therefore, are definitely inappropriate for the estimation of the error performance of multi-hop systems operating over non-identical Nakagami fading. This was also stipulated by the authors of [8].

Overall, the aforementioned numerical results unambiguously confirm the high accuracy and huge utility of the proposed framework for error probability analysis of AF multi-hop systems over not necessarily identical Nakagami-m fading conditions.

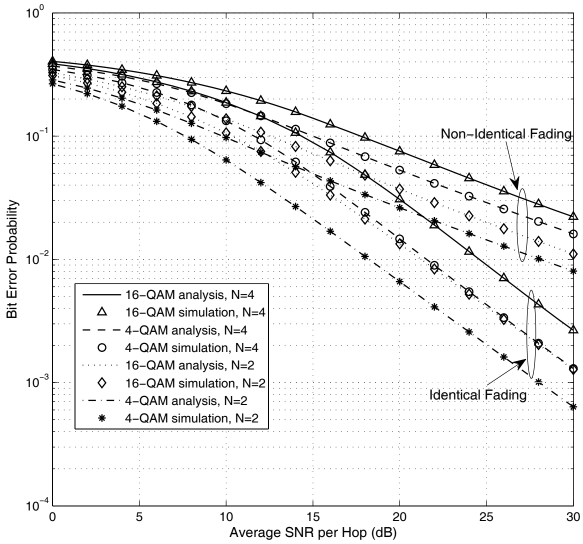

Fig. 1. Bit error probabilities for different QPSK and 16-QAM AF multihop transmission systems with variable-gain relays in identical and non identical]

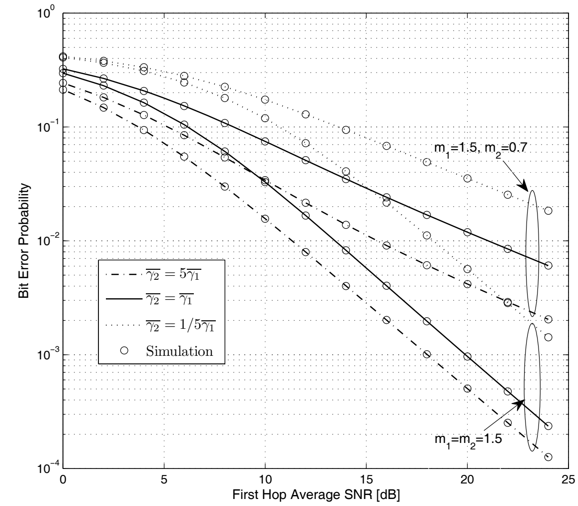

Fig. 3. Bit error probabilities for different BPSK AF multihop transmission systems with variable-gain relays over non identical Nakagami fading with unbalanced SNR.

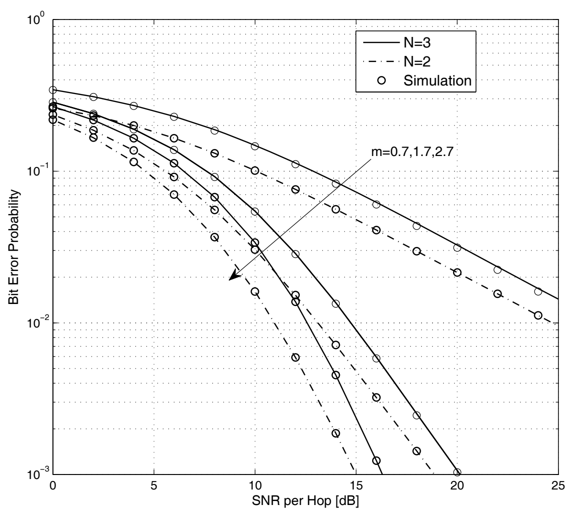

Fig. 2. Bit error probabilities vs. average SNR per hop for BFSK and different values of the fading parameter and number of hops $N = 2, 3$.

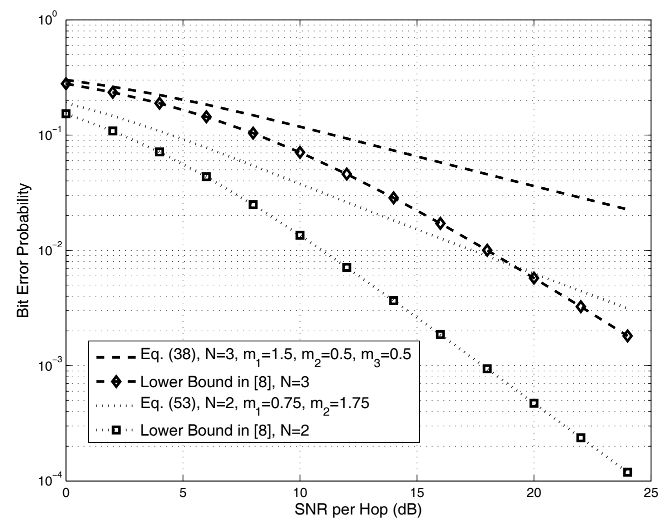

Fig. 4. Comparison between the exact bit error probability and the lower bound given in [8] for different BPSK AF multihop relaying systems with variable gain relays.

VI. Conclusion

In this paper, we have developed an analytical framework for the analysis of error probabilities of AF multihop variable gain relaying systems over Nakagami-m fading channels. The numerical examples show the accuracy and usefulness of the proposed new error probability expressions along three main contributions:

1) We have proposed new solutions for infinite integral forms involving the product of Bessel functions. Theses solutions have been shown useful to the evaluation of the error probabilities of multihop systems with arbitrary number of variable-gain relays operating over Nakagami-m fading. The obtained formulas establish a connection, hitherto unknown, between the error probabilities and the Lauricella's functions.
2) In the special case of an odd multiple of one half fading parameters, simpler expressions for the error probabilities, involving the Gauss hypergeometric function, have been obtained.
3) As far as the dual-hop case is concerned, it is shown in this special case of interest that the new error probability formulas reduce to previously known results in the literature. Moreover, these new formulas generalize previously known results pertaining to AF transmissions over non-identical fading with integer parameter.

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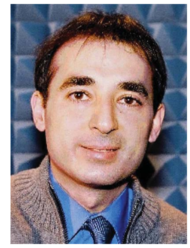

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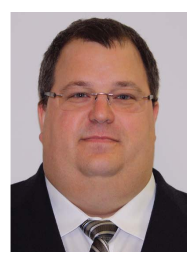

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