Cramér-Rao Lower Bounds for NDA SNR Estimates of Square QAM Modulated Transmissions

Faouzi Bellili, Alex Stéphenne, and Sofiène Affes

Abstract—**In this paper, we derive for the first time analytical expressions for the exact Cramér-Rao lower bounds on the variance of unbiased non-data-aided (NDA) signal-to-noise ratio (SNR) estimators of square QAM-modulated signals. The channel is assumed to be constant over the observation interval and the received signal is supposed to be corrupted by additive white Gaussian noise (AWGN). The derived expressions corroborate previous attempts to numerically compute the considered CRLBs. It will be shown that the NDA CRLBs differ widely from one modulation order to another especially at moderate SNR levels.**

Index Terms—**SNR estimation, Cramér-Rao lower bounds, non-data-aided estimation, QAM signals.**

I. INTRODUCTION

In modern communication systems, the SNR is an important measure of the channel quality. For instance, it provides necessary information for power control, link adaptation and adaptive modulation [1, 2], a few domains of application among many. Roughly speaking, depending if the *a priori* knowledge of the transmitted symbols is assumed or not, the SNR estimators can be categorized as data-aided (DA) or non-data-aided (NDA), respectively. On the other hand, SNR estimators are said to be envelope-based if the estimation process is based on the envelope of the received samples [3, 4, 5]. However, it is frequently of interest to obtain more accurate SNR estimates using the inphase/quadrature (I/Q) components of the received samples [3], and, in this case, the SNR estimators are called I/Q-based estimators.

A well-known common lower bound on the variance of any unbiased estimator is the Cramér-Rao lower bound (CRLB) and it has been widely used as a measure of the attainable precision of parameter estimates from a given set of observations. The CRLB for NDA envelope-based SNR estimates of QAM-modulated signals was derived in [5] and [6] for a known and an unknown noise variance, respectively. But, this CRLB does not represent the actual performance that can be achieved if we want to exploit all of the information contained in the I/Q components of the received samples. The I/Q CRLB for DA SNR estimates of QAM signals, for which all the transmitted symbols are supposed to be perfectly known to the receiver, was derived in [7]. On the other hand, the analytical expressions for the exact I/Q CRLBs for NDA SNR estimates were derived in [8], but only in the cases of BPSK and QPSK signaling, whereas, for higher-order QAM modulations, these analytical expressions have not yet been derived. They were only computed numerically or empirically in recent works (see [9] and [10]).

In this paper, considering square QAM constellations, the most popular, and using the I/Q components of the received signal, we derive analytical expressions for the CRLBs of NDA SNR estimates in AWGN channels. The final results introduced in this paper generalize the elegant CRLB expressions for the SNR estimates of BPSK- and QPSK-modulated signals presented in [8] to higher-order square QAM modulations.

The rest of this paper will be organized as follows. In section II, we will introduce the system model that will be used throughout the article. Section III will be dedicated to the results that are available in the open literature, regarding the I/Q CRLBs for NDA SNR estimates of QAM-modulated signals. Then, in section IV, we will derive analytical expressions for these CRLBs in the case of square QAM constellations. Finally, before moving to the concluding remarks in section VI, we will present and comment on some graphical representations of these analytical expressions in section V.

II. SYSTEM MODEL

Consider a traditional digital communication system broadcasting and receiving a QAM-modulated signal. The channel is supposed to be of a constant gain coefficient S over the observation interval. The received samples are assumed to be AWGN-corrupted with noise power $2\sigma^2$. Assuming an ideal receiver with perfect synchronization, the received signal at the output of the matched filter can be modelled as a complex signal as follows:

$$y(n) = S\ a(n)\ e^{j\phi} + w(n),\ n = 1, 2, \ldots, N, \qquad (1)$$

where $\{a(n)\}_{n=1,2,\cdots,N}$ are the N transmitted symbols and $\{y(n)\}_{n=1,2,\cdots,N}$ are the corresponding received samples. The complex noise components $\{w(n)\}_{n=1,2,\cdots,N}$ are assumed to be white and normally distributed with independent real and imaginary parts, each of variance σ^2. Moreover, the transmitted symbols, with equal *a priori* probability, are assumed to be independent and identically distributed (iid) and drawn from an M-ary square QAM constellation, where $M = 2^{2p}$ for any integer p. The parameter ϕ accounts for any non-random phase shift introduced by the channel. In addition, to derive standard CRLBs, which hold irrespectively of the transmission

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The authors are with INRS-EMT, 800, de la Gauchetière Ouest, Bureau 6900, Montreal, Qc, H5A 1K6, Canada. A. Stephenne is also with Huawei Technologies Canada, 411 Legget Drive, Ottawa, Ontario, K2K 3C9, Canada (e-mail: bellili@emt.inrs.ca, stephenne@ieee.org, affes@emt.inrs.ca).

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powers, the squared QAM constellation power is supposed to be normalized to one, i.e., $\mathrm{E}\{|a(n)|^2\} = 1$.

Using the multiple observations $\{y(n)\}_{n=1,2,\cdots,N}$, the true SNR, ρ, that we wish to estimate, is defined as

$$\rho = \frac{S^2\,\mathrm{E}\{|a(n)|^2\}}{2\sigma^2}, \quad (2)$$

$$= \frac{S^2}{2\sigma^2}. \quad (3)$$

From (3), we see that there are two unknown parameters which are involved in the derivation of the SNR CRLBs, which are: S and σ^2. Therefore, it is mathematically more convenient to use the following parameter vector:

$$\boldsymbol{\theta} = [S \quad \sigma^2]. \quad (4)$$

In addition, since using the decibel (dB) scale often provides easier interpretation of the performance of any SNR estimator, we will henceforth consider the following parameter transformation:

$$g(\boldsymbol{\theta}) = 10\log_{10}\left(\frac{S^2}{2\sigma^2}\right). \quad (5)$$

As shown in [11], the CRLB for parameter transformations is given by

$$\mathrm{CRLB}(\rho) = \frac{\partial g(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} I^{-1}(\boldsymbol{\theta}) \frac{\partial g(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}^T, \quad (6)$$

where the derivative of the parameter transformation, $\partial g(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$, is given by

$$\frac{\partial g(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \left[\frac{20}{\ln(10)S} \quad \frac{-10}{\ln(10)\sigma^2}\right], \quad (7)$$

and $\boldsymbol{I}(\boldsymbol{\theta})$ is the Fisher information matrix (FIM) defined as

$$\boldsymbol{I}(\boldsymbol{\theta}) = \begin{pmatrix} -\mathrm{E}\left\{\frac{\partial^2 \ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial S^2}\right\} & -\mathrm{E}\left\{\frac{\partial^2 \ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial S \partial \sigma^2}\right\} \\ -\mathrm{E}\left\{\frac{\partial^2 \ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial \sigma^2 \partial S}\right\} & -\mathrm{E}\left\{\frac{\partial^2 \ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial {\sigma^2}^2}\right\} \end{pmatrix}, \quad (8)$$

where $\boldsymbol{y}$ is a vector that contains the N data records, i.e., $\boldsymbol{y} = [y(1), y(2), \cdots, y(N)]^T$, and $P[\boldsymbol{y};\boldsymbol{\theta}]$ is the probability density function (pdf) of $\boldsymbol{y}$ parameterized by $\boldsymbol{\theta}$. The expectation $\mathrm{E}\{.\}$ is taken with respect to $\boldsymbol{y}$.

Usually, the derivation of the CRLB involves tedious algebraic manipulations. These mainly consist in the derivation of the FIM elements.

III. Background on the I/Q CRLBs for NDA SNR Estimators in QAM Transmissions

Although the problem of SNR estimation dates back to the 1960's [12], the derivation of the CRLBs for NDA SNR estimators was first considered in 2001 [8], but only for BPSK and QPSK signals. So far, for higher-order QAM modulations, these CRLBs have only been obtained numerically or computed empirically (cf. [9] and [10], respectively). However, their exact analytical expressions, as a function of the true SNR, have not been derived yet.

A. NDA I/Q CRLB for QPSK transmissions

The analytical expressions for the exact I/Q CRLB on the variance of unbiased NDA SNR estimators of QPSK-modulated signals, including phase distortion, were earlier derived in [8]. In fact, it was shown that the FIM is given by

$$\boldsymbol{I}(\boldsymbol{\theta}) = \frac{N}{\sigma^4}\begin{pmatrix} \sigma^2 - \sigma^2 f\left(\frac{\rho}{2}\right) & Sf\left(\frac{\rho}{2}\right) \\ Sf\left(\frac{\rho}{2}\right) & 1 - \frac{S^2}{\sigma^2} f\left(\frac{\rho}{2}\right) \end{pmatrix}, \quad (9)$$

where

$$f(\rho) = \frac{\exp(-\rho)}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{t^2}{\cosh(t\sqrt{2\rho})} e^{-\frac{t^2}{2}} dt. \quad (10)$$

Then, using (9) and (10), it was shown that

$$\mathrm{CRLB}_{\mathrm{QPSK}}(\rho) = \frac{100\left(\frac{2}{\rho} - f\left(\frac{\rho}{2}\right) + 1\right)}{N(\ln(10))^2\left(1 - f\left(\frac{\rho}{2}\right) - 2\rho f\left(\frac{\rho}{2}\right)\right)}. \quad (11)$$

From (11), it can be seen that the SNR CRLB was shown to be independent of the phase distortion ϕ introduced by the channel.

B. NDA I/Q CRLB for higher-order QAM transmissions

This problem has been recently addressed in [9] and [10], where the exact analytical expressions were not derived, but the CRLBs were approximately computed in two different ways. In fact, the major difficulty recognized by the authors of these two papers is the analytical derivation of the expected values, with respect to the received samples $y(n)$, of the second partial derivatives in (8). Indeed, in [10], this step was carried out using Monte Carlo simulations where samples of $y(n)$ were generated via computer simulation and the expected value computed empirically. However, in [9], the expectation (with respect to $\boldsymbol{y}$) involved in (8) was numerically carried out using a Gauss-Hermitian quadrature and the obtained numerical values for the FIM elements were used to evaluate the CRLB without any analytical expression.

IV. New Analytical Expressions for the Exact NDA I/Q CRLBs in Case of Square QAM Constellations

In this section, we will derive closed-form expressions for the FIM elements and hence for the exact CRLB of NDA SNR estimates when the transmitted symbols are drawn from any square QAM constellation. Under the assumptions made so far, for a general M-ary QAM constellation (i.e., square and non-square constellations), it can be shown that the pdf of the n^{th} received sample $y(n)$, parameterized by $\boldsymbol{\theta}$, $P[y(n);\boldsymbol{\theta}]$, is given by:

$$P[y(n);\boldsymbol{\theta}] = \frac{1}{2M\pi\sigma^2}\exp\left\{-\frac{I(n)^2 + Q(n)^2}{2\sigma^2}\right\} D_{\boldsymbol{\theta}}(n), \quad (12)$$

where

$$D_{\boldsymbol{\theta}}(n) = \sum_{c_k \in \mathcal{C}} \exp\left\{-\frac{S^2|c_k|^2}{2\sigma^2}\right\} \exp\left\{\frac{S\Re\{y^*(n)e^{j\phi}c_k\}}{\sigma^2}\right\}. \quad (13)$$

In these equations, $\Re\{.\}$ and the superscript $*$ refer, respectively, to the real part and the complex conjugate of any complex number; $\mathcal{C} = \{c_1, c_2, \cdots, c_M\}$ is the constellation alphabet; $I(n)$ and $Q(n)$ are, respectively, the inphase/real and quadrature/imaginary component/part of the corresponding received sample $y(n)$, which means $y(n) = I(n) + jQ(n)$. As stated in the previous section, it is mainly the complexity of the term $D_{\boldsymbol{\theta}}(n)$ that prevented the authors, in [9] and [10], to provide analytical expressions for the considered CRLBs. However, in this article, by only considering square QAM constellations, we notice that $D_{\boldsymbol{\theta}}(n)$, and therefore $P[y(n); \boldsymbol{\theta}]$, can be factorized. Consequently, we are able after tedious algebraic manipulations, which will be briefly outlined in the following, to provide analytical expressions for the FIM elements given by (8), and therefore exact analytical expressions for the considered CRLBs as a function of the true SNR ρ. Indeed, it can be seen that, for square QAM constellations (i.e., with $M = 2^{2p}$ for any $p \geq 1$), we have $\mathcal{C} = \{\pm(2i-1)d_p \pm j(2k-1)d_p\}_{i,k=1,2,\cdots,2^{p-1}}$ where $j^2 = -1$ and $2d_p$ is the intersymbol distance in the I/Q plane, which is appropriately selected to set the constellation power to a desired level. For a normalized-power square QAM constellation, d_p is computed using the following rule:

$$\frac{\sum_{k=1}^{2^{2p}} |c_k|^2}{2^{2p}} = 1, \tag{14}$$

which yields the following result:

$$d_p = \frac{2^{p-1}}{\sqrt{2^p \sum_{k=1}^{2^{p-1}} (2k-1)^2}}. \tag{15}$$

Now denoting by $\tilde{\mathcal{C}}$ the subset of the alphabet points that lie in the top right quadrant of the constellation, i.e., $\tilde{\mathcal{C}} = \{(2i-1)d_p + j(2k-1)d_p\}_{i,k=1,2,\cdots,2^{p-1}}$, we have $\mathcal{C} = \tilde{\mathcal{C}} \cup (-\tilde{\mathcal{C}}) \cup \tilde{\mathcal{C}}^* \cup (-\tilde{\mathcal{C}}^*)$ and we rewrite (13) as follows:

$$D_{\boldsymbol{\theta}}(n) = \sum_{\tilde{c}_k \in \tilde{\mathcal{C}}} \exp\left\{-\frac{S^2|\tilde{c}_k|^2}{2\sigma^2}\right\} \times \left(\exp\left\{\frac{S\Re\{y^*(n)e^{j\phi}\tilde{c}_k\}}{\sigma^2}\right\} + \exp\left\{\frac{S\Re\{y^*(n)e^{j\phi}(-\tilde{c}_k)\}}{\sigma^2}\right\} + \exp\left\{\frac{S\Re\{y^*(n)e^{j\phi}\tilde{c}_k^*\}}{\sigma^2}\right\} + \exp\left\{\frac{S\Re\{y^*(n)e^{j\phi}(-\tilde{c}_k^*)\}}{\sigma^2}\right\}\right). \tag{16}$$

Then using the fact that $e^x + e^{-x} = 2\cosh(x)$, we obtain:

$$D_{\boldsymbol{\theta}}(n) = 2\sum_{\tilde{c}_k \in \tilde{\mathcal{C}}} \exp\left\{-\frac{S^2|\tilde{c}_k|^2}{2\sigma^2}\right\}\left[\cosh\left(\frac{S\Re\{y^*(n)e^{j\phi}\tilde{c}_k\}}{\sigma^2}\right) + \cosh\left(\frac{S\Re\{y^*(n)e^{j\phi}\tilde{c}_k{}^*\}}{\sigma^2}\right)\right]. \tag{17}$$

Moreover, we have $\cosh(x) + \cosh(y) = 2\cosh(\frac{x+y}{2})\cosh(\frac{x-y}{2})$, and using the fact that $\tilde{c}_k + \tilde{c}_k^* = 2\Re\{\tilde{c}_k\}$ and $\tilde{c}_k - \tilde{c}_k^* = 2j\Im\{\tilde{c}_k\}$, (17) is rewritten as follows:

$$D_{\boldsymbol{\theta}}(n) = 4\sum_{\tilde{c}_k \in \tilde{\mathcal{C}}} \exp\left\{-\frac{S^2|\tilde{c}_k|^2}{2\sigma^2}\right\} \times \cosh\left(\frac{S\Re\{\tilde{c}_k\}\Re\{y^*(n)e^{j\phi}\}}{\sigma^2}\right) \times \cosh\left(\frac{S\Im\{\tilde{c}_k\}\Im\{y^*(n)e^{j\phi}\}}{\sigma^2}\right), \tag{18}$$

$$= 4\sum_{i=1}^{2^{p-1}}\sum_{k=1}^{2^{p-1}} \exp\left\{-\frac{S^2((2i-1)^2+(2k-1)^2)d_p^2}{2\sigma^2}\right\} \times \cosh\left(\frac{S(2i-1)d_p\Re\{y^*(n)e^{j\phi}\}}{\sigma^2}\right) \times \cosh\left(\frac{S(2k-1)d_p\Im\{y^*(n)e^{j\phi}\}}{\sigma^2}\right). \tag{19}$$

Finally, splitting the two sums in (19), it can be shown that $D_{\boldsymbol{\theta}}(n)$ is factorized as follows:

$$D_{\boldsymbol{\theta}}(n) = 4F_{\boldsymbol{\theta}}(U(n)) \times F_{\boldsymbol{\theta}}(V(n)), \tag{20}$$

where

$$F_{\boldsymbol{\theta}}(x) = \sum_{i=1}^{2^{p-1}} \exp\left\{-\frac{S^2(2i-1)^2 d_p^2}{2\sigma^2}\right\}\cosh\left(\frac{(2i-1)d_p S x}{\sigma^2}\right), \tag{21}$$

and

$$\begin{aligned} U(n) &= \Re\{y^*(n)e^{j\phi}\}, \\ &= I(n)\cos(\phi) + Q(n)\sin(\phi), \\ V(n) &= \Im\{y^*(n)e^{j\phi}\}, \\ &= I(n)\sin(\phi) - Q(n)\cos(\phi). \end{aligned} \tag{22}$$

Note that the proposed factorization is a generalization of the one used in [8] in the special case of QPSK constellation. Furthermore, injecting (20) in (12) and noticing that $I(n)^2 + Q(n)^2 = U(n)^2 + V(n)^2$, it can be shown that $P[y(n); \boldsymbol{\theta}]$ can be factorized as follows:

$$P[y(n); \boldsymbol{\theta}] = P[U(n); \boldsymbol{\theta}]P[V(n); \boldsymbol{\theta}], \tag{23}$$

where

$$P[U(n); \boldsymbol{\theta}] = \frac{\sqrt{2}}{\sqrt{M\pi\sigma^2}} e^{-\frac{U(n)^2}{2\sigma^2}} F_{\boldsymbol{\theta}}(U(n)), \tag{24}$$

$$P[V(n); \boldsymbol{\theta}] = \frac{\sqrt{2}}{\sqrt{M\pi\sigma^2}} e^{-\frac{V(n)^2}{2\sigma^2}} F_{\boldsymbol{\theta}}(V(n)). \tag{25}$$

On the other hand, since ϕ is assumed to be deterministic, then we have $P[y(n); \boldsymbol{\theta}] = P[y(n)^* e^{j\phi}; \boldsymbol{\theta}] = P[\Re\{y(n)^* e^{j\phi}\}, \Im\{y(n)^* e^{j\phi}\}; \boldsymbol{\theta}] = P[U(n), V(n); \boldsymbol{\theta}]$. Consequently, from (23), it follows that

$$P[U(n), V(n); \boldsymbol{\theta}] = P[U(n); \boldsymbol{\theta}]P[V(n); \boldsymbol{\theta}], \tag{26}$$

which means that the two real random variables U_n and V_n (whose realizations are $U(n)$ and $V(n)$, respectively) are independent and identically distributed according to (24) and (25), respectively. Moreover, since the transmitted symbols are assumed to be iid and the additive noise is white, then the corresponding received samples $\{y(n)\}_{n=1,2\cdots,N}$ are independent and the pdf of the received vector $\boldsymbol{y} = [y(1), y(2), \cdots, y(N)]$

is simply the product of the marginal pdfs $\{P[y(n),\boldsymbol{\theta}]\}_{n=1}^{N}$ such that:

$$P[\boldsymbol{y};\boldsymbol{\theta}] = \left(\frac{2}{M\pi\sigma^2}\right)^N \exp\left\{-\frac{\sum_{n=1}^{N} U(n)^2+V(n)^2}{2\sigma^2}\right\} \times \prod_{n=1}^{N} F_{\boldsymbol{\theta}}(U(n))F_{\boldsymbol{\theta}}(V(n)). \quad (27)$$

Finally, the log-likelihood function of the received samples reduces simply to

$$\ln(P[\boldsymbol{y};\boldsymbol{\theta}]) = N\ln\left(\frac{2}{M\pi\sigma^2}\right) - \sum_{n=1}^{N}\frac{U(n)^2+V(n)^2}{2\sigma^2} + \sum_{n=1}^{N}\ln\left(F_{\boldsymbol{\theta}}(U(n))\right) + \sum_{n=1}^{N}\ln\left(F_{\boldsymbol{\theta}}(V(n))\right). \quad (28)$$

As it can be seen from (28), due to the factorization of $D_{\boldsymbol{\theta}}(n)$, the log-likelihood function involves the sum of two analogous terms. This reduces the complexity of the derivation of the second partial derivatives and their expected values, although it remains tedious. The derivation of the first diagonal element of $\boldsymbol{I}(\boldsymbol{\theta})$ will be briefly outlined in the following and more details can be found in Appendix A.

First, it is worth noting that averaging with respect to the complex univariate random variable y_n (whose realization is $y(n)$) is equivalent to averaging with respect to the real bivariate random variable (U_n, V_n), i.e., $\mathrm{E}_{y_n}\{.\} = \mathrm{E}_{(U_n,V_n)}\{.\}$. Moreover, it can be easily shown that:

$$\mathrm{E}\left\{\frac{\partial^2\ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial S^2}\right\} = N\,\mathrm{E}_{(U_n,V_n)}\left\{\frac{\partial^2\ln(F_{\boldsymbol{\theta}}(U(n)))}{\partial S^2}\right\} + N\,\mathrm{E}_{(U_n,V_n)}\left\{\frac{\partial^2\ln(F_{\boldsymbol{\theta}}(V(n)))}{\partial S^2}\right\}. \quad (29)$$

But since $U(n)$ and $V(n)$ are independent and identically distributed, (29) reduces simply to:

$$\mathrm{E}\left\{\frac{\partial^2\ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial S^2}\right\} = 2N\,\mathrm{E}_{U_n}\left\{\frac{\partial^2\ln(F_{\boldsymbol{\theta}}(U(n)))}{\partial S^2}\right\} \quad (30)$$

$$= 2N\,\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(1)}(U(n))\right\} - 2N\,\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(2)}(U(n))\right\}, \quad (31)$$

where $H_{\boldsymbol{\theta}}^{(1)}(U(n))$ and $H_{\boldsymbol{\theta}}^{(2)}(U(n))$ are defined as follows:

$$H_{\boldsymbol{\theta}}^{(1)}(U(n)) = \frac{\frac{\partial^2 F_{\boldsymbol{\theta}}(U(n))}{\partial S^2}}{F_{\boldsymbol{\theta}}(U(n))}, \quad (32)$$

$$H_{\boldsymbol{\theta}}^{(2)}(U(n)) = \left(\frac{\frac{\partial F_{\boldsymbol{\theta}}(U(n))}{\partial S}}{F_{\boldsymbol{\theta}}(U(n))}\right)^2. \quad (33)$$

Moreover, we show in Appendix A that:

$$\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(1)}(U(n))\right\} = 0, \quad (34)$$

and

$$\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(2)}(U(n))\right\} = \frac{1}{2^{p-1}\sigma^2}F(\rho), \quad (35)$$

where $F(\rho)$ is defined as:

$$F(\rho) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\frac{f_\rho^2(t)}{h_\rho(t)}\ e^{-\frac{t^2}{2}}dt, \quad (36)$$

with

$$f_\rho(t) = \sum_{k=1}^{2^{p-1}} e^{-(2k-1)^2 d_p^2\rho} \times \Big[(2k-1)d_p t\sinh\left((2k-1)d_p\sqrt{2\rho}\ t\right) - (2k-1)^2 d_p^2\sqrt{2\rho}\ \cosh\left((2k-1)d_p\sqrt{2\rho}\ t\right)\Big], \quad (37)$$

$$h_\rho(t) = \sum_{k=1}^{2^{p-1}} e^{-(2k-1)^2 d_p^2\rho}\cosh\left((2k-1)d_p\sqrt{2\rho}\ t\right). \quad (38)$$

Therefore, we obtain:

$$\mathrm{E}\left\{\frac{\partial^2\ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial S^2}\right\} = -\frac{N}{2^{p-2}\sigma^2}F(\rho). \quad (39)$$

For the second diagonal element of $\boldsymbol{I}(\boldsymbol{\theta})$, it can be shown that:

$$\mathrm{E}\left\{\frac{\partial^2\ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial {\sigma^2}^2}\right\} = -N\frac{\partial^2\ln(\sigma^2)}{\partial {\sigma^2}^2} - N\frac{\partial^2\left(\frac{1}{\sigma^2}\right)}{\partial {\sigma^2}^2}\,\mathrm{E}_{U_n}\left\{U(n)^2\right\} + 2N\,\mathrm{E}_{U_n}\left\{\frac{\partial^2\ln(F_{\boldsymbol{\theta}}(U(n)))}{\partial {\sigma^2}^2}\right\}. \quad (40)$$

Note that $2N\,\mathrm{E}_{U_n}\left\{\frac{\partial^2\ln(F_{\boldsymbol{\theta}}(U(n)))}{\partial {\sigma^2}^2}\right\}$ in (40) is equivalent to $2N\,\mathrm{E}_{U_n}\left\{\frac{\partial^2\ln(F_{\boldsymbol{\theta}}(U(n)))}{\partial S^2}\right\}$ in (30) and it is hence derived in the same way. Moreover, we have:

$$\mathrm{E}_{U_n}\{U(n)^2\} = \int_{-\infty}^{+\infty}U(n)^2P[U(n),\boldsymbol{\theta}]dU(n) = \frac{\sqrt{2}}{\sqrt{M\pi\sigma^2}}\int_{-\infty}^{+\infty}U(n)^2e^{-\frac{U(n)^2}{2\sigma^2}}F_{\boldsymbol{\theta}}(U(n))dU(n) = \frac{\sqrt{2}}{\sqrt{M\pi\sigma^2}}\sum_{k=1}^{2^{p-1}}\sqrt{2\pi}\sigma(\sigma^2+S^2(2k-1)^2d_p^2). \quad (41)$$

Therefore, using these properties and straightforward developments, starting from (40), yield the following result:

$$\mathrm{E}\left\{\frac{\partial^2\ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial {\sigma^2}^2}\right\} = \frac{N}{2^{p-2}\sigma^4} \times \left[-2^{p-2} - \frac{S^2}{\sigma^2}[2^{p-2}+G(\rho)] + \rho\left(2^p - \rho A_4^{(p)}\right)\right], \quad (42)$$

where $A_4^{(p)}$ and $G(\rho)$ are given by:

$$A_4^{(p)} = 2^{2(p-2)}\frac{\sum_{k=1}^{2^{p-1}}(2k-1)^4}{\left(\sum_{k=1}^{2^{p-1}}(2k-1)^2\right)^2}, \quad (43)$$

$$G(\rho) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\frac{g_\rho^2(t)}{h_\rho(t)}\ e^{-\frac{t^2}{2}}dt, \quad (44)$$

and $g_\rho(t)$ is defined as follows:

$$g_\rho(t) = \sum_{k=1}^{2^{p-1}} e^{-(2k-1)^2 d_p^2 \rho} \times \Big[(2k-1)d_p t \sinh\Big((2k-1)d_p\sqrt{2\rho}\ t\Big) - (2k-1)^2 d_p^2 \sqrt{\frac{\rho}{2}}\ \cosh\Big((2k-1)d_p\sqrt{2\rho}\ t\Big)\Big]. \quad (45)$$

Equivalent derivations yield the following expression for the off-diagonal element of $\boldsymbol{I}(\boldsymbol{\theta})$:

$$\mathrm{E}\left\{\frac{\partial^2 \ln(P[\boldsymbol{y};\boldsymbol{\theta}])}{\partial S \partial \sigma^2}\right\} = -\frac{N}{2^{p-2}\sigma^4} S[2^{p-2} - H(\rho)], \quad (46)$$

where $H(\rho)$ is defined as:

$$H(\rho) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{f_\rho(t) g_\rho(t)}{h_\rho(t)}\ e^{-\frac{t^2}{2}} dt. \quad (47)$$

Therefore, the Fisher information matrix is given by:

$$\boldsymbol{I}(\boldsymbol{\theta}) = \frac{N}{2^{p-2}\sigma^4} \begin{pmatrix} \sigma^2 F(\rho) & S[2^{p-2} - H(\rho)] \\ S[2^{p-2} - H(\rho)] & I_{2,2} \end{pmatrix}, \quad (48)$$

where

$$I_{2,2} = 2^{p-2} + \frac{S^2}{\sigma^2}[2^{p-2} + G(\rho)] - \rho(2^p + \rho A_4^{(p)}). \quad (49)$$

Finally, injecting (7) and (48) in (6), we obtain the following analytical expression for the NDA I/Q CRLB:

$$\mathrm{CRLB}_{\mathrm{NDA}}(\rho) = \frac{25 \times 2^p}{N \ln^2(10)\rho} \times \frac{K(\rho)}{D(\rho)}, \quad (50)$$

where

$$K(\rho) = 2A_4^{(p)}\rho^2 - [F(\rho) + 4G(\rho) - 4H(\rho)]\rho - 2^{p-1}, \quad (51)$$

$$D(\rho) = F(\rho)[A_4^{(p)}\rho^2 + 2(2^{p-2} - G(\rho))\rho - 2^{p-2}] + 2[H(\rho) - 2^{p-2}]^2\rho. \quad (52)$$

It is worth noting that the above analytical expression for the CRLB, as function of the true SNR, generalizes to higher-order square QAM modulations the elegant CRLB expression derived in [8] for QPSK. In fact, it can be verified that the CRLB expression of QPSK-modulated signals, which was earlier derived in [8] and given by (11), can now be directly obtained from our newly derived general expression, after tedious manipulations, by taking $p = 1$. Moreover, as it can be seen from (50), for higher-order square QAM constellations, the CRLB does not depend also on the phase distortion ϕ introduced by the channel, as shown earlier in [8] for BPSK and QPSK. This general property was only explained intuitively in [9].

V. Graphical Representations

In this section we include the graphical representations of the lower bounds given by (50), for a fixed value of $N = 100$ received samples. Beforehand, we verify from Figs. 1, 2 and 3 that the three integrand functions $\frac{f_\rho(t)^2}{h_\rho(t)}$, $\frac{g_\rho(t)^2}{h_\rho(t)}$ and $\frac{f_\rho(t)g_\rho(t)}{h_\rho(t)}$ involved in (36), (44) and (47), respectively, take extremely small values as $|t|$ increases. The integrals over $[-\infty, +\infty]$ can be therefore accurately approximated by finite integrals over $[-T, T]$, for which the Riemann integration method can be adequately used. In our simulations, it should be noted that $T = 100$ and a summation step of 1 provided very accurate values for the infinite integrals and that the CRLB curves obtained in this paper are identical to those previously presented in [8, 9, 10]. In fact, Figs. 4 and 5 depict the SNR CRLBs in [dB^2] and the square root of the CRLB in [dB] (that corresponds to the standard deviation of the SNR estimators), respectively, for different modulation orders. It can be verified in the special cases of QPSK and 16-QAM constellations that the CRLB curves plotted using our analytical expressions are, respectively, similar to those presented in [8] (QPSK) and [10] (16-QAM).

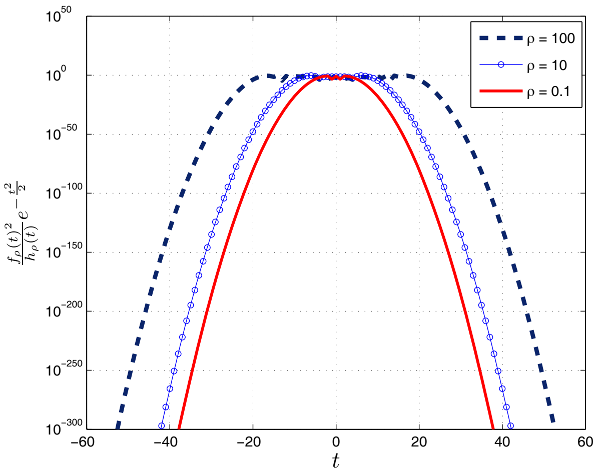
Furthermore, we see from Fig. 4 that, in the low SNR region, the NDA CRLBs deviate significantly from the DA CRLBs. Hence, it is in this SNR region where the DA techniques lend themselves as the only reliable solution to accurately estimate the SNR. However, for relatively high SNR values, we see that the NDA CRLBs reach the DA CRLBs over a range that varies from one modulation to another. But, ultimately, they coincide over one common SNR region since the DA CRLB itself has the same expression for all the modulation orders [8]:

$$\mathrm{CRLB}_{\mathrm{DA}}(\rho) = \frac{100}{N \ln^2(10)}\left(1 + \frac{2}{\rho}\right). \quad (53)$$

This means that, for sufficiently high SNR values, NDA SNR estimation techniques can exhibit performances that are equivalent to those that could be achieved if all the transmitted symbols were perfectly known. This is because, in this SNR region, the useful signal is not too much corrupted by the additive noise and, consequently, the signal and noise powers can be estimated quite adequately, even if the transmitted symbols are completely unknown to the receiver.

We see also that the NDA CRLB becomes very high as the SNR becomes very low. This means that in the low SNR region, unless the number of the available received samples N is very high, all the unbiased NDA techniques will fail to provide sufficiently accurate SNR estimates and DA SNR estimation techniques are, therefore, well motivated. On the other hand, NDA estimation techniques are also of great importance at relatively high SNR levels. In fact, as the NDA CRLB is inversely proportional to N, the number of the received samples can be increased in order to achieve a desired estimation accuracy, without any penalty on the throughput of the system. However, for the DA estimators, N cannot be increased without limiting, in counter part, the whole throughput of the system. On the other hand, we see in the moderate SNR region, that the CRLBs differ widely from one modulation order to another.

It is also worth noting that unlike the DA CRLB, the NDA CRLB becomes not strictly decreasing with the SNR as the number of levels in the constellation increases (i.e., M increases). In fact, there is a moving (with respect to the modulation order) range of SNR values over which the decreasing speed of the CRLB is slower than over the other ranges. This speed even reaches zero for 64-QAM and higher modulation



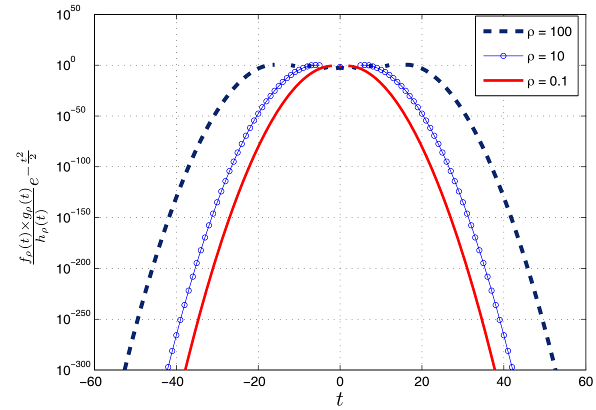


Fig. 3. Representation of the integrand function $\frac{f_\rho(t) \times g_\rho(t)}{h_\rho(t)} e^{-\frac{t^2}{2}}$.

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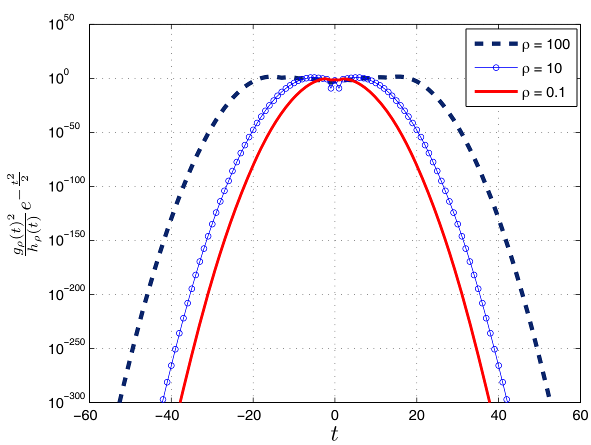

Fig. 2. Representation of the integrand function $\frac{g_\rho(t)^2}{h_\rho(t)} e^{-\frac{t^2}{2}}$

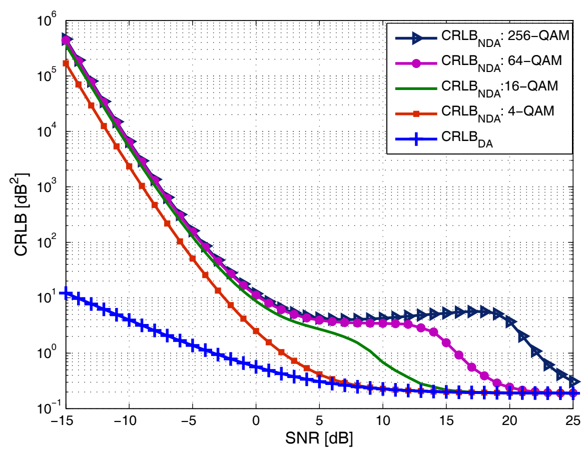

Fig. 4. Comparison of the SNR CRLBs for different modulation orders, $N = 100$.

orders for which the CRLB becomes even increasing over these specific ranges. This implies that over these ranges the received signal is not highly dependent on the parameter to be estimated which is the SNR in our case. It means that a slight change of the SNR, in this range, does not result in considerable variations of the received signal, which in turn does not bring much information about the SNR and the corresponding CRLB varies slowly. This behavior seems to be inherent to all constellations with non-constant modulus as it was also observed with the CRLBs for the NDA estimation of other parameters, from general QAM-modulated signals, such as the carrier frequency and the carrier phase [13].

VI. Conclusion

In this paper, we derived for the first time analytical expressions for the exact CRLBs on the variance of NDA SNR estimators of square QAM modulated signals as a function of the true SNR. The CRLB turns out to be inversely proportional to the number of independent data records and therefore it does not need to be recomputed as we move from one observation interval size to another. Moreover, our analytical expression corroborates the numerical and empirical approaches for the evaluation of these CRLBs which were proposed in [9] and [10], respectively. Finally, the derived expressions are of great value in that they allow analyzing the achievable performance and quantifying the performance of unbiased NDA SNR estimators operating on square QAM-modulated signals.

Appendix A - Proofs of (34) and (35)

In this appendix, we first detail the proof of (34) then of (35). In fact, we establish the first and second derivatives of $F_{\boldsymbol{\theta}}(U(n))$ with respect to S as follows:

$$\frac{\partial F_{\boldsymbol{\theta}}(U(n))}{\partial S} = \sum_{k=1}^{2^{p-1}} e^{-\frac{(2k-1)^2 d_p^2 S^2}{2\sigma^2}} \times \left[\frac{(2k-1)d_p U(n)}{\sigma^2} \sinh\left(\frac{(2k-1)d_p S U(n)}{\sigma^2}\right) - \frac{(2k-1)^2 d_p^2 S}{\sigma^2} \cosh\left(\frac{(2k-1)d_p S U(n)}{\sigma^2}\right)\right], \quad (54)$$

$$\frac{\partial^2 F_{\boldsymbol{\theta}}(U(n))}{\partial S^2} = \sum_{k=1}^{2^{p-1}} e^{-\frac{(2k-1)^2 d_p^2 S^2}{2\sigma^2}} \times \left[\frac{(2k-1)^2 d_p^2 U(n)^2}{\sigma^4} \cosh\left(\frac{(2k-1)d_p S U(n)}{\sigma^2}\right)\right.$$

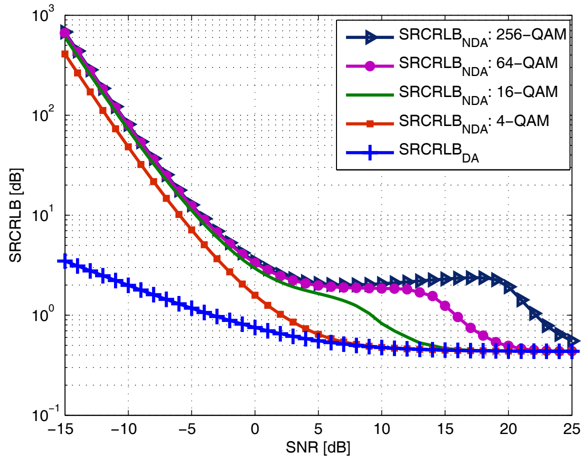

Fig. 5. Comparison of the the square root of the CRLBs (SRCRLBs) for different modulation orders, $N = 100$.

$$-\frac{2(2k-1)^3 d_p^3 S U(n)}{\sigma^4}\sinh\left(\frac{(2k-1)d_p S U(n)}{\sigma^2}\right) + \left(\frac{(2k-1)^4 d_p^4 S^2)}{\sigma^4} - \frac{(2k-1)^2 d_p^2}{\sigma^2}\right)\times \cosh\left(\frac{(2k-1)d_p S U(n)}{\sigma^2}\right)\Bigg]. \quad (55)$$

The expectation of $H_{\boldsymbol{\theta}}^{(1)}(U(n))$ with respect to $U(n)$ is calculated as follows:

$$\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(1)}(U(n))\right\} = \int_{-\infty}^{+\infty} H_{\boldsymbol{\theta}}^{(1)}(U(n))P[U(n),\boldsymbol{\theta}]dU(n), \quad (56)$$

with the pdf of $U(n)$, $P[U(n),\boldsymbol{\theta}]$, being given by (24). Therefore, plugging the expressions of $H_{\boldsymbol{\theta}}^{(1)}(U(n))$ and $P[U(n),\boldsymbol{\theta}]$ in (56), we obtain the following result:

$$\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(1)}(U(n))\right\} = \sum_{k=1}^{2^{p-1}} e^{-\frac{(2k-1)^2 d_p^2 S^2}{2\sigma^2}} \times \left[\frac{(2k-1)^2 d_p^2}{\sigma^4}\alpha_{\boldsymbol{\theta}}(k) - \frac{2(2k-1)^3 d_p^3 S)}{\sigma^4}\gamma_{\boldsymbol{\theta}}(k) + \left(\frac{(2k-1)^4 d_p^4 S^2}{\sigma^4} - \frac{(2k-1)^2 d_p^2}{\sigma^2}\right)\lambda_{\boldsymbol{\theta}}(k)\right], \quad (57)$$

where $\alpha_{\boldsymbol{\theta}}(k)$, $\gamma_{\boldsymbol{\theta}}(k)$ and $\lambda_{\boldsymbol{\theta}}(k)$ are defined as :

$$\alpha_{\boldsymbol{\theta}}(k) = \int_{-\infty}^{+\infty} U(n)^2 e^{-\frac{U(n)^2}{2\sigma^2}}\cosh\left(\frac{(2k-1)d_p S U(n)}{\sigma^2}\right)dU(n),$$
$$\gamma_{\boldsymbol{\theta}}(k) = \int_{-\infty}^{+\infty} U(n) e^{-\frac{U(n)^2}{2\sigma^2}}\sinh\left(\frac{(2k-1)d_p S U(n)}{\sigma^2}\right)dU(n),$$
$$\lambda_{\boldsymbol{\theta}}(k) = \int_{-\infty}^{+\infty} e^{-\frac{U(n)^2}{2\sigma^2}}\cosh\left(\frac{(2k-1)d_p S U(n)}{\sigma^2}\right)dU(n).$$

It can be shown that:

$$\alpha_{\boldsymbol{\theta}}(k) = \sqrt{2\pi}(\sigma^3 + (2k-1)^2 d_p^2 \sigma S^2)e^{\frac{(2k-1)^2 d_p^2 S^2}{2\sigma^2}}, \quad (58)$$
$$\gamma_{\boldsymbol{\theta}}(k) = \sqrt{2\pi}(2k-1)d_p \sigma S e^{\frac{(2k-1)^2 d_p^2 S^2}{2\sigma^2}}, \quad (59)$$
$$\lambda_{\boldsymbol{\theta}}(k) = \sqrt{2\pi}\sigma e^{\frac{(2k-1)^2 d_p^2 S^2}{2\sigma^2}}. \quad (60)$$

Then, injecting (58), (59) and (60) into (57), we obtain:

$$\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(1)}(U(n))\right\} = 0. \quad (61)$$

On the other hand, to show (35), we denote by $G_{\boldsymbol{\theta}}(U(n))$ the first derivative of $F_{\boldsymbol{\theta}}(U(n))$ with respect to S as given by (54). Therefore, we have:

$$\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(2)}(U(n))\right\} = \int_{-\infty}^{+\infty} H_{\boldsymbol{\theta}}^{(2)}(U(n))P[U(n),\boldsymbol{\theta}]dU(n), = \frac{\sqrt{2}}{\sqrt{M\pi\sigma^2}}\int_{-\infty}^{+\infty}\frac{G_{\boldsymbol{\theta}}(U(n))^2}{F_{\boldsymbol{\theta}}(U(n))}\times e^{-\frac{U(n)^2}{2\sigma^2}}dU(n). \quad (62)$$

We simplify (62) by changing $\frac{U(n)}{\sigma}$ by t and using $\rho = \frac{S^2}{2\sigma^2}$. Thus, we obtain the following result:

$$\mathrm{E}_{U_n}\left\{H_{\boldsymbol{\theta}}^{(2)}(U(n))\right\} = \frac{1}{2^{p-1}\sigma^2}F(\rho), \quad (63)$$

where $F(\rho)$ is defined as:

$$F(\rho) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\frac{f_\rho^2(t)}{h_\rho(t)}\ e^{-\frac{t^2}{2}}dt, \quad (64)$$

with

$$f_\rho(t) = \sum_{k=1}^{2^{p-1}} e^{-(2k-1)^2 d_p^2 \rho}\times \left[(2k-1)d_p t\sinh\left((2k-1)d_p\sqrt{2\rho}\ t\right) - (2k-1)^2 d_p^2\sqrt{2\rho}\ \cosh\left((2k-1)d_p\sqrt{2\rho}\ t\right)\right], \quad (65)$$
$$h_\rho(t) = \sum_{k=1}^{2^{p-1}} e^{-(2k-1)^2 d_p^2 \rho}\cosh\left((2k-1)d_p\sqrt{2\rho}\ t\right). \quad (66)$$

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Faouzi Bellili was born in Sbeitla, Kasserine, Tunisia, on June 16, 1983. He received the Diplôme d'Ingénieur in Signals and Systems (with Honors) from Tunisia Polytechnic School in 2007. In December 2009, he obtained his M.Sc. degree, with *exceptional* grade, at the Institut National de la Recherche Scientifique-Énergie, Matériaux, et Télécommunications (INRS-ÉMT), Université du Québec, Montréal, QC, Canada. He is currently persuing his PhD. degree INRS-EMT. His research focuses on statistical signal processing and array processing with an emphasis on parameters estimation for wireless communications. During his Msc., Mr. Bellili has authored/co-authered 6 international journal papers and more than 10 international conference papers. Mr. Bellili is selected by INRS as its candidate for the 2009-2010 competetion of the very prestegious Vanier Canada Graduate Scholarships program. He acts regularly as a reviewer for many international scientific journals and conferences. Mr. Bellili is the recipient of the National grant of excellence from the Tunisian Government.

Alex Stéphenne was born in Quebec, Canada, on May 8, 1969. He received the B.Ing. degree in electrical engineering from McGill University, Montreal, Quebec, in 1992, and the M.Sc. degree and Ph.D. degrees in telecommunications from INRS-Télécommunications, Université du Québec, Montreal, in 1994 and 2000, respectively. In 1999 he joined SITA Inc., in Montreal, where he worked on the design of remote management strategies for the computer systems of airline companies. In 2000, he became a DSP Design Specialist for Dataradio Inc., Montreal, a company specializing in the design and manufacturing of advanced wireless data products and systems for mission critical applications. In January 2001 he joined Ericsson and worked for over two years in Sweden, where he was responsible for the design of baseband algorithms for WCDMA commercial base station receivers. From June 2003 to December 2008, he is still working for Ericsson, but is now based in Montreal, where he is a researcher focusing on issues related to the physical layer of wireless communication systems. He is also an adjunct professor at INRS. His current research interests include wireless channel modeling/characterization/estimation as well as statistical signal processing, array processing and adaptive filtering for wireless telecommunication applications.

Alex has been a member of the IEEE since 1995 and a Senior member since 2006. He is a member of the "Ordre des Ingénieurs du Québec." He has served as a co-chair for the "Multiple antenna systems and space-time processing" track of the 2008-Fall IEEE Vehicular Technology Conference (VTC'08-Fall) in Calgary, as a co-chair of the Technical Program Committee (TPC) for VTC'2006-Fall in Montreal, and as a TPC member for VTC'05-Fall, Stockholm, Sweden. He acts regularly as a reviewer for many international scientific journals and conferences and for the funding organizations NSERC.

Research topics: Wireless Communications, Modeling/Characterization/Estimation of the Wireless Channel, Digital Signal Processing, CDMA and OFDM Radio Interfaces, statistical Signal Processing, Array Processing, Adaptive Filtering, Synchronization, Performance Analysis. Currently, Dr. Stéphenne is with Huawei Technologies Canada, Ottawa.

Sofiène Affes (S'94, M'95, SM'04) received the Diplôme d'Ingénieur in electrical engineering in 1992, and the Ph.D. degree with honors in signal processing in 1995, both from the ´Ecole Nationale Sup´erieure des T´el´ecommunications (ENST), Paris, France.

He has been since with INRS-EMT, University of Quebec, Montreal, Canada, as a Research Associate from 1995 till 1997, then as an Assistant Professor till 2000. Currently he is an Associate Professor in the Wireless Communications Group. His research interests are in wireless communications, statistical signal and array processing, adaptive space-time processing and MIMO. From 1998 to 2002 he has been leading the radio design and signal processing activities of the Bell/Nortel/NSERC Industrial Research Chair in Personal Communications at INRS-EMT, Montreal, Canada. Since 2004, he has been actively involved in major projects in wireless of PROMPT (Partnerships for Research on Microelectronics, Photonics and Telecommunications).

Professor Affes was the co-recipient of the 2002 Prize for Research Excellence of INRS. He currently holds a Canada Research Chair in Wireless Communications and a Discovery Accelerator Supplement Award from NSERC (Natural Sciences & Engineering Research Council of Canada). In 2006, Professor Affes served as a General Co-Chair of the IEEE VTC'2006-Fall conference, Montreal, Canada. In 2008, he received from the IEEE Vehicular Technology Society the IEEE VTC Chair Recognition Award for exemplary contributions to the success of IEEE VTC. He currently acts as a member of the Editorial Board of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and of the *Wiley Journal on Wireless Communications & Mobile Computing*.