Transactions Letters

Exact BER Performance of Asynchronous MC-DS-CDMA over Fading Channels

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***Abstract*—In this contribution an accurate average Bit Error Rate (BER) formula is derived for MC-DS-CDMA in the context of asynchronous transmissions and random spreading sequences. We consider a flat Nakagami-m fading channel for each subcarrier. Our analysis is based on the Characteristic Function (CF) and does not rely on any assumption concerning the statistical behavior of the interference. We develop a new closed-form expression for the conditional CF of the inter-carrier interference and provide a procedure for calculating the exact BER expressed in the form of a single numerical integration. The accuracy of the Standard Gaussian Approximation (SGA) technique is also evaluated. Link-level results confirm the accuracy of the SGA for most practical conditions.**

***Index Terms*—asynchronous MC-DS-CDMA; exact BER evaluation; Nakagami-fading; Characteristic Function.**

I. Introduction

MULTI-Carrier Direct-Sequence Code Division Multiple Access (MC-DS-CDMA) [1], [2] constitutes a particularly attractive design alternative for next-generation wireless communications, since it has numerous reconfigurable parameters, which may be adjusted for the sake of satisfying diverse design goals. The Bit-Error Rate (BER) is considered to be one of the most important performance measures for communication systems and hence it has been extensively studied. Both the Multiple Access Interference (MAI) and the Inter-Symbol Interference (ISI) affect the attainable performance of MC-DS-CDMA systems. Additionally, the MC-DS-CDMA system's performance is also affected by the Inter-Carrier Interference (ICI). When analyzing the BER performance of MC-DS-CDMA systems, the interference sources, namely, the MAI, the ISI and the ICI are commonly assumed to be Gaussian distributed [1], [4]. However, the accuracy of the Gaussian approximation technique depends on the specific configuration of the system. It is widely recognized that the Gaussian approximation techniques become less accurate when a low number of users is supported or when there is a dominant interferer, creating a near-far-scenario [5].

Therefore, the employment of an exact BER analysis dispensing with the previous assumptions concerning the distribution of the interfering sources is desirable. In order to avoid the limited accuracy of the Gaussian approximation of the interference, the BER can be calculated in the transform domain. More specifically, two widely used transforms of the decision variable's Probability Density Function (PDF) are its Fourier and Laplace transforms, corresponding to the Characteristic Function (CF) and the Moment Generating Function (MGF), respectively. The basic philosophy of exact BER calculation is that first the CF or MGF of the decision variable is derived, then the associated inverse transform is performed for calculating the BER, as detailed for example in [6]. Since the decision variable can be exactly described by its CF or MGF, the BER can be accurately evaluated using numerical integration techniques. For this reason, CF- and MGF-based methods have received considerable attention. A saddle point integration-based MGF oriented approach was proposed for computing the error probability of DS-CDMA systems communicating over both Rician [9] and Nakagami-fading [10] channels. This approach has been also applied for studying the performance of MC-DS-CDMA systems using deterministic spreading sequences [11]. The MGF-based method did not provide a closed-form expression of the ICI term. Recently, the CF method was employed to study the BER performance of DS-CDMA using random sequences in both Rayleigh [6] and Nakagami-fading [7],[8] channels. However, to the best of the authors' knowledge, the accurate BER analysis of asynchronous *Nakagami*-faded MC-DS-CDMA using *random* sequences is still an open problem. Hence in this paper, we will derive an accurate BER formula for Nakagami-faded MC-DS-CDMA in the context of asynchronous transmissions and random spreading sequences. Our analysis is based on the CF, and does not rely on any simplifying assumptions concerning the statistical behavior of the interference. A new closed-form expression, rather than an integral [11] is derived for the conditional CF of the ICI.

The outline of the paper is as follows. In Section II

Manuscript received September 4, 2007; revised November 22, 2007 and March 15, 2008; accepted April 21, 2008. The associate editor coordinating the review of this paper and approving it for publication was Y. Ma.

This paper was presented in part at the 6th International Workshop on Multi-Carrier Spread Spectrum (MCSS'07), and IEEE Wireless Communications and Networking Conference (WCNC'08).

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Digital Object Identifier 10.1109/TWC.2010.04.070986

our system and channel model are introduced, followed by the accurate analysis of Section III. The classic Gaussian approximation and our performance comparisons are provided in Section V, before concluding in Section VI.

II. System and Channel Model

A. MC-DS-CDMA Transmitter

We consider an asynchronous MC-DS-CDMA system using BPSK modulation, random spreading sequences and rectangular chip waveforms. The input information sequence of the k-th user is first converted into U parallel data sequences $b_u^k(t)$ for $u = 1, 2, \ldots, U$. The data sequence $b_u^k(t) = \sum_{i=-\infty}^{\infty} b_{u,i}^k \Pi_{T_s}(t - iT_s)$ consists of a sequence of mutually independent symbols of value +1 or -1 with equal probability, where $\Pi_{T_s}(t)$ is the rectangular symbol waveform that is defined over the interval $[0, T_s)$. After serial-to-parallel conversion, the u-th substream BPSK-modulates a subcarrier frequency f_u. Then, the U modulated subcarriers are superimposed, in order to form the complex-valued modulated signal. Finally, the frequency spectrum of the complex signal is spread due to multiplication in time with a spreading code. Therefore, the transmitted signal of the k-th user is given by:

$$s^k(t) = \sum_{u=1}^{U} \sqrt{2\mathcal{P}} b_u^k(t) a^k(t) \cos(2\pi f_u t + \phi_u^k), \qquad (1)$$

where $\mathcal{P}$ represents the transmitted power per subcarrier, while $a^k(t)$ and ϕ_u^k represent, respectively, the spreading-code segment and the phase angle introduced in the carrier modulation process. The spreading sequence can be expressed as $a^k(t) = \sum_{l=-\infty}^{\infty} a_l^k \Pi_{T_c}(t - lT_c)$, where a_l^k assumes values of +1 or -1 with equal probability, while $\Pi_{T_c}(t)$ is the rectangular chip waveform that is defined over the interval $[0, T_c)$, where $T_c = \frac{T_s}{L}$ is the chip duration, while L is the spreading factor.

For MC-DS-CDMA, the modulated subcarriers are orthogonal over the chip duration. Hence, the frequency corresponding to the u-th subcarrier is $f_u = f_p + u/T_c$, where f_p is the fundamental carrier frequency. Hence, the minimum spacing between two adjacent subcarriers equals $1/T_c$, which is a widely used assumption and is the case considered in this paper. When assuming a bandlimited chip waveform (such as square-root raised cosine), a minimum spacing of $2/T_c$ is required to guarantee the avoidance of subcarrier overlap [4].

B. Channel Model

We assume that the channel between the k-th transmitter and the corresponding receiver is a slowly flat fading channel for each subcarrier. We consider both Rayleigh and Nakagami-m fading channels in our study. Note that the Rayleigh distribution can be considered as a special case of the Nakagami-m distribution and in our analysis we will exploit the specific properties of the Rayleigh distribution in order to provide a compact closed-form expression. The channel impulse response for the k-th transmitted signal over the u-th subcarrier is given by

$$H_u^k(t) = h_u^k \delta(t - \tau_k) exp(-j\psi_u^k), \qquad (2)$$

where h_u^k, τ_k and ψ_u^k represent the attenuation factor, delay and phase-shift, respectively. The delay τ_k of the k-th user is assumed to be uniformly distributed over $[0, T_s)$.

The Nakagami-m distribution is a versatile statistical distribution, which is capable of modeling a variety of fading environments, such as land mobile, as well as indoor mobile multipath propagation channels and ionospheric radio links [12]. The PDF of the Nakagami-m fading amplitude is given by:

$$f_h(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} \exp(-\frac{m}{\Omega} x^2), \quad x > 0, \qquad (3)$$

where $\Omega = 2\sigma^2$ is the average fading power. The Nakagami-m fading family includes channels as diverse as the one-sided Gaussian fading associated with $m = 1/2$, the Rayleigh fading having $m = 1$, and no fading, corresponding to $m = \infty$. The distribution of the Nakagami-m fading phase ψ_u^k is assumed to be uniform over $[0, 2\pi)$.

We consider K asynchronous MC-DS-CDMA users, all of whom have the same number of subcarriers U and the same spreading factor L. The average power received from each user is also assumed to be the same. Consequently, the received signal may be written as:

$$r(t) = \sum_{k=1}^{K} \sum_{u=1}^{U} \sqrt{2\mathcal{P}} h_u^k b_u^k(t - \tau_k) a^k(t - \tau_k) \cos(2\pi f_u t + \varphi_u^k) + N(t), \qquad (4)$$

where $\varphi_u^k = \phi_u^k - \psi_u^k - 2\pi f_u \tau_k$, which is assumed to be an i.i.d. random variable having a uniform distribution over $[0, 2\pi)$, while $N(t)$ represents the Additive White Gaussian Noise (AWGN) with zero mean and double-sided power spectral density of $N_o/2$.

III. Accurate Performance Analysis

A. Decision Variable Statistics

Consider using a conventional single-user matched filter for coherently demodulating the desired user's signal. We assume, without loss of generality, that the reference user's index is $k = 1$ and that we have $\tau_1 = 0$, $\mathcal{P} = \Omega = 2$ and $T_c = 1$. The decision variable of the first user over the v-th subcarrier is:

$$Z_v = \int_0^{T_s} r(t) \times a^1(t) \cos(2\pi f_v t + \varphi_v^1) dt \qquad (5)$$

$$= D_v + \sum_{k=2}^{K} I^k + \sum_{k=2}^{K} \sum_{u=1, u \neq v}^{U} I_u^k + N_v, \qquad (6)$$

where $D_v = h_v^1 b_{v,0}^1 L$ is the desired output, I^k is the interference imposed by user k activating the same carrier v, I_u^k is the ICI imposed by the adjacent carriers ($u \neq v$) of user k, and N_v is the noise, which is a zero-mean Gaussian random variable with a variance $\sigma_n^2 = N_o L/4$.

1) Same Subcarrier Interference: The same carrier interference $\sum_{k=2}^{K} I^k$, where $I^k = h_v^k cos(\theta_v^k) W^k$ and $\theta_v^k = \varphi_v^1 - \varphi_v^k$, are identical to the interference that affects DS-CDMA. In their impressive study of random spreading sequences used for DS-CDMA, Lehnert and Pursley [13] simplified the expression of the random variable W^k. For a rectangular chip waveform, the random variable W^k was further simplified by Cheng and Beaulieu in [6] as follows:

$$W^k = P^k\nu_k + Q^k(1-\nu_k) + X^k + Y^k(1-2\nu_k), \quad (7)$$

where ν_k is a random variable (RV) uniformly distributed over $[0,1)$, while P^k and Q^k are symmetric Bernoulli RVs. Furthermore, X^k is a discrete RV that represents the sum of A independent symmetric Bernoulli RVs, where A equals the number of chip boundaries without transitions in User 1's spreading sequence. Y^k is a discrete RV that represents the sum of B independent symmetric Bernoulli RVs, where B equals the number of chip boundaries with transitions in User 1's spreading sequence. The random variables P^k, Q^k, X^k and Y^k conditioned on B are independent. Note that $A+B = L-1$ and the marginal PDFs of X^k and Y^k are given by:

$$P_{X^k}(j) = \begin{pmatrix} A \\ \frac{j+A}{2} \end{pmatrix} 2^{-A}, \quad (8)$$

$$j \in \mathcal{A} = \{-A, -A+2, \ldots, A-2, A\} \quad (9)$$

and

$$P_{Y^k}(j) = \begin{pmatrix} B \\ \frac{j+B}{2} \end{pmatrix} 2^{-B}, \quad (10)$$

$$j \in \mathcal{B} = \{-B, -B+2, \ldots, B-2, B\}. \quad (11)$$

2) Other Subcarrier Interference: The aim of this section is to simplify the expression of the ICI, which is given by $\sum_{k=2}^{K}\sum_{u=1,u\neq v}^{U} I_u^k$, where $I_u^k = h_u^k W_{u-v}^k$. We derive the random variable W_{u-v}^k as follows:

$$W_{u-v}^k = b_{u,-1}^k R_{k,1,u-v}(\tau_k,\theta_u^k) + b_{u,0}^k \hat{R}_{k,1,u-v}(\tau_k,\theta_u^k), \quad (12)$$

where $\theta_u^k = \varphi_v^1 - \varphi_u^k$, $R_{k,1,u-v}(\tau_k,\theta_u^k)$ and $\hat{R}_{k,1,u-v}(\tau_k,\theta_u^k)$ are the partial cross-correlation functions defined by:

$$R_{k,1,u-v}(\tau_k,\theta_u^k) = \int_0^{\tau_k} a^k(t-\tau_k)a^1(t)\cos(2\pi(f_u-f_v)t+\theta_u^k)dt,$$

$$\hat{R}_{k,1,u-v}(\tau_k,\theta_u^k) = \int_{\tau_k}^{T_s} a^k(t-\tau_k)a^1(t)\cos(2\pi(f_u-f_v)t+\theta_u^k)dt.$$

We follow the methodology used in [13] and invoke Equation (23) of [1] to simplify W_{u-v}^k as

$$\begin{aligned} W_{u-v}^k &= \cos(\theta_u^k + \pi\nu_k|u-v|) \\ &\times[(P^k-Q^k)\nu_k \mathrm{sinc}(\nu_k|u-v|) - 2Y^k\nu_k\mathrm{sinc}(\nu_k|u-v|)], \\ &= \cos(\theta_u^k + \pi\nu_k|u-v|)\mathcal{W}_{u-v}^k, \end{aligned} \quad (13)$$

where $(\theta_u^k + \pi\nu_k|u-v|)$ is a RV uniformly distributed over $[0,2\pi)$ and $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$ is the normalized sinc function. Note that X^k does not feature in the formulation of W_{u-v}^k, which will reduce the complexity of the BER calculation.

B. Same Subcarrier Interference Analysis

The next task is the determination of the conditional CF of the interference I^k incurred by user k activating the same subcarrier. The conditional CF $\Phi_{I^k|B}^N$ of the interference I^k encountered for transmission over a Nakagami-m fading channel has been studied in [8] for integer values of m and in [14] for arbitrary values of m. We use Eqs. (17)-(18) of [14] to evaluate $\Phi_{I^k|B}^N$ as follows:

$$\begin{aligned} \Phi_{I^k|B}^N(w) &= \frac{2^{-(N-1)}}{4}\sum_{i\in\mathcal{A}}\sum_{j\in\mathcal{B}}\begin{pmatrix} A \\ \frac{i+A}{2} \end{pmatrix}\begin{pmatrix} B \\ \frac{j+B}{2} \end{pmatrix} \\ &\times\Big[J_1^N(i+1,j)+J_1^N(i,j-1)+J_1^N(i,j+1)+J_1^N(i-1,j)\Big] \end{aligned} \quad (14)$$

where

$$J_1^N(i,j) = \begin{cases} {}_1\mathbb{F}_1\left(m,1,-\frac{i^2}{2m}w^2\right), & j=0 \\ \frac{1}{2j|w|}\{(i-j){}_2\mathbb{F}_2(m,\frac{1}{2};1,\frac{3}{2},-\frac{1}{2m}w^2(i-j)^2) \\ -(i+j){}_2\mathbb{F}_2(m,\frac{1}{2};1,\frac{3}{2},-\frac{1}{2m}w^2(i+j)^2)\}, & (j,w\neq 0) \end{cases}, \quad (15)$$

and ${}_p\mathbb{F}_q$ is the generalized hypergeometric function [15]. For the special case of Rayleigh fading (when $m=1$), we have

$$J_1^R(i,j) = \begin{cases} \exp\left(-\frac{i^2}{2}w^2\right), & j=0 \\ \frac{\sqrt{\frac{\pi}{2}}}{j|w|}\{Q(|w|(i-j)) - Q(|w|(i+j))\}, & (j,w\neq 0) \end{cases}, \quad (16)$$

and Q is the Gaussian Q function.

C. Other Subcarrier Interference Analysis

The only task that remains unsolved is the determination of the conditional CF of the interference I_u^k incurred by user k activating the other subcarriers. From the previous section, we have $I_u^k = h_u^k\cos(\theta_u^k + \pi\nu_k|u-v|)\mathcal{W}_{u-v}^k$, where h_u^k follows the Nakagami-m distribution, $(\theta_u^k + \pi\nu_k|u-v|)$ is the random phase uniformly distributed over $[0,2\pi)$ and $\mathcal{W}_{u-v}^k$ is defined in Eq. (13). Averaging over P_k, Q_k, Y_k (which is equivalent to averaging over all interferers' spreading sequences and data sequences), yields the CF of the ICI I_u^k given ν_k and B in the following form:

$$\Phi_{I_u^k|\nu_k,B}^N(w) = \frac{2^{-B}}{4}\sum_{j\in\mathcal{B}}\begin{pmatrix} B \\ \frac{j+B}{2} \end{pmatrix} \left\{\sum_{l=1,2,3,4} {}_1\mathbb{F}_1\left(m,1,-\frac{1}{2m}\sigma_l^2(j,\nu_k,u-v)w^2\right)\right\}, \quad (17)$$

where

$$\begin{aligned} \sigma_1^2(j,\nu_k,u-v) &= [2j\nu_k\mathrm{sinc}(\nu_k|u-v|)]^2, \\ \sigma_2^2(j,\nu_k,u-v) &= [2j\nu_k\mathrm{sinc}(\nu_k|u-v|)]^2, \\ \sigma_3^2(j,\nu_k,u-v) &= [2(1-j)\nu_k\mathrm{sinc}(\nu_k|u-v|)]^2, \\ \sigma_4^2(j,\nu_k,u-v) &= [2(1+j)\nu_k\mathrm{sinc}(\nu_k|u-v|)]^2. \end{aligned} \quad (18)$$

In the development above, we exploit the results of appendix A in [16]. The variables ν_k now appear in the argument of the function ${}_1\mathbb{F}_1$ and averaging can be carried out by using Eq. (7.512) of [15], yielding the CF of I_u^k given B as:

$$\Phi_{I_u^k|B}^N(w) = \int_0^1 \Phi_{I_u^k|\nu_k,B}^N(w)d\nu_k \quad (19)$$

$$= \frac{2^{-B}}{4}\sum_{j\in\mathcal{B}}\begin{pmatrix} B \\ \frac{j+B}{2} \end{pmatrix} \times\Big[2J_2^N(j)+J_2^N(j-1)+J_2^N(j+1)\Big] \quad (20)$$

where B varies from 0 to $L-1$, and

$$J_2^N(j) = \frac{1}{\pi}\Gamma(0.5)\Gamma(0.5)\ {}_2\mathbb{F}_2\left(\frac{1}{2}, m; 1, 1; -\frac{1}{2m}\frac{4j^2w^2}{\pi^2(u-v)^2}\right). \tag{21}$$

The detailed derivation of the above expressions is provided in the Appendix.

For the Rayleigh case ($m=1$), we have

$$J_2^R(j) = \exp\left(-\frac{j^2}{\pi^2(u-v)^2}w^2\right) I_0\left(-\frac{j^2}{\pi^2(u-v)^2}w^2\right), \tag{22}$$

with I_0 being the zeroth-order modified Bessel function of the first kind.

D. Exact BER Analysis

After evaluating the CF of the total interference, we can derive the average BER under asynchronous MC-DS-CDMA transmission conditions. Indeed, we applied the generalized equation (4) of [14] to calculate the BER for both Rayleigh and Nakagami-m fading. The CF of the total interference over a Nakagami-m fading channel is given by:

$$\Phi_{I|B}^N(w) = \Pi_{k=2}^K\left[\Phi_{I^k|B}^N(w)\Pi_{u=1,u\neq v}^U\Phi_{I_u^k|B}^N(w)\right]. \tag{23}$$

In the developments above, we exploited the fact that we transmit different data sequences over distinct subcarriers for a given user and hence assumed that the cross correlation terms from the different subcarriers are zero.

The conditional BER evaluated for transmission over a Nakagami-m fading channel can be expressed as [16] (assuming m takes only integer values):

$$\begin{aligned} P_{e|B} &= \frac{1}{2^m}\left[1-\frac{L}{\sqrt{L^2+m\sigma_n^2}}\right]^m \sum_{k=0}^{m-1} 2^{-k}\begin{pmatrix} m-1+k \\ k\end{pmatrix} \\ &\times\left[1+\frac{L}{\sqrt{L^2+m\sigma_n^2}}\right]^k + \frac{L}{\pi}\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\sqrt{\frac{2}{m}} \\ &\times\int_0^{+\infty}[1-\Phi_{I|B}^N(w)]\Phi_n(w)\ {}_1\mathbb{F}_1\left(m+\frac{1}{2}, \frac{3}{2}, -\frac{L^2w^2}{2m}\right)dw, \end{aligned} \tag{24}$$

where σ_n^2 is the variance of the background noise and $\Phi_n(w) = \exp(-\frac{\sigma_n^2 w^2}{2})$. Finally, the overall average BER is obtained by averaging $P_{e|B}$ over all spreading sequences:

$$P_e = 2^{-(L-1)}\sum_{B=0}^{L-1}\begin{pmatrix} L-1 \\ B\end{pmatrix} P_{e|B}. \tag{25}$$

IV. Standard Gaussian Approximation

Using the derivations found in [3], the average BER P_e over Nakagami-m fading approximated by the SGA can be shown to be (assuming that m takes only integer values):

$$P_e = \left[\frac{1-\mu}{2}\right]^m \sum_{k=0}^{m-1}\begin{pmatrix} m-1+k \\ k\end{pmatrix}\left[\frac{1+\mu}{2}\right]^k, \tag{26}$$

where

$$\mu = \sqrt{\gamma/(\gamma+m)} \tag{27}$$

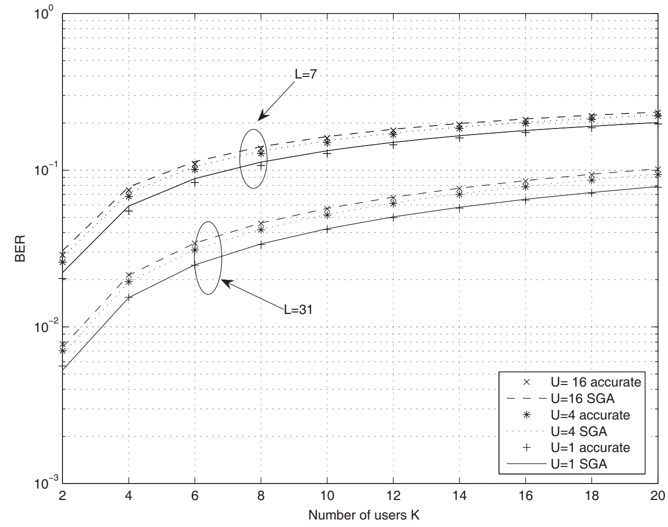

Fig. 1. BER versus the number of users K in an asynchronous MC-DS-CDMA system exposed to Rayleigh fading, using random spreading sequences and BPSK modulation. The length of the spreading sequence is $L=7$ and $L=31$. The number of subcarriers is $U=1,4,$ and 16. The average power of all subcarriers and users is equal and the background noise is ignored.

and

$$\gamma^{-1} = \frac{N_o}{4L} + \frac{2(K-1)}{3L} + (K-1)\left[\frac{1}{U}\sum_{u=1}^U\sum_{u=1,u\neq v}^U\frac{1}{\pi^2(u-v)^2L}\right]. \tag{28}$$

V. Numerical Results

This performance evaluation is not an approximation relying on central limit theorem arguments, but rather an arbitrarily accurate calculation, where the accuracy is controlled by the resolution of the numerical integration in Eq. (24). Section III and Appendix give the proof of our exact BER formula. In this section we will compare the results obtained from our accurate BER analysis to those generated by the SGA. Since the evaluation of the ICI effects on the performance of MC-DS-CDMA is the main objective of our analysis, we assumed first that the effect of noise is negligible. Figs. 1 and 2 show the average BER performance against the number of users in the context of Rayleigh and Nakagami-m ($m=2$) fading, respectively. We observe that the SGA provides good approximation to the accurate BER computed via the characteristic function. For Rayleigh fading, the SGA slightly over-estimates the BER. By contrast, when $m=2$, the SGA under-estimates the average BER. In Fig. 1, we see that the inaccuracy of the SGA becomes prevalent, when the number of users, the spreading factor, and/or the number of subcarriers decreases.

Figs. 3 and 4 illustrate the average BER performance versus the per-bit SNR, when the number of users is $K=4$. It compares the results obtained by the SGA to our accurate BER. Figs. 3 and 4 confirm, not surprisingly, that the presence of strong background noise improves the accuracy of the SGA method.

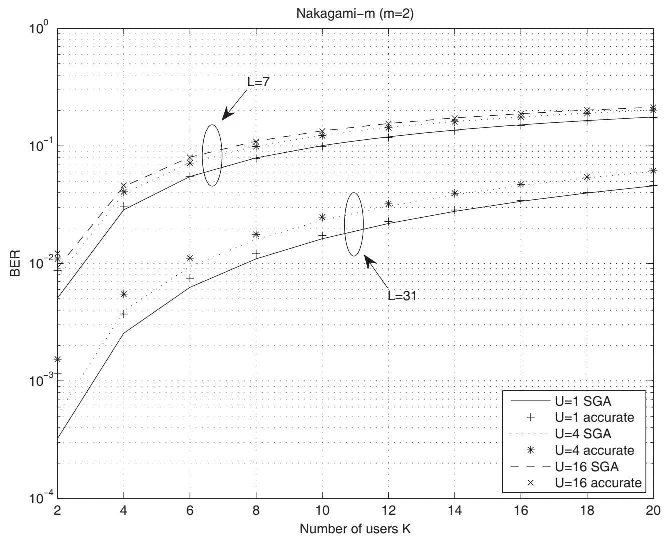

Fig. 2. BER versus the number of users K in an asynchronous MC-DS-CDMA system exposed to Nakagami-m fading ($m = 2$), using random spreading sequences and BPSK modulation. The length of the spreading sequence is $L = 7$ and $L = 31$. The number of subcarriers is $U = 1, 4,$ and 16. The average power of all subcarriers and users is equal and the background noise is ignored.

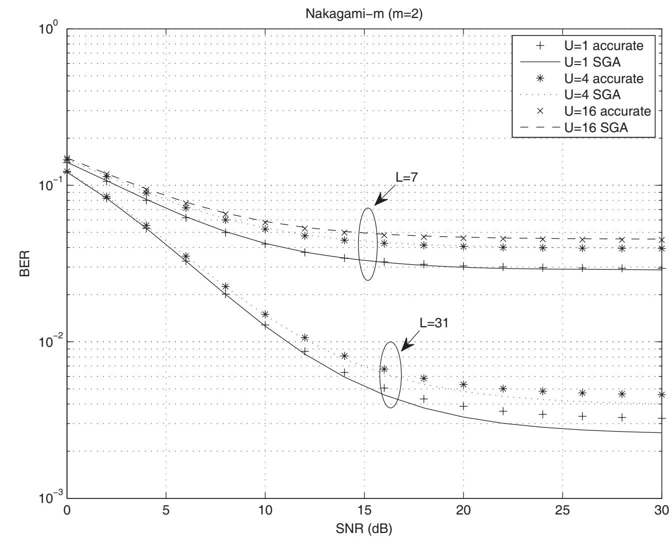

Fig. 4. BER versus the per-bit SNR in an asynchronous MC-DS-CDMA exposed to Nakagami-m fading ($m = 2$), when using random spreading sequences and BPSK modulation. The length of the spreading sequence is $L = 7$ and $L = 31$. The number of subcarriers is $U = 1, 4,$ and 16. The average power of all subcarriers and users is equal. The number of users is $K = 4$.

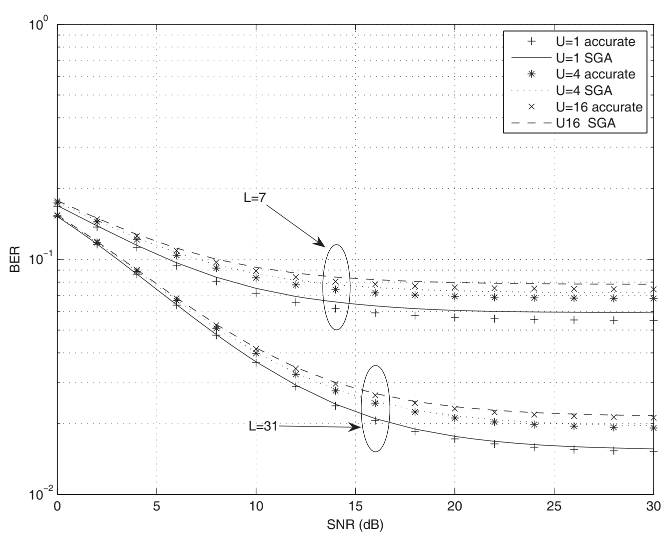

Fig. 3. BER versus the per-bit SNR in an asynchronous MC-DS-CDMA exposed to Rayleigh fading, when using random spreading sequences and BPSK modulation. The length of the spreading sequence is $L = 7$ and $L = 31$. The number of subcarriers is $U = 1, 4,$ and 9. The average power of all subcarriers and users is equal. The number of users is $K = 4$.

VI. Conclusion

We studied the accurate BER calculation of an asynchronous MC-DS-CDMA system using random spreading sequences and BPSK modulation. We considered a flat Nakagami-m fading channel with particular emphasis on the Rayleigh fading case. Using the CF approach, we derived a new closed-form expression for the conditional CF of the ICI and only a single integration was required for the BER calculation. For most practical scenarios the SGA remains fairly accurate, although it slightly over-estimates the average BER when the fading is severe while it under-estimates the average BER when the fading is benign.

Appendix

In this appendix, we derive $J_2^N(j)$ of Eq. (21). For $w \neq 0$, the integration of the first term in Eq. (17) gives

$$\begin{aligned}&\int_0^1 {}_1\mathbb{F}_1\left(m, 1, -\tfrac{1}{2m}\sigma_1^2(j,\nu_k,u-v)w^2\right)d\nu_k\\&= \int_0^1 {}_1\mathbb{F}_1\left(m,1,-\tfrac{1}{2m}[2j\nu_k \mathrm{sinc}(\nu_k|u-v|)]^2 w^2\right)d\nu_k\\&= \int_0^1 {}_1\mathbb{F}_1\left(m,1,-\tfrac{1}{2m}\tfrac{4j^2w^2}{\pi^2(u-v)^2}\sin^2(\pi\nu_k|u-v|)\right)d\nu_k.\end{aligned} \tag{29}$$

Then, we substitute the variable $\pi\nu_k|u-v|$ by θ_k and $[\sin(\theta_k)]^2$ by y as

$$\begin{aligned}&\int_0^1 {}_1\mathbb{F}_1\left(m,1,-\tfrac{1}{2m}\tfrac{4j^2w^2}{\pi^2(u-v)^2}\sin^2(\pi\nu_k|u-v|)\right)d\nu_k\\&= \tfrac{1}{\pi}\int_0^\pi {}_1\mathbb{F}_1\left(m,1,-\tfrac{1}{2m}\tfrac{4j^2w^2}{\pi^2(u-v)^2}\sin^2(\theta_k)\right)d\theta_k,\\&= \tfrac{1}{\pi}\int_0^1 {}_1\mathbb{F}_1\left(m,1,-\tfrac{1}{2m}\tfrac{4j^2w^2}{\pi^2(u-v)^2}y\right)\tfrac{1}{\sqrt{y}\sqrt{1-y}}dy.\end{aligned} \tag{30}$$

Using Eq. (7.512) of [15], we derive the previous integration as follows:

$$\begin{aligned}&\tfrac{1}{\pi}\int_0^1 {}_1\mathbb{F}_1\left(m,1,-\tfrac{1}{2m}\tfrac{4j^2w^2}{\pi^2(u-v)^2}y\right)\tfrac{1}{\sqrt{y}\sqrt{1-y}}dy\\&= \tfrac{1}{\pi}\Gamma(0.5)\Gamma(0.5)\,{}_2\mathbb{F}_2\left(\tfrac{1}{2},m;1,1;-\tfrac{1}{2m}\tfrac{4j^2w^2}{\pi^2(u-v)^2}\right).\end{aligned} \tag{31}$$

Acknowledgments

The financial support of the EPSRC, UK and of the EU under the auspices of the Optimix Project is gratefully acknowledged.

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