Moment-Based SNR Estimation over Linearly-Modulated Wireless SIMO Channels

Alex Stéphenne, Faouzi Bellili, and Sofiène Affes

Abstract—**In this paper, we develop a new method for signal-to-noise ratio (SNR) estimation when multiple antenna elements receive linearly-modulated signals in complex additive white Gaussian noise (AWGN) spatially uncorrelated between the antenna elements. We also derive extensions of other existing moment-based SNR estimators to the single-input multiple-output (SIMO) configuration. The new SIMO SNR estimation technique is non-data-aided (NDA) since it is a moment-based method and does not rely, therefore, on the *a priori* knowledge or detection of the transmitted symbols; it does not require the *a priori* knowledge of the modulation type or order. The new method is shown by Monte Carlo simulations to clearly outperform the best NDA moment-based SNR estimation methods in terms of normalized root mean square error (NRMSE) over QAM-modulated transmissions, namely the M_2M_4 method and the estimators referred to, in this paper, as the GT and the M_6 methods, even when we extend them to the SIMO configuration.**

Index Terms—**SNR, estimation, linear modulations, QAM, SIMO, flat fading, non-data-aided.**

I. Introduction

Many techniques for the optimal usage of radio resources are nowadays based on the *a priori* knowledge of the SNR [1, 2, 3]. For instance, in modern radio communication networks, a relevant concern is to determine to what extent the cell size may be reduced in order to reach higher capacity. The SNR knowledge offers a way to meet this challenge, in addition to offering many other advantages. In fact, SNR estimates can be required to control the power emissions of multiple channels in order to reduce the interference at the reception. SNR estimation is also often a requirement for many other applications such as equalization, handoff, dynamic allocation of resources and adaptive modulation.

Although the first basic SNR estimation algorithms date back to the 1960's [4, 5], tremendous efforts have been spent and reported since on the development of new SNR estimation techniques. This is in part due to the enormous worldwide success of cellular mobile radio systems, which resulted in an exponential growth in demand for wireless communications, and in part due to the rapid advances in microelectronics and microprocessors, which made sophisticated SNR estimation techniques feasible.

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Roughly speaking, estimation methods can be divided into two major categories, depending on whether they base the estimation process on the knowledge of the transmitted symbols or not. Methods that base the estimation only on the received signal and do not need the *a priori* knowledge of the transmitted symbols are called non-data-aided (NDA) methods, while data-aided (DA) methods assume the perfect knowledge of some transmitted symbols (for example, training sequences provided for synchronization and equalization), to facilitate the estimation process. The DA methods have the drawback of limiting the system throughput due to the transmission of known data. A subcategory of DA methods, the decision-directed (DD) techniques, base the estimation process on detected transmitted symbols. They require the transmission of fewer known data symbols but they can suffer from erroneous detections.

SNR estimates can be obtained from the inphase and quadrature (I/Q) components of the received signal or simply from its magnitude (i.e., envelope). They are, respectively, referred to as I/Q-based and envelope-based SNR estimators. So far, for linearly-modulated signals over flat-fading channels in the SISO configuration, various estimation techniques have been reported in the literature. These include the maximum-likelihood (ML) I/Q-based estimator [6, 7] and the ML envelope-based estimator [8]. In both cases, the analytical derivation of the NDA ML estimators was recognized to be mathematically intractable and the numerical computations of the ML SNR estimates were carried out using the iterative expectation-maximization (EM) algorithm. However, simpler and sufficiently accurate SNR estimators are also of practical interest, and as far as we know, the two best NDA techniques that meet this requirement and which are applicable for non-constant envelope modulations are the M_2M_4 method presented in [9], and the method introduced by Gao and Tepedelenlioglu in [10] referred to in this paper as the GT method. Another NDA SNR estimator which is optimally designed for constellations with two different amplitude levels was also recently introduced in [11]. This estimator makes use of the sixth-order moment of the received signal and is therefore referred to, in this paper, as the M_6 method. All three methods were derived for a single-input single-output (SISO) configuration. Therefore, we easily derive three enhanced SIMO versions of these SISO methods whereby we prove that the use of an antenna array results in a remarkable increase in their estimation accuracy. But the main contribution of this paper will be embodied by the development of a much more efficient moment-based SNR estimator for the SIMO configuration in complex additive white Gaussian noise (AWGN) temporally white and spatially uncorrelated between antenna elements[1]. The superiority of this new method will be confirmed by simulations against the enhanced SIMO versions of both the M_2M_4 [9], GT [10] and M_6 [11] methods.

The remainder of this paper is organized as follows. In section II, we begin by introducing the system model. In section III, we briefly introduce the main NDA moment-based SNR estimation methods and derive their extension to the SIMO configuration. Then, in section IV, we derive a new moment-based SIMO SNR estimation method, the main contribution of this paper. In section V, we assess the performance of the different SNR estimators by Monte Carlo simulations and confirm the superiority of the proposed M_4 method against the M_2M_4, GT and M_6 techniques over QAM-modulated transmissions. Finally, section VI concludes the paper.

II. System Model

We consider a digital communication system over a frequency-flat fading SIMO channel. The channel time-variations are assumed to be relatively slow so that the channel can be considered as constant over the estimation interval. We also assume that the noise components at all the N_a antenna elements can be adequately modelled by complex Gaussian variables, of average powers $N_i = 2\sigma_i^2$, which are temporally white and spatially uncorrelated between antenna elements. This is a valid assumption in many practical situations, for example when the noise is associated with a large number of interference sources or interference propagation paths. The thermal noise components that are added upon the arrival of the signal can also typically be assumed uncorrelated across the different antenna branches. Assuming an ideal receiver with perfect time synchronization, and considering the antenna element i ($i = 1, 2, \cdots, N_a$), the discrete input-output baseband relationship can be written as

$$y_i(n) = h_i\, x(n) + w_i(n), \quad n = 1, 2, \cdots, K, \tag{1}$$

where at time index n, $x(n)$ is the n^{th} transmitted linearly-modulated symbol and $y_i(n)$ is the n^{th} sample of the corresponding received signal, for the i^{th} antenna element. $w_i(n)$ is a complex AWGN component with zero mean and variance $N_i = 2\sigma_i^2$ at the same antenna element. Assumed constant during the observation interval and unknown to the receiver, $h_i = R_i e^{j\phi_i}$ is the complex channel coefficient for the antenna branch i, where R_i and ϕ_i stand for the real positive channel gain and the channel phase rotation, respectively. Besides their mutual spatial and temporal decorrelation, the noise components are also supposed to be uncorrelated with the transmitted symbols. Moreover, without loss of generality and for the sake of simplicity, from now on, the power of the constellation is assumed to be normalized to 1, which means $\mathrm{E}\{|x(n)|^2\}$=1, where $\mathrm{E}\{.\}$ denotes expectation.

Then, considering all the antenna branches, (1) can be conveniently rewritten in the following $N_a \times 1$ vector form

$$\mathbf{y}(n) = \mathbf{h}\, x(n) + \mathbf{w}(n), \tag{2}$$

[1]Of course, there is the more general the case where noise components are correlated. This is, however, the topic of a future investigation outside the scope of the current contribution.

where

$$\begin{aligned} \mathbf{y}(n) &= [y_1(n), y_2(n), \ldots, y_{N_a}(n)]^T, & (3)\\ \mathbf{h} &= [h_1, h_2, \ldots, h_{N_a}]^T, & (4)\\ \mathbf{w}(n) &= [w_1(n), w_2(n), \ldots, w_{N_a}(n)]^T. & (5) \end{aligned}$$

In the above mathematical model, the superscript T stands for the transpose operator. Given only the vectorial samples $\mathbf{y}(n)$, for $n = 1, 2 \ldots K$, where K is the size of the estimation interval, our purpose is to estimate the SNR at each antenna element i, which is given by:

$$\begin{aligned} \rho_i &= \frac{|h_i|^2 \mathrm{E}\{|x(n)|^2\}}{N_i}, & (6)\\ &= \frac{|h_i|^2}{N_i}, & (7)\\ &= \frac{P_i}{N_i}, & (8) \end{aligned}$$

where $P_i = |h_i|^2$ is the power of the channel component at the antenna element i.

III. Existing Methods and Their SIMO Extensions

In this section, first we present the classical M_2M_4 method [9], the GT method presented in [10], and the M_6 method recently proposed in [11], before deriving their direct extensions to the SIMO channel configuration when the noise components at all the N_a antenna elements are of equal average power (uniform noise). To the best of our knowledge, the resulting enhanced SIMO versions of these three methods, though straightforward, were first explicitly proposed and assessed in the framework of this work [12].

A. The M_2M_4 method and its SIMO extension

The M_2M_4 method is primarily based on the second- and fourth-order moments of the received signal. It was derived for the SISO configuration. Therefore, considering the i^{th} antenna element, the SNR estimate is given by [9]:

$$\widehat{\rho}_{i;M_2M_4} = \frac{\widehat{P}_{i;M_2M_4}}{\widehat{N}_{i;M_2M_4}}, \tag{9}$$

where

$$\widehat{P}_{i;M_2M_4} = \frac{1}{K_a + K_w - 4}\Big(\widehat{M}_{2;i}(K_w - 2) - \sqrt{(4 - K_a K_w)\widehat{M}_{2;i}^2 + \widehat{M}_{4;i}(K_a + K_w - 4)}\Big), \tag{10}$$

$$\widehat{N}_{i;M_2M_4} = \widehat{M}_{2;i} - \widehat{P}_{i;M_2M_4}, \tag{11}$$

where $\widehat{M}_{2;i} = \frac{1}{K}\sum_{n=1}^{K} |y_i(n)|^2$ and $\widehat{M}_{4;i} = \frac{1}{K}\sum_{n=1}^{K} |y_i(n)|^4$ and K_a and K_w are, respectively, the signal and noise kurtosis which are given by:

$$\begin{aligned} K_a &= \frac{\mathrm{E}\left\{|x(n)|^4\right\}}{(\mathrm{E}\left\{|x(n)|^2\right\})^2}, & (12)\\ K_w &= \frac{\mathrm{E}\left\{|w(n)|^4\right\}}{(\mathrm{E}\left\{|w(n)|^2\right\})^2}. & (13) \end{aligned}$$

It should be noted that $K_w = 2$ for AWGN noise and K_a depends on the modulation type. For instance, $K_a = 1$

for M-ary phase shift keying (PSK) signals and $K_a = 1 + 2/5[1 - 3/(M-1)]$ for M-ary square QAM (quadrature amplitude modulation) signals. It should also be noted that, in our simulation section, $\widehat{P}_{i;M_2M_4}$ was set to zero if it was not found to be real-valued after being computed using (10). However, $\widehat{N}_{i;M_2M_4}$ was set to $\epsilon = 10^{-3}$ if its value computed from (11) was found to be negative. These corrections are required because the estimated moments can be relatively noisy. Note that the signal kurtosis, K_a, which depends on the particular modulation used, is employed in (10) to compute the signal power $\widehat{P}_{i;M_2M_4}$. Thus, the *a priori* knowledge of the modulation type and order is required for the M_2M_4 method in order to estimate the SNR.

Despite the fact that the M_2M_4 method was primarily developed for SISO channels, it is relatively easy to modify so that it exploits the presence of multiple antenna elements in a SIMO scenario when the noise components at all the N_a antenna elements are of equal average power. To do so, we compute $\widehat{N}_i$ over each antenna branch using (11). Then, we average it over all antenna elements to have a more accurate value of the noise power, which is then used to compute the power estimates. We therefore have

$$\widehat{N}_{M_2M_4\_SIMO} = \frac{1}{N_a}\sum_{i=1}^{N_a}\widehat{N}_{i;M_2M_4}, \tag{14}$$

$$\widehat{P}_{i;M_2M_4\_SIMO} = \max\left(0, \widehat{M}_{2;i} - \widehat{N}_{M_2M_4\_SIMO}\right), \tag{15}$$

where the operator $\max\{.\}$ simply returns the maximum value of its arguments. The final SNR estimate, $\widehat{\rho}_{i;M_2M_4\_SIMO}$, over each antenna element i, is consequently given by

$$\widehat{\rho}_{i;M_2M_4\_SIMO} = \frac{\widehat{P}_{i;M_2M_4\_SIMO}}{\widehat{N}_{M_2M_4\_SIMO}}. \tag{16}$$

B. The GT method and its SIMO extension

In this section, we will present the GT method as introduced in [10]. Then we will derive its SIMO extension. Assume that $x(n)_{n=1,2,\cdots,K}$ is transmitted from a constellation that has Q different known amplitudes $A_1, A_2, \cdots, A_Q$ with Q different known probabilities $p_1, p_2, \ldots p_Q$ (e.g., $Q = 1$ for QPSK (quadrature PSK) modulation and $Q = 3$ for 16-QAM modulation). As explained in [10], the k^{th} moment, $M_{k;i}(\sigma^2, \rho_i) = \mathrm{E}\left\{|y_i(n)|^k\right\}$, of the received signal on the i^{th} antenna element is given by

$$M_{k;i}(\sigma^2, \rho_i) = \sum_{q=1}^{Q}\left[p_q(2\sigma^2)^{\frac{k}{2}}\Gamma\left(\frac{K}{2}+1\right)e^{(-\rho_i A_q^2)}\times {}_1F_1\left(\frac{K}{2}+1; 1; \rho_i A_q^2\right)\right], \tag{17}$$

where ${}_1F_1(.)$ and $\Gamma(.)$ are, respectively, the confluent hypergeometric and the gamma functions. From (17), it can be seen that the moments depend on two unknown parameters which are the SNR, ρ_i, and the noise variance $2\sigma^2$. Hence, to be able to estimate the SNR, we need at least estimates of two moments with different orders. For $k \neq l$, consider the following function of ρ_i:

$$f_{k;l}(\rho_i) = \frac{[M_{k;i}(\sigma^2, \rho_i)]^l}{[M_{l;i}(\sigma^2, \rho_i)]^k}. \tag{18}$$

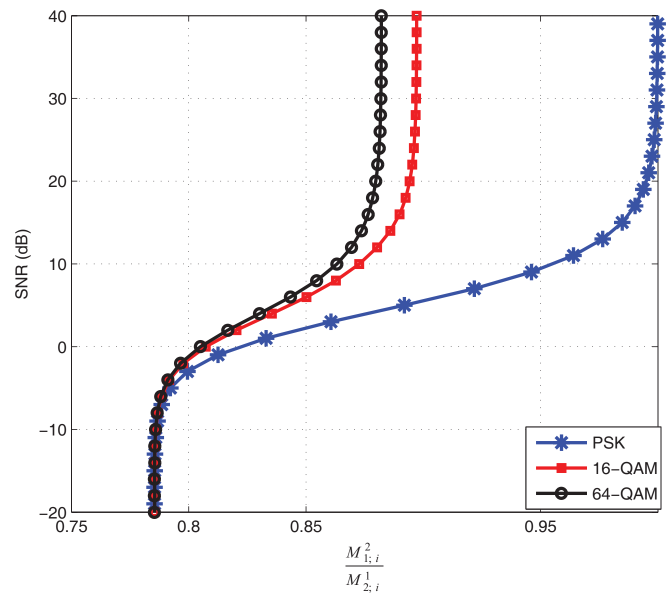

Fig. 1. $\widehat{\rho}_{i;GT_{1,2}} = f^{-1}(\widehat{M}^2_{1;i}/\widehat{M}^1_{2;i})$ for the GT estimator for different modulations.

Straightforward developments using the moments expression given in (17) show that $f_{k;l}$ no longer depends on σ, but only on the SNR ρ_i. Therefore, we can construct moment-based estimators of ρ_i expressed as

$$\rho_{i;GT_{k,l}} = f_{k;l}^{-1}\left(\frac{\widehat{M}^l_{k;i}}{\widehat{M}^k_{l,i}}\right), \tag{19}$$

where $\widehat{M}_{k;i} = \frac{1}{K}\sum_{n=1}^{K}|y_i(n)|^k$ is an estimate of $M_{k;i}$. Although the analytical inversion of $f_{k;l}(.)$ is often intractable, one can implement these estimators, in practice, by lookup tables. These lookup tables would simply consist in a number of samples of the function $f_{k;l}$. It should also be noted that, in our simulations section, the SNR estimates were obtained by linearly interpolating the values in the lookup tables.

Fig. 1 illustrates the function $\widehat{\rho}_{i;GT_{1,2}} = f^{-1}(\widehat{M}^2_{1;i}/\widehat{M}^1_{2;i})$ for PSK, 16-QAM and 64-QAM modulations. The lookup table that is needed to estimate the SNR with the GT method consists simply in a number of samples of the appropriate function for the selected modulation. We see from Fig. 1 that different modulation orders or types result in different lookup tables that will serve as a basis for the SNR estimation. Therefore, as stated before, similarly to the M_2M_4 method, one should have perfect *a priori* knowledge of the modulation type and order before being able to use the GT method.

Likewise, an extension of the SISO GT method to the SIMO configuration can be easily derived. In fact, noticing that $M_{2;i} = \mathrm{E}\{|y_i(n)|^2\} = P_i + N_i$ and using (8), we can simply write

$$\rho_i = \frac{M_{2;i} - N_i}{N_i}, \tag{20}$$

so that

$$N_i = \frac{M_{2;i}}{\rho_i + 1}. \tag{21}$$

Then, assuming that the noise components have the same average power, and averaging over all the antenna elements, we can obtain a more accurate estimate of the noise power as

$$\widehat{N}_{GT\_SIMO_{k,l}} = \frac{1}{N_a}\sum_{i=1}^{N_a} \frac{\widehat{M}_{2;i}}{\widehat{\rho}_{i;GT_{k,l}}+1}. \quad (22)$$

An estimate of each antenna element SNR can now be obtained by injecting (22) in (20), so that

$$\widehat{\rho}_{i;GT\_SIMO_{k,l}} = \max\left(0, \frac{\widehat{M}_{2;i} - \widehat{N}_{GT\_SIMO_{k,l}}}{\widehat{N}_{GT\_SIMO_{k,l}}}\right). \quad (23)$$

C. The M_6 method and its SIMO extension

The method recently derived in [11] and referred to in this paper as the M_6 method makes use of the sixth-order moment of the received signal. It is also a SISO method and can be therefore used antenna per antenna in any SIMO configuration. In fact, on each antenna element i, considering the normalized SNR, $z_i = \rho_i/(\rho_i+1)$, it was shown in [11] that:

$$\frac{D_i}{M_{2;i}^3} = (c_6 - 9c_4 + 12)z_i^3 + (9-b)(c_4-2)z_i^2, \quad (24)$$

where b is a coefficient that can be tuned depending on the constellation to reduce the variance of the estimator, c_p are the constellation moments denoted as $c_p = \mathrm{E}[|x(n)|^p]$ and D_i is given by:

$$D_i = M_{6;i} - 2(3-b)M_{2;i}^3 - bM_{2;i}M_{4;i}. \quad (25)$$

In (25), $M_{6;i}$ is the sixth-order moment of the received signal which can be estimated by:

$$\widehat{M}_{6;i} = \frac{1}{K}\sum_{n=1}^{K} |y_i(n)|^6. \quad (26)$$

Then solving for z_i from (24), one can obtain an SNR estimator as

$$\widehat{\rho}_{i;M6} = \frac{\hat{z}_i}{1-\hat{z}_i}. \quad (27)$$

Note also that the M_6 estimator relies on the *a priori* knowledge of the modulation type and order as the constellation moments c_4 and c_6 are involved in (24).

With simple manipulations, this method can also be extended to SIMO configurations. In fact, taking into account the fact that $M_{2;i} = S_i + N_i$ and using the following relations:

$$\gamma = (c_6 - 6) - b(c_4 - 2), \quad (28)$$
$$\alpha = (9-b)(c_4-2), \quad (29)$$

it can be shown that:

$$D_i = \gamma M_{2;i}^3 + (\alpha - 3\gamma)M_{2;i}^2 N_i + (3\gamma - 2\alpha)M_{2;i}N_i^2 + (\alpha - \gamma)N_i^3. \quad (30)$$

Therefore, resolving for N_i from (30), one can obtain an estimate, $\widehat{N}_{i;M6}$, of the noise power on each antenna element i. Then, in the presence of uniform noise across the antenna elements (i.e., $N_i = N$ for $i = 1, 2, \cdots, N_a$), the obtained noise power estimates can be averaged over the different antenna branches to yield a more accurate estimate of the mutual noise power

$$\widehat{N}_{M6\_SIMO} = \frac{1}{N_a}\sum_{i=1}^{N_a} \widehat{N}_{i;M6}, \quad (31)$$

and consequently the useful signal power on the i^{th} antenna branch is also more accurately estimated as

$$\widehat{S}_{i;M6\_SIMO} = \widehat{M}_{2;i} - \widehat{N}_{M6\_SIMO}. \quad (32)$$

Finally, the enhanced SIMO M_6 per-antenna SNR estimates are given by:

$$\widehat{\rho}_{i;M6\_SIMO} = \frac{\widehat{S}_{i;M6\_SIMO}}{\widehat{N}_{M6\_SIMO}}. \quad (33)$$

Recall that the M_2M_4, GT and M_6 methods were originally derived for SISO systems and that they can be hence applied antenna per-antenna in SIMO configurations without, however, any exploitation of the very rich spatial diversity offered by such systems. Their enhanced SIMO versions, as derived in this paper, offer one way to take *partially* advantage of this diversity, but only in the case where the noise, even if spatially correlated, has uniform power across the antenna elements. In the next section, we will derive a new SIMO SNR estimation method that *fully* exploits the spatial diversity even when the noise power is not equal over all the antenna branches.

IV. New SIMO SNR Estimation Method

Unlike the previously introduced SNR estimation methods, the main idea of the new SNR estimation technique is to *fully* exploit the correlation of the received signal between the different antenna pairs. It is an NDA method and therefore does not impinge upon the channel throughput. It is based on the following fourth-order moment:

$$\bar{M}_{4;i,k} = E[y_i(n+1)y_i(n)^* y_k(n+1)^* y_k(n)]. \quad (34)$$

Since the method presented in this section depends exclusively on a fourth-order moment of the received signal, we will simply refer to it as the M_4 method. Moreover, due to the temporal whiteness of the noise model, it can be shown that (34) reduces simply to

$$\bar{M}_{4;i,k} = |M_{2;i,k}|^2, \quad (35)$$

where $M_{2;i,k}$ is a second-order I/Q-based moment given by

$$M_{2;i,k} = \mathrm{E}[y_i(n)y_k^*(n)], \quad (36)$$
$$= \begin{cases} P_i + N_i, & \text{if } i = k \\ h_i h_k^*, & \text{otherwise.} \end{cases} \quad (37)$$

Therefore, our M_4 SNR estimator is built upon the squared modulus of an I/Q-based second-order moment of the received signal and does not, hence, completely discard the phase information, contrarily to the previous M_2M_4, GT and M_6 approaches which rely on the envelope-based moments of the received signal. Note also that the new M_4 SNR estimator is robust to any frequency offset with slow time variations relative to the symbol period.

A. The M_4 estimator for nonuniform noise power across the antenna elements

Here, we assume that the antenna elements exhibit different noise powers $\{N_i\}_{i=1,2,\cdots,N_a}$ across antennas. Moreover, assuming the noise components to be temporally white and spatially uncorrelated and statistically independent from the transmitted data, the expression for $\bar{M}_{4;i,k}$ simply reduces to

$$\bar{M}_{4;i,k} = \begin{cases} (P_i + N_i)^2, & \text{if } i = k \\ P_i P_k, & \text{otherwise.} \end{cases} \quad (38)$$

And one can conveniently write in a matrix form

$$\bar{\mathbf{M}}_4 = E\left\{[\mathbf{y}(n+1) \odot \mathbf{y}(n)^*][\mathbf{y}(n+1) \odot \mathbf{y}(n)^*]^H\right\}, \quad (39)$$

$$= \begin{pmatrix} (P_1+N_1)^2 & P_1P_2 & \cdots & P_1P_{N_a} \\ P_2P_1 & (P_2+N_2)^2 & \cdots & P_2P_{N_a} \\ \vdots & \vdots & \ddots & \vdots \\ P_{N_a}P_1 & P_{N_a}P_2 & \cdots & (P_{N_a}+N_{N_a})^2 \end{pmatrix}. \quad (40)$$

In the case of nonuniform noise, it can be seen from (40) that there are $\frac{N_a(N_a+1)}{2}$ independent equations that can be exploited to find the $2N_a$ unknowns, $\{P_i\}_{i=1}^{N_a}$ and $\{N_i\}_{i=1}^{N_a}$, involved in the estimation of the per-antenna SNRs $\{\rho_i\}_1^{N_a}$. Therefore, to ensure the identifiability of the system, the number of antenna branches N_a must verify the following inequality:

$$\frac{N_a(N_a+1)}{2} \geq 2N_a. \quad (41)$$

Therefore the minimum antenna-array size necessary to estimate all the per-branch SNRs, using our M_4 estimator must be at least equal to three (i.e., $N_a \geq 3$).

Now resolving for P_1, P_2 and P_3 from the first three off-diagonal entries of (40), i.e., $\widehat{\bar{M}}_{4;1,2}$, $\widehat{\bar{M}}_{4;1,3}$ and $\widehat{\bar{M}}_{4;2,3}$, one can find the following results:

$$\widehat{P}_{1;M_4} = \sqrt{\frac{\widehat{\bar{M}}_{4;2,1}\widehat{\bar{M}}_{4;1,3}}{\widehat{\bar{M}}_{4;2,3}}}, \quad (42)$$

$$\widehat{P}_{2;M_4} = \sqrt{\frac{\widehat{\bar{M}}_{4;1,2}\widehat{\bar{M}}_{4;2,3}}{\widehat{\bar{M}}_{4;1,3}}}, \quad (43)$$

$$\widehat{P}_{3;M_4} = \sqrt{\frac{\widehat{\bar{M}}_{4;1,3}\widehat{\bar{M}}_{4;3,2}}{\widehat{\bar{M}}_{4;1,2}}}, \quad (44)$$

where $\{\widehat{P}_{i;M_4}\}_{i=1}^3$ are the estimates of $\{P_i\}_{i=1}^3$ and $\{\widehat{\bar{M}}_{4;i,k}\}_{i,k=1}^{N_a}$ are the estimates of $\{\bar{M}_{4;i,k}\}_{i,k=1}^{N_a}$ simply given by:

$$\widehat{\bar{M}}_{4;i,k} = \frac{\sum_{n=1}^{K-1} \Re[y_i(n+1)y_i^*(n)y_k^*(n+1)y_k(n)]}{K-1}, \quad (45)$$

where $\Re[.]$ returns simply the real part of any complex argument. Then the remaining useful signal powers $\{P_1\}_{i=4}^{N_a}$ can be estimated from the remaining off-diagonal elements as follows:

$$\widehat{P}_{i;M_4} = \frac{1}{i-1}\sum_{k=1}^{i-1} \frac{\widehat{\bar{M}}_{4;i,k}}{\widehat{P}_{k;M_4}}, \quad i = 4, 5, \cdots, N_a. \quad (46)$$

The estimates, $\{\widehat{N}_{i;M_4}\}_{i=1}^{N_a}$, of the different noise powers, $\{N_i\}_{i=1}^{N_a}$, can be directly deduced from the diagonal elements of (40) as follows:

$$\widehat{N}_{i,M_4} = \sqrt{\widehat{\bar{M}}_{4;i,i}} - \widehat{P}_{i;M_4}, \quad i = 1, 2, \cdots, N_a. \quad (47)$$

Finally, the estimates of the per-antenna SNRs, which are provided by the new M_4 method are given by:

$$\widehat{\rho}_{i;M_4} = \frac{\widehat{P}_{i;M_4}}{\widehat{N}_{i;M_4}}, \quad i = 1, 2, \cdots, N_a. \quad (48)$$

B. The M_4 estimator for uniform noise power across the antenna elements

In this section, we assume that the noise power is the same over all the antenna elements, a situation that is also frequently encountered in practice. Therefore, assuming $\{N_i = N\}_{i=1}^{N_a}$, $\bar{M}_{4;i,k}$ is given by

$$\bar{M}_{4;i,k} = \begin{cases} (P_i + N)^2, & \text{if } i = k \\ P_i P_k, & \text{otherwise.} \end{cases} \quad (49)$$

Hence, in this case, the matrix $\bar{\mathbf{M}}_4$ becomes

$$\bar{\mathbf{M}}_4 = \begin{pmatrix} (P_1+N)^2 & P_1P_2 & \cdots & P_1P_{N_a} \\ P_2P_1 & (P_2+N)^2 & \cdots & P_2P_{N_a} \\ \vdots & \vdots & \ddots & \vdots \\ P_{N_a}P_1 & P_{N_a}P_2 & \cdots & (P_{N_a}+N)^2 \end{pmatrix}. \quad (50)$$

In the case of uniform noise power, it can be seen from (50) that there are $\frac{N_a(N_a+1)}{2}$ independent equations that can be used to find the N_a+1 unknowns, $\{P_i\}_{i=1}^{N_a}$ and N, involved in the estimation of the per-antenna SNRs $\{\rho_i\}_1^{N_a}$. Therefore, to ensure the identifiability of the system, the number of antenna branches N_a must verify the following inequality:

$$\frac{N_a(N_a+1)}{2} \geq N_a + 1 \;\Rightarrow\; (N_a - 2)(N_a + 1) \geq 0$$
$$\Rightarrow\; N_a \geq 2.$$

On the other hand, from (49), we have

$$P_i = \sqrt{\bar{M}_{4;i,i}} - N. \quad (51)$$

Substituting (51) in the off-diagonal elements of (50), we obtain

$$\bar{M}_{4;i,k} = \left(\sqrt{\bar{M}_{4;i,i}} - N\right)\left(\sqrt{\bar{M}_{4;k,k}} - N\right), \quad \text{for i} \neq k. \quad (52)$$

Resolving (52) for N and taking the negative root so that P_i given by (51) is positive, one can find

$$N = \frac{1}{2}\left(\sqrt{\bar{M}_{4;i,i}} + \sqrt{\bar{M}_{4;k,k}} - \sqrt{\left(\sqrt{\bar{M}_{4;i,i}} - \sqrt{\bar{M}_{4;k,k}}\right)^2 + 4\bar{M}_{4;i,k}}\right). \quad (53)$$

In practice, $\bar{M}_{4;i,k}$, which should be real and non-negative, is unknown and should be estimated by simple sample averaging. Its estimate $\widehat{\bar{M}}_{4;i,k}$ is therefore given by

$$\widehat{\bar{M}}_{4;i,k} = \frac{\max\left(0, \sum_{n=1}^{K-1} \Re[y_i(n+1)y_i^*(n)y_k^*(n+1)y_k(n)]\right)}{K-1}. \quad (54)$$

As mentioned before, (53) is valid only for all $i \neq k$ antenna element pairs, and there are $\frac{1}{2}N_a(N_a-1)$ such pairs. Hence, to obtain a more accurate value of the noise power estimate $\widehat{N}$, which is assumed to be positive, we can average over all the pairs and use the following expression:

$$\widehat{N}_{M_4} = \frac{1}{N_a(N_a-1)} \times \max\left(0, \sum_{i=1}^{N_a}\sum_{k>i}^{N_a}\left[\sqrt{\widehat{M}_{4;i,i}}+\sqrt{\widehat{M}_{4;k,k}} - \sqrt{\left(\sqrt{\widehat{M}_{4;i,i}}-\sqrt{\widehat{M}_{4;k,k}}\right)^2+4\widehat{M}_{4;i,k}}\right]\right). \quad (55)$$

Once the noise power is estimated, the signal power over each antenna element, i.e., P_i for $i = 1, 2, \ldots, N_a$, which should be always real and non-negative, can be estimated using (51), so that we have

$$\widehat{P}_{i;M_4} = \max\left(0, \sqrt{\widehat{M}_{4;i,i}} - \widehat{N}_{M_4}\right). \quad (56)$$

Finally our new SNR estimates (one per antenna element i) are given by:

$$\widehat{\rho}_{i;M_4} = \frac{\widehat{P}_{i;M_4}}{\widehat{N}_{M_4}}. \quad (57)$$

With this new SNR estimation technique, note that the same equations are used regardless of the modulation type or order. Therefore, our new method does not require the *a priori* knowledge of the modulation type or order to estimate the SNR, in contrast to other NDA estimation methods. This is because the receiver has more than one single-receiving antenna element. SIMO configurations provide indeed multiple independent noisy measurements of the same input $x(n)$ from which much more information about $x(n)$ can be extracted. They hence offer a precious degree of flexibility for applications that implement adaptive modulation.

V. Simulations

In this section, under complex-valued baseband equivalent channels, we will assess the performance of our new M_4 SNR estimation method and compare it with the performance of the M_2M_4, GT and M_6 methods. Monte Carlo simulations will be run over 10000 realizations. The normalized root mean square error (NRMSE), defined in (58), will be used as a performance measure for the different estimators:

$$\text{NRMSE}(\hat{\rho}_i) = \frac{\sqrt{\text{E}\{(\rho_i - \hat{\rho}_i)^2\}}}{\rho_i}. \quad (58)$$

We use also, unless specified otherwise, an array of $N_a = 3$ receiving antenna elements and choose an observation interval of $K = 2000$ symbols. For the sake of accuracy, the required lookup table for the GT method, used in our simulations, was chosen very large (with SNR ranging from -100 dB to 100 dB in a step of 0.02 dB). Linear interpolation is then used for the numerical inversion of the GT function, to ensure fairness in the comparison. It should be noted that, in all the presented figures, GT12 refers to the GT estimator that is built upon the first- and second-order moments of the received signal.

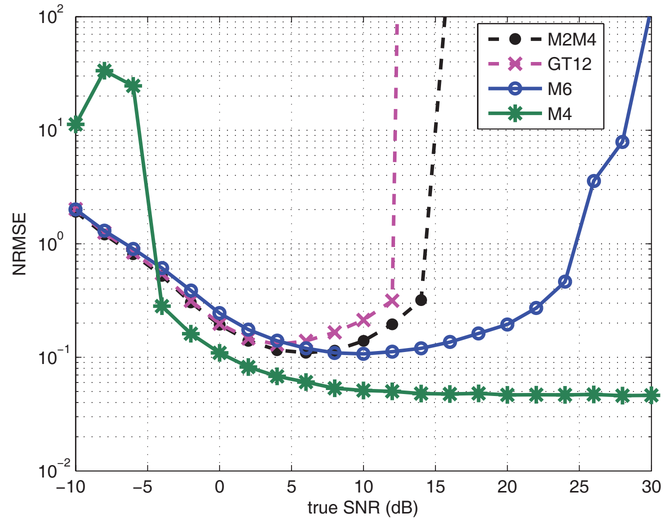

Fig. 2. SNR NRMSE on one of the 3 antennas, nonuniform noise power across the antenna branches, 16-QAM.

To begin with, Fig. 2 illustrates the performance behavior of the different considered estimators for 16-QAM constellation, when the antenna elements experience different noise powers. The M_2M_4, GT and M_6 methods are hence applied in their original SISO versions antenna-wise. It is seen that the M_6 estimator outperforms the classical M_2M_4 and the GT methods when the SNR exceeds 5dB, but the three methods fail to estimate the SNR when it becomes relatively high. However, the M_4 estimator clearly outperforms all these methods starting from $\rho = -4$ dB.

In the following, in order to compare the performance behavior of the M_4 estimator against the enhanced SIMO versions of the other methods, the noise power will be assumed to be the same across the antenna elements, a condition that is necessary only for the SIMO versions of the M_2M_4, GT and M_6 methods.

A. *Modulated received signal components of equal instantaneous power at all antenna elements*

The transmitted signal reaches the mobile via one or more unresolvable paths after scattering and multiple reflections with structures such as buildings. The resulting paths arrive with different angles at the different antenna elements. In this subsection, we suppose that the angle spread is very small so that the received signal experiences the same attenuation at the different antenna branches. The channel coefficients therefore have exactly the same magnitude, hence the same attenuation factor, over the estimation interval. They differ only in terms of the phase rotation. Under these assumptions, the different antenna branches experience exactly the same SNR.

Fig 3 depicts the performance behavior of the different estimators in the presence of uniform noise over all the diversity branches, still with 16-QAM. It can be seen that the existing SISO SNR estimators exhibit the same performance in the low SNR region. However, comparing these SISO estimators and their enhanced SIMO versions, we notice that the use of an antenna array leads to a remarkable increase in estimation accuracy over all the SNR values. Yet, both SISO and SIMO versions are unable to provide sufficiently

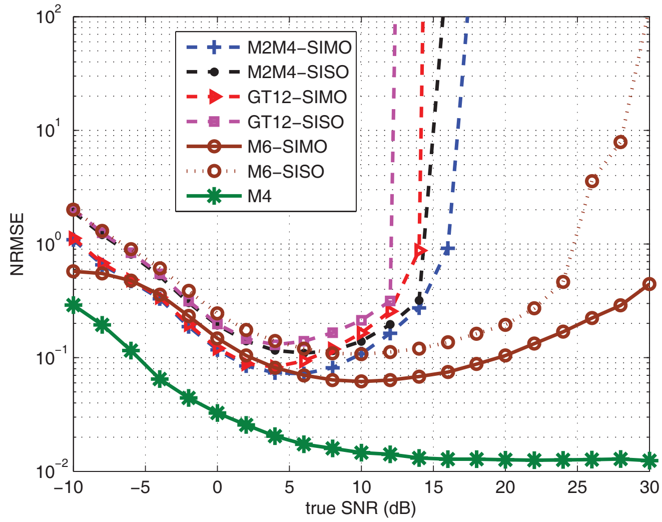

Fig. 3. SNR NRMSE on one of the 3 antennas with the same experienced SNR, uniform noise power across the antenna branches, 16-QAM.

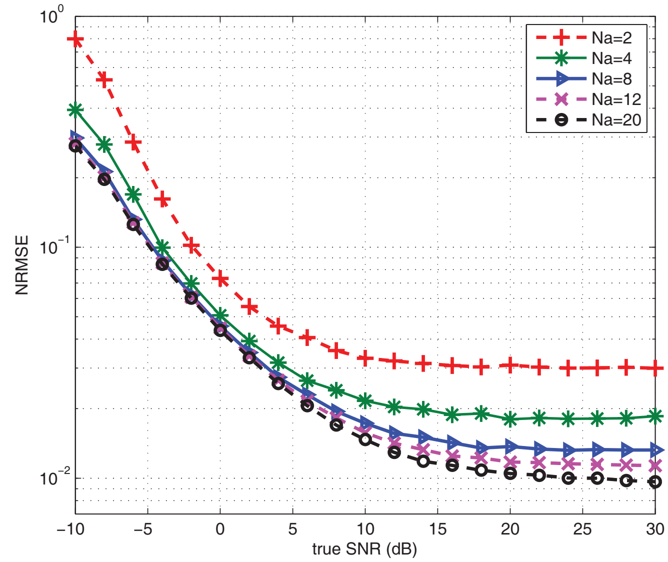

Fig. 4. SNR NRMSE of M_4 for different numbers of antenna elements when all the antenna elements experience the same SNR, 16-QAM.

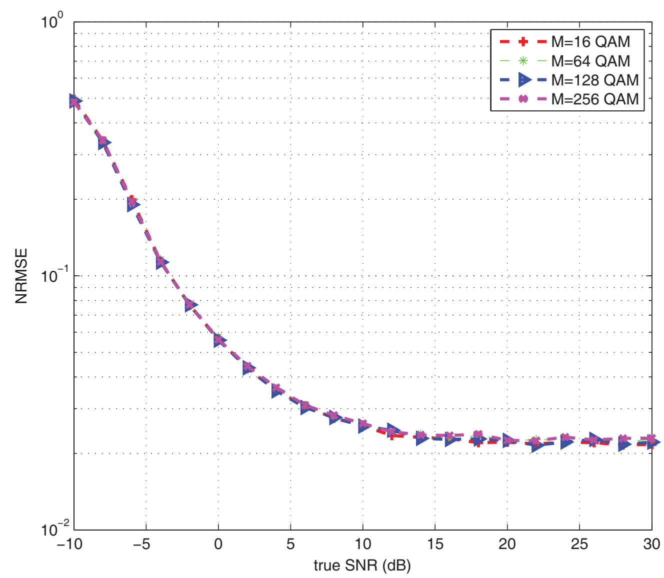

Fig. 5. SNR NRMSE of M_4 on one of the 3 antennas with the same experienced SNR for different modulation orders.

accurate estimates for high SNR values. This is because it is quite difficult to estimate the power of the weakest signal among two signals added together, especially, when the power difference between them increases. In fact, the estimation error of the desired power strongly corrupts the SNR estimate at low SNRs and so does the estimation error of the noise power at high SNR values. Consequently, since the noise power appears at the denominator, the effect of the estimation error is more pronounced at high SNR values. In contrast, the new M_4 estimator effectively reduces the noise estimation error by considering cross-moments over multiple antennas at a non-zero lag. Therefore, as it can be seen from Fig. 3, the new M_4 estimator stands out as the best estimator over the entire SNR range and is more reliable to accurately estimate the SNR even when it is high. But ultimately, for higher SNR values, the SNR estimate, even with our method, will break down[2]. This is hardly surprising since dividing by the estimate of an additive component that tends to zero, when the overall signal power remains approximately the same, will eventually result in numerical instability. With our M_4 method, the NRMSE does become high only for extremely high SNR values (i.e., above 140 dB).

The impact of increasing the antenna-array size, N_a, on the performance of our new M_4 method is also depicted in Fig. 4 where it is seen that, as expected, the estimation accuracy increases as the number of antenna elements increases. Moreover, beyond $N_a = 2$, the new estimator exhibits almost the same performance at low SNR values and increasing the number of receiving antennas results in a slight advantage over the medium and high SNR ranges. Thus, our M_4 method properly exploits spatial diversity with a small antenna-array size.

Fig. 5 depicts the performance of our new M_4 estimator on one of the 3 antenna elements experiencing the same SNR for common modulation orders (16, 64, 128 and 256-QAM). It clearly shows the robustness of the new method to an increasing modulation order since exactly the same performance is obtained. Therefore, it is well geared for integration in next-generation wireless transceivers that transmit data at high bit rates using adaptive modulation schemes. The performance of our estimator will be subsequently compared to the M_2M_4 and GT estimators, for 64-QAM, in a fading propagation environment.

B. Modulated received signal components of unequal average power at the antenna elements

In wireless communications, SIMO channels would be fading and the SNR is not therefore expected to be the same on all the antenna elements as it was assumed in the previous simulation scenarios. Hence, we consider now a more general case in which the received signal is assumed to have been affected by different attenuations at the different antenna elements. But we still assume that the channel corresponding to a given

[2]This was validated by simulations but the results were not included in this paper for the sake of briefness and lack of space.

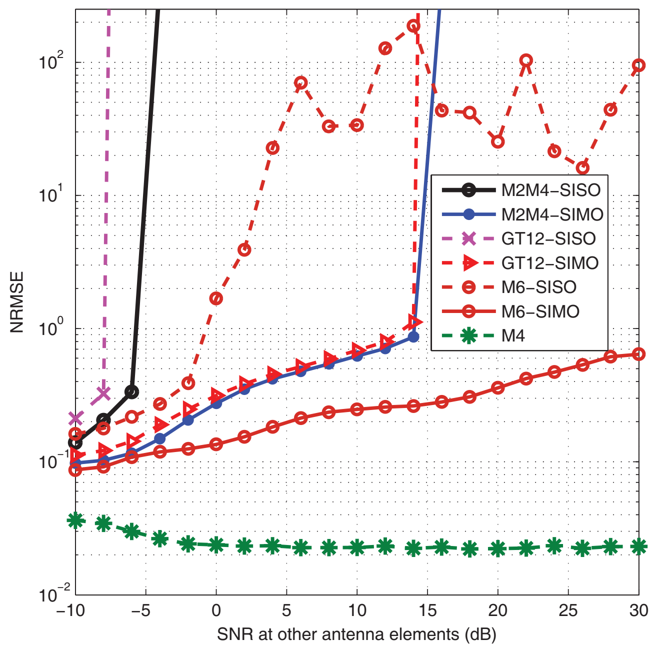

Fig. 6. SNR NRMSE on the high SNR antenna element, 64-QAM.

antenna element is time-invariant over the observation interval. The next considered scenario will suppose that one of the 3 antennas has 20 dB higher SNR than the SNR experienced on the other branches.

Fig. 6 illustrates the estimated NRMSE on the antenna element that experiences high SNR, as a function of the true SNR on the other antennas. We notice again, that the SIMO versions of the M_2M_4 and GT methods are better than their SISO versions for all the considered SNR values. However, they both appear to be unable to estimate the SNR when it exceeds about 14 dB. We see also that the M_6 estimator provides improved performance over the M_2M_4 and GT methods. However, it becomes unstable as the SNR becomes relatively high, where its enhanced SIMO version turns out to be more stable. In contrast, it is obvious that the new M_4 method stands out as the the best technique since it estimates the SNR more accurately over the entire range.

Fig. 7 illustrates the NRMSE at one of the antenna elements with equal SNR, still for 64-QAM. We notice, in this case, that the SISO versions are better for SNR values in the medium range (from -4 dB to 12 dB). This is due to the contribution of the noise estimate associated with the high SNR antenna element which is very inaccurate, as it can be seen in Fig. 6. Both the SISO and SIMO versions of the classical M_2M_4 and GT methods fail to estimate the SNR beyond about 12 dB. However, the M_4 method still has the most satisfactory behavior over the entire range, with clearly superior performance.

Note that similar performance improvements were observed for other modulation orders like 128 and 256-QAM, but were not included here for lack of space. Finally, we can state that in all considered cases, the new M_4 estimator clearly stands out as the best method and the only viable alternative for NDA SNR estimation over a wide SNR range in linearly-modulated

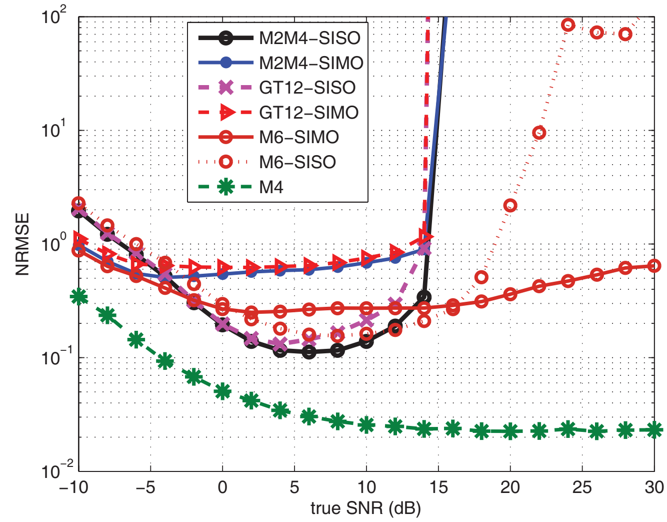

Fig. 7. SNR NRMSE on one of the low SNR antenna elements, 64-QAM.

transmissions[3].

VI. Conclusion and Discussion

In this paper, we derived a new SIMO SNR estimator for arbitrary linear modulations, named M_4. This new method exploits the space diversity provided by SIMO wireless communication systems and it is a low-complexity but efficient technique for per-antenna NDA SNR estimation. It assumes that the noise components at all the receiving antenna elements can be adequately modelled by complex Gaussian variables, which are temporally white and spatially uncorrelated between antenna elements. Its performance was compared with those achieved by the most accurate existing NDA moment-based SNR estimation methods, namely the M_2M_4, GT and M_6 methods, and their new SIMO extensions developed in this paper. The clear superiority of the new M_4 method against these three methods was unambiguously demonstrated over QAM-modulated transmissions. One additional advantage of the proposed M_4 estimator is that it does not require the *a priori* knowledge of the modulation type or order, making it well suited for next-generation or current wireless systems that implement adaptive modulation. Although the new M_4 method was presented in this paper in the context of multi-antenna systems, it is worth nothing that it could be directly applied to other systems having other types of diversity, besides or in addition to spatial diversity. For instance, the new method is directly applicable to SISO CDMA systems with path diversity, as managed, for example, by RAKE receivers. In that context, one would simply consider SNR per path instead of SNR per antenna element. It could also be applicable to a multi-carrier SIMO system over multiple adjacent subcarriers, with a per-subcarrier SNR instead of a per-antenna SNR.

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[3]Note also that Monte Carlo simulations show that our M_4 estimator exhibits a lower bias compared to the other considered methods. But these results were not included here for lack of space.

REFERENCES

[1] D. G. Brennan, "Linear diversity combining techniques," *Proc. IRE,* vol. 47, pp 1075-1102, June 1959.

[2] J. Hagenauer and P. Hoeher, "A Viterbi algorithm with soft-decision outputs and its applications," in *Proc. IEEE Global Telecommunications Conf.*, pp. 1680-1686, Nov. 1989.

[3] P. A. Wintz and E. J. Luecke, "Performance of optimum and suboptimum synchronizers," *IEEE Trans. Commun.*, vol. 17, pp. 380-389, June 1969.

[4] N. Nahi and R. Gagliardi, "On the estimation of signal-to-noise ratio and application to detection and tracking systems," University of Southern California, Los Angeles, EE Report 114, July 1964.

[5] T. Benedict and T. Soong, "The joint estimation of signal and noise from sum envelope," *IEEE Trans. Inf. Theory,* vol. 13, pp. 447-457, July 1967.

[6] A.Wiesel, J. Goldberg, and H. Messer, "Non-data-aided signal-to-noise ratio estimation," in *Proc. IEEE Inter. Conf. on Commun.*, vol. 1, pp. 197-201, Apr. 2002.

[7] A. Das (Nandan), "NDA SNR estimation: CRLBs and EM based estimators," in *Proc. IEEE TENCON'08*, pp. 1-6, Nov. 2008.

[8] R. Lopez-Valcarce, C. Mosquera, and W. Gappmair, "Iterative envelope-based SNR estimation for nonconstant modulus constellations," in *Proc. IEEE SPAWC'07,* pp. 1-5, June 2007.

[9] D. R. Pauluzzi and N. C. Beaulieu, "A comparison of SNR estimation techniques for the AWGN channel," *IEEE Trans. Commun.,* vol. 48, pp. 1681-1691, Oct. 2000.

[10] P. Gao and C. Tepedelenlioglu, "SNR estimation for non-constant modulus constellations," *IEEE Trans. Signal Process.,* vol. 53, no. 3, pp. 865-870, Mar. 2005.

[11] R. Lopez-Valcarce and C. Mosquera, "Sixth-order statistics-based non-data-aided SNR estimation," *IEEE Commun. Lett.,* vol. 11, no. 4, pp. 351-353, Apr. 2007.

[12] A. Stéphenne, F. Bellili, and S. Affes, "Moments-based SNR estimation for SIMO wireless communication systems using arbitrary QAM," in *Proc. 41st Asilomar Conference on Signals, Systems and Computers,* pp. 601-605, Nov. 2007.

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