Performance Analysis of Mobile Radio Systems over Composite Fading/Shadowing Channels with Co-located Interference

Imène Trigui, Amine Laourine, Sofiène Affes, and Alex Stéphenne

***Abstract*—This paper presents an analytical framework for performance evaluation of mobile radio systems operating in composite fading/shadowing channels in the presence of co-located co-channel interference. The desired user and the interferers are subject to Nakagami fading superimposed on gamma shadowing. The paper starts by presenting generic closed-form expressions for the signal-to-interference ratio (SIR) probability density function (pdf). From this pdf, closed-form expressions for the outage probability, the average bit error rate and the channel capacity are obtained in both cases of statistically identical interferers and multiple interferers with different parameters. The newly derived closed-form expressions of the aforementioned metrics allow us to easily assess the effects of the different channel and interference parameters. It turns out that the system performance metrics are predominantly affected by the fading parameters of the desired user, rather than by the fading parameters of the interferers.**

***Index Terms*—Average bit error rate, channel capacity, co-located co-channel interference, outage probability, Nakagami fading, shadowing.**

I. INTRODUCTION

ACCURATE system planning and performance evaluation need to take into account the presence of channel propagation impairments such as large-scale fading, which arises from shadowing, and small-scale fading due to multipath propagation [1]. In an interference-limited environment, co-channel interference, which is due to the aggressive frequency reuse in neighboring cells, should also be considered as a corruptive effect. Each interfering signal is also subject to multipath and shadow fading. To asses the impact of the aforementioned impairments on system evaluation metrics such as the outage probability, the bit error rate and the channel capacity, closed-form and tractable expressions are highly desirable. Nevertheless, difficulties arise when using the conventional compound Nakagami-lognormal [2] channel model, since its composite pdf is not in closed form. A three-parameter compound pdf was recently proposed as a substitute to the Nakagami-lognormal model, known as the generalized K-distribution [3]. This model leads to a closed-form solution for the density function of the desired signal power, simplifying the performance analysis.

The outage probability in the presence of co-channel interference has been assessed several times. For fading-only channels, a closed-form expression for the probability of outage was recently presented in [4]. But so far, to the best of our knowledge, no closed-from expressions for the outage probability over composite channels have been obtained. In [5], J. C. Lin et al. conducted an outage analysis for microcellular mobile radio systems that operate in shadowed Rician/Nakagami fading environments. They provided closed-form expressions for the outage probability in the absence of shadowing for independent identically distributed Nakagami faded interferers, but numerical integration was required to solve the shadowed case. The compound Nakagami-m fading gamma shadowing model was considered by I. M. Kostic in [6]. Nevertheless, the latter did not provide a closed-form expression for the probability of outage and its evaluation was done numerically. The effect of co-channel interference on the bit error rate of digital mobile radio systems in a Nakagami-fading channel was studied in [7]. So far no closed-form expression for the bit error rate has been proposed for composite channels with co-channel interference. Recently, in [8], the channel capacity of the Rayleigh lognormal channel with co-channel interference has been assessed using different lognormal approximation methods, but the final expression was not in closed form.

Due to the high degree of difficulty in resolving the problem it raises with respect to the current state of the art on this topic, and in order to obtain easy-to-compute closed-form expressions for the aforementioned system performance metrics, the co-channel interferers are constrained to have equal short-term mean power, typically the case of spatially co-located interferers[1]. This is, unarguably, a non negligible scenario since it can depict the meaningful case of a multi-antenna terminal (e.g., a relay or a mobile in a cellular system) or co-located cluster of single- or multiple-antenna terminals [e.g., cluster of nodes in wireless sensor networks (WSN), cluster of secondary users in a cognitive radio (CR) system, etc.] that interferes on a given desired communication link.

The remainder of this paper is organized as follows. The next section describes in more details our propagation and co-channel interference models. In Section III, the statistical properties of the SIR are assessed and are used to derive closed-form expressions for the aforementioned performance metrics. The effects of a combined shadowing/fading channel and co-channel interference are analyzed subsequently in

Manuscript received September 16, 2008; revised February 11, 2009; accepted April 19, 2009. The associate editor coordinating the review of this letter and approving it for publication was Y. Ma.

I. Trigui, S. Affes, and A. Stéphenne are with the Wireless Lab, Institut National de la Recherche Scientifique, Centre Energie, Matériaux, et Télécommunications, 800, de la Gauchetière Ouest, Bureau 6900, Montréal, H5A 1K6, Qc, Canada (e-mail: {itrigui, affes}@emt.inrs.ca, stephenne@ieee.org).

A. Laourine is with Cornell University, School of Electrical and Computer Engineering, Ithaca, NY (e-mail: al496@cornell.edu).

Digital Object Identifier 10.1109/TWC.2009.081250

[1]Note that the identical local mean power interferers assumption has been adopted in previous woks [9-11], but without our constraint that interferers are spatially co-located. The derived results are bounds on the performance of an interference-limited cellular system in the presence of both fading and shadowing.

Section IV. There, we show that the number of interferers and the shadowing spread have the most significant effect on the system performance, and that the latter is predominantly affected by the fading parameter of the desired user rather than by the fading parameters of the interferers. The final section summarizes our main results and concludes this paper.

II. Channel and System Models

In this section, we first outline the models for the different propagation impairments and co-channel interference affecting the studied cellular system.

A. Channel Model

In slowly-varying flat fading multipath channels, the envelope Z of the received signal is commonly modeled by a Nakagmi-m distribution [12]

$$p_Z(z) = \frac{2(\frac{m}{\Omega})^m}{\Gamma(m)} z^{2m-1} e^{(-\frac{mz^2}{\Omega})}, \quad z \geq 0, \tag{1}$$

denoted by $Z \sim N(m, \Omega)$, where $\Omega = E(Z^2)$ is the local mean received power, m is the fading severity parameter ($m \geq 1/2$), and $\Gamma(\cdot)$ is the gamma function. In urban macrocell systems, the link quality also suffers from shadowing caused by the variability associated with large-scale environmental obstacles. This induces a fluctuation of the mean power Ω about a constant area mean power P. Empirical studies have shown that Ω has a lognormal distribution. However, the lognormal pdf is often difficult to exploit when further analysis is required. Therefore, as an approximation to the lognormal pdf, and as was done in [3], we propose to use the gamma pdf, denoted by $G(\lambda, \Omega_s)$ and defined by

$$p_\Omega(x) = \frac{1}{\Gamma(\lambda)} (\frac{\lambda}{\Omega_s})^\lambda x^{\lambda-1} e^{-\frac{\lambda}{\Omega_s}x}, \quad x > 0, \quad \lambda > 0. \tag{2}$$

In (2), the parameter λ inversely reflects the shadowing severity and Ω_s is the gamma shadow mean power. Relations between the parameters of the lognormal pdf and the gamma pdf can be obtained by a moment-matching technique. In [6], the authors obtained

$$\lambda = 1/(e^{\sigma^2} - 1) \quad \text{and} \quad \Omega_s = P\sqrt{\lambda + 1/\lambda}. \tag{3}$$

where P is related to the path loss and σ is the lognormal shadow standard deviation.

B. Interference and System Model

We consider a radio system in which the desired and interfering signals are subject to slowly-varying flat Nakagami type fading and gamma-distributed shadowing. For analytical tractability and in order to obtain easy-to-compute closed-form expressions which provide useful insights, we adopt the two following assumptions: 1) equal short-term average power interferers, which is valid in the case where these interferers are approximately at the same distance from the receiver such as a single multi-antenna interferer or an interfering cluster of co-located terminals; and 2) the effect of thermal noise is neglected [8], which is reasonable for interference-limited systems. Note that the identical local mean power interferers assumption has been adopted by many other authors, without our constraint that interferers are co-located, e.g., M.S. Alouini et al. in [9-11]. Indeed, the latter claimed that the equal local mean power interferers assumption is suitable for the two limiting cases that can bound the performance of an interference-limited cellular system in the presence of fading and shadowing. These two limiting cases correspond to the case when the interferers are on the cell edges closest to the desired user cell (worst-case interference scenario) or when they are at the farthest edges (best-case interference scenario).

Under the assumptions that the system is interference limited and the co-channel interferers have equal short-term average power, the received SIR can be written as

$$\gamma = \frac{S_d}{I} = \frac{Z_d^2 \omega_d}{\omega_I \sum_{i=1}^N Z_i^2}, \tag{4}$$

where ω_d and ω_I are the short-term average signal powers of the desired user and interferers. In (4), Z_d and Z_i denote the channels' gains of the desired user and the i-th interfering signal, respectively. For a Nakagami fading type, we have $Z_d \sim N(m, \omega_d)$ and $Z_i \sim N(m_i, \omega_I)$. When multipath fading is superimposed on shadowing, typically the scenario in congested downtown areas with a high number of slow-moving pedestrians and vehicles, ω_d and ω_I are random variables assumed in this paper to be gamma distributed with $\omega_d \sim G(\lambda, \Omega_d)$ and $\omega_I \sim G(\lambda_I, \Omega_I)$.

From [13], the pdf of the desired signal power S_d is given by

$$f_{S_d}(x) = \frac{2(\frac{m\lambda}{\Omega_d})^{\frac{m+\lambda}{2}}}{\Gamma(m)\Gamma(\lambda)} x^{\frac{m+\lambda-2}{2}} K_{\lambda-m}\left(2\sqrt{\frac{m\lambda x}{\Omega_d}}\right), \tag{5}$$

where $K_\alpha(\cdot)$ is the modified Bessel function of the second kind and order α [14]. The pdf of the interfering signal I can be obtained as

$$f_I(y) = \int_0^\infty f_{I/w_I}(y/w) f_{w_I}(w) dw, \tag{6}$$

where f_{I/w_I} is the pdf of the interference I given the shadowing w_I. From (6), f_{I/w_I} is equal to the pdf of N squared Nakagami random variables. In our study, we will distinguish two scenarios, namely, statistically identical and statistically non-identical mutually independent interferers.

1) Multiple i.i.d. interferers: When the interfering signals are subject to statistically identical fading processes, typically the case of a multi-antenna interferer with closely spaced antennas, Z_is are i.i.d. with $m_i = m_I, \quad i = 1, 2, ..., N$ and, $f_I(y)$ is given by [13]

$$f_I(y) = \frac{2(\frac{m_I\lambda_I}{\Omega_I})^{\frac{Nm_I+\lambda_I}{2}}}{\Gamma(Nm_I)\Gamma(\lambda_I)} y^{\frac{Nm_I+\lambda_I-2}{2}} K_{\lambda_I - Nm_I}\left(2\sqrt{\frac{m_I\lambda_I y}{\Omega_I}}\right). \tag{7}$$

This comes from the fact that the sum of N gamma distributed RVs of fading parameter m_I and average power ω is a gamma-distributed RV with fading parameter Nm_I and average power $N\omega$. By averaging over the gamma shadowing distribution, we get the interference pdf in (7).

2) Multiple non-i.i.d. interferers: When the antennas of the multi-antenna interferer are sufficiently distant (e.g., a cluster of co-located nodes in a WSN or cluster of secondary users in a CR system, etc.), the interfering signals will travel through statistically non-identical fading channels Z_i each with a fading severity parameter m_i. Recently a closed-form expression of the sum of squared non i.i.d. Nakagami RVs has been derived by G. Karagiannidis et al. in [15]. Using the proposed pdf, the conditional probability of the interference with respect to the shadowing w can be written as

$$\begin{aligned} f_{I/w}(y/w) = &\sum_{i=1}^{N}\sum_{k=1}^{mi} \\ &EL(i,k,\{m_q\}_{q=1}^{N},\{\tfrac{w}{m_q}\}_{q=1}^{N},\{l_q\}_{q=1}^{N-2}) f_{Y_i}(y,k,\tfrac{w}{m_q}), \end{aligned} \quad (8)$$

where

$$f_{Y_i}(y,m_i,\eta_i) = \frac{y^{m_i-1}}{\eta_i^{m_i}(m_i-1)!}e^{(-\frac{y}{\eta_i})}, \quad (9)$$

is the Erlang distribution and EL is a function given in [15]. After some manipulations we find

$$\begin{aligned} EL(i,k,\{m_q\}_{q=1}^{N},\{\tfrac{w}{m_q}\}_{q=1}^{N},\{l_q\}_{q=1}^{N-2}) = w^{d_i} \\ EL(i,k,\{m_q\}_{q=1}^{N},\{\tfrac{1}{m_q}\}_{q=1}^{N},\{l_q\}_{q=1}^{N-2}), \end{aligned} \quad (10)$$

where $d_i = m_i - \sum_{i=1}^{N} m_i + \sum_{s=1}^{N-1} m_{s+U(s-i)}$ and $U(\cdot)$ is the well known unit step function defined as $U(x \geq 0) = 1$ and zero otherwise. It can be easily shown that $d_i = 0$ for $i = 1, ..., N$. Consequently, $f_{I/w_I}(y/w)$ reduces to

$$\begin{aligned} f_{I/w_I}(y/w) = &\sum_{i=1}^{N}\sum_{k=1}^{m_i} \\ &EL(i,k,\{m_q\}_{q=1}^{N},\{\tfrac{1}{m_q}\}_{q=1}^{N},\{l_q\}_{q=1}^{N-2}) \frac{y^{k-1}m_i^k}{w^k(k-1)!}e^{(-\frac{ym_i}{w})}, \end{aligned} \quad (11)$$

The interference pdf is obtained by averaging (11) over the pdf of the shadowing w_I given by $G(\lambda_I, \Omega_I)$ in (2)

$$\begin{aligned} f_I(y) = &\frac{(\frac{\lambda_I}{\Omega_I})^{\lambda_I} y^{k-1}}{\Gamma(\lambda_I)}\sum_{i=1}^{N}\sum_{k=1}^{m_i}\frac{EL(i,k,\{m_q\}_{q=1}^{N},\{\frac{1}{m_q}\}_{q=1}^{N},\{l_q\}_{q=1}^{N-2})}{(\frac{1}{m_i})^k\Gamma(k)} \\ &\int_0^\infty w^{\lambda_I-k-1}e^{-\frac{ym_i}{w}-\frac{\lambda_I}{\Omega_I}w}dw. \end{aligned} \quad (12)$$

Using [14, eq. 3.471.9], the pdf of the interfering signal in the non i.i.d. case is shown to be given by

$$\begin{aligned} f_I(y) = &\frac{2(\frac{\lambda_I}{\Omega_I})^{\frac{\lambda_I}{2}}}{\Gamma(\lambda_I)}\sum_{i=1}^{N}\sum_{k=1}^{m_i}\frac{EL(i,k,\{m_q\}_{q=1}^{N},\{1/m_q\}_{q=1}^{N},\{l_q\}_{q=1}^{N-2})}{(\frac{1}{m_i})^{\frac{k+\lambda_I}{2}}(\frac{\lambda_I}{\Omega_I})^{\frac{-k}{2}}\Gamma(k)} \\ &y^{\frac{\lambda_I+k-2}{2}} K_{\lambda_I-k}\left(2\sqrt{\frac{y\lambda_I m_i}{\Omega_I}}\right). \end{aligned} \quad (13)$$

Finally, it should be noted that the pdf in (13) is only applicable for integer values of m_i. To simplify the notation, we will omit later on the argument of the function EL.

III. Performance Analysis

In this section, the statistical properties of the desired and interfering signals, analyzed in the previous section, will facilitate the performance analysis that follows.

A. Outage Probability

The outage probability is defined as the probability that the output SIR falls below a given threshold y_{th}, i.e.,

$$P_{out} = \int_0^{y_{th}} f_\gamma(y)dy, \quad (14)$$

where $f_\gamma(y)$ is the pdf of the SIR given by $\gamma = S_d/I$. The pdf of the SIR can be derived as

$$f_\gamma(y) = \int_0^\infty f_I(z)f_{S_d}(yz)zdz. \quad (15)$$

1) Multiple i.i.d. interferers: Substituting (5) and (7) into (15) and changing the variable of integration to $x = \sqrt{z}$, the integral in (15) will be given by

$$\begin{aligned} f_\gamma(y) = Ay^{\frac{m+\lambda-2}{2}} &\int_0^\infty z^{Nm_I+m+\lambda_I+\lambda-1} \\ &K_{\lambda_I-Nm_I}\left(2\sqrt{\frac{m_I\lambda_I}{\Omega_I}}z\right) K_{\lambda-m}\left(2\sqrt{\frac{m\lambda y}{\Omega_d}}z\right)dz, \end{aligned} \quad (16)$$

where $A = \frac{8\left(\frac{m\lambda}{\Omega_d}\right)^{\frac{m\lambda}{2}}\left(\frac{m_I\lambda_I}{\Omega_I}\right)^{\frac{Nm_I+\lambda_I}{2}}}{\Gamma(m)\Gamma(\lambda)\Gamma(Nm_I)\Gamma(\lambda_I)}$. By the help of [14, Eq. 6.576.4] and after some manipulations, (15) can be expressed in closed form as

$$\begin{aligned} f_\gamma(y) = &\left(\frac{m\lambda}{m_I\lambda_I\rho}\right)^\lambda y^{\lambda-1}\frac{B(Nm_I+\lambda,m+\lambda_I)}{B(\lambda_I,\lambda)B(Nm_I,m)} \\ &{}_2F_1(\lambda+\lambda_I,Nm_I+\lambda;\lambda_I+Nm_I+m+\lambda;1-\tfrac{m\lambda y}{m_I\lambda_I\rho}), \end{aligned} \quad (17)$$

where ${}_2F_1(a,b;c;z)$ is the Gauss Hypergeometric function [16] and $\rho = \frac{\Omega_d}{\Omega_I}$ is the average SIR.

Using [14, eq. 7.811.4], one can easily verify that $\int_0^\infty f_\gamma(y)dy = 1$. Using the same formulas, the nth moment of the SIR γ is derived as

$$\begin{aligned} E[\gamma^n] &= \int_0^\infty y^n f_\gamma(y)dy \\ &= \left(\frac{Nm_I\lambda_I}{m\lambda}\rho\right)^n \frac{\Gamma(n+\lambda)\Gamma(n+m)\Gamma(\lambda_I-n)\Gamma(Nm_I-n)}{\Gamma(m)\Gamma(\lambda)\Gamma(Nm_I)\Gamma(\lambda_I)}. \end{aligned} \quad (18)$$

From (14), and using the transformation [14, Eq. 9.131.2], and the integration formulas given by

$$\int z^{\alpha-1}{}_2F_1(a,b;c;z)dz = \frac{z^\alpha}{\alpha}{}_3F_2(a,b,\alpha;c,\alpha+1;z), \quad (19)$$

the outage probability is obtained after several manipulations as

$$\begin{aligned} P_{out} = &\left(\frac{m\lambda}{m_I\lambda_I\rho}\right)^\lambda y_{th}^\lambda \frac{\Gamma(Nm_I+\lambda)\gamma(m-\lambda)}{\lambda\Gamma(Nm_I+m)B(\lambda,\lambda_I)B(Nm_I,m)} \\ &{}_3F_2(\lambda+\lambda_I,Nm_I+\lambda,\lambda;\lambda_I-m,\lambda+1;\tfrac{m\lambda}{m_I\lambda_I\rho}y_{th}) \\ &+\left(\frac{m\lambda}{m_I\lambda_I\rho}\right)^m y_{th}^m \frac{\Gamma(m+\lambda_I)\Gamma(\lambda-m)}{m\Gamma(\lambda_I+\lambda)B(\lambda,\lambda_I)B(Nm_I,m)} \\ &{}_3F_2(Nm_I+m,m+\lambda_I,m;m-\lambda+1,m+1;\tfrac{m\lambda}{m_I\lambda_I\rho}y_{th}), \end{aligned} \quad (20)$$

where ${}_3F_2(a,b,c;a_1,b_1;z)$ is the Generalized Hypergeometric function [16].

Let $c = \frac{m\lambda}{m_I\lambda_I\rho}$. If $cy_{th} \ll 1$, which is usually the case, ${}_3F_2(\cdot,\cdot,\cdot;\cdot,\cdot;cy_{th}) \to 1$. To corroborate this assumption, let us consider the special case where $\lambda = \lambda_I = \infty$ (no shadowing). In this case, we have $c = \frac{m}{m_I\rho}$, and upon taking the limit $(\lambda \longrightarrow \infty)$, the first term of (20) evaluates to zero, while the second term, using the asymptotic expression of the gamma function [16], $\Gamma(x) \approx \sqrt{2\pi}x^{x-\frac{1}{2}}e^{-x}/(x \to \infty)$ yields

$$P_{out} \approx \frac{(\frac{m}{m_I\rho}y_{th})^m}{mB(Nm_I,m)}, \quad \lambda = \lambda_I \to \infty, \quad \rho/y_{th} \gg 1. \quad (21)$$

As a check on (21), the special case of Rayleigh fading ($m = m_I = 1$) would result in $P_{out} \approx y_{th}/\rho$, which agrees with [17, eq. 29] for $\rho/y_{th} \gg 1$.

2) Multiple non-i.i.d. interferers: Substituting (5) and (13) into (15), and making the same calculus steps performed in the case of i.i.d. interferers, we obtain the SIR pdf for the non-i.i.d. case as

$$\begin{aligned} f_\gamma(y) = &\frac{(\frac{m\lambda}{\lambda_I\rho})^\lambda}{\Gamma(\lambda_I)\Gamma(m)}y^{\lambda-1}\sum_{i=1}^{N}\sum_{k=1}^{m_i}\frac{EL\Gamma(m+\lambda_I)}{m_i^\lambda}\frac{B(\lambda+\lambda_I,m+k)}{B(\lambda,k)} \\ &{}_2F_1(\lambda+\lambda_I,\lambda+k;m+\lambda+\lambda_I+k;1-\tfrac{m\lambda}{\lambda_I m_i\rho}y). \end{aligned} \quad (22)$$

Using [14, eq. 7.811.4], we find that the nth moment of the output SIR, in the presence of N non i.i.d interferers, is given by

$$E[\gamma^n] = \left(\frac{\lambda_I\rho}{m\lambda}\right)^n \frac{\Gamma(m+n)\Gamma(\lambda+n)}{\Gamma(\lambda_I)\Gamma(m)\Gamma(\lambda)} \sum_{i=1}^{N}\sum_{k=1}^{m_i} ELm_i^n \frac{\Gamma(k-n)\Gamma(\lambda_I-n)}{\Gamma(k)}. \tag{23}$$

Using [14, Eq. 9.131.2] and the integration formulas in (19), we find that the probability of outage in the presence of N non-i.i.d. interferers is

$$\begin{aligned} P_{out} = & \frac{(\frac{m\lambda}{\lambda_I})^{\lambda}}{\lambda\Gamma(\lambda_I)\Gamma(m)} y_{th}^{\lambda} \sum_{i=1}^{N}\sum_{k=1}^{m_i} \frac{EL}{m_i^{\lambda}} \frac{\Gamma(\lambda+\lambda_I)\Gamma(m-\lambda)}{B(\lambda,k)} \\ & {}_3F_2(\lambda+\lambda_I, \lambda+k, \lambda, \lambda-m+1, \lambda+1, \tfrac{m\lambda}{m_i\lambda_I\rho} y_{th}) \\ & + \frac{(\frac{m\lambda}{\lambda_I})^{m}}{m\Gamma(\lambda_I)\Gamma(\lambda)} y_{th}^{m} \sum_{i=1}^{N}\sum_{k=1}^{m_i} \frac{EL}{m_i^{m}} \frac{\Gamma(m+\lambda_I)\Gamma(\lambda-m)}{B(m,k)} \\ & {}_3F_2(m+k, m+\lambda_I, m, m-\lambda+1, m+1, \tfrac{m\lambda}{m_i\rho} y_{th}). \end{aligned} \tag{24}$$

As done in the case of i.i.d. interferers, the asymptotic representation of the outage probability [i.e., in the absence of shadowing ($\lambda = \lambda_I = \infty$)], is given by

$$P_{out} = \frac{(m\frac{y_{th}}{\rho})^m}{m} \sum_{i=1}^{N}\sum_{k=1}^{m_i} \frac{EL}{m_i^m B(m,k)}, \ \lambda = \lambda_I \to \infty, \ \rho/y_{th} \gg 1. \tag{25}$$

B. Average Bit Error Probability

The average bit error probability constitutes probably the most important performance measure of a digital communication system and is given by

$$P_{ae} = \int_0^{\infty} P_e(y) f_{\gamma}(y) dy, \tag{26}$$

where $P_e(y)$ is the conditional error probability (CEP) having generic expressions for different sets of modulation schemes. For binary modulations, the CEP is given by [1]

$$P_e(\gamma) = \frac{\Gamma(b, a\gamma)}{2\Gamma(b)}, \tag{27}$$

where $\Gamma(\alpha, x) = \int_x^{\infty} z^{\alpha-1} e^{-z} dz$ denotes the complementary incomplete Gamma function, and $(a, b) = (1, 0.5)$ for binary shift keying (BPSK), $(a, b) = (0.5, 0.5)$ for coherent frequency shift keying (BFSK) and $(a, b) = (1, 1)$ for differential BPSK (DBPSK). Recognizing that,

$$P_e(\gamma) = \frac{1}{2\Gamma(b)} G_{1,2}^{2,0}\left(a\gamma \,\middle|\, \begin{matrix} 1 \\ 0, b \end{matrix}\right), \tag{28}$$

where $G_{a,b}^{c,d}$ is the Meijer-G function [16], we obtain the average bit error probability expressions in the following cases.

1) Multiple i.i.d. interferers: By inserting (17) and (28) in (26), the hypergeometric function is firstly transformed using

$${}_2F_1(a, b; c; 1-z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)} G_{2,2}^{2,2}\left(z \,\middle|\, \begin{matrix} 1-a, 1-b \\ 1, c-a-b \end{matrix}\right). \tag{29}$$

Then using [14, eq. 7.813.1], the average bit error probability in the case of multiple i.i.d. interfering signals is found to be given by

$$P_{ae} = \frac{G_{3,4}^{4,2}\left(\frac{m_I\lambda_I}{m\lambda} a\rho \,\middle|\, \begin{matrix} -\lambda, 1-m, 1 \\ 0, b, \lambda_I, Nm_I \end{matrix}\right)}{2\Gamma(b)\Gamma(m)\Gamma(Nm_I)\Gamma(\lambda)\Gamma(\lambda_I)}. \tag{30}$$

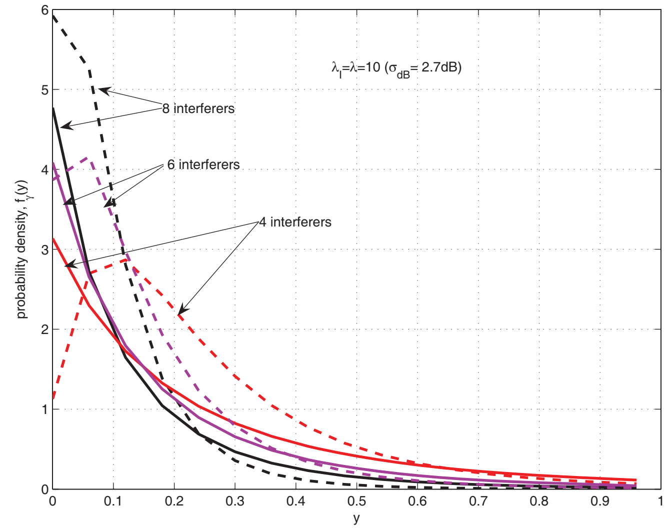

Fig. 1. SIR pdf for different values of the number of non i.i.d. co-channel interferers, for a severely faded desired user ($m = 1$: solid lines) and for a lightly faded desired user ($m = 4$: dashed lines), $\sigma_{dB} = 8.686\ \sigma$.

2) Multiple non-i.i.d. interferers: After substituting (22) and (28) into (26) and using the same calculus steps performed in the i.i.d. interferers case, we find that

$$P_{ae} = \frac{1}{2\Gamma(b)\Gamma(\lambda_I)\Gamma(m)\Gamma(\lambda)} \sum_{i=1}^{N}\sum_{k=1}^{m_i} \frac{EL}{\Gamma(k)} G_{3,4}^{4,2}\left(\frac{m_i\lambda_I}{m\lambda} a\rho \,\middle|\, \begin{matrix} -\lambda, 1-m, 1 \\ 0, b, \lambda_I, k \end{matrix}\right). \tag{31}$$

C. Channel Capacity

We consider an adaptive transmission scheme where optimal rate adaptation with constant transmit power is applied. This scheme entails variable-rate transmission relative to the channel, but is rather practical since the transmit power remains constant. The channel capacity is known to be given by [18] as

$$C = \int_0^{\infty} \ln_2(1+y) f_{\gamma}(y) dy, \tag{32}$$

where $f_{\gamma}(\cdot)$ is the pdf of the SIR.

1) Multiple i.i.d. interferers: After inserting (17) in (32), the logarithm function is firstly transformed as

$$\ln(1+z) = G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right). \tag{33}$$

After using the classical Meijer's integral from two G functions [16], the channel capacity in the presence of N i.i.d interferers is shown to be given by

$$C = \frac{G_{4,4}^{4,3}\left(\frac{m\lambda}{m_I\lambda_I\rho} \,\middle|\, \begin{matrix} 1-\lambda_I, 1-Nm_I-\lambda_I+\lambda, 0, 1 \\ \lambda, m, 0, 0 \end{matrix}\right)}{\ln(2)\Gamma(m)\Gamma(Nm_I)\Gamma(\lambda)\Gamma(\lambda_I)}. \tag{34}$$

2) Multiple non-i.i.d. interferers: By inserting (17) in (32), the same calculus steps, performed in the case of i.i.d. interferers, allow us to derive the channel capacity in the non i.i.d. case as

$$C = \frac{1}{\ln(2)\Gamma(\lambda_I)\Gamma(m)\Gamma(\lambda)} \sum_{i=1}^{N}\sum_{k=1}^{m_i} \frac{EL}{\Gamma(k)} G_{4,4}^{4,3}\left(\frac{m\lambda}{m_i\lambda_I\rho} \,\middle|\, \begin{matrix} 1-\lambda_I, 1-k, 0, 1 \\ \lambda, m, 0, 0 \end{matrix}\right). \tag{35}$$

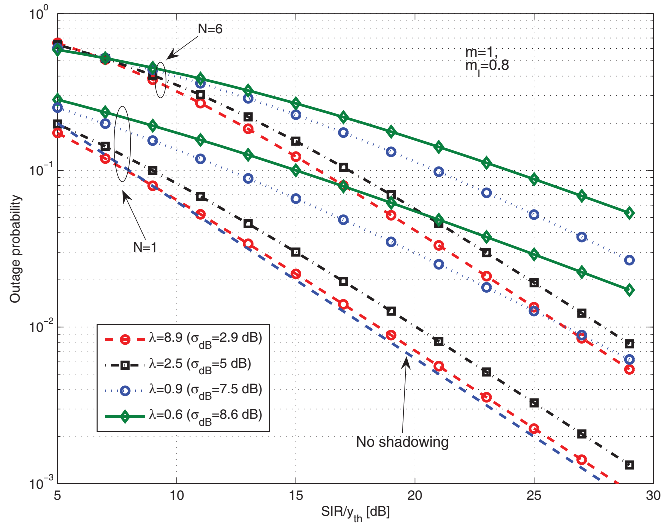

Fig. 2. Outage probability versus the inverse normalized threshold SIR/y_{th} for different shadowing scenarios in severely faded channel in the presence of $N = 1$ and $N = 6$ i.i.d. co-channel interferers.

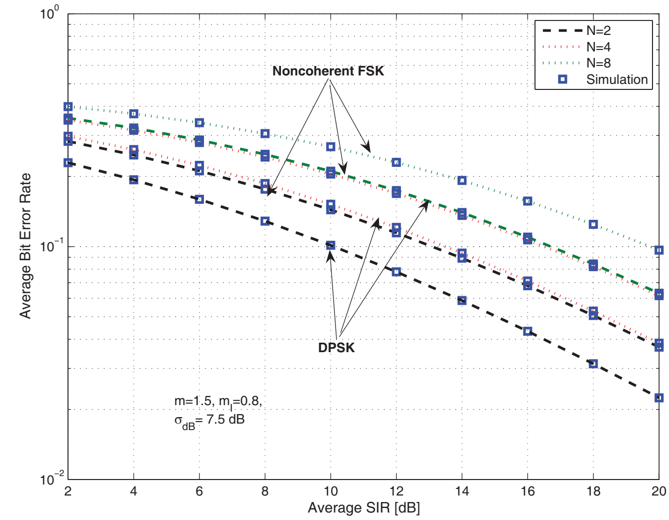

Fig. 4. Average bit error probability versus the average SIR for DPSK and noncoherent FSK in frequent heavy-shadowed and faded environment in the presence of different numbers of i.i.d. co-channel interferers.

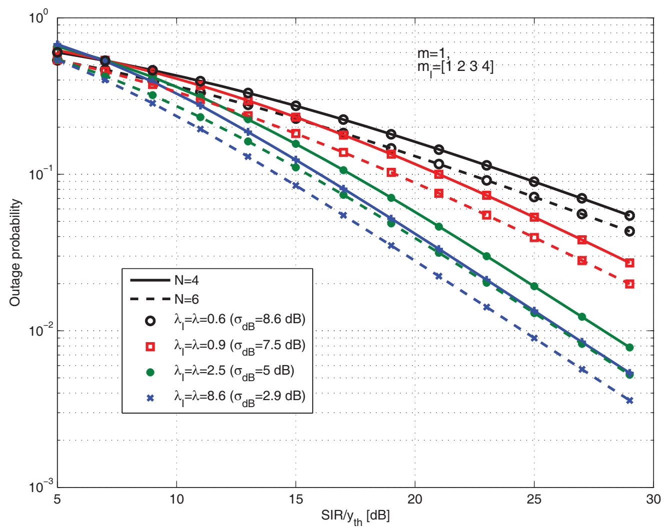

Fig. 3. Outage probability versus the inverse normalized threshold SIR/y_{th} for different shadowing scenarios in severely faded channel in the presence of $N = 4$ and $N = 6$ non i.i.d. co-channel interferers.

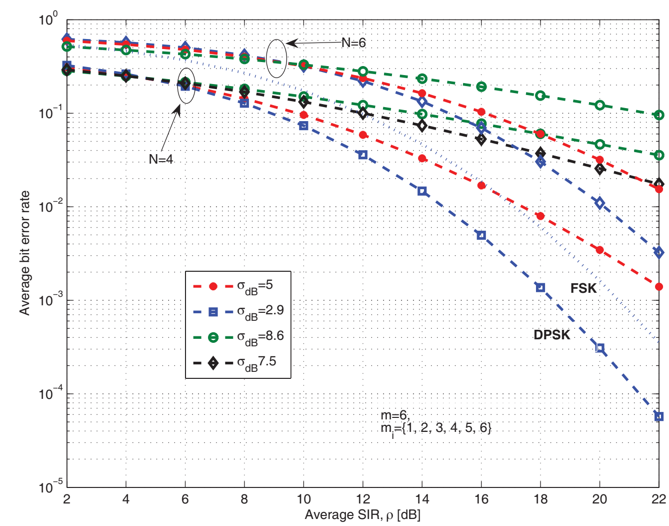

Fig. 5. Average bit error probability versus the average SIR in the presence of different numbers of non-i.i.d. co-channel interferers.

IV. Numerical Results

To gain a better understanding as to how the fading, shadowing and co-channel interference affect the outage probability, some plots are presented in this section for both i.i.d and non i.i.d. interferers. Without loss of generality, we assume identical shadowing statistics for both desired and interfering signals, which is a quite reasonable assumption [5]. In Fig. 1, we plot the SIR pdf for different numbers of co-channel interferers and for different levels of the desired user fading severity. As one can see from these numerical results, the number of co-channel interferers and the desired fading severity are dominant factors in determining the outage probability. Fig. 2 illustrates the effect of shadowing and co-channel interference on the probability of outage in severely faded microcellular environments $(m > m_I)$. To gain more insight into the effects of shadowing, the curve without a marker reports the outage performance without shadowing $(\lambda \to \infty)$. A general observation is that shadowing degrades the probability of outage. Nevertheless, an important phenomenon to be noticed, for low SIR/y_{th}, is that more severe shadowing can cause lower outage probability. A similar observation has been noted for lognormal shadowing in [5]. In Fig. 3, we show plots of the outage probability in the case of non i.i.d interferers (only integer values of the fading parameter are considered). The observations made before, in the case of i.i.d. co-channel interferers, are also valid in this case.

The average bit error probability for DPSK and FSK modulations, in the presence of different numbers of i.i.d co-channel interferers, is illustrated in Fig. 4. We notice that the BEP performance highly degrades as N increases. Fig. 5 also depicts the bit error rate performance this time in the presence of non i.i.d. co-channel interferers and for different shadowing spread values. As expected, shadowing deteriorates the BEP.

Fig. 6 depicts the capacity versus the SIR for different values of the shadowing spread, the fading severity of the

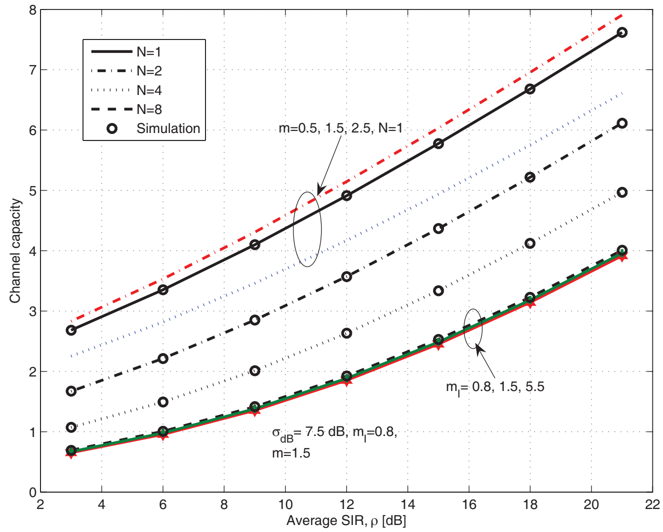

Fig. 6. Channel capacity versus the average SIR in the presence of different numbers of i.i.d. co-channel interferers for different desired user and interfering users fading scenarios in severely shadowed environment.

desired user and the number of i.i.d. interferers as a function of the SIR. As expected, the channel capacity deteriorates as the shadowing, the fading severity and the number of interferers increase. Observe that the system performance is insensitive to changes in the fading severity of interfering signals. This phenomenon demonstrates that the number of interferers and the shadowing spread have the most significant effect on the channel capacity. Therefore, the system performance is predominantly affected by the fading parameter of the desired user rather than by the fading parameters of the interferers.

V. Conclusion

We analyzed the performances of interference-limited radio mobile systems that operate in composite channels. The analysis is sufficiently general to include the combined effects of Nakagami fading, gamma shadowing, identical and non-identical co-located co-channel interference. Based on the SIR pdf preliminarily obtained in this work, we then derived closed-form expressions for the outage probability, the average bit error rate and the channel capacity. It turns out that these metrics are predominantly affected by the fading parameters of the desired user, rather than by the fading parameters of the interferers.

References

[1] M. K. Simon and M. S. Alouni, *Digital Communication over Fading Channels.* New York, NY: John Wiley & Sons Inc., 2nd ed., 2004.

[2] M. J. Ho and G. L. Stuber, “Co-channel interference of microcellular systems on shadowed Nakagami fading channels,” in *Proc. IEEE Veh. Technol. Conf. (VTC93)*, pp. 568-571, May 1993.

[3] P. S. Bithas, N. C. Sagias, P. T. Mathiopoulos, G. K. Karagiannidis, and A. A. Rontogiannis, “On the performance analysis of digital communications over generalized-K fading channels,” *IEEE Commun. Lett.*, vol. 10, no. 5, pp. 353-355, May 2006.

[4] J. M. Romero-Jerez and A. J. Goldsmith, “Antenna array processing in fading and interference: an interference-cancellation vs. diversity comparative performnce,” *IEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 920-632, Mar. 2008.

[5] J. C. Lin, W. C. Kao, Y. T. Su, and T. H. Lee, “Outage and coverage considerations for microcellular mobile systems in a shadowed-Rician/shadowed-Nakagami environment,” *IEEE Trans. Veh. Technol.*, vol. 48, no. 1, pp. 66-75, Jan. 1999.

[6] I. M. Kostiè, “Analytical approach to performance analysis for channel subject to shadowing and fading,” *IEE Proc. Commun.,* vol. 152, no. 6, pp. 821-827, Dec. 2005.

[7] V. A. Aalo and J. Zhang, “On the effect of cochannel interference on average error rates in Nakagami-fading channels,” *IEEE Commun. Lett.*, vol. 3, no. 5, pp. 136-138, May 1999.

[8] J. Wu, N .B Mehta, A. F. Molisch, and J. Zhang, “Spectral efficiency of channel-aware scheduler in non-identical composite links with interference,” in *Proc. ICC’07,* pp. 5218-5223, June 2007.

[9] M.-S. Alouini and A. J. Goldsmith, “Area spectral efficiency of cellular mobile radio systems,” *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1047-1066, July 1999.

[10] M. O. Hasna, M.-S. Alouini, A. Bastami, and E. S. Ebbini, “Performance analysis of cellular mobile systems with successive co-channel interference cancellation,” *IEEE Trans. Wireless Commun.*, vol. 2, pp. 29-40, Jan. 2003.

[11] M. Kang, M.-S. Alouini, and L. Yang, “Outage probability and spectrum efficiency of cellular mobile radio systems with smart antennas,” *IEEE Trans. Commun.,* vol. 50, pp. 1871-1877, Dec. 2002.

[12] M. Nakagami, “The m-distribution: a general formula of intensity distribution of rapid fading,” *Statistical Methods in Radio Wave Propagation.* Oxford, U.K.: Pergamon, pp. 3-36, 1960.

[13] P. M. Shankar, “Outage probabilities in shadowed fading channels using a compound statistical model,” *IEE Proc.-Commun,* vol. 152, no. 6, pp. 828-832, Dec. 2005.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 7th ed. San Diego, CA: Academic, 1994.

[15] G. K. Karagiannidis, N. C. Sagias, and T. A. Tsiftsis, “Closed-form statistics for the sum of squared Nakagami-m variates and its applications,” *IEEE Commun. Lett.*, vol. 54, no. 8, pp. 1353-1359, Aug. 2006.

[16] “The Wolfram functions site.” [Online]. Available: http://mathworld.wolfram.com.

[17] A. A. Abu-Dayya and N. C. Beaulieu, “Outage probabilities of diversity cellular systems with cochannel interference in Nakagami fading,” *IEEE Trans. Veh. Technol.,* vol. 41, no. 4, pp. 343-355, Nov. 1992.

[18] M.-S. Alouini and A. J. Goldsmith, “Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques,” *IEEE Trans. Veh. Technol.,* vol. 48, no. 4, pp. 1165-1181, July 1999.