Transmit Diversity for FSO/RF-Based Multiuser Networks

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***Abstract*—In this paper, we consider multi-aperture multiuser dual-hop amplify-and-forward (AF) free-space optical/radio frequency (FSO/RF) communication systems. The FSO-RF channels are assumed to follow Málaga-$\mathcal{M}$/shadowed κ-μ fading models with pointing errors on the FSO links. Under the assumption of transmit aperture selection at the source and opportunistic user scheduling at the destination, we derive exact closed-form expressions for the outage probability and average symbol error probability (SEP). Additionally, the system performance is studied at the high signal-to-noise ratio (SNR) regime where an approximate expression for the system outage probability is derived, in addition to deriving the system diversity order and coding gain.**

***Index Terms*—Mixed FSO-RF relay network, Málaga-$\mathcal{M}$ distribution, shadowed κ-μ fading, aperture selection, multiuser diversity, millimeter wave.**

I. Introduction

Free-space optical (FSO) networks are known by their cost-efficiency, high data rate capabilities, and immunity to interference. They represent an efficient/promising solution that guarantees low-cost backhaul to address the immerse of massive data traffic from small cells in ultra-dense networks [2]. In this context, relay-assisted FSO-based backhaul framework and radio frequency (RF)-based access links, where relays are utilized as optical to RF "converter" to assist the communications of small cells, is being offered as a strong candidate for 5G networks.

Recently, studying the performance and different scenarios of mixed FSO/RF systems has attracted a lot of research interest. The performance of an amplify-and-forward (AF) mixed RF/FSO relay network over Nakagami-m and Gamma-Gamma fading channels was studied in [3]-[5]. Considering κ-μ and η-μ fading models for the RF link and a Gamma-Gamma fading model for the FSO link, Zhang $et\ al.$ studied in [6] the outage probability and error rate performances of a dual-hop mixed RF/FSO relay network. Promoting the mixed RF/FSO relay system to a millimeter-wave (mmWave) scenario was done in [7] where the authors considered a mmWave Rician distributed RF channel and a Málaga-$\mathcal{M}$ distributed FSO channel in their derivations. On the other hand, the authors in [8] investigated the performance of a mixed FSO/RF relay network assuming Málaga-$\mathcal{M}$/shadowed κ-μ fading models in their analysis. They derived exact and asymptotic (i.e. at high signal-to-noise ratio (SNR) values) closed-form expressions for the system outage probability and channel capacity. In [9], the authors investigated the performance of an AF mixed RF/FSO relay network with multiple antennas at the source and multiple apertures at the destination. Considering the presence of multiple users at the RF side, the authors in [10], [11], and [12] studied the performance of mixed RF-FSO relay networks when the RF links and FSO link were assumed to follow Nakagami-m and Gamma-Gamma fading models, respectively.

Although the previous studies in literature on multi-aperture multiuser mixed FSO/RF systems are insightful, they have been successfully tractable only for Rayleigh and Nakagami-m fading models for the RF links and Gamma-Gamma fading for the FSO links. Such infatuation with Rayleigh and Nakagami-m fading models, however, limited legitimacy since these channel distributions may fail to capture and represent new and more realistic fading environments. Furthermore, the utilization of Rayleigh and Nakagami-m fading models is even more pessimistic in future femtocells where line-of-sight (LOS) components may be created by reflections in close proximity to the base stations (BSs) and/or users or may appear in mmWave networks [13]. From a FSO channel model perspective, although many researchers considered before the dual-aperture FSO system scenario, they have merely considered log-normal [14] or Gamma-Gamma [15] fading channels. Moreover, it can be seen that none of them has been even developed in the challenging context of mixed FSO/RF scenario. Besides, due to their complexity, obtaining closed-form analytical solutions for the outage probability and symbol error probability (SEP) in [14]-[15] is considered to be very hard.

Trying to contribute in this topic of research and consider more efficient/general FSO fading models, this work presents a performance analysis framework where versatile multiple-parameter fading models are incorporated into a tractable performance analysis of multiuser RF relay network encompassing FSO-based backhauling through the implementation of aperture selection. These fading models include Málaga-$\mathcal{M}$ distribution for the FSO link and shadowed κ-μ model for the RF links. In addition to their elegance, these models are governed by more than two tunable parameters which makes them able to capture a broad range of turbulence conditions/shadowed LOS and non-LOS (NLOS) cases that cover most notably the compelling cases of mixed FSO/mmWave,

Work supported by the NSERC CREATE PERSWADE Program and a Discovery Accelerator Supplement Award from NSERC. A full version of this work has been published in [1].

whence their practical significance.

II. System and Channel Models

Consider a dual-hop mixed FSO-RF multiuser relay network comprising an optical source (S) equipped with L photo-detectors, an AF relay node (R), and K single-antenna RF users U_k ($k = 1, \ldots, K$). The relay node R is assumed to have a single photo detector from one side and multiple antennas from the other side. The BS communicates with the users U_k via an AF relay R with no direct links between them. Among L apertures at the source, a single aperture that maximizes the pre-processing SNR is selected and used for transmission. The l-th FSO link irradiance is assumed to follow a Málaga-$\mathcal{M}$ distribution with pointing errors impairments. Using AF relaying, all the M transmit antennas at the relay are used for MRT (maximum ratio transmission), while a single user that maximizes the postprocessing SNR is selected for reception. The channel gains between the j-th transmit antenna at the relay and the k-th user are assumed to be independent identically-distributed (i.i.d.) shadowed κ-μ random variables (rvs) over the M antennas at the relay with fading parameters m_k, κ_k, μ_k, and mean $\Omega_k = \mathbb{E}\{h_{j,k}\}$, $k = 1, 2, \ldots, K$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. Henceforth, we use the shorthand notation $Z \sim \mathcal{S}(m, \kappa, \mu)$ to denote that the RV Z follows a shadowed κ-μ distribution with parameters m, κ, and μ.

A. Transmit Aperture Selection (TAPS)

The selection of the suitable transmit aperture at the serving optical station is implemented to maximize the received signal at the relay. The TAPS-based SNR is then given by

$$\gamma_{\mathcal{F}} = \mu_r \left(\Xi \max_{l=1,\ldots,L} f_l \right)^r, \qquad (1)$$

where f_l is the optical irradiance of the l-th aperture, r is the parameter that describes the detection technique at the relay (i.e. $r = 1$ is associated with heterodyne detection and $r = 2$ is associated with IM/DD), μ_r refers to the electrical SNR of the FSO link and Ξ represents the impairments due to the pointing errors assumed to be equal for all the apertures.

The l-th aperture irradiation f_l follows a Málaga-$\mathcal{M}$ distribution for which the complementary cumulative distribution function (ccdf) is obtained using [16, Eq. (24)] as

$$F_{f_l}^{(c)}(x) = A\sqrt{\pi} \sum_{k=1}^{\beta} \sum_{j=0}^{\alpha-k-\frac{1}{2}} \frac{\psi_{k,j}}{\left(\sqrt{\frac{\alpha\beta}{\mu\beta+\Omega}}\right)^{\alpha+k}} \times \Gamma\left(\alpha + k - j - \frac{1}{2}, 2\sqrt{\frac{\alpha\beta x}{g\beta + \phi}}\right), \qquad (2)$$

where $A = \alpha^{\frac{\alpha}{2}} \left(g\beta/(g\beta + \phi)\right)^{\beta+\frac{\alpha}{2}} g^{-1-\frac{\alpha}{2}}$ whereby α, β, g, and ϕ are the fading parameters related to the atmospheric turbulence conditions, $\psi_{k,j} = \frac{a_k(\alpha-k-\frac{1}{2}+j)!2^{\frac{1}{2}-\alpha-k-j}}{(\alpha-k-\frac{1}{2}-j)j!}$ with $a_k = \binom{\beta-1}{k-1}(g\beta+\phi)^{1-\frac{k}{2}}\left((g\beta+\phi)/\alpha\beta\right)^{\frac{\alpha+k}{2}}(\phi/g)^{k-1}(\alpha/\beta)^{\frac{k}{2}}$, and $\Gamma(\cdot,\cdot)$ stands for the upper incomplete Gamma function [17, Eq. (8.350.2)]. Upon substituting the incomplete Gamma function in (2) by its series expansion $\Gamma(n, z) = \Gamma(n)e^{-z} \sum_{j=0}^{n-1} \frac{z^m}{m!}$ [17, Eq. (8.352.2)] and applying the multinomial expansion, the cdf of the first hop SNR $\gamma_{\mathcal{F}}$ can be obtained as

$$\begin{aligned} F_{\gamma_{\mathcal{F}}}(\gamma) &= \mathbb{E}_{\Xi}\left\{\left[F_{f_l}\left(\frac{1}{\Xi}\left(\frac{\gamma}{\mu_r}\right)^{\frac{1}{r}}\right)\right]^L\right\} \\ &= 1 - \sum_{l=1}^{L}\sum_{\Upsilon}\tau_l \mathbb{E}_{\Xi}\left\{e^{-2l\sqrt{\frac{\alpha\beta\left(\frac{x}{\mu_r}\right)^{\frac{1}{r}}}{\Xi(g\beta+\phi)}}}\left(\frac{\gamma}{\mu_r}\right)^{\frac{\delta_l}{2r}}\left(\frac{1}{\Xi}\right)^{\frac{\delta_l}{2}}\right\}, \end{aligned} \qquad (3)$$

where $\sum_{\Upsilon} = \sum_{\Upsilon_{l,\beta}} \sum_{\Upsilon_{l_p,\alpha-l_p+\frac{1}{2}}} \sum_{\Upsilon_{l_{pq}\alpha+l_p-l_{pq}-\frac{1}{2}}}$, whereby $\Upsilon_{z,l} = \{(z_1, \ldots, z_l) : z_i \geqslant 0, \sum_{i=1}^{l} z_i = z\}$; $\delta_l = \sum_{l=0}^{\alpha+l_p-l_{pq}-\frac{3}{2}} l t_{p_{q_{l+1}}}$ and

$$\tau_t = \prod_{p=1}^{\beta} \prod_{q=1}^{\alpha-t_p+\frac{1}{2}} \prod_{r=1}^{\alpha+t_p-t_{pq}-\frac{1}{2}} \frac{(A\sqrt{\pi})^{t_{p_{q_r}}}\binom{L}{t}(-1)^{t+1}t!}{\prod_{k=1}^{\alpha+t_p-t_{pq}-\frac{1}{2}} t_{p_{q_k}}} \times \left(\psi_{t_p,t_{p_{q_r}},t_{p_q}}\left(\frac{\alpha\beta}{g\beta+\phi}\right)^{\frac{t_{p_{q_r}}-\alpha-t_p-1}{2}}\right)^{t_{p_{q_r}}}. \qquad (4)$$

Recalling that $f_{\Xi}(x) = \frac{\xi^2}{A_0^{\xi^2}} x^{\xi^2-1}, \quad 0 \leqslant x \leqslant A_0$, where ξ is the ratio between the equivalent beam radius and the pointing error displacement standard deviation (i.e. jitter) at the relay (for negligible pointing errors $\xi \to \infty$), A_0 defines the pointing loss [16]. From (3), the TAPS-based SNR cdf with pointing errors is obtained as

$$\begin{aligned} F_{\gamma_{\mathcal{F}}}(\gamma) &\overset{(a)}{=} 1 - \frac{\xi^2}{A_0^{\xi^2}} \sum_{l=1}^{L}\sum_{\Upsilon}\tau_l\left(\frac{\gamma}{\mu_r}\right)^{\frac{\delta_l}{2r}} \int_0^{A_0} x^{\xi^2-\frac{\delta_l}{2}-1} \\ &\quad \times \mathcal{H}_{0,1}^{1,0}\left[2l\sqrt{\frac{\alpha\beta\left(\frac{\gamma}{\mu_r}\right)^{\frac{1}{r}}}{x(g\beta+\phi)}}\,\middle|\,\begin{matrix} - \\ (0,1) \end{matrix}\right]dx, \end{aligned} \qquad (5)$$

where (a) follows from substituting the exponential function using $e^{-\sqrt{z}} = \frac{1}{\sqrt{\pi}}\mathcal{H}_{0,2}^{2,0}\left[\frac{z}{4}\,\middle|\,\begin{matrix} - \\ (0,1),(\frac{1}{2},1)\end{matrix}\right]$, with $\mathcal{H}_{p,q}^{m,n}[\cdot]$ standing for the Fox's H-function [18, Eq. (1.2)]. Applying [18, Eq. (2.53)] to (5) yields

$$\begin{aligned} F_{\gamma_{\mathcal{F}}}(\gamma) &= 1 - \frac{\xi^2 r}{\sqrt{\pi}} \sum_{l=1}^{L}\sum_{\Upsilon}\tau_l\left(\frac{\gamma}{\widetilde{\mu}_r}\right)^{\frac{\delta_l}{2r}} \\ &\quad \times \mathcal{H}_{1,3}^{3,0}\left[\frac{B^{2r}l^{2r}\gamma}{\widetilde{\mu}_r}\,\middle|\,\begin{matrix} (\xi^2+1-\frac{\delta_l}{2}, r) \\ (\xi^2-\frac{\delta_l}{2}, r), (0, r), (\frac{1}{2}, r)\end{matrix}\right], \end{aligned} \qquad (6)$$

where $B = \sqrt{\alpha\beta/g\beta+\phi}$ and $\widetilde{\mu}_r = \mu_r A_0^r$ is the average SNR of the FSO link.

B. User Selection

The post-user selection SNR is given by

$$\gamma_{\mathcal{R}} = \max_{k=1,\ldots,K}\{\gamma_k\}, \qquad (7)$$

where $\gamma_k = \bar{\gamma}_k \sum_{j=1}^{M} h_{j,k}$ is the combined SNR per user, and $\bar{\gamma}_k$ is the average SNR per user. From (7) we can write that

$\gamma_k \sim \mathcal{S}(\kappa, m_k = M\tilde{m}_k, \mu_k = M\tilde{\mu}_k, 1/\bar{\gamma}_k)$ with a cdf given by

$$F_{\gamma_k}(\gamma) = 1 - \sum_{j=0}^{N_k} C_{j,k} e^{-\frac{\gamma}{\Omega_{j,k}}} \sum_{r=0}^{m_{j,k}-1} \frac{1}{r!}\left(\frac{\gamma}{\Omega_{j,k}}\right)^r, \quad (8)$$

where N_k and the set of parameters $\{C_{j,k}, m_{j,k}, \Omega_{j,k}\}$ are expressed in terms of $\bar{\gamma}_k, \kappa_k, \mu_k$, and m_k. If $\mu > m$, then $M = \mu$, $m_j = \mu - m - j + 1$, for $0 \le j \le \mu - m$ and $m_j = \mu - j + 1$ for $\mu - m < j \le \mu$. Otherwise $M = m - \mu$ and $m_j = m - j$. For other coefficients Ω_j and C_j, please refer to [19, Table 1] for more details. The ccdf of $\gamma_{\mathcal{R}}$ is obtained as

$$\begin{aligned} F^{(c)}_{\gamma_{\mathcal{R}}}(\gamma) &= 1 - \prod_{k=1}^{K}\left[1 - \sum_{j=0}^{N_k} C_{j,k} e^{-\frac{\gamma}{\Omega_{j,k}}} \sum_{r=0}^{m_{j,k}-1} \frac{1}{r!}\left(\frac{\gamma}{\Omega_{j,k}}\right)^r\right] \\ &\overset{(a)}{=} \sum_{\mathcal{S}\backslash\varnothing} \sum_{\mathcal{J}} \sum_{\mathcal{R}} (-1)^{Q-1}\, C_{\mathcal{SJR}}\, \gamma^{r_{\mathcal{J},\mathcal{R}}}\, e^{-E_{\mathcal{S}\mathcal{J}}\gamma}, \quad (9) \end{aligned}$$

where in (a) $\mathcal{S} = \{s_1, s_2, \ldots, s_Q\}$ is a subset of R, $\sum_{\mathcal{S}}$ is the sum over all possible $\mathcal{S}$, $\sum_{\mathcal{J}} = \sum_{j_{s_1}=1}^{N_{s_1}} \ldots, \sum_{j_{s_Q}=1}^{N_{s_Q}}$, and $\sum_{\mathcal{R}} = \sum_{r_1=1}^{m_{j_{s_1}}-1} \ldots, \sum_{r_Q=1}^{m_{j_{s_Q}}-1}$. Also, we denote $r_{\mathcal{J},\mathcal{R}} = \sum_{p=1}^{Q} r_p$, $E_{\mathcal{S},\mathcal{J}} = \sum_{p=1}^{Q} \Omega^{-1}_{j_{s_p}, s_p}$, and $C_{\mathcal{S},\mathcal{J},\mathcal{R}} = \prod_{i=1}^{Q} C_{j_{s_i}, s_i} \prod_{p=0}^{Q} \frac{\Omega^{-r_p}_{j_{s_p}, s_p}}{r_p!}$. Notice that when $\mathcal{S} = \varnothing$ is an empty set, we have $r_{\mathcal{J},\mathcal{R}} = 0$, $E_{\mathcal{S}\mathcal{J}} = 0$, and $C_{\mathcal{SJR}} = 1$.

III. Analytical End-to-End Performance Analysis

A. Outage Probability

The outage probability is defined as the probability that the end-to-end (e2e) SNR $\gamma_{eq} = \left(\frac{1}{\gamma_{\mathcal{F}}} + \frac{1}{\gamma_{\mathcal{R}}}\right)^{-1}$ falls below a predetermined threshold γ_{th}

$$\begin{aligned} P_{\text{out}}(\gamma_{th}) &= \Pr(\gamma_{eq} < \gamma_{th}) \\ &= 1 - \gamma_{th} \int_1^\infty \Pr\left(\gamma_{\mathcal{R}} > \gamma_{th} + \frac{\gamma_{th}}{u-1}\right) f_{\gamma_{\mathcal{F}}}(u\gamma_{th}) du, \quad (10) \end{aligned}$$

where $f_{\gamma_{\mathcal{F}}}$ is obtained from differentiating (6) and the term $\Pr(\gamma_{\mathcal{R}} > \gamma) = F^{(c)}_{\gamma_{\mathcal{R}}}(\gamma)$ is already obtained in (9).

The outage probability of mixed TAPS FSO/multiuser relay networks in Málaga-$\mathcal{M}$ and shadowed κ-μ fading is obtained as

$$\begin{aligned} P_{\text{out}}(\gamma_{th}) = 1 - &\frac{\xi^2 r}{\sqrt{\pi}} \sum_{l=1}^{L} \sum_{\Upsilon} \frac{\tau'_l}{B^{\delta_l} l^{\delta_l}} \sum_{\mathcal{S}\backslash\varnothing} \sum_{\mathcal{J}} \sum_{\mathcal{R}} (-1)^{Q-1} \\ &\times C_{\mathcal{SJR}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} E^k_{\mathcal{SJ}} \gamma_{th}^{k + r_{\mathcal{J},\mathcal{R}} - 1} \\ \times \mathcal{H}^{1,0:2,5:1,2}_{0,1:3,1:1,1} &\left[\begin{matrix} \frac{B^{2r} l^{2r} \gamma_{th}}{\tilde{\mu}_r} \\ -1 \end{matrix} \middle| \begin{matrix} (-1:1,1) \\ - \\ (\xi, \Xi); (k + r_{\mathcal{J},\mathcal{R}}, 1) \\ (\delta, \Delta), (-1,1); (0,1), (k + r_{\mathcal{J},\mathcal{R}} - 1, 1) \end{matrix} \right], \end{aligned} \quad (11)$$

where $\mathcal{H}[\cdot,\cdot]$ denotes the Fox's H-function of two variables [20, Eq. (1.1)], $(\xi, \Xi) = (0,1), (\xi^2 + 1, r)$ and $(\delta, \Delta) = (\xi^2, r), (\frac{\delta_l}{2}, r), (\frac{1}{2} + \frac{\delta_l}{2}, r), (1,1)$.

Proof: By plugging (9) and the probability density function (pdf) of the FSO TAPS SNR, obtained from differentiating (6) with respect to γ by applying [18, Eq. (1.69)], into (10) and relabeling $u = \frac{u-1}{u}$ after using the Taylor series expansion of the exponential function, we obtain

$$\begin{aligned} P_{\text{out}}(\gamma_{th}) &= 1 - \frac{\xi^2 r}{\sqrt{\pi}} \sum_{l=1}^{L} \sum_{\Upsilon} \frac{\tau'_l}{B^{\delta_l} l^{\delta_l}} \sum_{\mathcal{S}\backslash\varnothing} \sum_{\mathcal{J}} \sum_{\mathcal{R}} C_{\mathcal{SJR}} \\ &\times \sum_{k=0}^{\infty} \frac{(-1)^{k+Q}}{k!} E^k_{\mathcal{SJ}} \gamma_{th}^{k + r_{\mathcal{J},\mathcal{R}} - 1} \int_0^1 \frac{u^{-k - r_{\mathcal{J},\mathcal{R}}}}{(1-u)^2} \\ &\times \mathcal{H}^{3,1}_{2,4}\left[\frac{B^{2r} l^{2r} \gamma_{th}}{\tilde{\mu}_r (1-u)} \middle| \begin{matrix} (\xi, \Xi) \\ (\delta, \Delta) \end{matrix}\right] du. \end{aligned} \quad (12)$$

Then resorting to [18, Eq. (2.66)] yields (11) after some manipulations.

Special Cases: It can be shown for $L = K = 1$ (i.e. Málaga-$\mathcal{M}$/shadowed κ-μ fading) that (11) simplifies to [8, Eq. (19)]. For $L = 1$, $m_k = \mu_k, \mu_k = 2\mu, \kappa_k = (1 - \eta_k)/2\eta_k$, and $k = 1, \ldots, K$, (i.e. Málaga-$\mathcal{M}$/multiuser η-μ fading), the cdf in (11) simplifies to [10, Eq. (21)] when $g = 0, \phi = 1$, and $\kappa_k \to \infty$ (i.e. Gamma-Gamma/multiuser Nakagami-m fading).

B. Exact Average Symbol Error Probability

The average SEP performance at the best user is

$$\mathcal{B} = a \int_0^\infty \sqrt{\frac{b}{t}}\, \mathcal{J}_{-1}\left(2\sqrt{bt}\right) \mathcal{M}_{\gamma_{\mathcal{R}}^{-1}}(t)\, \mathcal{M}_{\gamma_{\mathcal{F}}^{-1}}(t)\, dt, \quad (13)$$

where $\mathcal{M}_{1/\gamma_X}(s)$, $X \in \{\mathcal{F}, \mathcal{R}\}$ is the MGF of the inverse SNR of the first and second hops, respectively. Moreover, a and b are parameters for different M-ary modulations, and $\mathcal{J}_{-1}$ is the Bessel function of the first kind [17]. In what follows, $\mathcal{M}_{1/\gamma_X}(s)$ are obtained directly from the ccdfs as $M_{1/\gamma_X}(s) = s\int_0^\infty t^{-2} e^{-s/t} F^c_X(t) dt$, where $F^{(c)}_X(t)$, $X \in \{\mathcal{R}, \mathcal{F}\}$. In the FSO link, exploiting (6) and recognizing that $\exp(-x) = \mathrm{H}^{1,0}_{0,1}\left[x \middle| \begin{matrix} - \\ (0,1) \end{matrix}\right]$ [18, Eq. (1.125)], $M_{1/\gamma_{\mathcal{F}}}(s)$ is expressed using the Fox's H-function properties [18, Eq. (1.58)] and applying [18, Eq. (2.3)], thereby yielding

$$\begin{aligned} M_{\gamma_{\mathcal{F}}^{-1}}(s) &= \frac{\xi^2 r}{s\sqrt{\pi}} \sum_{l=1}^{L} \sum_{\Upsilon} \frac{\tau'_l}{B^{\frac{\delta_l}{2}} l^{\delta_l}} \\ &\times \mathcal{H}^{4,0}_{1,4}\left[\frac{B^r l^{2r} s}{\tilde{\mu}_r} \middle| \begin{matrix} (\xi^2 + 1 - \frac{\delta_l}{2}, r) \\ (\delta, \Delta) \end{matrix}\right]. \end{aligned} \quad (14)$$

The expression of $M_{1/\gamma_{\mathcal{R}}}(s)$ is obtained from (9) by relying on the very same approach adopted in (14) and applying [18, Eq. (2.3)] as

$$\begin{aligned} M_{\gamma_{\mathcal{R}}^{-1}}(s) &= \sum_{\mathcal{S}\backslash\varnothing} \sum_{\mathcal{J}} \sum_{\mathcal{R}} (-1)^{Q-1} \frac{s\, C_{\mathcal{SJR}}}{E_{\mathcal{SJ}}^{r_{\mathcal{J},\mathcal{R}}-1}} \\ &\times \mathcal{H}^{2,0}_{0,2}\left[E_{\mathcal{SJ}} s \middle| \begin{matrix} - \\ (r_{\mathcal{J},\mathcal{R}} - 1, 1), (0,1) \end{matrix}\right]. \end{aligned} \quad (15)$$

Substituting the last expression with (14) into (13) and applying [20, Eq. (2.1)] with $J_{-1}(z) = \mathrm{H}^{1,0}_{0,2}\left[\frac{z^2}{4} \middle| \begin{matrix} - \\ (-\frac{1}{2}, 1), (\frac{1}{2}, 1) \end{matrix}\right]$ [18,

$$
\mathcal{B} = \frac{\xi^2 ar}{\sqrt{\pi}} \sum_{l=1}^{L} \sum_{\Upsilon} \frac{\tau_l'}{B^{\frac{\delta_l}{2}} l^{\delta_l}} \sum_{\mathcal{S}\backslash\varnothing} \sum_{\mathcal{J}} \sum_{\mathcal{R}} (-1)^{Q-1} \frac{C_{\mathcal{S}\mathcal{J}\mathcal{R}}}{E_{\mathcal{S}\mathcal{J}}^{r_{\mathcal{J},\mathcal{R}}-1}} \mathcal{H}_{2,[0:1],0,[2:4]}^{1,0,0,2,4} \left[\begin{array}{c|c} \frac{E_{\mathcal{S}\mathcal{J}}}{b} & (0,1),(1,1) \\ & -, (\xi^2+1-\frac{\delta_l}{2}, r) \\ \frac{B^r l^{2r}\gamma}{\widetilde{\mu}_r b} & - \\ & (r_{\mathcal{J},\mathcal{R}}-1,1),(0,1),(\delta,\Delta) \end{array} \right]. \quad (16)
$$

Eq. (1.127)], yield the exact error probability expression as shown in (16) at the top of this page.

Special Cases: As a special case, it can be shown that (16) provides an exact alternative expression to the approximate one for the SEP obtained in [21, Eq. (22)] if we set $L=1$, $m_k=\mu_k, \mu_k = 2\mu, \kappa_k = (1-\eta_k)/2\eta_k$, and $k=1,\ldots,K$, (i.e. Málaga-$\mathcal{M}$/multiuser η-μ fading). For $K=1$, $m=\mu=1$, the SEP in (16) simplifies to [9, Eq. (32)] obtained at high SNR for multi-aperture Málaga-$\mathcal{M}$/Rayleigh fading channels. For $L=K=1$, $g=0, \phi=1$, and $m=\mu$ (i.e. Gamma-Gamma/Nakagami-m fading), the SEP in (16) simplifies to [3, Eq. (39)].

IV. Asymptotic Analysis

A. Outage Probability

The outage probability can be approximated at high SNR values as

$$
P_{\text{out}}^{\infty} \approx F_{\gamma_{\mathcal{F}}}^{\infty}(\gamma_{th}) + F_{\gamma_{\mathcal{R}}}^{\infty}(\gamma_{th}). \quad (17)
$$

The asymptotic outage probability with the use of opportunistic user scheduling at the RF hop and TAPS at the FSO hop is derived as

$$
\begin{aligned}
P_{\text{out}}^{\infty} &\approx (\mathcal{G}_c \bar{\gamma})^{-\mathcal{G}_d} && (18)\\
&\approx \frac{(\mu(1+\kappa))^{K\mu}}{\left(\frac{\mu\kappa+m}{m}\right)^{Km} (\mu-1)!^K} \left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{K\mu} + \\
&\quad \Lambda \left(\frac{\gamma_{th}}{\widetilde{\mu}_r}\right)^{\min\left(\frac{\xi^2}{r}, \frac{L\beta}{r}, \frac{L\alpha}{r}\right)}, && (19)
\end{aligned}
$$

where

$$
\frac{\Lambda}{\xi^2} = \begin{cases} 2\sum_{p=1}^{L} \binom{L}{p} \frac{(-A\sqrt{\pi})^p}{(2Bp)^{-2\xi^2}} \widetilde{\sum}_k \widetilde{\sum}_i \widetilde{\sum}_j \Xi_1, & \delta = \frac{\xi^2}{r}; \\ \left(\frac{A\sqrt{\pi} 2^{2\beta-1} \psi_{\beta,\alpha-\beta-\frac{1}{2}}}{B^{\alpha-\beta}(\xi^2-\beta)}\right)^L, & \delta = \frac{L\beta}{r}; \\ \left(\frac{A\sqrt{\pi} 2^{2\alpha-1} \psi_{\beta,\beta-\alpha-\frac{1}{2}}}{B^{\alpha-\beta}(\xi^2-\alpha)}\right)^L, & \delta = \frac{L\alpha}{r}, \end{cases} \quad (20)
$$

where $\Xi_1 = \prod_{t=1}^{p} \frac{\Psi_{k_t,i_t}\Gamma(\alpha+k_t-i_t-\frac{1}{2})}{B^{\alpha+k_t} p^{j_t} j_t!} \Gamma\left(\sum_{t=1}^{p} j_t - 2\xi^2\right)$ and $\delta = \min\left(\frac{\xi^2}{r}, \frac{L\beta}{r}, \frac{L\alpha}{r}\right)$.

Proof: See Appendix.

Comparing (18) with (19), the achievable diversity gain is expressed as $\mathcal{G}_d = \min\{\frac{\xi^2}{r}, \frac{L\beta}{r}, \frac{L\alpha}{r}, K\mu\}$. This result shows that the diversity order is dependent of the worst channel link between the RF and FSO links [8]. In the special case where $m=\mu$, the first term in the RHS of (19) reduces to the asymptotic cdf of the post-user selection SNR in Nakagami-m fading given by $\frac{m^{Km}}{\Gamma(m)^K}\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{Km}$, and $\mathcal{G}_d$ coincides with a previously obtained result in [12] when $L=1$. It is worthwhile to mention here that the MRT scheme employed at the M-antenna relay improves the diversity gain which is in fact given by $KM\tilde{\mu}$, where $\tilde{\mu}$ being the effective fading parameter between the M-antennas relay and the selected user as defined after (7).

1) MmWave Multiuser Mixed FSO-RF Systems: By setting $\mu=1$ and letting $m \longrightarrow \infty$, the κ-μ shadowed distribution specializes to the Rician fading [19]. The latter has been shown to be appropriate model for near LOS conditions and has been well established for different mmWave-based applications [13]. Therefore, it is quite straightforward to obtain the outage probability and SEP of mmWave multiuser hybrid FSO-RF systems by using the results of Sections III and IV. In particular, the asymptotic outage probability in mixed FSO/multiuser mmWave RF relay networks is obtained from (19) as

$$
\begin{aligned}
P_{out}^{MMv} &\overset{(a)}{=} \Lambda\left(\frac{\gamma_{th}}{\widetilde{\mu}_r}\right)^{\min\{\frac{\xi^2}{r}, \frac{L\beta}{r}, \frac{Ln}{r}\}} \\
&\quad + e^{-KK_R}\left(\frac{(1+K_R)\gamma_{th}}{\widetilde{\bar{\gamma}}}\right)^K, \quad (21)
\end{aligned}
$$

where $K_R > 0$ is the Rice factor of the selected user. Moreover, (a) follows by letting $\mu=1$, $\kappa=K_R$ in (19) while applying $\lim\limits_{m \to \infty}\left(1+\frac{x}{m}\right)^{-m} = e^{-x}$. From (21), we observe that the Rician factor only affects the SNR gain, while the diversity gain stems form multiplexing K users over Rayleigh fading. Note that, in practice, the κ-μ shadowed distribution converges rapidly to the Rician distribution, i.e., for $m \approx 15-20$.

B. Asymptotic Average SEP

The Using (19), the asymptotic MGF is given by

$$
M_{\gamma_{ub}}^{\infty}(s) \approx \Lambda_1 \frac{\Gamma(1+\min\{K\mu,\delta\})}{s^{\min\{K\mu,\delta\}}} \left(\frac{1}{\bar{\gamma}}\right)^{\min\{K\mu,\delta\}}. \quad (22)
$$

Hence, the asymptotic error probability $\mathcal{B} = \frac{\beta_M}{\pi}\sum_{p=1}^{\tau_M}\int_0^{\pi/2} M_{\gamma_{ub}}\left(\frac{a_p^2}{2\sin^2(\theta)}\right)d\theta$ where β_M, a_p, and τ_M are modulation-dependent parameters, is expressed as

$$
\mathcal{B}^{\infty} = \frac{\beta_M \Psi}{\pi} \sum_{p=1}^{\tau_M} \frac{\Lambda_1 \Gamma(1+\min\{K\mu,\delta\})}{\left(\frac{1}{a_p^2\bar{\gamma}}\right)^{-\min\{K\mu,\delta\}}}, \quad (23)
$$

where $\Psi = \int_0^{\pi/2} \sin^2(\theta)^{\mathcal{G}_d} d\theta = \frac{\sqrt{\pi}\Gamma(\mathcal{G}_d+\frac{1}{2})}{2\Gamma(\mathcal{G}_d+1)}$.

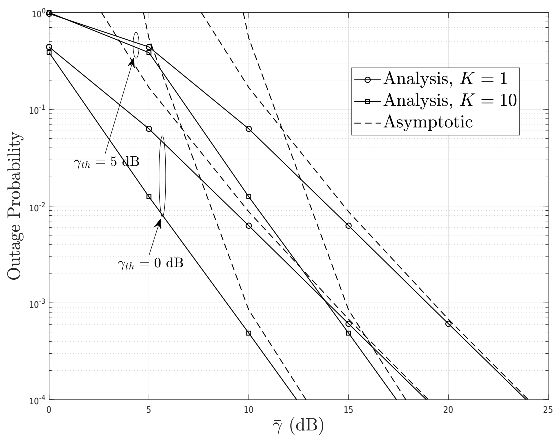

Fig. 1. The outage probability of user selection in mixed Málaga-$\mathcal{M}$/mmWave relay networks for different values of the Rice factor $K_R = \{1, 10\}$ and threshold $\gamma_{th} = \{0, 5\}$ dB.

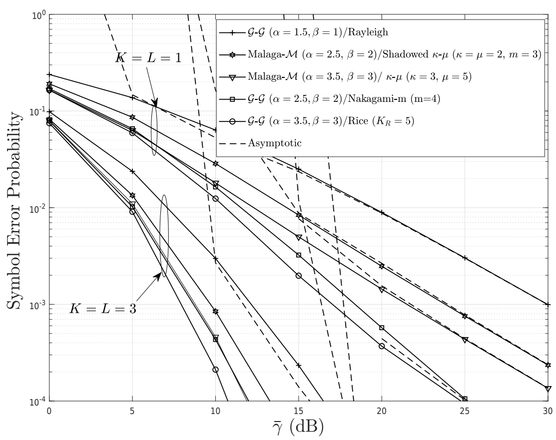

Fig. 2. The average SEP of user selection in mixed Málaga-$\mathcal{M}$/shadowed κ-μ relay networks with $L = K = \{1, 3\}$ and QPSK modulation.

V. Numerical Results

In this section, numerical examples are shown to substantiate the accuracy of the new unified mathematical framework derived in section III and to confirm its potential for analyzing multiuser FSO-RF networks with transmit diversity (TAPS).

Fig. 1 shows the outage probability of a mmWave multiuser AF relay network with aperture selection versus the average SNR for different values of $\gamma_{th} = 0, 5$ dB. We verify that the analytical curves for the shadowed κ-μ distribution with $\kappa = K_R$, $\mu = 1$ and sufficiently large m, set to $m = 15$ for sufficient numerical accuracy, perfectly matches those plotted by Monte-Carlo simulations in the Rician case. The exact match with Monte-Carlo simulation results confirms the precision of our theoretical analysis. Moreover, we notice that the exact and asymptotic expansion in (21) agree very well at high SNRs. Fig. 3 depicts the average SEP per formance of mixed TAPS/user selection relay systems in Málaga-$\mathcal{M}$ and shadowed κ-μ fading channels both for $L = K = \{1, 3\}$. In the legend, we have identified some particular turbulence and fading distribution cases that simply stem from the general Málaga and κ-μ shadowed fading scenarios, respectively. In particular, when $g = 0$ and $\phi = 1$, (16) reduces to the SEP for Gamma-Gamma ($\mathcal{G}$-$\mathcal{G}$) TAPS/SNR-based selection over shadowed κ-μ fading. The latter, includes as special cases κ-μ ($m \to \infty$), Nakagami-m ($\mu = m$ and $\kappa \to 0$), Rayleigh ($\mu = m = 1$ and $\kappa \to 0$), and Rice ($\mu = 1, \kappa = K$ and $m \to \infty$), to name a few [19].

VI. Conclusion

Different processing designs and network conditions for mixed FSO-RF relay networks were presented in this paper. Additionally, a very generic two-hop propagation model over Málaga-$\mathcal{M}$ optical channels with pointing errors on the first hop and shadowed κ-μ distributed radio channels on the second hop was considered. The performance of mixed FSO-RF control-access schemes for multiuser networks was firstly studied using TAPS/max-SNR user selection. Also, a new analytical results for some key performance metrics, namely the outage probability and the average SEP were derived. Asymptotic approximations were also derived for these performance measures, in addition to the system diversity order and coding gain. We were then able to extend our treatment to cover the compelling case of mmWave user selection made possible owing the suitability of the shadowed κ-μ distribution in modeling LOS channels. The results showed that, under weak atmospheric turbulence conditions, the system performance is dominated by the RF channels and achieves a full diversity order of $K\mu$. Whereas, the performance is dominated under severe atmospheric turbulence conditions by the FSO channel and its diversity order is proportional to the minimum value of L times the turbulence fading parameters and the pointing errors parameter.

VII. Appendix

Asymptotic $F_{\gamma_{\mathcal{X}}}, X \in \{\mathcal{F}, \mathcal{R}\}$

The asymptotic performance of a dual-hop relaying system depends on the behavior of $f_{\gamma_X}(y)$, $X \in \{\mathcal{F}, \mathcal{X}\}$ at $y = 0_+$. Assume that $f_{\gamma_X}(y)$ accepts a Taylor series expansion at $y \to 0_+$ as $f_{\gamma_X}(y) \underset{y \to 0_+}{\approx} a_X y^{b_X} + o(y^{b_X})$, where a_X and b_X are appropriate constants. Then the corresponding per hop cdf can be approximated as $F_{\gamma_X}(y) \underset{y \to 0_+}{\approx} a_X/(b_x+1) y^{b_X+1} + o(y^{b_X})$.

1) Calculation of $F^{\infty}_{\gamma_{\mathcal{F}}}(\gamma)$: Before delving in the asymptotic expansion of the first-hop FSO TAPS cdf, it is more convenient to reexpress $F_{\gamma_{\mathcal{F}}}$ in (5) as follows

$$
\begin{aligned}
F_{\gamma_{\mathcal{F}}}(\gamma) \; &= \; 1 - 2\xi^2 \sum_{p=1}^{L} (-1)^{p-1} (A\sqrt{\pi})^p \widetilde{\sum_k} \widetilde{\sum_i} \widetilde{\sum_j} \\
&\prod_{t=1}^{p} \frac{2^{j_t} \Psi_{k_t, i_t}}{B^{\alpha + k_t - j_t}} \frac{\Gamma(\alpha + k_t - i_t - \frac{1}{2})}{j_t!} (2Bp)^{\left(2\xi^2 - \sum_{t=1}^{p} j_t\right)} \\
&\times \left(\frac{\gamma}{\widetilde{\mu}_r}\right)^{\frac{\xi^2}{r}} \Gamma\left(\sum_{t=1}^{p} j_t - 2\xi^2, 2Bp \sqrt{\left(\frac{\gamma}{\widetilde{\mu}_r}\right)^{\frac{1}{r}}} \right), \qquad (24)
\end{aligned}
$$

where $B = \sqrt{\frac{\alpha\beta}{g\beta+\phi}}$, $\widetilde{\sum}_k = \sum_{k_1}^{\beta} \ldots \sum_{k_p}^{\beta}$, $\widetilde{\sum}_i = \sum_{i_1}^{\alpha-k_1-\frac{1}{2}} \ldots \sum_{i_p}^{\alpha-k_p-\frac{1}{2}}$, and $\widetilde{\sum}_j = \sum_{j_1}^{\alpha+k_1-i_1-\frac{3}{2}} \ldots \sum_{j_p}^{\alpha+k_p-i_p-\frac{3}{2}}$. For large values of $\widetilde{\mu}_r$ it holds that $\Gamma(a,z) \underset{z\to 0}{\approx} \Gamma(a) - \frac{z^a}{a}$. Then keeping in mind that $K_{-\nu}(z) \underset{\nu \notin \mathbb{Z}}{=} K_\nu(z)$ in [16, Eq. (24)], the asymptotic expansion of the FSO TAPS SNR corresponding to its pdf series expansion from (24) is obtained as

$$F_{\gamma_{\mathcal{F}}}(\gamma) \underset{\widetilde{\mu}_r\to\infty}{\simeq} \Lambda \left(\frac{\gamma}{\widetilde{\mu}_r}\right)^{\min\left(\frac{\xi^2}{r}, \frac{L\beta}{r}, \frac{Ln}{r}\right)}, \tag{25}$$

where Λ is given in (20).

2) Calculation of $F_{\gamma_{\mathcal{R}}}^{\infty}(\gamma)$: In the RF side, the users are assumed to have identical channels i.e., $N_k = N$ and the set of parameters $\{C_{j,k}, m_{j,k}, \Omega_{j,k}\} = \{C_j, m_j, \Omega_j\}$ for $k = 1 \ldots K$. Moreover, when $\mu \leq m$, we have

$$\Omega_j = \Omega = \frac{\mu\kappa + m}{m} \frac{\bar{\gamma}}{\mu(1+\kappa)}, \quad j = 1, \ldots, N \tag{26}$$

with $\bar{\gamma}$ being the average SNR. After applying the Taylor series expansion of exponentials ($e^{-x} \approx 1 - x$ as $x \to 0$), the cdf in (8) simplifies when $m \geq \mu$ to

$$\begin{aligned} F_{\gamma_b}(\gamma) &\overset{(a)}{\simeq} \left(\sum_{j=0}^{m-\mu} \frac{C_j}{(m-j-1)!}\left(\frac{\gamma}{\Omega}\right)^{m-j}\right)^K \\ &\underset{\bar{\gamma}\to\infty}{\simeq} \left(\frac{C_{m-\mu}}{(\mu-1)!}\right)^K \left(\frac{\gamma}{\Omega}\right)^{K\mu}, \end{aligned} \tag{27}$$

where (a) follows after recognizing that $\sum_{j=0}^{m-\mu} C_j = 1$ with $C_i = \binom{m-\mu}{i}\left(\frac{\mu\kappa+m}{m}\right)^{-i}\left(\frac{\mu\kappa+m}{\mu\kappa}\right)^{m-\mu-i}$.

References

[1] I.Trigui, S. Affes, A. M. Salhab, M-S. Alouini, "Multi-User Mixed FSO-RF Systems With Aperture Selection Under Poisson Field Interference", *IEEE Access*, vol. 7, pp. 73764-73781, May 2019.

[2] X. Ge, S. Tu, G. Mao, C.-X. Wang, and T. Han, "5G ultra-dense cellular networks," *IEEE Wireless Commun.*, vol. 23, no. 1, pp. 72–79, Feb. 2016

[3] E. Zedini, I. S. Ansari, and M.-S. Alouini, "Performance analysis of mixed Nakagami-m and Gamma-Gamma dual-hop FSO transmission systems," *IEEE/OSA Photon. J.*, vol. 7, no. 1, Feb. 2015.

[4] S. Anees and M. R. Bhatnagar, "Performance of an amplify-and-forward dual-hop asymmetric RF-FSO communication system," *IEEE/OSA J. Opt. Commun. and Netw.*, vol. 7, no. 2, pp. 124–135, Feb. 2015.

[5] E. Soleimani-Nasab and M. Uysal, "Generalized performance analysis of mixed RF/FSO cooperative systems," *IEEE Trans. Wireless Commun.*, vol. 15, no. 1, pp. 714–727, Jan. 2016.

[6] J. Zhang, L. Dai, Y. Zhang, and Z. Wang, "Unified performance analysis of mixed radio frequency/free-space optical dual-hop transmission systems," *IEEE/OSA J. Lightw. Technol.*, vol. 33, no. 11, pp. 2286–2293, Jun. 2015.

[7] P. V. Trinh, T. C. Thang, and A. T. Pham, "Mixed mmWave RF/FSO relaying systems over generalized fading channels with pointing errors," *IEEE/OSA Photon. J.*, vol. 9, no. 1, Feb. 2017.

[8] I. Trigui, N. Cherif, and S. Affes, "Relay-assisted mixed FSO/RF systems over Málaga-$\mathcal{M}$ and κ-μ shadowed fading channels," *IEEE Wireless Commun. Lett.*, vol. 6, no. 5, pp. 682–685, Oct. 2017.

[9] L. Yang, M. O. Hasna, and X. Gao, "Performance of mixed RF/FSO with variable gain over generalized atmospheric turbulence channels," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 9, pp. 1913–1924, Sep. 2015.

[10] A. M. Salhab, F. Al-Qahtani, R. M. Radaydeh, S. A. Zummo, and H. Alnuweiri, "Power allocation and performance of multiuser mixed RF-FSO relay networks with opportunistic scheduling and outdated channel information," *IEEE/OSA J. Lightw. Technol.*, vol. 34, no. 13, pp. 3259–3272, Jul. 2016.

[11] A. M. Salhab, "Performance of multiuser mixed RF-FSO relay networks with generalized order user scheduling and outdated channel information," *Arabian J. for Science and Engineering*, vol. 40, no. 9, pp. 2671–2683, Jul. 2015.

[12] A. A. El-Malek, A. M. Salhab, S. A. Zummo, and M.-S. Alouini, "Security-reliability trade-off analysis for multiuser SIMO mixed RF-FSO relay networks with opportunistic user scheduling," *IEEE Trans. Wireless Commun.*, vol. 15, no. 9, pp. 5904–5918, Sep. 2016.

[13] Y. Azar, G. N. Wong, K. Wang, R. Mayzus, J. K. Schulz, H. Zhao, F. Gutierrez, D. Hwang, and T. S. Rappaport, "28 GHz propagation measurements for outdoor cellular communications using steerable beam antennas in New York city," in *Proc. IEEE Int. Conf. Commun. (ICC),* pp. 5143–5147, Jun. 2013.

[14] C. Abou-Rjeily and M. Bkassiny, "On the achievable diversity orders over non-severely faded lognormal channels," *IEEE Commun. Lett.*, vol. 14, no. 8, pp. 695–697, Aug. 2010

[15] H. AlQuwaiee, H.-C. Yang, and M.-S. Alouini. "On the asymptotic capacity of dual-aperture FSO systems with generalized pointing error model," *IEEE Trans. Wireless Commun.*, vol. 15, no. 9, pp. 6502–6512, Sep. 2016.

[16] A. Jurado-Navas, J. M. Garrido-Balsells, J. F. Paris, M. Castillo-Vazquez, and A. Puerta-Notario, "Impact of pointing errors on the performance of generalized atmospheric optical channels," *Optics Express,* vol. 20, no. 11, pp. 12550–12562, May 2012.

[17] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, 1994.

[18] A. M. Mathai, R. K. Saxena, and H. J. Haubold, *The H-function: Theory and Applications*, *Springer Science & Business Media*, 2009.

[19] L. Moreno-Pozas, F. J. Lopez-Martinez, J. F. Paris, and E. Martos-Naya, "The κ-μ shadowed fading model: Unifying the κ-μ and η-μ distributions," *IEEE Trans. Veh. Technol.,* vol. 65, no. 12, pp. 9630–9641, Dec. 2016.

[20] P. Mittal and K. Gupta, "An integral involving generalized function of two variables," *Proceedings of the Indian Academy of Sciences-Section A*, vol. 75, no. 3, pp. 117–123, 1972.

[21] L. Yang, M. O. Hasna, and I. S. Ansari, "Unified performance analysis for multiuser mixed η-μ and $\mathcal{M}$-distribution dual-hop RF/FSO systems," *IEEE Trans. Wireless Commun.*, vol. 65, no. 8, pp. 3601–3613, Aug. 2017.