BLIND MAXIMUM LIKELIHOOD JADE IN MULTIPATH ENVIRONEMENT USING IMPORTANCE SAMPLING*

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ABSTRACT

In this paper, we tackle the problem of joint angles and time delays estimation (JADE) in a non-data aided (NDA) scenario where the transmitted signal is unknown at the receiver. We do so by applying the maximum likelihood (ML) in order to obtain the best performance achievable. The importance sampling (IS) technique is used to reduce the multi-dimensionality of the maximization problem without recurring to an iterative option. Computer simulations show that the new ML IS solution approaches the DA techniques and the Cramér-Rao lower bound (CRLB) at medium and high SNR levels.

1. INTRODUCTION

The joint angles and time delays estimation (JADE) problem finds application in diverse domains ranging from indoor positioning [1,2], RADAR systems [3], etc., to broadband wireless communications [4]. So far, a number of JADE techniques have been reported in the literature and except for the MI-MUSIC approach proposed [5], all the existing solutions are either geared toward data-aided estimation or rely on an estimate of the channel matrix. Roughly speaking, they can be broadly categorized into two major categories: subspace-based and ML-based estimators. Most of the subspace-based techniques are built upon the well-known MUSIC and ESPRIT algorithms [4,6,7]. In practice, subspace-based approaches are more attractive due to their reduced computational load. However, they are usually suboptimal and suffer from severe performance degradation for low SNR levels. ML approaches [3,8,9], however, are known to enjoy higher accuracy. In the specific JADE context, to the best of our knowledge, the ML approach has not been addressed so far.

In this paper, we develop a non-iterative ML estimator for the JADE problem under the NDA scenario that is based on the importance sampling (IS) concept. In this work, we design a separable (i.e., factorisable) joint angle-delay *pseudo-pdf* which allows a very easy generation of the required vector realizations. Computer simulations show that the technique has a comparable performance when compared to the fully DA technique in terms of estimation accuracy.

The rest of the paper is organized as follows: In Section 2, we introduce the system model. In Section 3, we derive the new NDA ML solution for the underlying estimation problem. In Section 4, we provide the required details regarding the IS technique. In Section 5, we use exhaustive computer simulations to assess the performance of the proposed approach. Finally, we draw out some concluding remarks in Section 6.

The notations adopted in this paper are as follows. Vectors and matrices are represented in lower- and upper-case bold fonts, respectively. Moreover, $\{.\}^T$ and $\{.\}^H$ denote the conjugate and Hermitian (i.e., transpose conjugate) operators. The Euclidean norm of any vector is denoted as $\|.\|$. For any matrix $\mathbf{X}$, $[\mathbf{X}]_q$ and $[\mathbf{X}]_{l,k}$ denote its q^{th} column and $(l,k)^{th}$ entry, respectively. For any vector $\mathbf{x}$, diag$\{\mathbf{x}\}$ refers to the diagonal matrix whose elements are those of $\mathbf{x}$. Moreover, $\{.\}^*$, $\angle\{.\}$, and $|.|$ return the conjugate, angle, and modulus of any complex number, respectively. $\lfloor x \rfloor$ is the ceil function defined as $\lfloor x \rfloor = \max\{n \in \mathbb{Z} | n \leq x\}$. Finally, j is the pure imaginary number (i.e., $j^2 = -1$), and the notation $\triangleq$ is used for definitions.

2. SYSTEM MODEL

Consider a single input multiple output (SIMO) system equipped with P antenna elements at the receiver side. The transmitted signal goes through a multipath channel consisting of Q different paths that impinge on the receiving antenna array from Q different angles $\{\theta_q\}_{q=1}^{Q}$, respectively. Each path is also characterized by a propagation delay $\{\tau_q\}_{q=1}^{Q}$ assumed to be unknown but constant and a path gain $\{\gamma_q\}_{q=1}^{Q}$. Note that $(\tau_1, \tau_2, \cdots, \tau_Q) \subset [0, \tau_{\max}]^Q$ where $\tau_{\max \leq T}$. At the destination, the received signal at the output of the p^{th} antenna element is given by:

$$y_p(t) = \sum_{q=1}^{Q} \gamma_q e^{j\varphi_p(\theta_q)} \sum_{k=0}^{K-1} c_k h(t - \tau_q - kT) + w_p(t), \quad (1)$$

where $\varphi_p(\theta)$ is a real-valued angular transformations that depend on the array geometry and $w_p(t)$ is an additive white Gaussian noise (AWGN) with zero mean and variance σ^2. $h(t)$ is the shaping pulse, T is the symbol duration and $\mathbf{c} = [c_0, c_2, \ldots, c_{K-1}]^T$ is the vector containing the K transmitted symbols. The latter are generated randomly from a $M-$ary constellation alphabet, $\mathcal{C}^M$. Moreover, we assume that all the transmitted symbols are unknown at the receiver side. The received signal is sampled at the rate $\zeta = \frac{T}{T_s}$ where T_s is the sampling period, leading to the following samples at the

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output of the p^{th} antenna element:

$$y_p(n) = \sum_{q=1}^{Q} \gamma_q e^{j\varphi_p(\theta_q)} \sum_{k=0}^{K-1} c_k h(t_n - \tau_q - kT) + w_p(n), \quad (2)$$

where $\{t_n\}_{n=1}^{N}$ are the time samples. We now group all the unknown multipath parameters in the following three vectors: $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_Q]^T$, $\boldsymbol{\tau} = [\tau_1, \tau_2, \cdots, \tau_Q]^T$, and $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \cdots, \gamma_Q]^T$. We further gather the samples collected across all the antenna elements at a given time instant into a single vector, $\mathbf{y}(n) = [y_1(n), y_2(n), \cdots, y_P(n)]^T$, given by:

$$\mathbf{y}(n) = \sum_{q=1}^{Q} \gamma_q \mathbf{a}(\theta_q) \sum_{k=0}^{K-1} c_k h(t_n - \tau_q - kT) + \mathbf{w}(n), \quad (3)$$

where $\mathbf{w}(n) = [w_1(n), w_2(n), \cdots, w_P(n)]^T$ is the corresponding noise vector and $\mathbf{a}(\varphi) \triangleq \left[1,\ e^{j\varphi_1(\theta)},\ e^{j\varphi_2(\theta)}, \cdots,\ e^{j\varphi_P(\theta)}\right]^T$ is the array *steering vector* defined for any direction θ. Our goal in the remainder of this paper is to jointly estimate the parameters $\{\theta_q\}_{q=1}^{Q}$ and $\{\tau_q\}_{q=1}^{Q}$ using K symbols that are unknown to the receiver. To that end, we gather all the receiving samples in a single matrix $\mathbf{Y} = [\mathbf{y}(1), \mathbf{y}(2), \ldots, \mathbf{y}(N)]$ as follows:

$$\mathbf{Y} = \mathbf{A}(\boldsymbol{\theta})\text{diag}\{\boldsymbol{\gamma}\}\mathbf{H}(\boldsymbol{\tau})\mathbf{C} + \mathbf{W}, \quad (4)$$

where $\mathbf{W}$ is the $(P \times N)$ noise matrix with elements $[\mathbf{W}]_{p,n} = w_p(t_n)$, $\mathbf{C} = \mathbf{I}_N \otimes \mathbf{c}$ is a $(KN \times N)$ matrix of the transmitted symbols, and $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ..., \mathbf{a}(\theta_Q)]$ is a $(P \times Q)$ matrix containing the Q steering vectors. The delayed samples of the pulse shape $h(t)$ are gathered into a $(Q \times KN)$ matrix, $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_N]$, where $\mathbf{H}_n$ for $n = 1, ..., N$ is given by:

$$\mathbf{H}_n \triangleq \begin{pmatrix} h(nT_s - \tau_1) & \ldots & h(nT_s - \tau_1 - (K-1)T) \\ h(nT_s - \tau_2) & \ldots & h(nT_s - \tau_2 - (K-1)T) \\ \vdots & \vdots & \vdots \\ h(nT_s - \tau_Q) & \ldots & h(nT_s - \tau_Q - (K-1)T) \end{pmatrix}.$$

3. BLIND JADE

We start by deriving the log-likelihood function (LLF) for the estimation problem that depends on all the unknown parameters $\boldsymbol{\theta}$, $\boldsymbol{\tau}$, $\boldsymbol{\gamma}$, $\mathbf{c}$, and σ^2. Then, we focus on the *compressed* likelihood function (CLF) that depends solely on the delays and the angles. Since the noise components are Gaussian distributed and assumed to be spatially and temporally white, it follows that the LLF that depends on $\boldsymbol{\tau}$, $\boldsymbol{\theta}$, $\boldsymbol{\gamma}, \mathbf{c}$, and σ^2 is:

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\gamma}, \mathbf{c}, \sigma^2) = -PN\ln(\pi\sigma^2) - \frac{1}{\sigma^2}\sum_{n=1}^{N}\left\|\mathbf{y}(n) - \sum_{q=1}^{Q}\gamma_q\mathbf{a}(\theta_q)\sum_{k=0}^{K-1}c_k h(nT_s - kT - \tau_q)\right\|^2. \quad (5)$$

We can reduce the number of unknown parameters that characterize the LLF by maximizing it with respect to the noise variance. To do so, we differentiate te LLF in (5) with respect to σ^2 and inject it back into (5) to obtain the LLF that depends only on $\boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\gamma}$ and $\mathbf{c}$:

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\gamma}, \mathbf{c}) = \sum_{n=1}^{N}\left\|\mathbf{y}(n) - \sum_{q=1}^{Q}\gamma_q\mathbf{a}(\theta_q)\sum_{k=0}^{K-1}c_k h(nT_s - kT - \tau_q)\right\|^2. \quad (6)$$

Now, owing to the Parseval's identity, the LLF can be expressed in the frequency domain as follows:

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\gamma}, \mathbf{c}) \approx \sum_{n=1}^{N}\left\|\mathbf{y}(\omega_n) - \sum_{q=1}^{Q}\gamma_q\mathbf{a}(\theta_q)\sum_{k=0}^{K-1}c_k h(\omega_n)e^{-j\omega_n(kT+\tau_q)}\right\|^2, \quad (7)$$

where $\left\{\omega_n = \frac{n-1}{NT_s}\right\}_{n=1}^{N}$ is the n^{th} frequency bin and $\mathbf{y}(\omega_n)$ and $h(\omega_n)$ are the DFTs of $\mathbf{y}(n)$ and $h(n)$, respectively. The frequency samples of the delayed signals can be written in the following form:

$$\boldsymbol{\Phi}_n(\boldsymbol{\tau}) \triangleq h(\omega_n)\text{diag}\{\mathbf{f}_n(\boldsymbol{\tau})\} \otimes \mathbf{f}_n^{(K)^T}, \quad (8)$$

where $\mathbf{f}_n(\boldsymbol{\tau}) = [e^{-j\omega_n\tau_1}, e^{-j\omega_n\tau_2}, \cdots, e^{-j\omega_n\tau_Q}]^T$ and $\mathbf{f}_n^{(K)} = [1, e^{-j\omega_n kT}, \cdots, e^{-j\omega_n(K-1)T}]^T$. It follows that the LLF in (7) can be written in a matrix form as:

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\gamma}) \approx \left\|\mathbf{y} - \left[\mathbf{I}_N \otimes \mathbf{A}(\boldsymbol{\theta})\right]\boldsymbol{\Phi}(\boldsymbol{\tau})\bar{\boldsymbol{\gamma}}\right\|^2, \quad (9)$$

where $\mathbf{y} = \left[\mathbf{y}(\omega_1)^T\mathbf{y}(\omega_2)^T \cdots \mathbf{y}(\omega_N)^T\right]^T$, $\bar{\boldsymbol{\gamma}} = \boldsymbol{\gamma} \otimes \mathbf{c}$, and $\boldsymbol{\Phi}(\boldsymbol{\tau}) = \left[\boldsymbol{\Phi}_1(\boldsymbol{\tau})^T\ \boldsymbol{\Phi}_2(\boldsymbol{\tau})^T \cdots \boldsymbol{\Phi}_N(\boldsymbol{\tau})^T\right]^T$ is a $(NQ \times KQ)$ matrix. By differentiating the LLF in (9) and setting the result to zero, we obtain the ML estimate of $\bar{\boldsymbol{\gamma}}$ which is given by:

$$\widehat{\bar{\boldsymbol{\gamma}}}_{\text{MLE}} = \Big[\underbrace{\left[\mathbf{I}_N \otimes \mathbf{A}(\boldsymbol{\theta})\right]\boldsymbol{\Phi}(\boldsymbol{\tau})}_{\triangleq\,\mathbf{D}}\Big]^{\dagger}\mathbf{y}, \quad (10)$$

where $\mathbf{D}^{\dagger}$ is the Moore-Penrose pseudo-inverse of $\mathbf{D}$ given by $\mathbf{D}^{\dagger} = \left(\mathbf{D}^H\mathbf{D}\right)^{-1}\mathbf{D}$. Injecting $\widehat{\bar{\boldsymbol{\gamma}}}_{\text{MLE}}$ back into (9) leads to the CLF which depends only on $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$:

$$\mathcal{L}_c(\boldsymbol{\theta}, \boldsymbol{\tau}) = \mathbf{y}^H\mathbf{D}(\mathbf{D}^H\mathbf{D})^{-1}\mathbf{D}^H\mathbf{y}, \quad (11)$$

and the joint ML estimates of $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$ are hence obtained as the solution to the following optimization problem:

$$[\widehat{\boldsymbol{\theta}},\ \widehat{\boldsymbol{\tau}}] = \underset{\boldsymbol{\theta}, \boldsymbol{\tau}}{\text{argmax}}\ \mathcal{L}_c(\boldsymbol{\theta}, \boldsymbol{\tau}). \quad (12)$$

Note that the CLF, $\mathcal{L}_c(\boldsymbol{\theta}, \boldsymbol{\tau})$, is a non-linear function of the delays and the angles. Hence, finding its maximum *analytically* is intractable. Therefore, to find a *non-ietrative* solution to the estimation problem in (12), we use the maximization theorem introduced by Pincus in [10]. The latter, when applied to our estimation problem, leads to the following ML estimates (MLEs) of the delays and angles:

$$\widehat{\tau}_q = \int\cdots\int \tau_q\, \bar{\mathcal{L}}_c(\boldsymbol{\theta}, \boldsymbol{\tau})d\boldsymbol{\theta}d\boldsymbol{\tau}, \quad q = 1,\ 2, \cdots, Q \quad (13)$$

$$\widehat{\theta}_q = \int\cdots\int \theta_q\, \bar{\mathcal{L}}_c(\boldsymbol{\theta}, \boldsymbol{\tau})d\boldsymbol{\theta}d\boldsymbol{\tau}, \quad q = 1,\ 2, \cdots, Q \quad (14)$$

where $\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})$ is the *normalized* CLF defined as:

$$\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau}) \triangleq \frac{e^{\rho_0 \mathcal{L}_c(\boldsymbol{\theta},\boldsymbol{\tau})}}{\int\cdots\int e^{\rho_0 \mathcal{L}_c(\boldsymbol{\theta},\boldsymbol{\tau})} d\boldsymbol{\theta} d\boldsymbol{\tau}}. \quad (15)$$

Intuitively, as ρ_0 tends to infinity, $\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})$ becomes a Dirac delta function centered at the true maximum of $\mathcal{L}_c(\boldsymbol{\theta},\boldsymbol{\tau})$. By noticing that $\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})$ in (15) is nonnegative and integrates to one, the *normalized* CLF can be considered as a pdf. Taking this observation under consideration, the MLEs in (13) and (14) can be seen as statistical expectations, i.e., for $q = 1, 2, \cdots, Q$, which leads to the following results:

$$\widehat{\tau}_q = \frac{1}{R}\sum_{r=1}^{R} \tau_q^{(r)} \quad \text{and} \quad \widehat{\theta}_q = \frac{1}{R}\sum_{r=1}^{R} \theta^{(r)}, \quad (16)$$

where $\{\boldsymbol{\tau}^{(r)}\}_{r=1}^{R}$ and $\{\boldsymbol{\theta}^{(r)}\}_{r=1}^{R}$ are R random realizations generated according to $\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})$.

In the case where the number of paths is unknown, it needs to be estimated before proceeding to AoAs and TDs acquisition. A heuristic approach that allows the exact estimation of Q is proposed in [8].

4. IMPORTANCE SAMPLING CONCEPT

Since $\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})$ is $2Q-$dimensional and extremely non-linear, one interesting solution to generate the required realizations is by using a different distribution than the one of interest [11-13]. The importance sampling concept allows us to practically apply such idea to generate $\{\boldsymbol{\tau}^{(r)}\}_{r=1}^{R}$ and $\{\boldsymbol{\theta}^{(r)}\}_{r=1}^{R}$. In fact, we can rewrite (13) and (14) in the following equivalent forms, for $q = 1, 2, \cdots, Q$:

$$\widehat{\tau}_q = \int\cdots\int \tau_q \frac{\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})}{\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})} \bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau}) d\boldsymbol{\theta} d\boldsymbol{\tau}, \quad (17)$$

$$\widehat{\theta}_q = \int\cdots\int \theta_q \frac{\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})}{\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})} \bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau}) d\boldsymbol{\theta} d\boldsymbol{\tau}, \quad (18)$$

where $\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})$ is a new function to be chosen as close as possible to $\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})$ and allowing at the same time an easy generation of the required realizations $\{\boldsymbol{\tau}^{(r)}\}_{r=1}^{R}$ and $\{\boldsymbol{\theta}^{(r)}\}_{r=1}^{R}$. In this case, both $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$ can be alternatively considered as jointly distributed according to $\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})$. Hence, the MLEs in (17) and (18) are rewritten as:

$$\widehat{\tau}_q = \frac{1}{R}\sum_{r=1}^{R} \eta\big(\boldsymbol{\theta}^{(r)},\boldsymbol{\tau}^{(r)}\big)\tau_q^{(r)}, \ \widehat{\theta}_q = \frac{1}{R}\sum_{r=1}^{R} \eta\big(\boldsymbol{\theta}^{(r)},\boldsymbol{\tau}^{(r)}\big)\theta_q^{(r)}, (19)$$

where $\eta(\boldsymbol{\theta},\boldsymbol{\tau})$ is defined as the following ratio:

$$\eta(\boldsymbol{\theta},\boldsymbol{\tau}) \triangleq \frac{\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})}{\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})}. \quad (20)$$

The importance function $\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})$ must be separable in terms of the Q angle-delay pairs $\{(\theta_q,\tau_q)\}_{q=1}^{Q}$. In other words, we aim to find a function $\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})$ that can be written as follows:

$$\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau}) = \prod_{q=1}^{Q} \bar{g}_q(\theta_q,\tau_q). \quad (21)$$

Consequently, each of $\{(\theta_q,\tau_q)\}_{q=1}^{Q}$ are generated independently using their corresponding pdfs $\{\bar{g}_q(\theta_q,\tau_q)\}_{q}^{Q}$ instead of using $\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})$ to generate $Q-$dimensional random vectors $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$. However, we have always to keep in mind that $\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau})$ should be as close as possible to the original CLF, $\bar{\mathcal{L}}_c(\boldsymbol{\theta},\boldsymbol{\tau})$, in order to obtain the best achievable performance. Since $\mathbf{I}_N \otimes \left[\mathbf{A}(\boldsymbol{\theta})^H\mathbf{A}(\boldsymbol{\theta})\right]$ is a block-diagonal matrix, it follows that:

$$\mathbf{D}^H\mathbf{D} \ = \ \sum_{n=1}^{N} \boldsymbol{\Phi}_n(\boldsymbol{\tau})^H \mathbf{A}(\boldsymbol{\theta})^H \mathbf{A}(\boldsymbol{\theta}) \boldsymbol{\Phi}_n(\boldsymbol{\tau}). \quad (22)$$

The $\{l^{th}\}_{l=1}^{KQ}$ column of $\mathbf{A}(\boldsymbol{\theta})\boldsymbol{\Phi}_n(\boldsymbol{\tau})$ can be written as:

$$[\mathbf{A}(\boldsymbol{\theta})\boldsymbol{\Phi}_n(\boldsymbol{\tau})]_l = \mathbf{a}(\theta_{l'})h(\omega_n)e^{-j\omega_n(\tau_{l'}+l''T)}, \quad (23)$$

where $\{l' = \lfloor (l-1)/K \rfloor + 1\}$ and l'' is the remainder of dividing $l-1$ by the number of symbols K (i.e., $l'' = (l-1)\text{mod}K$). It follows that the $(l,m)^{th}$ entry of $\mathbf{D}^H\mathbf{D}$ is given by:

$$[\mathbf{D}^H\mathbf{D}]_{l,m} = \Big(\sum_{n=1}^{N} |h(\omega_n)|^2 e^{j\omega_n(\tau_{l'}-\tau_{m'}+(l''-m'')T)}\Big) \times \Big(\sum_{p=1}^{P} e^{j(\varphi_p(\theta_{m'})-\varphi_p(\theta_{l'}))}\Big), \quad (24)$$

where $m' = \lfloor (m-1)/K \rfloor + 1$ and $m'' = (m-1) \bmod K$. Note that the diagonal elements of $\mathbf{D}^H\mathbf{D}$ have the same expression:

$$[\mathbf{D}^H\mathbf{D}]_{l,l} = P\sum_{n=1}^{N} |h(\omega_n)|^2. \quad (25)$$

Due to the destructive superposition (for $l \neq m$) of the complex exponentials in (24), the off-diagonal entries of $\mathbf{D}^H\mathbf{D}$ are expected to be very small compared to its diagonal ones [2,14]. Thus we obtain the following useful approximation:

$$\mathbf{D}^H\mathbf{D} \approx PE_h\mathbf{I}_{KQ}, \quad (26)$$

where $E_h = \sum_{n=1}^{N} |h(\omega_n)|^2$ is the energy of the pulse shaping filter. Next, we inject it in (11) to obtain the following approximation for the CLF:

$$\mathcal{L}_c(\boldsymbol{\theta},\boldsymbol{\tau}) \approx \frac{1}{PE_h}\sum_{q=1}^{Q}\sum_{k=1}^{K} I_k(\theta_q,\tau_q), \quad (27)$$

in which $I_k(\theta,\tau)$ is the periodogram of the signal at the k^{th} symbol period given by:

$$I_k(\theta,\tau) = \left|\sum_{p=1}^{P} e^{j\varphi_p(\theta)} \sum_{n=1}^{N} h(\omega_n) y_p^*(\omega_n) e^{-j2\pi\omega_n(\tau+(k-1)T)}\right|^2. (28)$$

Owing to the decomposition of the *approximate* CLF in (27) into separate contributions pertaining each to one of the Q angle-delay pairs at each symbol period, we exploit it below as a new pdf:

$$\bar{\mathcal{G}}(\boldsymbol{\theta},\boldsymbol{\tau}) = \frac{\exp\Big\{\rho_1 \sum_{q=1}^{Q}\sum_{k=1}^{K} I_k(\theta_q,\tau_q)\Big\}}{\int\cdots\int \exp\Big\{\rho_1 \sum_{q=1}^{Q}\sum_{k=1}^{K} I_k(\theta'_q,\tau'_q)\Big\} d\boldsymbol{\theta}' d\boldsymbol{\tau}'} (29)$$

The factor $\frac{1}{PE_h}$ involved in (27) is absorbed within the new design parameter, $\rho_1 \neq \rho_0$.

Due to the linear decomposition in (27), $\bar{\mathcal{G}}(\boldsymbol{\theta}, \boldsymbol{\tau})$ is *separable* in terms of the angle-delay pairs. Indeed, it can be easily shown that $\bar{\mathcal{G}}(\boldsymbol{\theta}, \boldsymbol{\tau})$ factorizes as follows:

$$\bar{\mathcal{G}}(\boldsymbol{\theta}, \boldsymbol{\tau}) \;=\; \prod_{q=1}^{Q} \bar{g}_{\theta,\tau}(\theta_q, \tau_q), \tag{30}$$

where:

$$\bar{g}_{\theta,\tau}(\theta, \tau) = \frac{e^{\rho_1 \sum_{k=1}^{K} I_k(\theta_q, \tau_q)}}{\iint e^{\rho_1 \sum_{k=1}^{K} I_k(\theta', \tau')} d\theta' d\tau'}. \tag{31}$$

From (30), the Q angle-delay pairs, $\left\{(\theta_q, \tau_q)\right\}_{q=1}^{Q}$, are independent and identically distributed (iid). Therefore, to generate each couple of realizations $\boldsymbol{\theta}^{(r)}$ and $\boldsymbol{\tau}^{(r)}$ according to the multidimensional distribution $\bar{\mathcal{G}}(\boldsymbol{\theta}, \boldsymbol{\tau})$, one can easily generate, independently, Q couples of realizations $(\theta_q^{(r)}, \tau_q^{(r)})$ according to the common bivariate distribution $\bar{g}_{\theta,\tau}(\theta, \tau)$ and forms the realizations vector $\boldsymbol{\theta}^{(r)} = \left[\theta_1^{(r)}, \theta_1^{(r)}, \cdots, \theta_Q^{(r)}\right]$ and $\boldsymbol{\tau}^{(r)} = \left[\tau_1^{(r)}, \tau_1^{(r)}, \cdots, \tau_Q^{(r)}\right]$. As far the generation of the couple $(\theta_q^{(r)}, \tau_q^{(r)})$ is concerned, we start by computing the marginal pdf, $\bar{g}_\theta(\theta)$, that is evaluated as follows:

$$\bar{g}_\theta(\theta) = \int \bar{g}_{\theta,\tau}(\theta, \tau) d\tau. \tag{32}$$

The latter is used to generate the R angle realizations, $\left\{\boldsymbol{\theta}^{(r)} = \left[\theta_1^{(r)}, \theta_2^{(r)}, \cdots, \theta_Q^{(r)}\right]^T\right\}_{r=1}^{R}$. The generated angles can be used to compute the q^{th} conditional delay pdf:

$$\bar{g}_{\tau|\theta}\left(\tau|\theta = \theta_q^{(r)}\right) = \frac{\bar{g}_{\tau,\theta}\left(\tau, \theta_q^{(r)}\right)}{\bar{g}_\theta\left(\theta_q^{(r)}\right)}, \text{for } q = 1, 2, \cdots, \ Q. \tag{33}$$

The conditional pdf in (33) is used to generate the R angle realizations, $\left\{\boldsymbol{\tau}^{(r)} = \left[\tau_1^{(r)}, \tau_1^{(r)}, \cdots, \tau_Q^{(r)}\right]\right\}_{r=1}^{R}$.

After generating the realizations for all the Q paths, the TDs and AoAs are estimated by using the *circular* sample mean which are given by:

$$\widehat{\tau}_q = \tau_{\max}\left(\frac{1}{2\pi}\angle\left[\sum_{r=1}^{R}\eta\left(\boldsymbol{\theta}^{(r)}, \boldsymbol{\tau}^{(r)}\right)\exp\left\{j2\pi\left(\frac{\tau_q^{(r)}}{\tau_{\max}} - \frac{1}{2}\right)\right\}\right] + \frac{1}{2}\right), \tag{34}$$

$$\widehat{\theta}_q = \frac{1}{2}\angle\left[\sum_{r=1}^{R}\eta\left(\boldsymbol{\theta}^{(r)}, \boldsymbol{\tau}^{(r)}\right)\exp\left\{j\left(2\theta_q^{(r)} - \pi\right)\right\}\right] + \frac{\pi}{2}. \tag{35}$$

Recall that the weighting coefficient $\eta\left(\boldsymbol{\theta}^{(r)}, \boldsymbol{\tau}^{(r)}\right)$ was earlier defined in (20). In order to greatly reduce the computational load of this coefficient, we can use the following *normalized* weighting coefficient:

$$\begin{aligned}\overline{\eta}\left(\boldsymbol{\theta}^{(r)}, \boldsymbol{\tau}^{(r)}\right) &= \exp\Bigg\{\rho_0\mathcal{L}_c\left(\boldsymbol{\theta}^{(r)}, \boldsymbol{\tau}^{(r)}\right) - \rho_1\textstyle\sum_{q=1}^{Q} I\left(\theta_q^{(r)}, \tau_q^{(r)}\right) \\ &\quad - \max_{1\le r\le R}\left[\rho_0\mathcal{L}_c\left(\boldsymbol{\theta}^{(r)}, \boldsymbol{\tau}^{(r)}\right) - \rho_1\sum_{q=1}^{Q} I\left(\theta_q^{(r)}, \tau_q^{(r)}\right)\right]\Bigg\}. \end{aligned} \tag{36}$$

5. SIMULATION RESULTS

In the following we assess the performance of the proposed IS-based ML estimator in terms of the root mean square error (RMSE) with a total number of Monte-Carlo runs $M_c = 500$. In our simulation, we consider a SIMO system composed of one source, one destination and $Q = 2$ paths. The transmitted sequence consists of 128 symbols taken from a QPSK constellation. The pulse shaping function is a SRRC with a roll-off factor $\alpha = 0.5$. The design parameter, ρ_1, is set to 60, and ρ_0, which must be sufficiently high, is set to 8000.

In Fig. 1, we compare the proposed technique against the DA CRLB [6] and the DA algorithm (i.e., DA IS) presented in [8]. We observe that NDA ML IS performs nearly the same as

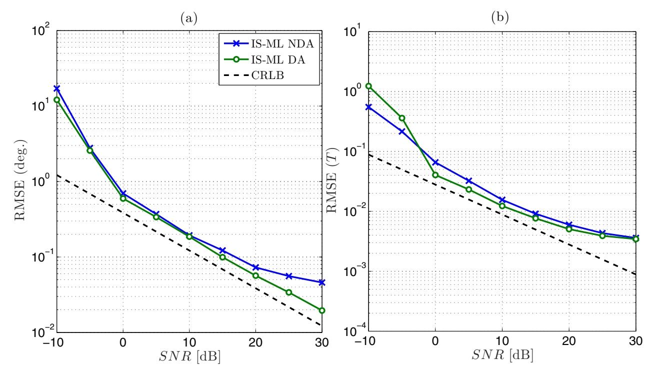

Fig. 1. RMSE for TDs and AoAs joint estimation vs SNR with $K = 128$ symbols, $\alpha = 0.51$, $Q = 2$, and QPSK: (a) RMSE of the AoAs, and (b) RMSE of the delays.

the DA technique at medium and high SNR values. However, we can observe some performance deterioration at low SNR. The latter stems from the fact that the periodogram in (27) exhibits higher secondary lobes due to the relatively stronger presence of noise. At extremely low SNR, the amplitudes of the secondary lobes can be reduced by adjusting the parameter ρ_1. However, at high SNR, larger values of ρ_1 can affect the main lobes of the periodogram leading to some performance deterioration. Hence, the value of ρ_1 should be carefully selected to achieve best trade-off between the low and high SNR regimes.

6. CONCLUSION

In this paper, we proposed a new JADE ML estimator that takes advantage of the powerful importance sampling concept. The new estimator achieves the global maximum of the likelihood function in a blind scenario. Simulation results showed that clearly, the new IS-based ML estimator can achieve the same performance as the DA techniques over a wide range of useful SNR values.

7. REFERENCES

[1] C. Nerguizian, C. Despins, and S. Affes, "Method and system for indoor geolocation using an impulse response fingerprint-

ing technique," U.S. Patent 7630943 B2, Dec. 8, 2009.

[2] S. Yoon, K. Lee, Y. Yun, and I. Rhee, "ACMI: FM-Based indoor localization via autonomous fingerprinting," *IEEE Trans. Mobile Computing.*, vol. 15, no. 6, pp. 1318-1332, June 2016.

[3] M. Wax and A. Leshem, "Joint estimation of time delays and directions of arrival of multiple reflections of a known signal", *IEEE Trans. Sig. Process.*, vol. 45, no. 10, pp. 2477-2477, Oct. 1997.

[4] Y. Wang, J. Chen, and W. Fang, "TST-MUSIC for joint DOA-delay estimation", *IEEE Trans. Sig. Process.*, vol. 49, no. 4, pp. 721-729, Apr. 2001.

[5] X. Zhang, G. Feng, and D. Xu, "Blind direction of angle and time delay estimation algorithm for uniform linear array employing multi-invariance MUSIC," *Prog. Electromagn. Res. Lett.*, vol. 13, no. 2, pp. 11-20, Feb. 2010.

[6] M. van der Veen, A. Vanderveen, and A. Paulraj, "Joint angle and delay estimation using shift-invariance techniques," *IEEE Trans. Sig. Process.*, vol. 46, no. 2, pp. 405-418, Feb. 1998.

[7] A. Gaber and A. Omar "A study of wireless indoor positioning based on joint TDOA and DOA estimation using 2-D matrix pencil algorithms and IEEE 802.11ac," *IEEE Trans. Wireless Commun.*, Vol. 14, No. 5, pp. 2440-2454, May 2015.

[8] F. Bellili, S. Ben Amor, S. Affes, and A. Ghrayeb, "Maximum likelihood joint angle and delay estimation from multipath and multicarrier transmissions with application to indoor localization over IEEE 802.11ac radio," *IEEE Trans. Mobile Comput.*, vol. 17, no. 1, pp. 201-217, 2018.

[9] B. H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus, and I. K. Pedersen, "Channel parameter estimation in mobile radio environments using the SAGE algorithm," *IEEE J. Sel. Areas Commun.*, vol. 17 , no.3, pp. 434-450, Mar. 1999.

[10] M. Pincus "A closed form solution for certain programming problems", *Oper. Res.,* pp. 690-694, 1962.

[11] A. Masmoudi, F. Bellili, S. Affes, and A. Ghrayeb, "Maximum likelihood time delay estimation from single- and multicarrier DSSS multipath MIMO transmissions for future 5G networks," *IEEE Trans. Wireless Commun.*, vol. 16, no. 8, pp. 4851- 4865, Aug. 2017.

[12] A. Masmoudi, F. Bellili, S. Affes, and A. Stéphenne, "A maximum likelihood time delay estimator in a multipath environment using importance sampling", *IEEE Trans. Sig. Process.*, vol. 61, no. 1, pp. 182-193, January 2013.

[13] H. Wang and S. Kay, "Maximum likelihood angle-doppler estimator using importance sampling," *IEEE Trans. Aerospace and Electronic Systems.*, vol. 46, no. 2, pp. 610-622, Apr. 2010. 5082-5092, Oct. 2008.

[14] A. Masmoudi, F. Bellili, S. Affes, and A. Stéphenne, "A non-data-aided maximum likelihood time delay estimator using importance sampling," *IEEE Trans. Signal Process.*, vol. 59, no. 10, pp. 4505-4515, Oct. 2011.