

Distributed Zero-Forcing AF Beamforming for Energy-Efficient Communications in Networked Smart Cities

Slim Zaidi*, Oussama Ben Smida[†], Sofiène Affes[†], and Shahrokh Valaee*

*ECE Department, University of Toronto, Toronto, Ontario, M5S 3G4, Canada, Email: {slim.zaidi, valaee}@utoronto.ca

[†]INRS-EMT, Université du Québec, Montreal, QC, H5A 1K6, Canada, Email: {oussama.ben.smida,affes}@emt.inrs.ca

Abstract—In this paper, amplify-and-forward beamforming (AFB) is considered to establish a reliable communication between devices in networked smart cities. All sources send their data during the first time slot while the cooperative terminals forward a properly weighted version of their received signals during the second. These zero-forcing AFB (ZFB) weights are properly selected to maximize the desired power while completely canceling the interference signals. We show, however, that their implementation requires a huge terminals' information exchange, making ZFB unsuitable for smart cities where the overhead and power restrictions are very stringent. To address this issue, we exploit the asymptotic expression at large K of the ZFB weights whose computation requires much less information exchange and, further, well-approximate their original counterparts. The performance of the proposed beamforming is analyzed and compared to ZFB and monochromatic (i.e., single-ray) AFB (MB) whose design neglects the presence of scattering.

Index Terms—Amplify-and-forward (AF) beamforming, smart cities, scattering, beampattern, cost and power efficiencies, overhead.

I. INTRODUCTION

One of the main challenges in the smart cities is establishing a low-cost, energy-efficient, and reliable communication between devices. As such, many cooperative communication schemes have been introduced to significantly reduce the energy required at each terminal through cooperation [6]-[14]. Indeed, in a cooperative networks, cooperative terminals play often a central role in the signal transmission flow by processing the received signals from the source and then forwarding them to the destination. Many techniques have been so far developed to process the signals at terminals. Among them is amplify-and-forward beamforming (AFB) which reduces processing at each terminal to a simple multiplication of the received signal by properly selected beamforming weights, thereby avoiding decoding or other sophisticated techniques that are excessively complex [13]-[17]. These weights are crucial not only to achieving predefined objectives, but also to complying with real-world constraints and restrictions, making their design an active research subject. When the total transmit power is fixed, AFB can achieve up to K -fold gain in the received power

at the intended direction [5], [11], [14]. As such, not only the communication range is substantially extended, but also each terminal decreases its transmission power inversely proportional to K , thereby preserving their energy resources.

The AFB has aroused an increased interest due to its practical benefits. [6] has introduced the AFB concept and analyzed the behavior of its beampattern when terminals are uniformly distributed. Beampattern characteristics has been also evaluated in [7] when the terminals are Gaussian distributed. [8] analyzes the beampattern properties for several terminal distributions, [9] and [10] have respectively proposed terminals selection schemes aiming to achieve narrower mainbeam and minimum sidelobes effects. [11] has studied the robustness of AFB against the terminals asynchrony and [12] has proposed new synchronization methods. [13] and [14] has summarized the different beamforming techniques and their required synchronization approaches, respectively.

Despite their importance, all these contributions assume single-ray (i.e., monochromatic) channels which is often not valid assumption due to the presence of scattering in real-world applications. Characterized by its angle spread (AS), such a phenomenon generates several rays from the original signal, thereby forming a multi-ray (i.e., polychromatic) channel [15]-[22]. [20] has studied the scattering effect on monochromatic AFB (MB) whose design neglects such a phenomenon. It has been shown that the MB performance significantly deteriorate when AS increases. Aiming to address this issue, [21] and [22] have developed AFB techniques that reaches optimality only in lightly- to moderately-scattered environments (i.e., AS values up to 17 deg). It has been shown that their performance severely deteriorate in highly-scattered environments, especially with the presence of interference.

In order to cope with real-world conditions, many works adopt the optimal AFB (OB) since it is the sole design able to handle both interference and scattering [3]-[2] [21]-[24]. Unfortunately, each OB weight depends often not only on the terminal's CSI, but also on the other terminals' CSI. Since terminals are autonomous and,

hence, do not have enough knowledge about the other CSIs, they have to exchange their local information to be able to compute their respective weights. This results in both huge overhead and terminal power depletion, making OB unsuitable for smart cities which are subject to stringent overhead and power restrictions. This critical impediment motivates us to design new AFB technique able to approach OB performance at very low overhead and power costs.

In this paper, OB is considered to establish a communication, through K terminals, from a source to a receiver in the presence of both scattering and interference. All sources send their data to the terminals during the first time slot while the latter forward a properly weighted version of their received signals during the second. These ZFB weights are properly selected to maximize the desired power while completely canceling the interference signals. We show, however, that they are dependent on information locally unavailable at each terminal, making the conventional ZFB unsuitable for smart cities due to the prohibitive power and overhead costs its implementation may incur. To address this issue, we exploit the asymptotic expression at large K of the ZFB weights that is locally computable at every terminal and, further, well-approximate their original counterparts. The performance of the proposed beamforming is analyzed and compared to ZFB and MB whose design neglects the presence of scattering.

II. SYSTEM MODEL

We consider a wireless networks of K terminals, each equipped with a single antenna, a receiver Rx , and M sources including one desired source and $M_I = M - 1$ interfering terminals. The terminals are assumed to be randomly distributed over a disc of radius R . It is also assumed that the channel from the source to the destination experiences severe pathloss attenuation and, hence, they are unable to communicate directly. Let (A_m, ϕ_m) and (r_k, ψ_k) denote the polar coordinates of the m -th source and the k -th terminal, respectively. $(A_1, \phi_1 = 0)$ is assumed to be the location of the desired source, without any loss of generality. We also assume that the m -th source is located in the far-field region and, hence, $A_m \gg R$.

The following assumptions are also adopted in this work:

i) L_m rays are the generated from the m -th source signal to form a polychromatic channel [18]-[23]. The l -th ray has a complex amplitude $\alpha_{l,m}$ and an angle deviation $\theta_{l,m}$ from the nominal direction ϕ_m . The $\alpha_{l,m}$ s are independent and identically distributed (i.i.d) random variables (RV)s with a zero-mean and a variance $1/L_m$. The $\theta_{l,m}$ s are i.i.d. zero-mean random variables with a symmetric pdf $p_m(\theta)$ and variance σ_m^2 . The latter are known as scattering distribution and angular spread

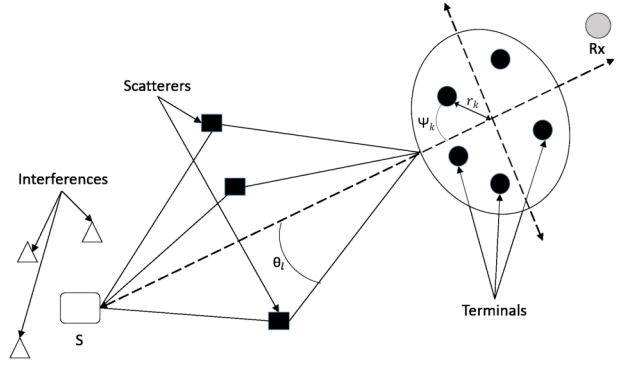


Fig. 1. System model.

(AS), respectively. All $\theta_{l,m}$ s and $\alpha_{l,m}$ s are mutually independent.

ii) The k -th terminal's forward channel $[\mathbf{f}]_k$ is a circular Gaussian RV with a zero-mean and a unit-variance.

iii) Noises at both the receiver and terminals are zero-mean Gaussian RVs with variances σ_n^2 and σ_v^2 , respectively. All sources' signals are narrow band zero-mean RVs statistically independent from noises and channels.

iv) Each terminal has a perfect knowledge of its own location and forward channel, the wavelength λ , and the total number of terminals K . It is however not privy to other terminals information (i.e., locations and channels).

i) along with $A_m \gg R$ imply that the m -th source's backward channel is

$$[\mathbf{g}_m]_k = \sum_{l=1}^{L_m} \alpha_{l,m} e^{-j \frac{2\pi}{\lambda} r_k \cos(\phi_m + \theta_{l,m} - \psi_k)}. \quad (1)$$

Please note that (1) generalizes the steering vector well known in the array-processing literature [6]-[9], [15], [22]. Indeed, (1) reduces to

$$[\mathbf{g}_m^{(1)}]_k = e^{-j(2\pi/\lambda)r_k \cos(\phi_m - \psi_k)}, \quad (2)$$

in scattering-free environments where $\sigma_m = 0$.

III. ZERO-FORCING AF BEAMFORMER

The desired source communicates with Rx using a dual-hop communication mode. The m -th source sends then its signal s_m to the network during the first time slot. The received signal vector \mathbf{y} at terminal is

$$\mathbf{y} = \mathbf{g}_1 s_1 + \mathbf{G}_I \mathbf{s}_I + \mathbf{v}, \quad (3)$$

where $\mathbf{G}_I = [\mathbf{g}_2 \dots \mathbf{g}_M]$, $\mathbf{s}_I = [s_2 \dots s_M]^T$, and \mathbf{v} denotes the vector of noises at terminals. During the second time slot, the k -th terminal forwards its received signal after multiplying with the complex conjugate of the beamforming weight w_k . Rx receives then

$$\begin{aligned} r &= \mathbf{f}^H (\mathbf{w}^* \odot \mathbf{y}) + n \\ &= \mathbf{w}^H (\mathbf{f} \odot (\mathbf{g}_1 s_1 + \mathbf{G}_I \mathbf{s}_I) + \mathbf{f} \odot \mathbf{v}) + n \\ &= s_1 \mathbf{w}^H \mathbf{h}_1 + \mathbf{w}^H \mathbf{H}_I \mathbf{s}_I + \mathbf{w}^H (\mathbf{f} \odot \mathbf{v}) + n, \end{aligned} \quad (4)$$

where n is the receiver's noise, $\mathbf{h}_1 = \mathbf{f} \odot \mathbf{g}_1$, and $\mathbf{H}_1^H = [\mathbf{f} \odot \mathbf{g}_2 \dots \mathbf{f} \odot \mathbf{g}_I]$. Various designing approaches may be adopted to derive the beamforming weights. Among them is the zero-forcing approach aiming to maximize the desired power while putting nulls at the interfering sources directions. This translates to the following convex optimization problem:

$$\mathbf{w}_{\text{ZF}} = \arg \max |\mathbf{w}^H \mathbf{h}_1|^2 \quad \text{Subject to } \mathbf{w}^H \mathbf{H}_1 = 0, \quad (5)$$

where \mathbf{w}_{ZF} is the beamforming vector associated with the prospective zero-forcing AF beamformer. It can be shown that \mathbf{w}_{ZF} is given by

$$\mathbf{w}_{\text{ZF}} = \frac{1}{K} \left(\mathbf{h}_1 - \mathbf{H}_1 (\mathbf{H}_1 \mathbf{H}_1^H)^{-1} \mathbf{H}_1^H \mathbf{h}_1 \right). \quad (6)$$

Therefore, the implementation of \mathbf{w}_{ZF} requires that the k -th terminal be able to compute its corresponding weight

$$[\mathbf{w}_{\text{ZF}}]_k = \frac{1}{K} \left([\mathbf{h}_1]_k - \sum_{m=1}^M [\mathbf{H}_1]_{km} \left[(\mathbf{H}_1 \mathbf{H}_1^H)^{-1} \mathbf{H}_1^H \mathbf{h}_1 \right]_m \right). \quad (7)$$

A straightforward inspection of (7) reveals that $\left[(\mathbf{H}_1 \mathbf{H}_1^H)^{-1} \mathbf{H}_1^H \mathbf{h}_1 \right]_m$ depends on the coordinates as well as the forward channels of all collaborating terminals. This implies that the latter must exchange their locally available information in order to compute their weights, resulting in a prohibitive overhead that may hinder the system spectral and power efficiencies. Consequently, the conventional ZFB in (6) is unsuitable for an implementation in smart cities where the overhead and power restrictions are very stringent.

IV. PROPOSED AF BEAMFORMER

To address the aforementioned critical issue, we propose in this work to substitute, in \mathbf{w}_{ZF} , $(\mathbf{H}_1 \mathbf{H}_1^H)^{-1} \mathbf{H}_1^H \mathbf{h}_1$ by its asymptotic approximations at large K

$$\begin{aligned} \lim_{K \rightarrow \infty} (\mathbf{H}_1 \mathbf{H}_1^H)^{-1} \mathbf{H}_1^H \mathbf{h}_1 \\ &= \left(\frac{1}{K} \lim_{K \rightarrow \infty} \mathbf{H}_1 \mathbf{H}_1^H \right)^{-1} \frac{1}{K} \lim_{K \rightarrow \infty} \mathbf{H}_1^H \mathbf{h}_1 \\ &= \mathbf{\Pi}^{-1} \boldsymbol{\beta}. \end{aligned} \quad (8)$$

Actually, this was motivated by the exponential growth of wireless devices foreseen in the future smart cities. In what follows, we will prove that both $\mathbf{\Pi}$ and $\boldsymbol{\beta}$ depend on the information locally available at each terminal, thereby paving the way towards the implementation of

ZFB in such networks. Let us first start by $\mathbf{\Pi}$. According to the definition of \mathbf{H}_1 , we have

$$\begin{aligned} [\mathbf{\Pi}]_{ij} &= \frac{1}{K} \lim_{K \rightarrow \infty} [\mathbf{H}_1 \mathbf{H}_1^H]_{ij} \\ &= \sum_{l, l'=1}^L \alpha_l \alpha_{l'}^* \lim_{K \rightarrow \infty} \frac{1}{K} \sum_{k=1}^K |[\mathbf{f}]_k|^2 \\ &\quad e^{-j \frac{2\pi}{\lambda} r_k (\cos(\phi_j + \theta_l - \psi_k) - \cos(\phi_i + \theta_{l'} - \psi_k))} \end{aligned} \quad (9)$$

Exploiting the strong law of large numbers (LLN), we obtain

$$[\mathbf{\Pi}]_{ij} = \sum_{l, l'=1}^L \alpha_l \alpha_{l'}^* \mathbb{E}_{r_k, \psi_k, [\mathbf{f}]_k} \left\{ |[\mathbf{f}]_k|^2 e^{-j \frac{2\pi}{\lambda} r_k (\cos(\phi_j + \theta_l - \psi_k) - \cos(\phi_i + \theta_{l'} - \psi_k))} \right\}, \quad (10)$$

where \mathbb{E}_{r_k, ψ_k} stands to be the expectation taken with respect to the random variables r_k s, ψ_k s, $[\mathbf{f}]_k$ s. In order to compute $[\mathbf{\Pi}]_{ij}$, one must have then a prior knowledge on terminals' distribution. Assuming that the latters are uniformly distributed in a disc whose radius R , one can show that

$$[\mathbf{\Pi}]_{ij} = \sum_{l, l'=1}^L \alpha_l \alpha_{l'}^* \Delta(\phi_i - \phi_j + \theta_l - \theta_{l'}), \quad (11)$$

where

$$\Delta(\phi) = 2 \frac{J_1 \left(4\pi \frac{R}{\lambda} \sin(\phi/2) \right)}{4\pi \frac{R}{\lambda} \sin(\phi/2)}. \quad (12)$$

It follows from (11) and (12) that $\mathbf{\Pi}$ solely depends on information locally available at every collaborating terminal and, hence, is locally computable at the latter. Following the above steps, one can also obtain

$$\begin{aligned} [\boldsymbol{\beta}]_i &= \frac{1}{K} \lim_{K \rightarrow \infty} [\mathbf{H}_1^H \mathbf{h}_1]_i \\ &= \sum_{l, l'=1}^L \alpha_l \alpha_{l'}^* \Delta(\phi_{i'} - \theta_l + \theta_{l'}). \end{aligned} \quad (13)$$

As could be observed from (13), $\boldsymbol{\beta}$ is also independent of any information locally unavailable at every terminal, making it computable at the latter. In this paper, we propose then to use as AF beamforming vector

$$\mathbf{w}_P = \frac{1}{K} (\mathbf{h}_1 - \mathbf{H}_1 \mathbf{\Pi}^{-1} \boldsymbol{\beta}). \quad (14)$$

in lieu of \mathbf{w}_{ZF} . In contrast with the latter, using the proposed AF beamformer, all collaborating terminals are able to derive their corresponding weights without requiring any information exchange, improving thereby the system spectral and power efficiencies.

V. PERFORMANCE ANALYSIS

In order to verify the efficiency of the proposed AFB, we analyze in this section the power received at Rx from the desired source and the $M - 1$ interfering ones. Let $P_{\mathbf{w}_P}(\phi_m) = |\mathbf{w}_P^H \mathbf{h}_m|^2$ denote the power received from the m -th source. $P_{\mathbf{w}_P}(\phi_m)$ is unfortunately a complex combination of several random variables, making its analysis a tedious task. In this work, we propose to study the behaviors of the received power $\bar{P}_{\mathbf{w}_P}(\phi_m) = E_{r_k, \psi_k, [\mathbf{f}]_k} \left\{ |\mathbf{w}_P^H \mathbf{h}_m|^2 \right\}$ averaged over all the terminals' forward channels and positions. As such, we introduce the following theorem:

Theorem: For any given $p_m(\theta)$ and σ_m , $m = 1, \dots, M$, $\bar{P}_{\mathbf{w}_P}(\phi_m)$ can be expressed as shown in (15) on the top of the next page where the scalars Σ_0 and Σ_1 as well the vectors Σ_2 , Σ_3 , and Σ_4 are complex functions of the sources directions and their angular deviations.

Proof: See Appendix.

It follows from (15) that for large K the desired power boils down to

$$\bar{P}_{\mathbf{w}_P}(0) = 4 \left| \Sigma_1(\theta, \phi_1, \phi_1) - \Sigma_4^H(\theta, \phi_I, \phi_1) \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right|^2, \quad (16)$$

From (16), one can readily show that $\bar{P}_{\mathbf{w}_P}(0) < \bar{P}_{\mathbf{w}_{ZF}}(0) = 1$. This implies that the desired power received using the proposed AFB is less than that received using the original ZFB. Actually, this slight loss is nothing but the cost of using the asymptotic approximation at large K when designing \mathbf{w}_P in Section IV.

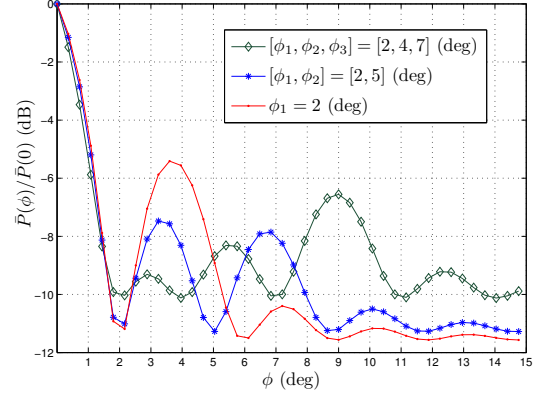
Let us now derive the power received from the n -th interfering terminal. According to the definition of $\Sigma_4^H(\theta, \phi_I, \phi_1)$ in Appendix A, we have

$$\begin{aligned} [\Sigma_4(\theta, \phi_I, \phi_n)]_i &= \sum_{l, l'=1}^L \alpha_l \alpha_{l'}^* \Delta(\theta_l - \theta_{l'} + \phi_n - \phi_i) \\ &= [\mathbf{\Pi}]_{ni}. \end{aligned} \quad (17)$$

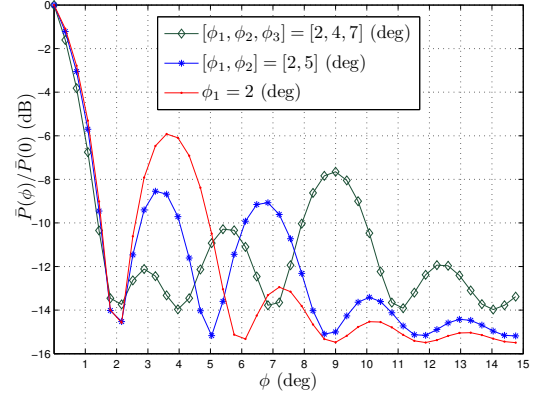
This implies that $\Sigma_4(\theta, \phi_I, \phi_n) = e_n \mathbf{\Pi}$ where e_n is the vector having 1 in its n -th entry and zeros elsewhere. Exploiting this propriety, one can easily prove that

$$\Sigma_1(\theta, \phi_1, \phi_n) = \Sigma_4^H(\theta, \phi_I, \phi_n) \mathbf{\Pi}^{-1} \boldsymbol{\beta}. \quad (18)$$

It follows from (15) and (18) that the n -th interference power is given by (19). Although efficient, our proposed technique does not totally remove the interference, in contrast with the conventional ZFB technique that satisfies the constraint in (5) while being unsuitable for implementation in smart cities. Actually, this is again the cost of using the approach introduced in Section IV when designing \mathbf{w}_P . Nevertheless, for large K , we have $\bar{P}_{\mathbf{w}_P}(\phi_n) \approx 0$. Consequently, the proposed AFB is able to cancel all interference, when the number of collaborating terminals is large enough.



(a) $K = 20$.



(b) $K = 50$.

Fig. 2. The normalized received power $\bar{P}(\phi_*)/\bar{P}(0)$ versus ϕ_* for $\sigma = 35$ (deg), $R/\lambda = 10$, and different sets of interfering sources.

VI. SIMULATION RESULTS

This section verifies the efficiency of the proposed AFB using computer simulations. All empirical average quantities are calculated over 10^6 random realizations of r_k , ψ_k , $[\mathbf{f}]_k$ for $k = 1, \dots, K$ and $\alpha_{l,m}$, $\theta_{l,m}$ for $l = 1, \dots, L_m$. In all simulations, all sources have the same unit power, $\sigma_n^2 = \sigma_v^2 = 1$, and $L_m = 6$. We also consider that all rays have equal power $1/L_m$ (i.e., $E\{|\alpha_{l,m}|^2\} = 1/L_m$) and $\theta_{l,m}$ s are uniformly distributed random variables with variance σ^2 .

Fig. 2 displays the normalized received power $\bar{P}_{\mathbf{w}_P}(\phi_m)/\bar{P}_{\mathbf{w}_P}(0)$ at Rx versus ϕ_* for $\sigma = 35$ (deg), $R/\lambda = 10$, and different sets of interfering sources. In Figs. 2(a) and 2(b), we consider $K = 20$ and $K = 50$, respectively. Both figures show that, using the proposed AFB, the received power at Rx has a peak at the desired source direction (i.e., $\phi = 0$) and minimums at the interfering sources' directions. This proves that it is able to enhance any signal received from the desired source while reducing any interference. Furthermore, from Fig. 2, the proposed AFB guarantees that each interference does not exceed 10 (dB) below the

$$\begin{aligned} \bar{P}_{\mathbf{w}_P}(\phi_m) &= \frac{2}{K} \left(\Sigma_0(\theta, \phi_1, \phi_m) - 2\text{Re} \left(\Sigma_2^H(\theta, \phi_I, \phi_1, \phi_m) \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right) + \boldsymbol{\beta}^H \mathbf{\Pi}^{-1} \Sigma_3(\theta, \phi_I, \phi_I, \phi_m) \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right) \\ &\quad + 4 \left(1 - \frac{1}{K} \right) \left| \Sigma_1(\theta, \phi_1, \phi_m) - \Sigma_4^H(\theta, \phi_I, \phi_m) \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right|^2. \end{aligned} \quad (15)$$

$$\bar{P}_{\mathbf{w}_P}(\phi_n) = \frac{2}{K} \left(\Sigma_0(\theta, \phi_1, \phi_n) - 2\text{Re} \left(\Sigma_2^H(\theta, \phi_I, \phi_1, \phi_n) \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right) + \boldsymbol{\beta}^H \mathbf{\Pi}^{-1} \Sigma_3(\theta, \phi_I, \phi_I, \phi_n) \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right). \quad (19)$$

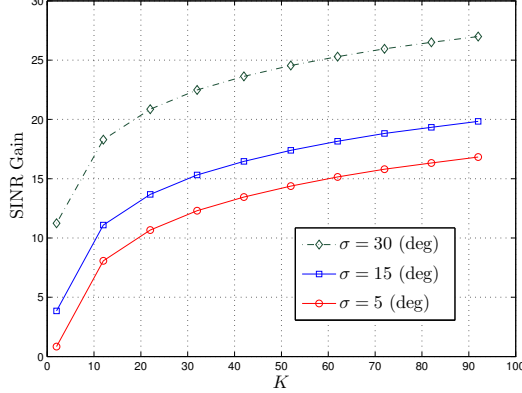


Fig. 3. The SINR gain of the proposed AFB over MB for $\sigma = 5, 15, 30$ (deg).

desired power floor. All these observations corroborate the analytical results of Section V.

Fig. 3 shows the SINR gain achieved by the proposed AFB over MB, whose design neglects scattering and, hence, assumes single-ray (i.e., monochromatic) backward channels (i.e., $[\mathbf{g}_m]_k = e^{-j \frac{2\pi}{\lambda} r_k \cos(\phi_m - \psi_k)}$). From this figure, our proposed AFB achieves a substantial SINR gain over its counterpart. Such a gain increases with both K and σ and can reach 25 (dB). This is expected since MB does not capture all the channel information, thereby hindering its efficiency. As σ increases, the non line-of-sight NLoS component of the channel becomes more dominant and, hence, the performance of the monochromatic (i.e., LoS) design severely deteriorates. This is in contrast with the proposed AFB whose design captures all the channel information.

VII. CONCLUSION

In this paper, OB was considered to establish a communication from a source to a receiver in the presence of both scattering and interference. All sources send their data during the first time slot while the terminals forward a properly weighted version of their received signals during the second. These ZFB weights are properly selected to maximize the desired power while completely canceling the interference signals. We showed that they

are dependent on information locally unavailable at each terminal, making the conventional ZFB unsuitable for smart cities due to the prohibitive power and overhead costs its implementation may incur. To address this issue, we exploited the asymptotic expression at large K of the ZFB weights that is locally computable at every terminal and, further, well-approximate their original counterparts. The performance of the proposed beamforming is analyzed and compared to ZFB and MB whose design neglects the presence of scattering.

APPENDIX

The power received at Rx from the m -th source located at ϕ_m is defined as

$$\begin{aligned} \bar{P}(\phi_m) &= \mathbb{E} \left\{ |\mathbf{w}^H(\mathbf{f} \odot \mathbf{g}_m)|^2 \right\} = \frac{1}{K^2} \left(\mathbb{E} \left\{ \mathbf{h}_1^H \mathbf{h}_m \mathbf{h}_m^H \mathbf{h}_1 \right\} \right. \\ &\quad - \mathbb{E} \left\{ \mathbf{h}_1^H \mathbf{h}_m \mathbf{h}_m^H \mathbf{H}_1 \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right\} - \mathbb{E} \left\{ \left(\mathbf{h}_1^H \mathbf{h}_m \mathbf{h}_m^H \mathbf{H}_1 \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right)^* \right\} \\ &\quad \left. + \mathbb{E} \left\{ \left(\mathbf{H}_1 \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right)^H \mathbf{h}_m \mathbf{h}_m^H \left(\mathbf{H}_1 \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right) \right\} \right) \\ &= \frac{1}{K^2} \left(\mathbb{E}(\Gamma_1) - \mathbb{E}(\Gamma_2) - \mathbb{E}(\Gamma_2^*) + \mathbb{E}(\Gamma_3) \right), \end{aligned} \quad (20)$$

where the expectation is taken over r_k s, ψ_k s, and $[\mathbf{f}]_k$ s and $\Gamma_1 = \mathbf{h}_1^H \mathbf{h}_m \mathbf{h}_m^H \mathbf{h}_1$, $\Gamma_2 = \mathbf{h}_1^H \mathbf{h}_m \mathbf{h}_m^H \mathbf{H}_1 \mathbf{\Pi}^{-1} \boldsymbol{\beta}$, and $\Gamma_3 = \left(\mathbf{H}_1 \mathbf{\Pi}^{-1} \boldsymbol{\beta} \right)^H \mathbf{h}_m \mathbf{h}_m^H \mathbf{H}_1 \mathbf{\Pi}^{-1} \boldsymbol{\beta}$.

Let us first start by deriving the expression of Γ_1 as follows in (21). On the other hand, we have

$$\begin{aligned} &e^{-j \frac{2\pi}{\lambda} r_p \left[\cos(\phi_m + \theta_l - \psi_p) - \cos(\phi_1 + \theta_{l'} - \psi_p) \right]} \times \\ &e^{-j \frac{2\pi}{\lambda} r_p \left[\cos(\phi_1 + \theta_{l_1} - \psi_p) - \cos(\phi_m + \theta_{l_2} - \psi_p) \right]} \\ &= e^{-j \frac{4\pi}{\lambda} r_p (x \sin(x' - \psi_p) + y \sin(y' - \psi_p))}, \end{aligned} \quad (22)$$

where $x = \sin((\phi_m + \theta_l - \phi_1 - \theta_{l'})/2)$, $x' = (\phi_m + \theta_l + \phi_1 + \theta_{l'})/2$, $y = \sin((\phi_1 + \theta_{l_1} - \phi_m - \theta_{l_2})/2)$, and $y' = (\phi_1 + \theta_{l_1} + \phi_m + \theta_{l_2})/2$. Using the fact that r_p s and ψ_p s are mutually independent random variables with pdfs

$$f_{r_p}(r) = \frac{2r}{R}, \quad 0 < r < R \quad (23)$$

$$f_{\psi_p}(\psi) = \frac{1}{2\pi}, \quad -\pi \leq \psi < \pi, \quad (24)$$

$$\begin{aligned}
\Gamma_1 &= \left(\sum_{p=1}^K \sum_{l,l'=1}^L \alpha_l \alpha_{l'}^* e^{-j \frac{2\pi}{\lambda} r_p [\cos(\phi_m + \theta_l - \psi_p) - \cos(\phi_1 + \theta_{l'} - \psi_p)]} \right) \times \\
&\quad \left(\sum_{k=1}^K \sum_{l_1, l_2=1}^L \alpha_{l_1} \alpha_{l_2}^* e^{-j \frac{2\pi}{\lambda} r_k [\cos(\phi_1 + \theta_{l_1} - \psi_k) - \cos(\phi_m + \theta_{l_2} - \psi_k)]} \right) \\
&= \sum_{p=1}^K \sum_{l, l', l_1, l_2=1}^L \alpha_l \alpha_{l'}^* \alpha_{l_1} \alpha_{l_2}^* e^{-j \frac{2\pi}{\lambda} r_p [\cos(\phi_m + \theta_l - \psi_p) - \cos(\phi_1 + \theta_{l'} - \psi_p) + \cos(\phi_1 + \theta_{l_1} - \psi_p) - \cos(\phi_m + \theta_{l_2} - \psi_p)]} \\
&+ \sum_{p=1}^K \sum_{k=1, k \neq p}^K \sum_{l, l', l_1, l_2=1}^L \alpha_l \alpha_{l'}^* \alpha_{l_1} \alpha_{l_2}^* e^{-j \frac{2\pi}{\lambda} r_p [\cos(\phi_m + \theta_l - \psi_p) - \cos(\phi_1 + \theta_{l'} - \psi_p)]} \\
&\times e^{-j \frac{2\pi}{\lambda} r_k [\cos(\phi_1 + \theta_{l_1} - \psi_k) - \cos(\phi_m + \theta_{l_2} - \psi_k)]}. \tag{21}
\end{aligned}$$

respectively, we show that

$$\begin{aligned}
\mathbb{E}_{\psi_p} &\left(e^{-j \frac{4\pi}{\lambda} r_p (x \sin(x' - \psi_p) + y \sin(y' - \psi_p))} \right) \\
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j \frac{4\pi}{\lambda} r_p (x \sin(x' - \psi_p) + y \sin(y' - \psi_p))} d\psi_p \\
&= I_0 \left(-j \frac{4\pi}{\lambda} r_p \sqrt{x^2 + y^2 + 2xy \cos(x' - y')} \right) \\
&= J_0 \left(\frac{4\pi}{\lambda} r_p \sqrt{x^2 + y^2 + 2xy \cos(x' - y')} \right). \tag{25}
\end{aligned}$$

By averaging the latter expression over r_p , we obtain

$$\begin{aligned}
\mathbb{E}_{r_p, \psi_p} &\left(e^{j \frac{4\pi}{\lambda} r_p (x \sin(x' - \psi_p) + y \sin(y' - \psi_p))} \right) \\
&= \mathbb{E}_{r_p} \left(J_0 \left(\frac{4\pi}{\lambda} r_p \sqrt{x^2 + y^2 + 2xy \cos(x' - y')} \right) \right) \\
&= \int_0^R \sum_{p=0}^{\infty} \frac{(-1)^p}{(p!)^2} \left(\frac{4\pi}{2\lambda} \right)^{2p} \sqrt{x^2 + y^2 + 2xy \cos(x' - y')}^{2p} \\
&\times (r_p)^{2p} \left(\frac{2r_p}{R} \right) dr_p \\
&= \sum_{p=0}^{\infty} \frac{(-1)^p}{p!(p+1)!} \left(\frac{4\pi R \sqrt{x^2 + y^2 + 2xy \cos(x' - y')}}{2\lambda} \right)^{2p} \\
&= \frac{2\lambda J_1 \left(\frac{4\pi R \sqrt{x^2 + y^2 + 2xy \cos(x' - y')}}{\lambda} \right)}{4\pi R \sqrt{x^2 + y^2 + 2xy \cos(x' - y')}} \\
&= \Delta(\gamma_0(\theta, \phi_1, \phi_m)), \tag{26}
\end{aligned}$$

where

$$\gamma_0(\theta, \phi_1, \phi_m) = \arcsin \left((x^2 + y^2 + 2xy \cos(x' - y'))^{1/2} \right). \tag{27}$$

Using (26) in (21) yields

$$\begin{aligned}
\mathbb{E}(\Gamma_1) &= 2K \Sigma_0(\theta, \phi_1, \phi_m) + 4K(K-1) \\
&\quad \Sigma_1(\theta, \phi_1, \phi_m) \Sigma_1^*(\theta, \phi_1, \phi_m), \tag{28}
\end{aligned}$$

where

$$\Sigma_0(\theta, \phi_1, \phi_m) = \sum_{l, l', l_1, l_2=1}^L \alpha_l \alpha_{l'}^* \alpha_{l_1} \alpha_{l_2}^* \Delta(2\gamma_0(\theta, \phi_1, \phi_m)), \tag{29}$$

and

$$\Sigma_1(\theta, \phi_1, \phi_m) = \sum_{l, l'=1}^L \alpha_l \alpha_{l'}^* \Delta(\theta_l - \theta_{l'} + \phi_m - \phi_1). \tag{30}$$

Following the above approach, we can also obtain

$$\begin{aligned}
\mathbb{E}(\Gamma_2) &= 2K \Sigma_2^H(\theta, \phi_I, \phi_1, \phi_m) \mathbf{\Pi}^{-1} \boldsymbol{\beta} + 4K(K-1) \\
&\quad \Sigma_1(\theta, \phi_1, \phi_m) \Sigma_4^H(\theta, \phi_I, \phi_m) \mathbf{\Pi}^{-1} \boldsymbol{\beta}. \tag{31}
\end{aligned}$$

where

$$[\Sigma_2(\theta, \phi_I, \phi_1, \phi_m)]_i = \sum_{l, l', l_1, l_2=1}^L \alpha_l \alpha_{l'}^* \alpha_{il_1} \alpha_{l_2}^* \Delta(2\gamma_1(\theta, \phi_i, \phi_1, \phi_m)), \tag{32}$$

$$\begin{aligned}
\gamma_1(\theta, \phi_i, \phi_1, \phi_m) &= \arcsin \left((x^2 + y_1^2 + 2xy_1 \cos(x' - y_1'))^{1/2} \right), \quad y_1 = \sin(\phi_i + \theta_{l_1} - \phi_m - \theta_{l_2})/2, \quad y_1' = \\
&(\phi_i + \theta_{l_1} + \phi_m + \theta_{l_2})/2, \quad \text{and } [\Sigma_4(\theta, \phi_I, \phi_m)]_i = \\
&\sum_{l, l'=1}^L \alpha_l \alpha_{l'}^* \Delta(\theta_l - \theta_{l'} + \phi_m - \phi_i).
\end{aligned}$$

As far as $\mathbb{E}(\Gamma_3)$ is concerned, it can be expressed as

$$\begin{aligned}
\mathbb{E}(\Gamma_3) &= 2K \boldsymbol{\beta}^H \mathbf{\Pi}^{-1} \Sigma_3(\theta, \phi_I, \phi_m) \mathbf{\Pi}^{-1} \boldsymbol{\beta} \\
&+ 4K(K-1) \boldsymbol{\beta}^H \mathbf{\Pi}^{-1} \Sigma_4(\theta, \phi_I, \phi_m) \\
&\quad \Sigma_4^H(\theta, \phi_I, \phi_I, \phi_m) \mathbf{\Pi}^{-1} \boldsymbol{\beta}, \tag{33}
\end{aligned}$$

where

$$[\Sigma_3(\theta, \phi_I, \phi_I, \phi_m)]_{ij} = \sum_{l, l', l_1, l_2=1}^L \alpha_l \alpha_{j l'}^* \alpha_{l_1} \alpha_{l_2}^* \Delta(2\gamma_2(\theta, \phi_i, \phi_j, \phi_m)), \tag{34}$$

$$\begin{aligned}
\gamma_2(\theta, \phi_i, \phi_j, \phi_m) &= \arcsin \left((x_1^2 + y_1^2 + 2x_1 y_1 \cos(x_1' - y_1'))^{1/2} \right), \quad x_1 = \sin(\phi_m + \theta_l - \phi_j - \theta_{j l'})/2, \quad x_1' = (\phi_m \\
&+ \theta_l + \phi_j + \theta_{l'})/2, \quad y_1 = \sin(\phi_i + \theta_{l_1} - \phi_m - \theta_{l_2})/2,
\end{aligned}$$

$$y'_1 = (\phi_i + \theta_{l_1} + \phi_m + \theta_{l_2})/2.$$

Substituting (28), (31), and (33) in (20) yields to (15).

REFERENCES

- [1] H. Shen, W. Xu, S. Jin, and C. Zhao, "Joint transmit and receive beamforming for MIMO downlinks with channel uncertainty," *IEEE Trans. Veh. Tech.*, vol. 63, pp. 2319-2335, June 2014.
- [2] Z. Yi and I. Kim, "Joint optimization of relay-precoders and decoders with partial channel side information in cooperative networks," *IEEE J. Select. Areas Com.*, vol. 25, pp. 447-458, February 2007.
- [3] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE Trans. Inf. Theory*, vol. 55, pp. 2499-2517, June 2009.
- [4] H.-B. Kong, C. Song, H. Park, and I. Lee, "A new beamforming design for MIMO AF relaying systems with direct link," *IEEE Trans. Com.*, vol. 62, pp. 2286-2295, July 2014.
- [5] L. Dong, A. P. Petropulu, and H. V. Poor, "A cross-layer approach to collaborative beamforming for wireless ad hoc networks," *IEEE Trans. Sig. Process.*, vol. 56, pp. 2981-2993, July 2008.
- [6] H. Ochiai, P. Mitran, H. Poor, and V. Tarokh, "Collaborative beamforming for distributed wireless ad hoc sensor networks," *IEEE Trans. Sig. Process.*, vol. 53, pp. 4110-4124, November 2005.
- [7] M. F. A. Ahmed and S. A. Vorobyov, "Collaborative beamforming for wireless sensor networks with Gaussian distributed sensor nodes," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 638-643, February 2009.
- [8] J. Huang, P. Wang, and Q. Wan, "Collaborative beamforming for wireless sensor networks with arbitrary distributed sensors," *IEEE Commun. Lett.*, vol. 16, pp. 1118-1120, July 2012.
- [9] K. Zariifi, A. Ghayeb, and S. Affes, "Distributed beamforming for wireless sensor networks with improved graph connectivity and energy efficiency," *IEEE Trans. Sig. Process.*, vol. 58, pp. 1904-1921, March 2010.
- [10] M. Ahmed and S. Vorobyov, "Sidelobe control in collaborative beamforming via node selection," *IEEE Trans. Sig. Process.*, vol. 58, pp. 6168-6180, December 2010.
- [11] R. Mudumbai, G. Barriac, and U. Madhow, "On the feasibility of distributed beamforming in wireless networks," *IEEE Trans. Wireless Comm.*, vol. 6, pp. 1754-1763, May 2007.
- [12] D. R. Brown and H. Poor, "Time-slotted round trip carrier synchronization for distributed beamforming," *IEEE Trans. on Sig. Process.*, vol. 56, pp. 5630-5643, November 2008.
- [13] L. C. Godara, "Application of antenna arrays to mobile communications, Part II: Beam-forming and direction-of-arrival considerations," *Proc. IEEE*, vol. 85, pp. 1195-1245, August 1997.
- [14] R. Mudumbai, D. R. Brown, U. Madhow, and H. Poor, "Distributed transmit beamforming: challenges and recent progress," *IEEE Com. Mag.*, vol. 47, pp. 102-110, February 2009.
- [15] D. Astly and B. Ottersten, "The effects of local scattering on direction of arrival estimation with MUSIC," *IEEE Trans. Sig. Process.*, vol. 47, pp. 3220-3234, December 1999.
- [16] O. Besson, P. Stoica, and A. Gershman, "Simple and accurate direction of arrival estimator in the case of imperfect spatial coherence," *IEEE Trans. Sig. Process.*, vol. 49, pp. 730737, April 2001.
- [17] S. Shahbazpanahi, S. Valaee, and A. Gershman, "A covariance fitting approach to parametric localization of multiple incoherently distributed sources," *IEEE Trans. Sig. Process.*, vol. 52, pp. 592-600, March 2004.
- [18] M. Souden, S. Affes, and J. Benesty, "A two-stage approach to estimate the angles of arrival and the angular spreads of locally scattered sources," *IEEE Trans. Sig. Process.*, vol. 56, pp. 1968-1983, May 2008.
- [19] M. Bengtsson and B. Ottersten, "Low-complexity estimators for distributed sources," *IEEE Trans. Sig. Process.*, vol. 48, pp. 2185-2194, August 2000.
- [20] A. Amar, "The effect of local scattering on the gain and beamwidth of a collaborative beamforming for wireless sensor networks," *IEEE Trans. Wireless Com.*, vol. 9, pp. 2730-2736, September 2010.
- [21] S. Zaidi and S. Affes, "Distributed collaborative beamforming in the presence of angular scattering," *IEEE Trans. Com.*, vol. 62, pp. 1668-1680, May 2014.
- [22] S. Zaidi and S. Affes, "Distributed collaborative beamforming design for maximized throughput in interfered and scattered environments," *IEEE Trans. Com.*, vol. 63, pp. 4905-4919, December 2015.
- [23] S. Zaidi and S. Affes, "SNR and throughput analysis of distributed collaborative beamforming in locally-scattered environments," *Wiley J. Wireless Com. and Mobile Comput.*, vol. 12, pp. 1620-1633, December 2012.
- [24] M. Zeng, R. Zhang, and S. Cui, "On design of collaborative beamforming for two-way relay networks," *IEEE Trans. Sig. Process.*, vol. 59, pp. 2284-2295, May 2011.