Dual-hop Málaga-$\mathcal{M}$ FSO Systems with Pointing Errors

Nesrine Cherif, *IEEE*, Imène Trigui, *IEEE*, and Sofiène Affes, Senior Member, *IEEE*,
INRS-EMT, 800, de la Gauchetière Ouest, Bureau 6900, Montréal, H5A 1K6, Qc, Canada.
{nesrine.cherif, itrigui, affes}@emt.inrs.ca

***Abstract*—In this paper, we unify the performance analysis for the outage probability (OP) and the bit-error rate (ABER) of relay-assisted dual-hop free-space optics (FSO) transmission under heterodyne detection and intensity modulation/direct detection (IM/DD). The FSO link experiences the generalized Málaga-$\mathcal{M}$ distribution with pointing errors. under the assumption of fixed and variable gain relaying schemes, we derive the analytical closed-form expressions for the performance metrics in terms of bivariate Fox-H function (FHF). Finally, we present some Monte-carlo simulation results to corroborate the new derived expressions.**

***Index Terms*— Fox-H function(FHF), free-space optics (FSO), Málaga-$\mathcal{M}$ distribution, pointing errors.**

I. Introduction

Free-space optics (FSO) system captured a huge interest in the next generation wireless networks due to the good features that it presents. More specifically, FSO link provides a very high data rates and a strong immunity to interference in the Terahertz bandwidth among others [1], [2]. However, although all its advantages, FSO transmission is inhibited by the poor link reliability especially in long distances due to atmospheric turbulence-induced fading and its high sensitivity to weather conditions [3], [4]. In addition, beam deviation from its original path caused by small earthquakes, wind loads and thermal expansion results a further performance deterioration of the FSO link [5]. This misalignment is widely referred in the literature as pointing errors [5]. These main weaknesses of the FSO link, i.e., atmospheric turbulence-induced fading and pointing errors, severely affect the FSO link's quality [6].

Th atmospheric turbulence-induced fading channel modeling is widely investigated in the literature [4], [7], where lognormal and Gamma-Gamma ($\mathcal{G}$-$\mathcal{G}$) are presented as the most common distributions for the FSO link [4]. For instance, the lognormal distribution is an effective channel model only under weak turbulence conditions [4]. However, the $\mathcal{G}$-$\mathcal{G}$ model is suitable for small and large scale atmospheric fluctuations [7]. Recently, Navas *et al* [8] derived the Málaga-$\mathcal{M}$ distribution as a new generalized statistical model for wireless optical communications that unifies almost all the existing FSO channel models in the literature discovered so far. The Málaga-$\mathcal{M}$ statistical distribution is a versatile model with its ability to reflect a wide range of optical fluctuations and offers an attractive mathematical tractability for performance analysis [9].

The short distance coverage, the turbulence-induced fading and the pointing errors can seriously cause FSO link outage and failure. In an attempt to overpass all listed constraints, dual-hop FSO systems have been investigated in some recent works using amplify-and-forward (AF) and decode-and-forward relaying protocols [10], [11]. To the best of authors' knowledge, almost all existing works assume restrictive turbulence channel characterization where either weak turbulence model (lognormal) or strong turbulence model ($\mathcal{G}$-$\mathcal{G}$) is considered [10]–[12]. For instance, [12] provides a complete performance analysis of dual-hop FSO systems with pointing errors operating over $\mathcal{G}$-$\mathcal{G}$ turbulence channels using fixed-gain relaying. In this work, we extend and complete the efforts done in this research line while considering the generalized Málaga-$\mathcal{M}$ statistical model for the FSO link that encompasses all linear turbulence-induced fading channels derived so far including the lognormal and the $\mathcal{G}$-$\mathcal{G}$ channel models [8], [9], [13]. In this paper, assuming fixed and variable gain relaying and taking into account the effect of pointing errors while considering both heterodyne and IM/DD detection techniques, we studied the performance of dual-hop FSO systems over Málaga-$\mathcal{M}$ fading channels by deriving new analytical expressions of the outage probability (OP) as well as the average bit-error rate (ABER) all in terms of bivariate FHF.

The rest of the paper is organized as follows. In Section II we present the channel and system model of the FSO link under Málaga-$\mathcal{M}$ with pointing errors and accounting for both detection techniques (heterodyne and IM/DD). Section III quantify the performance in respect of the OP and ABER of the system under consideration

Work supported by the Discovery Grants and the CREATE PERSWADE (www.create-perswade.ca) programs of NSERC, a Discovery Accelerator Supplement (DAS) Award from NSERC, and the NSERC SPG Project on Advanced Signal Processing and Networking Techniques for Cost-Effective Ultra-Dense 5G Networks.

978-1-5386-3531-5/17/$31.00

employing CSI-assisted relaying. In Section IV, we investigate the same performance metrics using fixed-gain relaying scheme. Section V provides some Monte-carlo simulation results to validate the obtained analytical closed-form expressions. Finally, a brief conclusion is given in Section VI.

II. Channel and System Model

The considered system consists of a relay-assisted AF FSO communication, where the transmission between the optical transmitter T and the optical receiver is assisted by a relay R. Using AF relaying protocol, the relay R amplify the received optical signal and retransmit it to the destination D.

The i-th FSO link $i \in \{1,2\}$ (the first FSO link is associated with T-R and the second FSO link is associated with T-D) undergoes a Málaga-$\mathcal{M}$ statistical model with pointing errors accounting for heterodyne and IM/DD detection techniques where the CDF of its instantaneous SNR, γ_i, is expressed by [13]

$$F_{\gamma_i}(x) = 1-\frac{\xi_i^2 A_i}{\Gamma(\alpha_i)}\sum_{k=1}^{\beta_i}\frac{b_{i,k}}{\Gamma(k)} \mathrm{G}_{2,4}^{4,0}\left[B_i\left(\frac{x}{\mu_{r_i}}\right)^{\frac{1}{r_i}}\middle|\begin{matrix}\xi_i^2+1,1\\0,\xi_i^2,\alpha_i,k\end{matrix}\right], \quad (1)$$

where ξ_i is the ratio between the equivalent beam radius and the pointing error displacement standard deviation (i.e., jitter) for the i-th FSO link (for negligible pointing errors $\xi_i \to +\infty$) [5],

$$A_i = \alpha_i^{\frac{\alpha_i}{2}}\left[\frac{g_i\beta_i}{g_i\beta_i+\Omega_i}\right]^{\beta_i+\frac{\alpha_i}{2}} g_i^{-1-\frac{\alpha_i}{2}}, \quad (2)$$

$$\text{with } g_i = 2b_{0_i}(1-\rho_i), \quad (3)$$

$$b_{i,k} = \binom{\beta_i-1}{k-1}(g_i\beta_i+\Omega_i)^{1-\frac{k}{2}}\left[\frac{g_i\beta_i+\Omega_i}{\alpha_i\beta_i}\right]^{\frac{\alpha_i+k}{2}} \left(\frac{\Omega_i}{g_i}\right)^{k-1}\left(\frac{\alpha_i}{\beta_i}\right)^{\frac{k}{2}}, \quad (4)$$

$$B_i = \frac{\alpha_i\beta_i h_i(g_i+\Omega_i)}{(g_i\beta_i+\Omega_i)}, \quad \text{with } h_i = \frac{\xi_i^2}{1+\xi_i^2} \quad (5)$$

where α_i, β_i, g_i and Ω_i are the fading parameters related to the atmospheric turbulence conditions of the i-th FSO link [8], [13], whereby in g_i (3) , $2b_{0_i}$ is the average power of the LOS term and ρ_i represents the amount of scattering power coupled to the LOS component ($0 \leqslant \rho_i \leqslant 1$). Moreover in (1), $\mathrm{G}_{p,q}^{m,n}[\cdot]$ and $\Gamma(\cdot)$ stand for the Meijer-G function [14, Eq.(9.301)] and the incomplete Gamma function [14, Eq.(8.310.1)], respectively. Furthermore, r_i is the parameter that describes the detection technique at the relay. For instance, $r_i = 1$ is referred to heterodyne detection, however $r_i = 2$ is referred to IM/DD detection technique. Moreover in (2), μ_{r_i} denotes the electrical SNR of the i-th FSO link [13]. Specifically, for $r = 1$,

$$\mu_{1_i} = \mu_{\text{heterodyne}_i} = \mathbb{E}[\gamma_i] = \bar{\gamma}_i, \quad (6)$$

and for $r = 2$, it becomes [13, Eq.(8)]

$$\mu_{2_i} = \mu_{\text{IM/DD}_i} = \frac{\mu_{1_i}\alpha_i\xi_i^2(\xi_i^2+1)^{-2}(\xi_i^2+2)(g_i+\Omega_i)}{(\alpha_i+1)[2g_i(g_i+2\Omega_i)+\Omega_i^2(1+\frac{1}{\beta_i})]} \quad (7)$$

The end-to-end SNR of a CSI-assisted relaying FSO system is explicitly given by [15, Eq.(7)]

$$\gamma = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}. \quad (8)$$

However, under fixed-gain relaying scheme, the end-to-end SNR is expressed as [16, Eq.(2)]

$$\gamma = \frac{\gamma_1\gamma_2}{\gamma_2+\mathrm{C}}, \quad (9)$$

where C determines the relay's fixed gain.

III. Performance Analysis under CSI-assisted relaying

A. Outage Probability (OP)

In dual-hop AF relaying FSO communication, the quality of service is guaranteed by keeping the instantaneous end-to-end SNR, γ, above a threshold γ_{th}. More specifically, the OP in dual-hop AF FSO transmission is given by

$$P_{out} = F_\gamma(\gamma_{th}). \quad (10)$$

Due to the intractability of the end-to-end SNR in (8), we resort to an upper bound given by [17, Eq.(20)] as $\gamma = \min(\gamma_1,\gamma_2) > \gamma_1\gamma_2/(\gamma_1+\gamma_2+1)$, whose CDF can be expressed as $F_\gamma(x) = 1-F_{\gamma_1}^{(c)}(x)F_{\gamma_2}^{(c)}(x)$, where $F_{\gamma_1}^{(c)}$ and $F_{\gamma_2}^{(c)}$ stand for the complementary CDF of γ_1 and γ_2, respectively. Hence, using (1), the CDF of CSI-assisted relaying FSO transmission in Málaga-$\mathcal{M}$ with pointing errors under both heterodyne and IM/DD techniques can be obtained as

$$F_\gamma(x) = 1-\prod_{i=1}^{2}\frac{\xi_i^2 A_i}{\Gamma(\alpha_i)}\sum_{k=1}^{\beta_i}\frac{b_{i,k}}{\Gamma(k)} \mathrm{G}_{2,4}^{4,0}\left[B_i\left(\frac{x}{\mu_{r_i}}\right)^{\frac{1}{r_i}}\middle|\begin{matrix}\xi_i^2+1,1\\0,\xi_i^2,\alpha_i,k\end{matrix}\right], \quad (11)$$

$$\overset{(a)}{=} 1-\frac{\xi_1^2\xi_2^2 r_1 r_2 A_1 A_2}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\sum_{k=1}^{\beta_1}\sum_{h=1}^{\beta_2}\frac{b_{1,k}}{\Gamma(k)}\frac{b_{2,h}}{\Gamma(h)} \mathrm{H}_{0,0:2,4:2,4}^{0,0:4,0:4,0}\left[\begin{matrix}\frac{B_1^{r_1}x}{\mu_{r_1}}\\ \frac{B_2^{r_2}x}{\mu_{r_2}}\end{matrix}\middle|\begin{matrix}-\\-\\(\lambda_1,\Lambda_1)\\(\lambda_2,\Lambda_2)\\(\upsilon_1,\Upsilon_1)\\(\upsilon_2,\Upsilon_2)\end{matrix}\right], \quad (12)$$

where (a) follows from applying [18, Eqs. (1.111) and (1.59)] and [19, Eq.(1.4)]. Moreover in (12), $\mathrm{H}[\because]$ stands for the bivariate Fox-H function (FHF) [19, Eq.(1.1)], whose efficient Mathematica code may be found in [20, Table I], whereby $(\lambda_1, \Lambda_1) = (\xi_1^2 + 1, r_1), (1, r_1)$; $(\lambda_2, \Lambda_2) = (0, r_1), (\xi_1^2, r_1), (\alpha_1, r_1), (k, r_1)$; $(\upsilon_1, \Upsilon_1) = (\xi_2^2 + 1, r_2), (1, r_2)$; and $(\upsilon_2, \Upsilon_2) = (0, r_2), (\xi_2^2, r_2), (\alpha_2, r_2), (h, r_2)$.

Asymptotic Outage Probability: By applying the Meijer-G function's asymptotic expansion [17, Eq.(55)] to (11), the asymptotic OP can be expressed at high per hop normalized average SNR $\frac{\mu_{r_i}}{\gamma_{th}} \to +\infty$ as

$$P_{out}^{\infty} = \frac{\xi_1^2 \xi_2^2 r_1 r_2 A_1 A_2}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \sum_{k=1}^{\beta_1} \sum_{h=1}^{\beta_2} \frac{b_{1,k}}{\Gamma(k)} \frac{b_{2,h}}{\Gamma(h)} \sum_{l=1}^{6} \xi_l \left(\frac{\gamma_{th}}{\bar{\mu}}\right)^{\Xi_l}, \tag{13}$$

where the coefficients ξ_l are easily obtained by applying [17, Eq.(55)]; $\Xi = \{\frac{\xi_1^2}{r_1}, \frac{\alpha_1}{r_1}, \frac{k}{r_1}, \frac{\xi_2^2}{r_2}, \frac{\alpha_2}{r_2}, \frac{h}{r_2}\}$ and $\bar{\mu} = \mu_{r_1} = \mu_{r_2}$.

Diversity Order: The asymptotic OP of the system under consideration can be rewritten as $P_{out}^{\infty} \simeq (G_c SNR)^{-G_d}$, where G_c and G_d stand for the coding gain and the diversity order of the system, respectively. Hence, based on (13), the diversity order of dual-hop FSO system in Málaga-$\mathcal{M}$ turbulence fading and under heterodyne and IM/DD techniques is equal to

$$G_d = \min\left\{\frac{\xi_1^2}{r_1}, \frac{\alpha_1}{r_1}, \frac{\beta_1}{r_1}, \frac{\xi_2^2}{r_2}, \frac{\alpha_2}{r_2}, \frac{\beta_2}{r_2}\right\}. \tag{14}$$

It may useful to mention that a similar result is obtained in [12, Eq.(24)].

B. Average bit-error rate (ABER)

To further investigate the performance of dual-hop AF FSO systems, we study the ABER which is a very important metric that reflects the usefulness of a wireless communication. Under different modulation schemes, the ABER is expressed as [12, Eq.(19)]

$$\overline{P}_e = \frac{\delta}{2\Gamma(p)} \sum_{j=1}^{n} q_j^p \int_0^{\infty} e^{-q_j x} x^{p-1} F_\gamma(x) \mathrm{d}x, \tag{15}$$

where the set of parameters δ, n, p and q_j accounts for varying modulations schemes (See [12, Table I]).

The ABER in CSI-assisted relaying AF FSO systems in Málaga-$\mathcal{M}$ turbulence-induced fading channel and under heterodyne and IM/DD detection techniques is given by

$$\overline{P}_e = \frac{\delta n}{2} - \frac{\delta \xi_1^2 \xi_2^2 r_1 r_2 A_1 A_2}{2\Gamma(p)\Gamma(\alpha_2)\Gamma(\alpha_2)} \sum_{j=1}^{n} \sum_{k=1}^{\beta_1} \sum_{h=1}^{\beta_2} \frac{b_{1,k}}{\Gamma(k)} \frac{b_{2,h}}{\Gamma(h)} \mathrm{H}_{1,0:2,4:2,4}^{0,1:4,0:4,0} \left[\begin{array}{c|c} & (1-p,1,1) \\ & - \\ \frac{B_1^{r_1}}{\mu_{r_1} q_j} & (\lambda_1, \Lambda_1) \\ \frac{B_2^{r_2}}{\mu_{r_2} q_j} & (\lambda_2, \Lambda_2) \\ & (\upsilon_1, \Upsilon_1) \\ & (\upsilon_2, \Upsilon_2) \end{array} \right]. \tag{16}$$

Proof: Plugging (12) into (15) and resorting to [19, Eq.(2.2)] completes the proof. ■

IV. Performance Analysis under Fixed-Gain Relaying

A. OP

The OP of fixed-gain relaying AF FSO transmission accounting for both detection techniques: heterodyne and IM/DD can be expressed as

$$F_\gamma(x) = \frac{\xi_1^2 \xi_2^2 A_1 A_2 r_1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \sum_{k=1}^{\beta_1} \sum_{h=1}^{\beta_2} \frac{b_{1,k}}{\Gamma(k)} \frac{b_{2,h}}{\Gamma(h)} \mathrm{H}_{1,0:4,3:1,4}^{0,1:1,3:4,0} \left[\begin{array}{c|c} & (1,1,1) \\ & - \\ \frac{\mu_{r_1}}{B_1^{r_1} x} & (\theta_1, \Theta_1) \\ \frac{B_2^{r_2} \mathrm{C}}{\mu_{r_2}} & (\theta_2, \Theta_2) \\ & (\phi_1, \Phi_1) \\ & (\phi_2, \Phi_2) \end{array} \right], \tag{17}$$

where $(\theta_1, \Theta_1) = (1 - \xi_1^2, r_1), (1 - \alpha_1, r_1), (1 - k, r_1), (1, r_1)$; $(\theta_2, \Theta_2) = (0, r_1), (1, 1), (-\xi_1^2, r_1)$; $(\phi_1, \Phi_1) = (\xi_2^2 + 1, r_2)$ and $(\phi_2, \Phi_2) = (0, 1), (\xi_2^2, r_2), (\alpha_2, r_2), (h, r_2)$.

Proof: The CDF of the end-to-end SNR γ with fixed-gain relaying scheme can be derived using [16, Eq.(8)] as

$$F_\gamma(x) = \int_0^{\infty} F_{\gamma_1}\left(x\left(\frac{\mathrm{C}}{y} + 1\right)\right) f_{\gamma_2}(y) \mathrm{d}y, \tag{18}$$

where F_{γ_1} is the first-hop FSO link's CDF that can be rewritten from (1) as

$$F_{\gamma_1}(x) = \frac{\xi_1^2 A_1 r_1}{\Gamma(\alpha_1)} \sum_{k=1}^{\beta_1} \frac{b_{1,k}}{\Gamma(k)} \mathrm{H}_{2,4}^{3,1}\left[\frac{B_1^{r_1} x}{\mu_{r_1}} \middle| \begin{array}{c} (1, r_1), (\xi_1^2+1, r_1) \\ (\xi_1^2, r_1), (\alpha_1, r_1), (k, r_1), (0, r_1) \end{array}\right], \tag{19}$$

where $\mathrm{H}[\cdot]$ is the Fox-H function [18, Eq.(1.2)], and f_{γ_2} is the PDF of the second-hop FSO link obtained from differentiating (19) over x as

$$f_{\gamma_2}(x) = \frac{\xi_2^2 A_2}{\Gamma(\alpha_2) x} \sum_{h=1}^{\beta_2} \frac{b_{2,h}}{\Gamma(h)} \mathrm{H}_{1,3}^{3,0}\left[\frac{B_2^{r_2} x}{\mu_{r_2}} \middle| \begin{array}{c} (\xi_2^2+1, r_2) \\ (\xi_2^2, r_2), (\alpha_2, r_2), (h, r_2) \end{array}\right]. \tag{20}$$

Finally, plugging (19) and (20) into (18) and utilizing the Mellin-Barnes representation of the bivariate FHF in [18, Eq.(2.57)] along some additional manipulations yield the closed-form expression in (17). ■

B. ABER

The ABER of fixed-gain relaying AF FSO transmission under heterodyne and IM/DD techniques with

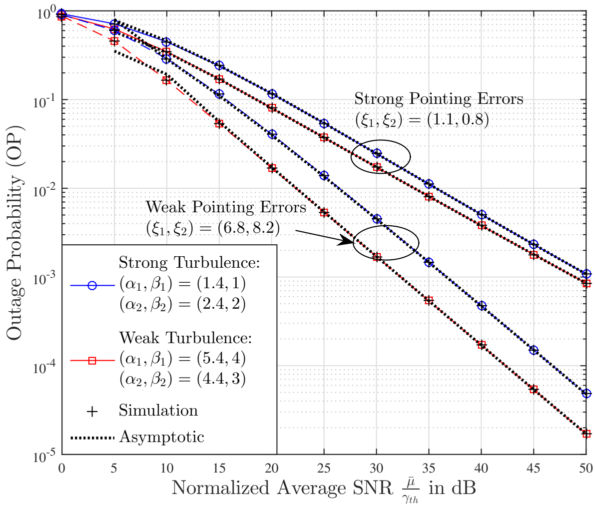

Fig. 1. OP of CSI-assisted relaying AF FSO transmission under different turbulence and pointing errors severities.

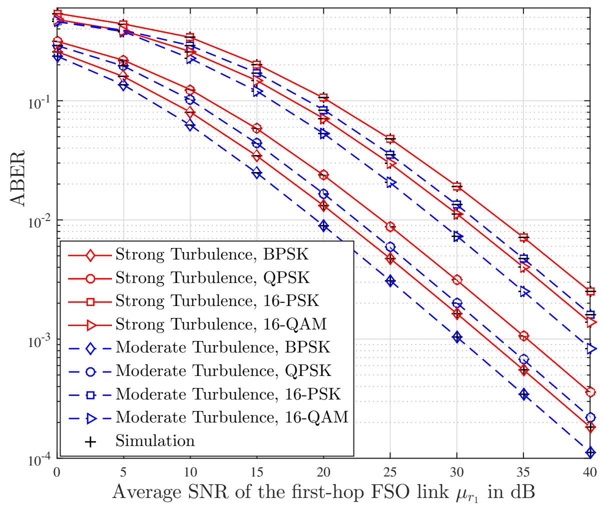

Fig. 3. ABER of CSI-assisted relaying AF FSO transmission under different turbulence effects and modulation schemes for $\mu_{r_2} = 20$ dB.

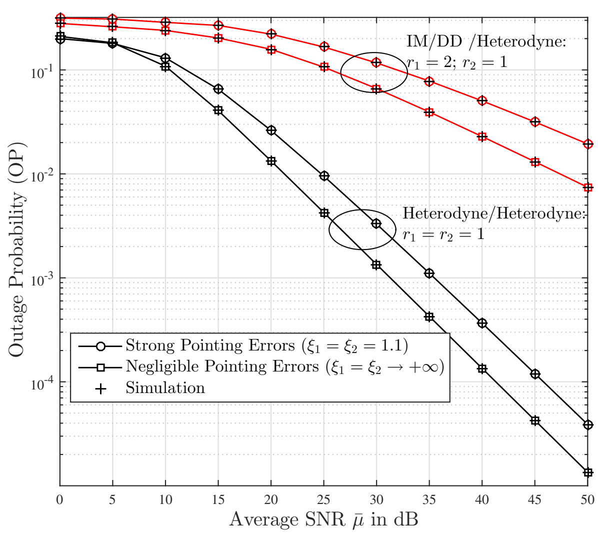

Fig. 2. OP of fixed-gain relaying AF FSO transmission under different pointing errors severities ξ_i and detection techniques.

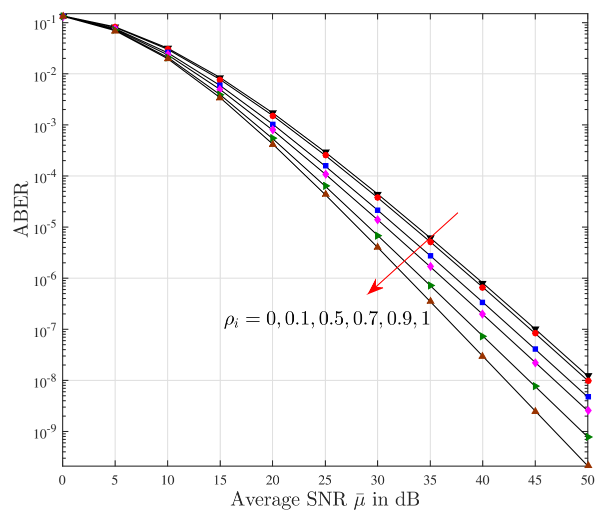

Fig. 4. ABER of fixed-gain relaying AF FSO transmission for different values of ρ_i.

pointing errors impairments can be expressed as

$$\overline{P}_b = \frac{\delta \xi_1^2 \xi_2^2 A_1 A_2 r_1}{2\Gamma(p)\Gamma(\alpha_1)\Gamma(\alpha_2)} \sum_{j=1}^{n} \sum_{k=1}^{\beta_1} \sum_{h=1}^{\beta_2} \frac{b_{1,k}}{\Gamma(k)} \frac{b_{2,h}}{\Gamma(h)} \mathrm{H}^{0,1:2,4:4,0}_{1,0:4,4:1,4} \left[\begin{array}{c|c} \frac{\mu_{r_1} q_j}{B_1^{r_1}} & (1,1,1) \\ & - \\ & (\theta_1, \Theta_1) \\ \frac{B_2^{r_2} \mathrm{C}}{\mu_{r_2}} & (p,1), (\theta_2, \Theta_2) \\ & (\phi_1, \Phi_1) \\ & (\phi_2, \Phi_2) \end{array} \right]. \quad (21)$$

Proof: (21) follows after plugging (17) into the Mellin-barnes representation of the bivariate FHF in (15) while resorting to the Mellin transform of the FHF in [18, Eq.(2.8)]. ■

V. Numerical Results

In this section, we show some Monte-carlo simulations to prove the accuracy of the derived closed-form expressions for the OP and the ABER under CSI-assisted and fixed-gain relaying schemes. Without loss of generality we consider $(\alpha_i, \beta_i) = (2.4, 2)$ for strong turbulence and $(\alpha_i, \beta_i) = (5.4, 4)$ for moderate turbulence conditions. We associate strong and negligible pointing errors to $\xi_i = 1.1$ and $\xi_i \to +\infty$, respectively. Unless stated otherwise, all the simulations were carried with the following parameters: $\rho_i = 0.1$, $g_i = 0.45$, $\Omega_i = 0.71$ for $i \in \{1, 2\}$, and $\mathrm{C} = 1.7$. The correctness of our newly derived closed-form expressions are confirmed by Monte-carlo simulations.

Fig.1 investigates the impacts of asymmetric turbulence-induced fading and pointing errors on the system performance. As expected, the OP deteriorates by decreasing the pointing error displacement standard deviation, i.e., for smaller ξ, or decreasing the turbulence fading parameter, i.e., smaller α and β. At high SNR, the asymptotic expansion in (13) matches very well its exact counterpart, which confirms the validity of our mathematical analysis for different parameter settings. It can be observed through simulations that the performance deterioration due to strong turbulence is more severe under weak pointing errors rather than strong pointing errors conditions. For instance, the curves under strong pointing errors have the same slopes thereby inferring the equality between their diversity order. More specifically, using (14), the diversity order of the system is equal to 0.64 under severe pointing errors.

Fig.2 shows the OP for different FSO detection techniques and pointing errors severities using fixed-gain relaying. Regardless of the pointing errors, employing IM/DD detection in one FSO link yields a considerable performance degradation compared to all heterodyne-based FSO transmission since heterodyne detection employs more complicated coherent receivers.

Fig.3 depicts the ABER of CSI-assisted relaying FSO system operating over Málaga-$\mathcal{M}$ fading under varying turbulence conditions. An expected trend in Fig.3 is noticed, where using the same modulation scheme, the performance under moderate turbulence conditions is better than its counterparts under strong turbulence. Moreover, under similar turbulence conditions, 16-QAM modulation secure a better performance than 16-PSK modulation, as expected.

Fig.4 demonstrates the end-to-end performance in terms of ABER using BPSK binary modulation for different values of ρ_i. We can see form the figure that higher value of ρ_i guarantee a better overall system's performance. This is an expected observation since increasing ρ_i reduces the atmospheric-induced fading whereby yielding a better performance [13].

VI. Conclusion

In this paper, the end-to-end performance of relay-assisted AF FSO transmission in Málaga-$\mathcal{M}$ turbulence fading with pointing errors is investigated. Results shows that severe atmospheric turbulence and strong pointing errors are detrimental for the system's performance. Moreover, the diversity order is related to the the minimum value of the atmospheric turbulence and pointing error parameters.

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