Optimal Anchors Placement Strategy for Super Accurate Nodes Localization in Anisotropic Wireless Sensor Networks

Invited Paper

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***Abstract*—In this paper, we develop a novel optimal anchors placement strategy tailored for anisotropic WSNs. By resorting to the well-known particle swarm optimization (PSO), we derive the optimal anchors' positions that minimize the average location estimation error (LEE). We show that our placement strategy provides substantial accuracy gains if used instead of the conventional ones and that it is able to reduce not only the average LEE but also the LEE itself and, hence, guarantees high accuracy for any WSN configuration.**

***Index Terms*—Optimal anchors placement, wireless sensor networks (WSN)s, localization algorithms, anisotropic environments, particle swarm optimization (PSO).**

I. Introduction

Recent advances in wireless communications and low-power circuits technologies have led to proliferation of wireless sensor networks (WSNs). A WSN is a set of small and low-cost sensor nodes often equipped with small batteries. The latter are often deployed in a random fashion to sense or collect from the surrounding environments some physical phenomena such as temperature, light, pressure, etc. [1]-[3]. Since power is a scarce resource in such networks, sensors usually resort to multi-hop transmission in order to send their gathered data to an access point (AP). However, the received data at the latter are often fully or partially meaningless if the location from where they have been measured is unknown [4], making sensors' localization an essential task in WSNs. Many localization algorithms available in the literature [5]-[13] were designed to comply with such networks. To properly localize each sensor, most of these algorithms require the distance between the latter and at least three position-aware nodes called hereafter anchors[1]. Since it is very likely in WSNs that some sensors be unable to directly communicate with all anchors, the distance between each anchor-sensor pair is usually estimated using their shortest path. This distance is in fact approximated by the sum of the distances between any consecutive intermediate nodes located on this path. Several approaches have been so far developed to estimate these distances. Although efficient, they were unfortunately unable to guarantee high accuracy, especially in anisotropic environments where the shortest multi-hop path between each anchor-sensor pair is often much longer than the actual distance separating them. This is actually due to the fact that the accuracy of any localization algorithm is governed not only by the distance estimation (DE) efficiency, but also the position of the anchors themselves. Significant research endeavors have been recently devoted to developing anchor placement strategies able to guarantee high sensor localization accuracy [14]-[22]. In [15], it has been proven that perimeter placement is the optimal strategy in isotropic environments free of obstacles (e.g., mountains, coverage holes, etc.). In [18], this strategy was investigated and compared in accuracy performance to other strategies in anisotropic environments. It was shown in [18] and [19] that the perimeter placement performs poorly in anisotropic environments. Some attempts to derive the optimal anchors positions in such environments have been made in [20]-[22] without providing significant accuracy gains.

In this paper, we develop a novel optimal anchors placement strategy properly tailored for anisotropic WSNs. By resorting to the well-known particle swarm optimization (PSO), we derive the optimal anchors' positions that minimize the average location estimation error (LEE). We show that our placement strategy provides substantial accuracy gains if used instead of the conventional ones and that it is able to reduce not only the average LEE but also the LEE itself and, hence, guarantees high accuracy for any WSN configuration.

The rest of this paper is organized as follows: Section II describes the network model. Section III introduces the average LEE and proves its adequacy to anchor-based localization. Section IV proposes a novel optimal anchors placement strategy. Simulation results are discussed in Section V and concluding remarks are made in Section VI.

II. Network model

Fig. 1 illustrates a network model of M anchors and N sensors deployed in a 2-D square area S. The anchors are aware of their positions while the sensors are oblivious to this information. These sensors are assumed to be uniformly distributed in S. All anchor and sensor nodes are assumed to have the same transmission capability (i.e., range) denoted by R. Each node is able to directly communicate with any other node located in the disc having that node as a center and R as a radius, while it communicates in a multi-hop fashion with

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[1]In practice, an anchor node refers to a sensor, base station, or a nearby access point (AP) with known position. This information is usually acquired using global positioning system (GPS) technology, configured, or manually entered into the node memory prior to deployment.

the nodes located outside. As shown in Fig. 1, the anchors are marked with red triangles and the senors are marked with blue circles. If two nodes are able to directly communicate, they are linked with a dashed line that represents one hop.

Let us denote by (a_i, b_i) , $i = 1, \ldots, M$ the coordinates of the anchor nodes and (x_i, y_i) , $i = 1, \ldots, N$ those of the regular ones.

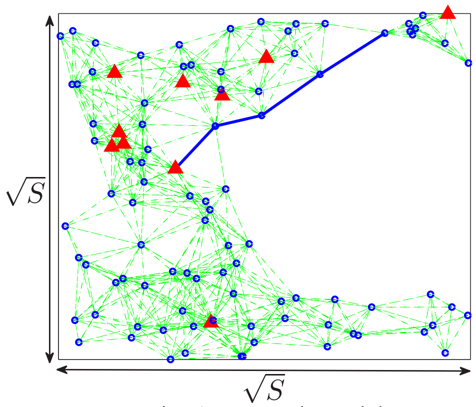


Fig. 1: Network model.

In what follows, we propose an efficient anchor placement strategy able to significantly enhance the accuracy of any anchor-based localization algorithm. To this end, one should first determine the metric which properly gauges the accuracy of such algorithms. From this perspective, Section III presents a new metric and proves its adequacy to anchor-based localization algorithms.

III. Average location estimation error (LEE)

As a first step of any anchor-based localization algorithm, the k-th anchor broadcasts its coordinate (a_k, b_k) in the network. The regular nodes receive these information either directly or through multi-hop communication. Once the i-th regular node obtains all anchors' coordinates and computes their corresponding distances, either heuristically or analytically, it derives its own position by solving the following nonlinear equations system:

$$\left\{ \begin{array}{rl} (a_1 - \hat{x}_i)^2 + (b_1 - \hat{y}_i)^2 = & \hat{d}_{i-1}^2 \\ (a_2 - \hat{x}_i)^2 + (b_2 - \hat{y}_i)^2 = & \hat{d}_{i-2}^2 \\ \vdots & \vdots \\ (a_M - \hat{x}_i)^2 + (b_M - \hat{y}_i)^2 = & \hat{d}_{i-M}^2 \end{array} \right. , \quad (1)$$

where $(\hat{x}_i, \hat{y}_i)$ are the estimated i-th sensor's coordinates and $\hat{d}_{i-k}$ is its estimated distance to the k-th anchor. After some rearrangements aiming to linearize the above system , we obtain

$$\boldsymbol{\Upsilon} \hat{\boldsymbol{\alpha}}_i = -\frac{1}{2} \boldsymbol{\kappa_i}, \quad (2)$$

where $\hat{\boldsymbol{\alpha}}_i = [\hat{x}_i, \hat{y}_i]^T$,

$$\boldsymbol{\Upsilon} = \begin{bmatrix} a_1 - a_M & b_1 - b_M \\ a_2 - a_M & b_2 - b_M \\ \vdots & \vdots \\ a_{(M-1)} - a_M & b_{(M-1)} - b_M \end{bmatrix}, \quad (3)$$

and

$$\boldsymbol{\kappa_i} = \begin{bmatrix} \hat{d}_{i-1}^2 - \hat{d}_{i-M}^2 + a_M^2 - a_1^2 + b_M^2 - b_1^2 \\ \hat{d}_{i-2}^2 - \hat{d}_{i-M}^2 + a_M^2 - a_2^2 + b_M^2 - b_2^2 \\ \vdots \\ \hat{d}_{i-(M-1)}^2 - \hat{d}_{i-M}^2 + a_M^2 - a_{(M-1)}^2 + b_M^2 - b_{(M-1)}^2 \end{bmatrix}. \quad (4)$$

Since $\boldsymbol{\Upsilon}$ is a non-invertible matrix, $\hat{\boldsymbol{\alpha}}_i$ could be estimated with the pseudo-inverse of $\boldsymbol{\Upsilon}$ as follows:

$$\hat{\boldsymbol{\alpha}}_i = -\frac{1}{2} \left(\boldsymbol{\Upsilon}^T \boldsymbol{\Upsilon} \right)^{-1} \boldsymbol{\Upsilon}^T \boldsymbol{\kappa_i}. \quad (5)$$

Therefore, the i-th sensor is able to obtain an estimate of its coordinates as $\hat{x}_i = [\hat{\boldsymbol{\alpha}}_i]_1$, and $\hat{y}_i = [\hat{\boldsymbol{\alpha}}_i]_2$. Let $\mathcal{E}_{\mathrm{P},i}$ denote the i-th sensor's location estimation error (LEE) given by

$$\mathcal{E}_{\mathrm{P},i} = \|\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}}_i\|^2, \quad (6)$$

where $\boldsymbol{\alpha}_i = [x_i, y_i]^T$ is a vector whose entries are the true i-th sensor coordinates. From (6), $\mathcal{E}_{\mathrm{P},i}$ is an excessively complex function of the random variables $(x_i, y_i), i = 1, \ldots, N$, d_{i-k} and $\hat{d}_{i-k}, k = 1, \ldots, M$ and, hence, a random quantity of its own. Optimizing the anchors' locations using such a metric would not only be a tedious task, but it would also result in locations strongly dependent on the sensors' coordinates. Recall here that such information are not available. A much more appealing metric would be then the average LEE $\bar{\mathcal{E}}_{\mathrm{P}}(N) = \mathrm{E}\{\mathcal{E}_{\mathrm{P},i}\}$ where the expectation is taken with respect to all the sensors' coordinates. Actually, $\bar{\mathcal{E}}_{\mathrm{P}}(N)$ could be differently defined as

$$\bar{\mathcal{E}}_{\mathrm{P}}(N) = \mathrm{E}\left\{\mathcal{G}_{\mathrm{P}}^{\mathrm{Net}}(N)\right\}, \quad (7)$$

where

$$\mathcal{G}_{\mathrm{P}}^{\mathrm{Net}}(N) = \frac{1}{N} \sum_{i=1}^{N} \mathcal{E}_{\mathrm{P},i}, \quad (8)$$

refers to the global LEE through the network. Furthermore, using the strong law of large numbers, we show for large N that we have

$$\mathcal{G}_{\mathrm{P}}^{\mathrm{Net}}(N) \xrightarrow{p1} \bar{\mathcal{E}}_{\mathrm{P}}(N), \quad (9)$$

where $\xrightarrow{p1}$ stands for convergence with probability one. From (9), $\bar{\mathcal{E}}_{\mathrm{P}}(N)$ is not only the statistical average of $\mathcal{G}_{\mathrm{P}}^{\mathrm{Net}}(N)$, but also it approaches the latter for any given realization (i.e., any given $(x_i, y_i), i = 1, \ldots, N$). All the above proves unambiguously that $\bar{\mathcal{E}}_{\mathrm{P}}(N)$ is a meaningful and useful performance metric. It follows from (5) that

$$\mathcal{E}_{\mathrm{P},i} = \frac{1}{4} \left\| \left(\boldsymbol{\Upsilon}^T \boldsymbol{\Upsilon} \right)^{-1} \boldsymbol{\Upsilon}^T \boldsymbol{\delta}_i \right\|^2, \quad (10)$$

where $[\boldsymbol{\delta}_i] = [\epsilon_1 - \epsilon_M, \ldots, \epsilon_{M-1} - \epsilon_M]^T$ with $\epsilon_k = \hat{d}_{i-k}^2 - d_{i-k}^2$ being the squared-distance estimation error. $\mathcal{E}_{\mathrm{P},i}$ is then given by

$$\begin{aligned} \mathcal{E}_{\mathrm{P},i} &= \mathrm{Tr}\left(\left(\boldsymbol{\Upsilon}^T \boldsymbol{\Upsilon} \right)^{-1} \boldsymbol{\Upsilon}^T \boldsymbol{\delta}_i \boldsymbol{\delta}_i^T \boldsymbol{\Upsilon} \left(\boldsymbol{\Upsilon}^T \boldsymbol{\Upsilon} \right)^{-1} \right) \\ &= \mathrm{Tr}\left(\boldsymbol{\Omega} \boldsymbol{\delta}_i \boldsymbol{\delta}_i^T \right) \\ &= \sum_{k=1}^{M-1} \boldsymbol{\Omega}_{kk} \left([\boldsymbol{\delta}_i]_k \right)^2 + \sum_{k=1}^{M-1} \sum_{l=1, l \neq k}^{M-1} \boldsymbol{\Omega}_{kl} [\boldsymbol{\delta}_i]_l [\boldsymbol{\delta}_i]_k, \quad (11) \end{aligned}$$

where $\mathrm{Tr}(\mathbf{X})$ is the trace of the matrix $\mathbf{X}$ and $\mathbf{\Omega} = \mathbf{\Upsilon}\left(\mathbf{\Upsilon}^T\mathbf{\Upsilon}\right)^{-2}\mathbf{\Upsilon}^T$. Note in the second line of (11) that we exploit the cyclic property of the trace. Since $\epsilon_k, k = 1, \ldots, M$ are i.i.d random variables, we have from (11) the following

$$\bar{\mathcal{E}}_{\mathrm{P}}(N) = \sigma_\epsilon^2\left(2\mathrm{Tr}(\mathbf{\Omega}) + \sum_{k=1}^{M-1}\sum_{l=1, l\neq k}^{M-1}\mathbf{\Omega}_{kl}\right) = \sigma_\epsilon^2 F(\Omega). \quad (12)$$

Therefore, in order to reduce $\bar{\mathcal{E}}_{\mathrm{P}}(N)$ (i.e., improve the localization accuracy), one should minimize both σ_ϵ^2 and $F(\mathbf{\Omega}) = 2\mathrm{Tr}(\mathbf{\Omega}) + \sum_{k=1}^{M-1}\sum_{l=1,l\neq k}^{M-1}\mathbf{\Omega}_{kl}$, the former by use of accurate DE techniques [5]-[13] while $F(\mathbf{\Omega})$ requires the optimization of the anchors positions. In the next section, adopt $F(\Omega)$ as a new design cost function to develop a novel optimal anchors placement strategy.

IV. Proposed anchor placement strategy

In order to improve the localization accuracy in the anisotropic environments of our concern, one could compute the optimal set of anchors' positions $\mathcal{S}_{\mathrm{opt}}$ that satisfies

$$\begin{aligned} \mathcal{S}_{\mathrm{opt}} &= \arg\min F(\mathbf{\Omega}) \\ \text{s.t.} \quad & L_a \leq a_i \leq U_a \qquad i = 1, 2, \cdots, N_a \\ & L_b \leq b_i \leq U_b \qquad i = 1, 2, \cdots, N_a \\ & \|P_i - P_j\| \geq d_{min} \qquad \forall\, i \neq j \end{aligned} \quad (13)$$

where $P_i = [a_i, b_i]^T$ is the vector of the i-th anchors coordinates and L_a, L_b, U_a, and U_b are lower and upper bounds on all anchors coordinates. These bounds depend on the obstacle form and position. Please note that the first two constraints ensure that anchors be located within the obstacle surrounding area. Whereas the third constraint imposes a minimum distance d_{min} between the anchors and, hence, guarantees their deployment all over the available area.

Several effective optimization algorithms that require a moderate memory and reasonable computational resources have been proposed so far to solve such complex optimization problem, for instance the simulated annealing algorithm (SA), genetic algorithms (GA), artificial intelligence (AI), and particle swarm optimization (PSO) [23]. Due to its ease of implementation, high resolution, and speed of convergence, the latter has attracted a lot of attention in the research community and has been recently introduced as a promising tool for solving a wide range of optimization problems in different contexts such as UWB antenna design, data mining, acoustic communication, and localization [24]. However, despite their advantage, traditional PSO-based algorithms may easily fall into local optima, especially when solving a complex multimodal problem such as the one of our concern [25]. In order to overcome this issue, we propose in this paper a novel non-linear fitness-based inertia weight expression given by

$$\phi^k = w_{\max}\left(1 - \frac{(w_{\max} - w_{\min})\,\mu + w_{\min}}{1 + e^{\left(-2w_{\min}\frac{\min f_i^k - \max f_i^k}{\bar{f}_i^k}\right)}}\right), \quad (14)$$

where μ is a random variable uniformly distributed in the interval $[0, 1]$ and $\bar{f}_i^k$ is the average fitness value at the k-th generation. From (14), the value of the inertia weight will be then dynamically updated at each iteration in a non-linear manner according to the calculated fitness. This allows a shorter exploration time than with existing approaches such as the linear, random, constant, and chaotic ones [26]-[29]. Once we get ϕ^k, the velocity and position of each particle are updated using the following equations

$$V_i^{k+1} = \phi^k V_i^k + c_1\alpha\left(\rho_i^k - P_i^k\right) + +c_2\beta\left(\gamma_g^k - P_i^k\right), \quad (15)$$

and

$$P_i^{k+1} = P_i^k + V_i^{k+1}, \quad (16)$$

Algorithm 1 Optimal anchor nodes placement algorithm

% s_k is the set of anchor nodes%
Initialize the first two anchor nodes positions
$s_k = [0 \quad S; S \quad S]$
Initialize the cognitive and social scaling parameters c_1 and c_2, respectively
Initialize the maximum number of iterations k_{max}
Initialize γ_g in such a way that the fitness of γ_g is as close to infinity as possible
Initialize position and velocity boundaries
$m = 3$
for $m \leq N_a$ **do**
  $X_0 = s_k(m-1)$
  while Constraints criteria are not met **do**
    $k = 1$
    for each particle i **do**
      $P_i = |X_0 + rand(1,2)|$
      $V_i = V_{max} \times rand(1,2)$
      Compute $f(P_i)$
      if $f(P_i) < f(\gamma_g)$ **then**
        $\gamma_g = P_i$
        $f(\gamma_g) = f(P_i)$
      end if
    end for
    while $k \neq k_{max}v$ **do**
      $\phi^{k+1} \leftarrow$ Equation (14)
      for each particle i **do**
        $V_i^{k+1} \leftarrow$ Equation (15)
        $P_i^{k+1} \leftarrow$ Equation (16)
        Check the velocity and position boundaries
        Compute $f(P_i)$
        if $f(P_i) < f(\rho_i)$ **then**
          $\rho_i = P_i$
          $f(\rho_i) = f(P_i)$
        end if
        if $f(P_i) < f(\gamma_g)$ **then**
          $\gamma_g = P_i$
          $f(\gamma_g) = f(P_i)$
        end if
      end for
      $k = k + 1$
    end while
  end while
  $m = m + 1$
  $s_k = s_k \cup \{\gamma_g\}$
end for

where ρ_i^k is the best previous position of the i-th particle, γ_g^k is the best global position at the k-th generation, c_1 and c_2 are the cognitive and social scaling parameters, respectively, and α and β are two random variables uniformly distributed within the interval $[0, 1]$. The rest of the proposed PSO-based estimation algorithm of the optimal anchors positions with a minimum average LEE is summarized in Algorithm 1.

In the next section, we prove that placing the anchors in the positions obtained using our PSO-based algorithm can enhance localization accuracy in anisotropic environments substantially.

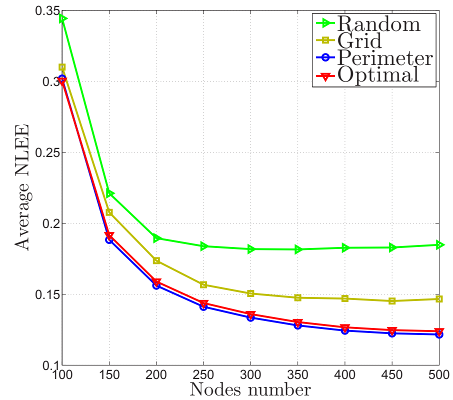
V. Simulations results

Monte-Carlo simulations are provided in this section to verify the efficiency of the proposed anchors placement strategy. These simulations are conducted to compare, under the same network settings, the latter with three commonly adopted benchmarks, namely the grid [14], perimeter [15], and random [13] placement strategies. All these strategies are tested using two localization algorithms: the well-known RAPS [8] and one of our recently developed algorithms [9]. All simulation results are obtained by averaging over 800 trials. In all simulations, nodes are uniformly deployed in a 2-D square area in the presence of a rectangle obstacle which makes the network topology C-shaped, except in Fig. 2 where we consider an isotropic environment. S and R are set to $50^2\ m^2$ and $10\ m$, respectively. M is set to 12, expect in Fig. 5 where it varies from 5% to 10%.

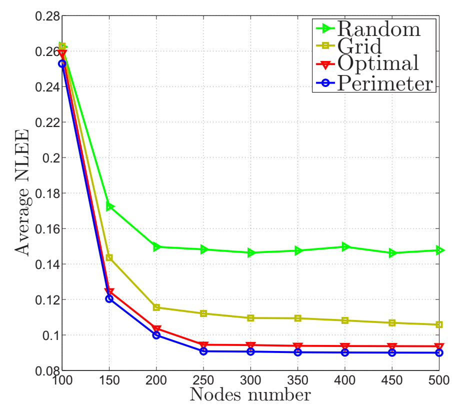
Figs. 2(a) and 2(b) plot the average R^2-normalized LEE (NLEE) achieved by RAPS [8] and our localization algorithm in [9] using the proposed anchor placement, grid, perimeter, and random strategies for different values of N in an isotropic environment. From these figures, the accuracy of both localization algorithms is improved using the proposed strategy instead of the grid and random strategies. Furthermore, the proposed strategy guarantees almost the same accuracy as the perimeter placement, which was previously proven to be the optimal one in any isotropic environment [15]. This validates the optimality of the proposed anchors placement strategy.

Figs. 3(a) and 3(b) display the average NLEE achieved by RAPS [8] and our localization algorithm in [9] using the proposed anchor placement, grid, perimeter, and random strategies for different values of N in an anisotropic environment. As could be observed from these figures, the lowest average NLEE is always achieved by the proposed strategy. The latter turns out to be until about 76.8%, 61.62%, and 50.64% more accurate than than grid, perimeter, and random strategies, respectively. This proves the superiority of the proposed PSO-based anchor placement strategy.

Figs. 4(a) and 4(b) plot the NLEE's standard deviation achieved by RAPS [8] and our localization algorithm in [9] using all anchors placement strategies for different values of N. From these figures, using any strategy, the NLEE's standard deviation decreases as expected when the node density increases. However, the one achieved by the proposed strategy approaches 0 as N grows large, in contrast to all its counterparts. Our strategy is actually able to minimize not only the average NLEE, but also the NLEE itself. This is a highly desirable feature, since it guarantees high accuracy for any WSN configuration.



(a) RAPS [8].

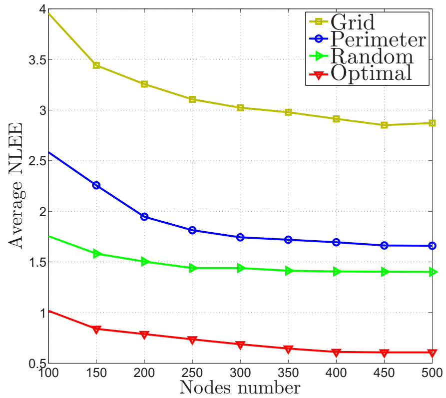


(b) Our localization algorithm in [9].

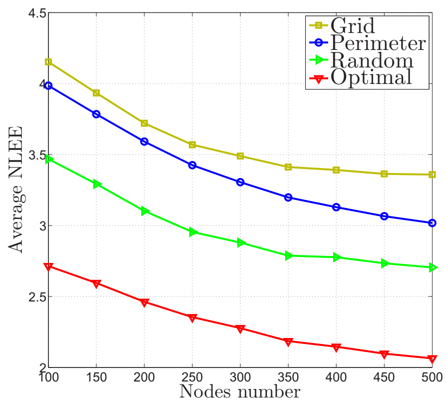
Fig. 2: Average NLEE achieved by RAPS [8] and our localization algorithm in [9] using the proposed anchor placement, grid, perimeter, and random strategies for different values of N in an isotropic environment.

Figs. 5(a) and 5(b) show the average NLEE achieved by RAPS [8] and our localization algorithm in [9] using the proposed anchor placement, grid, perimeter, and random strategies for different values of M with $N = 150$. As could be observed from these figures, the localization accuracy is improved as expected when he number of anchors is improved. However, the average NLEE achieved using our new anchor placement strategy remains the lowest, thereby further proving its high efficiency.

Figs. 6(a) and 6(a) illustrate the NLEE's CDF achieved by RAPS [8] and our localization algorithm in [9] using all the anchor placement strategies. With the proposed strategy, until 90% of the sensors could estimate their position with a NLEE less than 2 using the RAPS algorithm. In contrast, 62% achieve the same accuracy with the random strategy, 52% with the perimeter strategy, and only about 40% with the grid strategy. This highlights again the net advantage of the proposed PSO-based placement strategy against its counterparts in anisotropic environments.

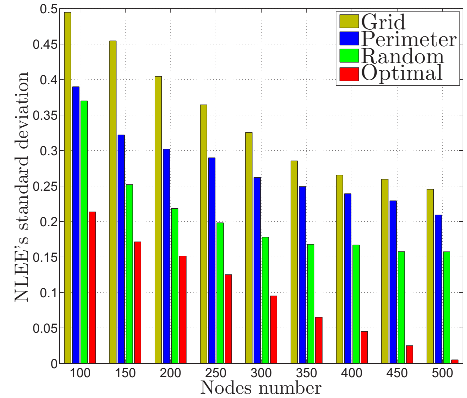


(a) RAPS [8].

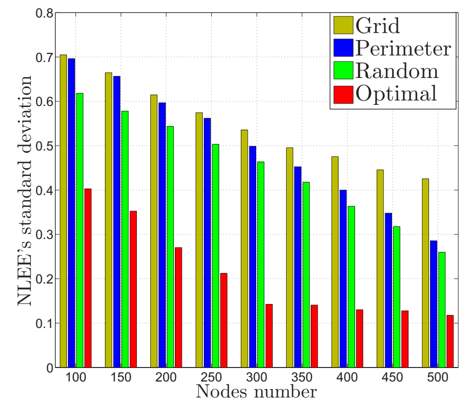


(b) Our localization algorithm in [9].

Fig. 3: Average NLEE achieved by RAPS [8] and our localization algorithm in [9] using the proposed anchor placement, grid, perimeter, and random strategies for different values of N in an anisotropic environment.



(a) RAPS [8].



(b) Our localization algorithm in [9].

Fig. 4: NLEE's standard deviation achieved by RAPS [8] and our localization algorithm in [9] using the proposed anchor placement, grid, perimeter, and random strategies for different values of N in an anisotropic environment.

VI. Conclusion

In this paper, we developed a novel optimal anchor placement strategy tailored for anisotropic WSNs. By resorting to the well-known particle swarm optimization (PSO), we derived the optimal anchors positions that minimize the average location estimation error (LEE). It was shown that our placement strategy provides substantial accuracy gains if used instead of conventional ones and that it is able to reduce not only the average LEE but also the LEE itself and, hence, guarantees high accuracy for any WSN configuration.

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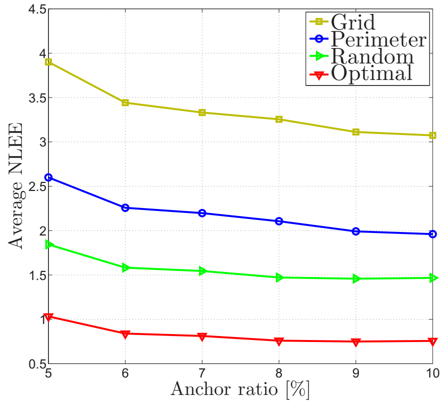
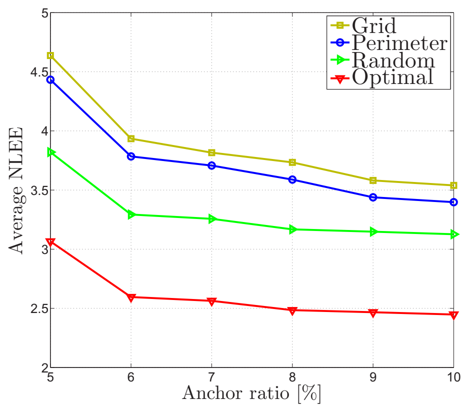
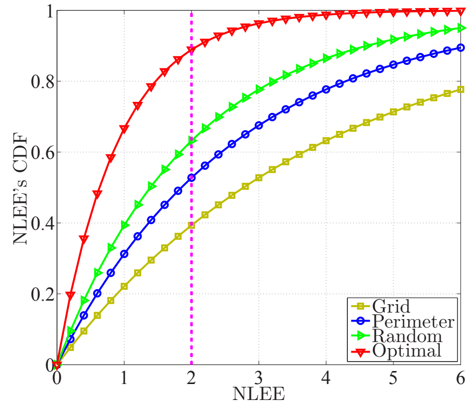


(a) RAPS [8].

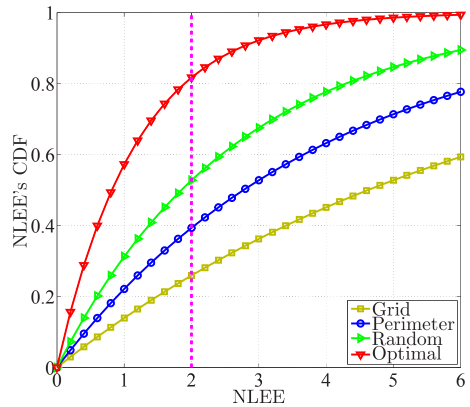


(b) Our localization algorithm in [9].

Fig. 5: Average NLEE achieved by RAPS [8] and our localization algorithm in [9] using the proposed anchor placement, grid, perimeter, and random strategies for different values of M with $N = 150$ in an anisotropic environment.



(a) RAPS [8].



(b) Our localization algorithm in [9].

Fig. 6: NLEE's CDF achieved by RAPS [8] and our localization algorithm in [9] the proposed anchor placement, grid, perimeter, and random strategies with $N = 150$ in an anisotropic environment.

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