Cost-effective and Accurate Nodes Localization in Heterogeneous Wireless Sensor Networks

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***Abstract*—In this paper, we propose a novel low-cost localization algorithm tailored for multi-hop heterogeneous wireless sensor networks (HWSNs) where nodes' transmission capabilities are different. This characteristic, if not taken into account when designing the localization algorithm, may severely hinder its accuracy. Assuming different nodes' transmission capabilities, we develop a novel approach to derive the expected hop progress (EHP). Exploiting the latter, we propose a localization algorithm that is able to accurately locate the sensor nodes owing to a new low-cost implementation. Furthermore, we develop a correction mechanism which complies with the heterogeneous nature of WSNs to further improve localization accuracy without incurring any additional costs. Simulations results show that the proposed algorithm, whether applied with or without correction, outperforms in accuracy the most representative WSN localization algorithms.**

***Index Terms*—Heterogeneous wireless sensor networks (WSNs), multi-hop, localization, low cost, energy harvesting (EH), EH-WSNs, expected hop progress (EHP).**

I. Introduction

Recent advances in wireless communications and low-power circuits technologies have led to proliferation of wireless sensor networks (WSNs). A WSN is a set of small and low-cost sensor nodes often equipped with small batteries. The latter are often deployed in a random fashion to sense or collect from the surrounding environments some physical phenomena such as temperature, light, pressure, etc. [1]- [3]. Since power is a scarce resource in such networks, sensor nodes usually recur to multi-hop transmission in order to send their gathered data to an access point (AP). However, the received data at the latter are often fully or partially meaningless if the location from where they have been measured is unknown [4], making the nodes' localization an essential task in multi-hop WSNs. Owing to the low-cost requirements of WSNs, unconventional paradigms in localization must yet be investigated. Many interesting solutions exist in the literature [5]-[12]. To properly localize each regular or position-unaware node, most of these algorithms require the distance between the latter and at least three position-aware nodes called hereafter anchors. Since it is very likely in multi-hop WSNs that some regular nodes be unable to directly communicate with all anchors, the distance between each anchor-regular nodes pair is usually estimated using their shortest path. The latter is obtained by summing the distances between any consecutive intermediate nodes located on the shortest path between the two nodes. Depending on the process used to estimate these distances, localization algorithms may fall into three categories: measurement-based, heuristic, and analytical [5]-[12].

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Measurement-based algorithms exploit the measurements of the received signals' characteristics such as the received signal strength (RSS) [5]. or the time of arrival (TOA) [6], etc. Using the RSS measurement, the distance between any sensors' pair could be obtained by converting the power loss due to propagation from a sensor to another based on some propagation laws. Unfortunately, due to the probable presence of noise and interference, the distance's estimate would be far from being accurate, thereby leading to unreliable localization accuracy. Using the TOA measurement, nodes require high-resolution clocks and extremely accurate synchronization between them. While the first requirement may dramatically increase the cost and the size of sensor nodes, the second results in severe depletion of their power due to the additional overhead required by such a process. Furthermore, in the presence of noise and/or multipath, the TOA measurement is severely affected thereby hindering nodes' localization accuracy. As far as heuristic algorithms [7] are concerned, they also have a major drawback. Indeed, most of these algorithms are based on variations of DV-HOP [7] whose implementation in multi-hop WSNs requires a correction factor derived in a non-localized manner and broadcasted in the network by each anchor. This causes an undesired prohibitive overhead and power consumption, thereby increasing the overall cost of the network.

Popular alternatives, more suitable for multi-hop WSNs, are the analytical algorithms [8]-[11] which evaluate theoretically the distance between any two consecutive intermediate nodes. The latter is in fact locally computable at each node, thereby avoiding unnecessary costs incurred if it is fully or partially computed at other nodes and then broadcasted in the network, such as in heuristic algorithms. In spite of their valuable contributions, the approaches developed so far in [8]-[11] to derive that distance are based on the unrealistic assumption that all nodes have the same transmission capabilities (i.e., the WSN is homogenous). However, due to the fact that these sensor nodes are designed using various technologies to achieve different tasks, their sensing as well as transmission capabilities are very-often different. Furthermore, if an energy harvesting (EH) technology is locally integrated at each node, which is the case in the most recently developed WSNs referred to hereafter as EH-WSNs, the available harvested power at nodes would then be random. This phenomenon actually results in the randomization of the nodes' transmission capabilities, since the latters are closely related to the nodes' available powers. During the localization process, it is then very likely that nodes' transmission capabilities be different. As the approaches in [8]-[11] assume the same transmission capability throughout the network, their localization accuracy substantially deteriorates in the so-called heterogeneous WSNs (HWSNs) making them unsuitable for such networks. To the best of our knowledge, there is no analytical algorithm that accounts so far for the heterogeneous nature of WSNs.

To bridge this gap, we propose in this paper a novel analytical algorithm tailored for multi-hop HWSNs where nodes have different transmission capabilities. Taking into account this characteristic, a novel approach is developed to accurately derive the distances between any consecutive nodes. Using the so-obtained distances, the proposed algorithm is able to accurately locate the nodes owing to a new low-cost implementation. Furthermore, we develop a correction mechanism which complies with the heterogeneous nature of WSNs to further improve localization accuracy without incurring any additional costs. Simulations results show that the proposed algorithm, whether applied with or without correction, outperforms in accuracy the most representative multi-hop WSNs localization algorithms.

The rest of this paper is organized as follows: Section II describes the system model and discusses the motivation of this work. Section III derives the distance between consecutive sensors using the novel proposed approach. A novel localization algorithm for HWSNs is proposed in section IV. Its implementation cost is discussed in Section VI. Simulation results are discussed in Section VII and concluding remarks are made in section VIII.

II. Network model and overview

The network model of our concern consists on N sensors deployed in a 2-D square area S. The i-th node could directly communicate with any node located in $D\left(i, Tc_i\right)$, the disc having this node as a center and its transmission capability Tc_i as a radius. Due to the heterogonous nature of WSNs, nodes are assumed here to have different transmission capabilities. It is also assumed that only a few nodes commonly known as anchors are aware of their positions. The other nodes, called hereafter position-unaware or regular nodes for the sake of simplicity, are oblivious to this information. Let N_a and $N_u = N - N_a$ denote the number of anchors and regular nodes, respectively. Without loss of generality, let $(x_i, y_i)\,,\ i = 1, \ldots, N_a$ be the coordinates of the anchors and $(x_i, y_i)\,,\ i = N_a + 1, \ldots, N$ those of the regular nodes. As a first step of any localization algorithm for multi-hop WSNs aiming to estimate the regular nodes' positions, the k-th anchor broadcasts through the network a message containing its position. If the $(i - N_a)$-th regular node (or the i-th node) is located outside the anchor coverage area, it receives this message through multi-hop transmission. For simplicity, let us assume that only one intermediate node j located over the shortest path between the k-th anchor and the i-th node is necessary (i.e., two-hop transmission). Assuming a high node density in the network, the distance d_{k-i} between the two nodes can be accurately approximated as [8]-[11]

$$d_{k-i} \simeq d_{k-j} + d_{j-i}, \tag{1}$$

where $d_{\star-*}$ is the effective distance between the $\star$-th and the $*$-th node. Two approaches have been so far developed to analytically estimate the distance d_{k-i} exploiting the aforementioned approximation [8]-[11].

In the first approach, the j-th node estimates the distance d_{k-j} using the number of common neighbors with the k-th node. However, n_{kj} cannot be accurately obtained since some neighbors of the j-th and/or k-th sensor/s are not able to communicate with the latters due to their weaker capabilities and, therefore, this approach is not suitable for HWSNs.

The second approach uses the fact that the minimum square error (MMSE) of the distance estimation is obtained if $\hat{d} = E\left(d\right)$ and, hence,

$$\hat{d}_{k-i} \simeq \bar{d}_{k-j} + \bar{d}_{j-i}, \tag{2}$$

where $\bar{d}_{k-j} = \mathrm{E}\left\{d_{k-i}\right\}$ is the expected hop progress (EHP) and $\bar{d}_{j-i} = \mathrm{E}\left\{d_{j-i}\right\}$ is the mean last hop (MLH). One of the well-known analytical expressions of EHP is the one developed in [8] as follows:

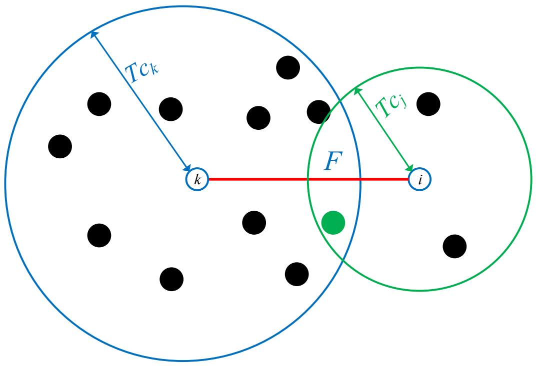
$$\bar{d}_{k-j} = \sqrt{3}\lambda \int_0^{Tc_k} x^2 e^{-\frac{1}{3}\lambda\pi\left(Tc_k^2 - x^2\right)} dx, \tag{3}$$

where λ is the node density, and x is the distance between the k-th and the i-th node. From (3), the EHP a priori depends only on the k-th node transmission capability Tc_k and, therefore, its computation does not supposedly require any knowledge of the j-th node transmission capability Tc_j. In what follows, and in contrast to (3), we will prove the EHP expression to be dependent on both Tc_k and Tc_j thereby revealing the expression derived in [8], as one example among too many others whose approaches are similar to the above but not discussed here for lack of space, to lack accuracy.

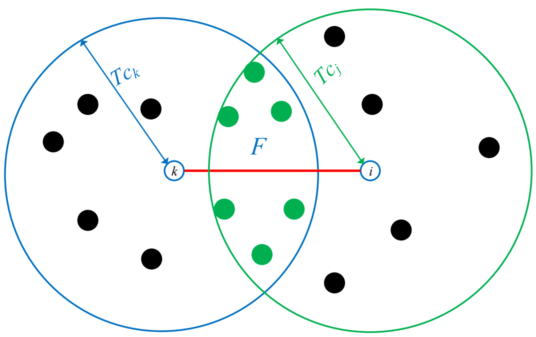
Let F be the potential forwarding area wherein the intermediate node j could be located. Since this node should, at the same time, be located in the k-th node coverage area and communicate directly with the i-th node using its transmission capability Tc_j, F is given by

$$F = D(k, Tc_k) \cap D(i, Tc_j). \tag{4}$$

It is noteworthy that the EHP is nothing but the mean of all distances between the k-th node and all the potential intermediate nodes located in F and, hence, the EHP strongly depends on F. As can be observed from Fig. 3, if the intermediate node transmission capability Tc_j increases, the potential forwarding area F increases to include potential intermediate nodes closer to the k-th anchor, thereby decreasing the EHP. Likewise, if Tc_j decreases, F decreases to exclude potential intermediate nodes closer to the k-th anchor and, hence, the EHP increases. Consequently, the EHP depends not only on Tc_k, but also on Tc_j. Let us now turn our attention to the MLH. It is obvious that the transmission capability of the i-th node does not have any effects on the last hop size d_{j-i}. Therefore, in contrast



(a) Small Tc_j



(b) Large Tc_j

Fig. 1. Effect of the intermediate sensor transmission capability.

with the EHP, the MLH depends only on the transmission capability of the transmitting node j. In the next section, novel approach is developed to accurately derive the expressions of both the MLH and the EHP. These results will be exploited in Section IV to propose a low-cost localization algorithm that complies with the heterogeneous nature of WSNs.

III. Analytical Evaluation of the MLH and EHP

In this section, expressions of both the MLH and the EHP are accurately derived. To this end, we consider the same scenario described in Section II. For the sake of clarity, in what follows, we denote by X, Y, and Z the random variables that represent d_{k-i}, d_{j-i}, and d_{k-j}, respectively.

A. MLH derivation

Since the i-th regular node could be located anywhere in $D(j, Tc_j)$ (the j-th node's coverage area) with the same probability, Y can be considered as a uniformly distributed random variable on $[0, Tc_j]$. Therefore, the MLH denoted hereafter by $h_{\text{last}}(Tc_j)$ is given by

$$h_{\text{last}}(Tc_j) = \int_0^{Tc_j} y f_Y(y) dy = \int_0^{Tc_j} \frac{y}{Tc_j} dy = \frac{Tc_j}{2}, \quad (5)$$

where $f_Y(y) = 1/Tc_j$ is the probability density function (pdf) of Y.

B. EHP derivation

In order to derive the EHP, one should first compute the conditional cumulative distribution function (CDF) $F_{Z|X}(z) = P(Z \leq z|x)$ of Z with respect to the random variable X. As can be shown from Fig. 2, $Z \leq z$ is guaranteed only if there are no nodes in the dashed area A. Therefore, the conditional CDF $F_{Z|X}(z)$ can be defined as

$$F_{Z|X}(z) = P(Z \leq z|x) = P(\mathrm{A}_0|F_1), \quad (6)$$

where $P(A_0|F_1)$ is the probability that the event $A_0 = \{$no nodes in the dashed area $A\}$ given $F_1 = \{$at least one node in the potential forwarding area $\}$ occurs. Since the nodes are uniformly deployed in S, the probability of having K nodes in F follows a Binomial distribution $\mathrm{Bin}(N, p)$ where $p = \frac{F}{S}$. For relatively large N and small p, it can be readily shown that $\mathrm{Bin}(N, p)$ can be accurately approximated by a Poisson distribution $\mathrm{Pois}(\lambda F)$. Using the Bayes' theorem, and for a large number of nodes N and small p, $F_{Z|X}(z)$ could be rewritten as

$$F_{Z|X}(z) = \frac{P(\mathrm{F}_1|A_0) P(\mathrm{A}_0)}{P(F_1)} = \frac{e^{-\lambda A}\left(1 - e^{-\lambda B}\right)}{\left(1 - e^{-\lambda F}\right)}, \quad (7)$$

where $B = F - A$. In the equation above, note that we use the fact that $P(\mathrm{F}_1|A_0)$ is the probability that at least one node is in B. As can be observed from (7), when $z = \alpha$, we have $B = 0$ and $A = F$ and, therefore, $F_{Z|X}(z) = 0$. This is expected since all potential intermediate nodes are located in the forwarding zone F where any node is at least at distance α from the k-th node (i.e., $P(Z \leq \alpha) = 0$). Furthermore, if $z = Tc_k$, it holds that $B = F$ and $A = 0$ and, hence, $P(Z \leq Tc_k) = 1$. This is also expected since all potential intermediate nodes are located in the k-th node's coverage area at distance Tc_k at most from the latter (i.e., $P(Z \leq Tc_k) = 1$). It should be noticed here that the properties above are not satisfied by any previously developed CDF expressions such as those in [8]-[9]. Exploiting some geometrical properties and

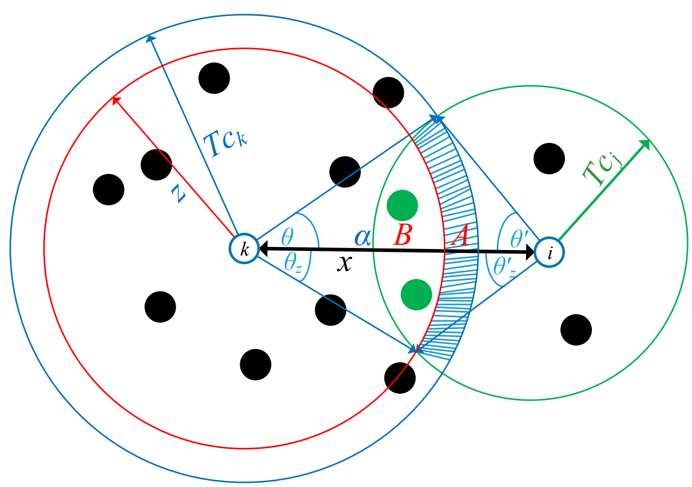
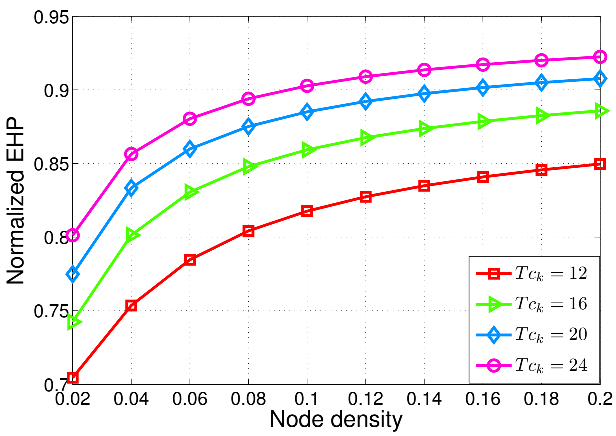


Fig. 2. EHP analysis.

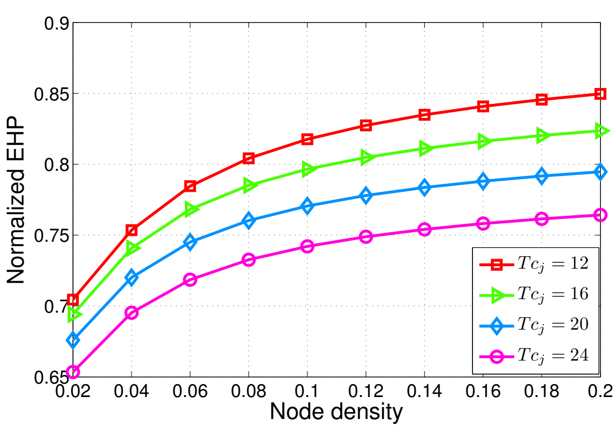
trigonometric transformations, one can easily obtain F and B. Using the latter, we derive the EHP $h(Tc_k, Tc_j)$ between the k-th and j-th nodes as

$$h(Tc_k, Tc_j) = \int_{Tc_k}^{Tc_k + Tc_j} \left(\alpha H_{Z|X}(\alpha) + \int_{\alpha}^{Tc_k} H_{Z|X}(z) dz\right) f_X(x)\, dx, \quad (8)$$

where $H(\star) = 1-F_{Z|X}(\star)$ and $\alpha = x - Tc_j$ and $f_X(x)$ is the pdf of X. Note that the latter can be considered as a uniform random variable over $[Tc_k, Tc_k + Tc_j]$ and, hence, $f_X(x)$ can be substituted there by $1/Tc_j$. To the best of our knowledge, a closed-form expression for the EHP in (8) does not exist. However, $h(Tc_k, Tc_j)$ can be easily implemented since it depends on finite integrals. As can be observed from (8), the proposed EHP depends on both Tc_k and Tc_j, in contrast to the previously proposed EHPs, such as in (3), which are only dependent on the sender node's transmission capability. It can be shown from Figs. 3(a) and 3(b) that the so-obtained



(a) Effect of Tc_k



(b) Effect of Tc_j

Fig. 3. Effect of the transmission capabilities on the EHP.

EHP decreases if Tc_j increases while it increases when Tc_k increases. This collaborates the discussion made above. These figures also show that the proposed EHP above increases with the node density. This is expected since it is very likely that the per-hop distance increases when the number of nodes located in F increases if, of course, both Tc_k and Tc_j are fixed.

IV. Proposed Localization for HWSNs

In this section, we propose a novel three-step localization algorithm for HWSNs.

A. Step 1: Initialization

In this step, the k-th anchor starts by broadcasting through the network a packet which consists of a header followed by a data payload. The packet header contains the anchor position (x_k, y_k), while the data payload contains $(Tc_k, \hat{d}_k)$, where Tc_k is the transmission capability of the k-th anchor and $\hat{d}_k$ is the estimated distance initialized to zero. If the packet is successfully received by a node, the latter estimates the EHP using the above approach, adds it to $\hat{d}_k$, stores the resulting value in its database and then, rebroadcasts the resulting packet after substituting Tc_k by its own transmission capability. Once this packet is received by another node, its database information is checked. If the k-th anchor information exists and the stored estimated distance is larger than that of the received one, the node updates the k-th anchor's information, then broadcasts the resulting packet after substituting the received transmission capability by its own. Otherwise, the node discards the received packet. However, when the node is oblivious to the k-th anchor position, it adds this information to its database and forwards the received packet after substituting the received transmission capability by its own. This mechanism will continue until each regular node in the network becomes aware of each anchor position as well as the distance from the latter to the last intermediate node before reaching that node. Note that the implementation of the proposed algorithm requires that each node broadcasts the anchor information not only with its estimated distance but also its transmission capability to allow the EHP computation at the next receiving node. In contrast, the implementation of existing algorithms in HWSNs requires the broadcast of the anchor information and the estimated distance only. Yet we will prove next in Section VI that the additional power cost that could be incurred a priori when broadcasting the transmission capabilities can be easily avoided by the proposed algorithm.

B. Step 2: Positions' computation

In this section, we will show how the so-received information can be exploited to get an initial guess of each regular node position. Using its available information, the $(i - N_a)$-th regular node (or the i-th node) computes an estimate of its distance to the k-th anchor as

$$\hat{d}_{k-i} = \hat{d}_k + h_{\text{last}}(Tc_{k+L}), \tag{9}$$

where $\hat{d}_k = \sum_{l=k}^{k+L-1} h\left(Tc_l, Tc_{l+1}\right)$, is the distance from the k-th anchor to the last intermediate node. In (9) and (IV-B), we assume for simplicity, yet without loss of generality, that L intermediate nodes exist over the shortest path between the k-th anchor and the $(i - N_a)$-th regular node and that the l-th intermediate node is the $(k + l)$-th node. Using its estimated distances to the N_a anchors as well as the latters' coordinates, the position of the i-th node could be deduced by solving the following nonlinear equations system: $(x_k - \hat{x}_i)^2 + (y_k - \hat{y}_i)^2 = \hat{d}^2_{k-i}$ for $k = 1, \ldots, N_a$. After some rearrangements that linearize the system above, we obtain

$$\boldsymbol{\Upsilon}\hat{\boldsymbol{\alpha}}_i = -\frac{1}{2}\boldsymbol{\kappa_i}, \tag{10}$$

where $\hat{\boldsymbol{\alpha}}_i = [\hat{x}_i, \hat{y}_i]^T$, $\boldsymbol{\Upsilon}$ is a $(Na-1)\times 2$ matrix with $[\boldsymbol{\Upsilon}]_{k1} = x_k - x_{N_a}$ and $[\boldsymbol{\Upsilon}]_{k2} = y_k - y_{N_a}$, and $\boldsymbol{\kappa}$ is a $(Na-1)\times 1$ vector with $[\boldsymbol{\kappa_i}]_k = d^2_{k-i} - d^2_{N_a-i} - x^2_k + x^2_{N_a} - y^2_k + y^2_{N_a}$. Since $\boldsymbol{\Upsilon}$ is a non-invertible matrix, $\hat{\boldsymbol{\alpha}}$ could be estimated with the pseudo-inverse of $\boldsymbol{\Upsilon}$ as $\hat{\boldsymbol{\alpha}}_i = -\frac{1}{2}\boldsymbol{\Upsilon}^T\left(\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^T\right)^{-1}\boldsymbol{\kappa}_i$. Therefore, the i-th regular node is able to obtain an initial guess of its coordinates as $\hat{x}_i = [\hat{\boldsymbol{\alpha}}_i]_1$, and $\hat{y}_i = [\hat{\boldsymbol{\alpha}}_i]_2$. It is also noteworthy from the definition of $\boldsymbol{\Upsilon}$ and $\boldsymbol{\kappa}$ that $\hat{x}_i$ and $\hat{y}_i$ are solely dependant on the anchors' coordinates $(x_k, y_k), k = 1, \ldots, N_a$ and the estimated distances $\hat{d}_{k-i}, , k = 1, \ldots, N_a$ which are all locally available at the $(N_a - i)$-th regular node. Unfortunately, errors are expected to occur when estimating the distance between each regular node-anchor pair, thereby hindering localization accuracy. As a third step of our proposed algorithm, we propose a correction mechanism aiming to reduce this error.

C. Step 3: Correction mechanism

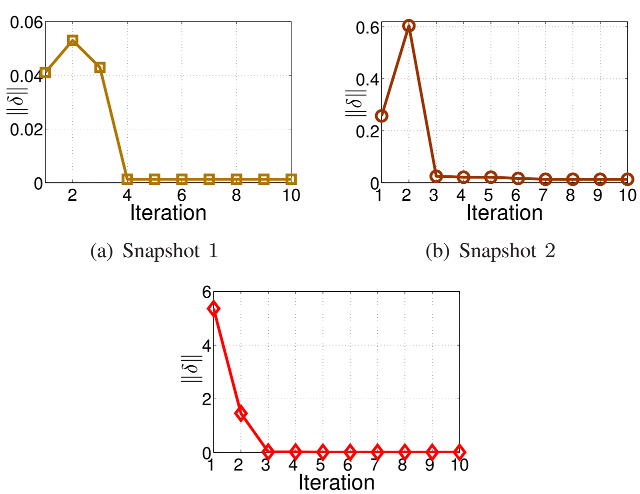
Let ϵ_{ki} denote the estimation error of the distance between the k-th anchor and the i-th regular node as

$$\epsilon_{ki} = \hat{d}_{k-i} - d_{k-i}, \tag{11}$$

where d_{k-i} is the true distance between the two nodes. As discussed above, this error hinders localization accuracy. As such, we have $x_i = \hat{x}_i + \delta_{x_i}$ and $y_i = \hat{y}_i + \delta_{y_i}$, where δ_{x_i} and δ_{y_i} are the location coordinates' errors to be determined. Retaining the first two terms of the Taylor series expansion of d_{k-i} and rewriting the result in a matrix form yields

$$\boldsymbol{\Gamma}_i\boldsymbol{\delta}_i = \boldsymbol{\zeta}_i - \boldsymbol{\epsilon}_i, \tag{12}$$

where $\boldsymbol{\Gamma}$ is a $N_a \times 2$ matrix with $[\boldsymbol{\Gamma}_i]_{k1} = \frac{\hat{x}_i - x_k}{\tilde{d}_{k-i}}$ and $[\boldsymbol{\Gamma}_i]_{k2} = \frac{\hat{y}_i - y_k}{\tilde{d}_{k-i}}$, $\boldsymbol{\epsilon}_i = [\epsilon_{1i}, \epsilon_{2i}, \ldots, \epsilon_{N_a i}]^T$, $\boldsymbol{\zeta}_i = \left(\hat{d}_{1-1} - \tilde{d}_{2-i}, \ldots \hat{d}_{N_a-i} - \tilde{d}_{N_a-i}\right)^T$, $\boldsymbol{\delta}_i = [\delta_{x_i}, \delta_{y_i}]^T$, and $\tilde{d}_{k-i} = \sqrt{(\hat{x}_i - x_k)^2 - (\hat{y}_i - y_k)^2}$. Many methods such as the weighted least squares (WLS) might be used to properly derive $\boldsymbol{\delta}_i$. Using WLS, the solution of (12) is given by :



(a) Snapshot 1 (b) Snapshot 2 (c) Snapshot 3

Fig. 4. Convergence of $\|\boldsymbol{\delta}\|$ vs. the number of iterations.

$$\boldsymbol{\delta}_i = \left(\boldsymbol{\Gamma}_i^T \mathbf{P}_i^{-1} \boldsymbol{\Gamma}_i\right)^{-1} \boldsymbol{\Gamma}_i^T \mathbf{P}_i^{-1} \boldsymbol{\zeta}_i, \tag{13}$$

where $\mathbf{P}_i$ is the covariance matrix of $\boldsymbol{\epsilon}_i$. Since ϵ_{ki} $k = 1, \ldots, N_a$ are independent random variables, $\mathbf{P}_i$ boils down to diag$\left\{\sigma^2_{1i}, \ldots, \sigma^2_{N_a i}\right\}$ where σ^2_{ki} is the variance of ϵ_{ki}. A straightforward inspection of (13) reveals that $\boldsymbol{\delta}_i$ depends on some locally available information as well as all $\sigma^2_{ki}, k = 1, \ldots, N_a$. Yet we will show in what follows that the derivation of $\sigma^2_{ki}, k = 1, \ldots, N_a$ requires a negligible additional power cost that could be easily avoided. Once we get $\boldsymbol{\delta}_i$, the value of $(\hat{x}_i, \hat{y}_i)$ is updated as $\hat{x}_i$=$\hat{x}_i + \delta_{x_i}$ and $\hat{y}_i = \hat{y}_i + \delta_{y_i}$. The computations are repeated until $\|\boldsymbol{\delta}_i\|$ approaches zero. In such a case, we have $x_i \simeq \hat{x}_i$ and $y_i \simeq \hat{y}_i$ and, hence, more accurate localization is performed. As can be observed from Fig. 4, the proposed correction mechanism converges after 5 iterations at most. Nevertheless, we will prove in Section VI that the proposed algorithm perfectly tailored for HWSNs, and it does not burden the overall cost of the WSN.

V. Variance Evaluation

This section aims to derive the expression of the variances σ^2_{ki}, $k = 1, \ldots, N_a$ which are required for the proposed algorithm's implementation. As such, two different methods, analytical and non-parametric, are proposed.

A. Analytical method

Assuming a high node density in the network, the distance d_{k-i} between two nodes can be rewritten as $d_{k-i} \simeq \sum_{l=k}^{k+L} d_{l,l+1}$, where L is the number of intermediate nodes over the shortest path and $d_{l,l+1}$ the distance between the l-th and $(l+1)$-th intermediate node. It follows from (9) and (V-A) that $\varepsilon_{ki} \simeq \sum_{l=k}^{k+L-1} e_l + e_{\text{last}}$, with $e_l = h\left(Tc_l, Tc_{l+1}\right) - d_{l-(l+1)}$ is the distance estimation error incurred during the $(l-k+1)$-th hop and $e_{\text{last}} = h_{\text{last}}(Tc_{k+L}) - d_{(k+L)-i}$ is the error incurred at the last hop. It can be readily shown that $\sigma^2_{ki} = \sum_{l=k}^{k+L-1} \sigma^2_l + {\sigma_{\text{last}}}^2$ where σ^2_l and ${\sigma_{\text{last}}}^2$ are the variances of e_l and e_{last}, respectively. Using the results developed in Section III, we obtain ${\sigma_{\text{last}}}^2 = \frac{Tc^2_{(k+L))}}{12}$, and

$$\begin{aligned}{\sigma_l}^2 = &\int_{Tc_l}^{Tc_l+Tc_{l+1}}\left(\alpha^2 H(\alpha) + 2\int_{\alpha}^{Tc_l} zH(z)dz\right)f_X(x)dx - \\ &\left(\int_{Tc_l}^{Tc_l+Tc_{l+1}}\left(\alpha H(\alpha) + \int_{\alpha}^{Tc_l} H(z)dz\right)f_X(x)dx\right)^2.\end{aligned} \tag{14}$$

Note that ${\sigma_l}^2$ could be obtained using any of the CDFs developed in Section III-B. As can be observed from the latter results, ${\sigma_{\text{last}}}^2$ is locally computable by the i-th node while ${\sigma_l}^2$ should be computed at the $(l+1)$-th intermediate node, added to the term $\sum_{m=k}^{l-1} \sigma^2_m$, then forwarded to the next intermediate node. This results in an additional few bits that must be transmitted by each node in the network. In what follows, we will prove that the additional power cost that could be incurred a priori when transmitting σ_l can be easily avoided by the proposed algorithm.

B. Non-parametric method

In the previous section, the analytical expression of σ_{ki}^2 was derived using the approximation in (V-A) which holds only for highly dense networks. However, if this assumption is not satisfied (i.e., lowly dense network), (V-A) would no longer be valid and, hence, σ_{ki}^2's expression would no longer be accurate enough. In such a case, to properly derive σ_{ki}^2, we propose to exploit the PDF of the distance estimation error ε_{ki} denoted by $f(\varepsilon)$. Unfortunately, to the best of our knowledge, there is no closed form solution for such a PDF. In this work, we propose to use a non-parametric technique to estimate it owing to some potential observations available at anchors. So far, many non-parametric techniques have been proposed in the literature such as the histogram and the well-known kernel density estimation (KDE) techniques. In this paper, we are only concerned by the latter which can estimate an arbitrary distribution without much observations. Such observations can in fact be easily obtained at the k-th anchor. Indeed, since this anchor is aware of all other anchor positions, it is able to derive the actual distances between it and the latters. Using (9), the k-th anchor could also obtain the estimated distances between it and the other anchors and, therefore, derive ε_{ki}. Hence, if N_a anchors exist in the network, the total number of available observations is $n_o = N_a(N_a - 1)$. Let $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{n_o}$ denote such observations. Using the KDE technique, $f(\varepsilon)$ can be then approximated by

$$\hat{f}(\varepsilon) = \frac{1}{p s_\varepsilon} \sum_{t=1}^{n_o} K\left(\frac{\varepsilon - \varepsilon_t}{s_\varepsilon}\right), \tag{15}$$

where s_ε is a smoothing parameter determined using the method in [13] and $K(\varepsilon)$ is the Gaussian kernel given by

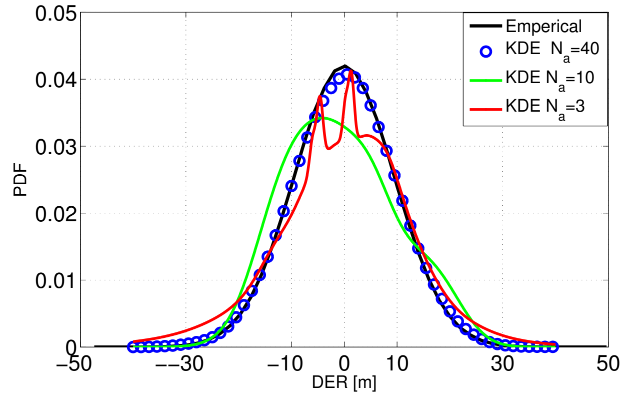
$$K(\varepsilon) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}\varepsilon^2). \tag{16}$$

As can be noticed from (15) and (16), the estimated PDF is computed by averaging the Gaussian density over all observations. Substituting (16) in (15) and using the resulting PDF to compute σ_{ki}^2 yields

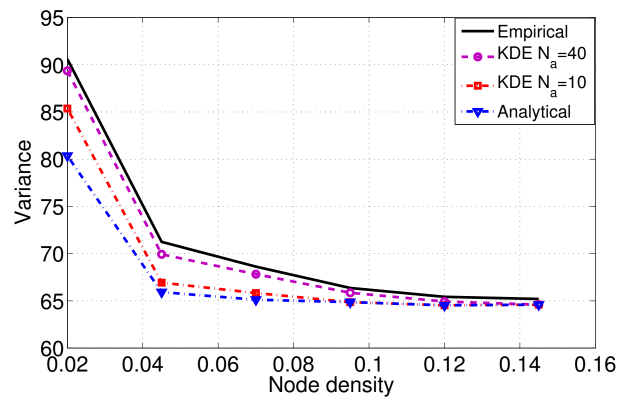
$$\sigma_{ki}^2 = \frac{\sum_{t=1}^{n_o} \left(X_t G_t - Y_t^2\right)}{\sum_{t=1}^{n_o} G_t^2}, \tag{17}$$

where $X_t = \left(s_\varepsilon^2 + \varepsilon_i^2\right) G_t - s_\varepsilon^2 \left((\varepsilon_t + 1)\, e^{-\frac{(1-\varepsilon_t)^2}{2s_\varepsilon^2}} + (\varepsilon_t - 1)\, e^{-\frac{(1+\varepsilon_t)^2}{2s_\varepsilon^2}} \right)$, $G_t = s_\varepsilon \sqrt{2\pi} \left(Q\left(\frac{\varepsilon_t - 1}{s_\varepsilon}\right) - Q\left(\frac{\varepsilon_t + 1}{s_\varepsilon}\right) \right)$, and $Y_t = \varepsilon_t G_t - s_\varepsilon^2 \left(e^{-\frac{(1-\varepsilon_t)^2}{2s_\varepsilon^2}} - e^{-\frac{(1+\varepsilon_t)^2}{2s_\varepsilon^2}} \right)$ with $Q(x)$ being the Q-function. Fig. 5(a) plots the empirical $f(\varepsilon)$ as well as $\hat{f}(\varepsilon)$ for different numbers of anchors. We see there that only a few anchors (i.e., few observations) are required to accurately estimate the localization errors' PDF. Furthermore, from Fig. 5(a), the estimated PDF approaches the empirical one, as N_a increases. This gives a sanity check for the proposed nonparametric method.

Nevertheless, in order to derive σ_{ki}^2 using this approach, each regular node needs to be aware of all observations. If this is not properly done, it will be very expensive in terms of power consumption, since each anchor would recur to a second broadcast to share its observations with the regular nodes. In order to circumvent this problem, we propose in what follows a power-efficient observation sharing protocol where anchors periodically broadcast their information. In fact, during the first time slot, only the first anchor should broadcast its own information while the $(N_a - 1)$ other anchors only execute the tasks described in Section IV-A. At the second time slot, the second anchor derives an estimation error observation using the information received from the first anchor, adds it to its packet and broadcasts the resulting packet in the network. Upon reception of this information, the rest of anchors derive and store a second observation. Two observations are then available at the third anchor which also broadcasts them in the network. This process will continue until each regular node becomes aware of a sufficient number of observations. Note that if N_a is large enough so that $(N_a - 1)$ observations are sufficient to accurately derive the PDF, only two time slots are required. Indeed, after the first time slot, $(N_a - 1)$ observations are available and can be simultaneously broadcasted by the $(N_a - 1)$ anchors in the network. In the next section, we will prove that each anchor could transmit few observations without incurring any power cost.



(a) DER's PDF using KDE.



(b) Global DER's variance.

Fig. 5. Distance estimation error (DER).

Fig. 5(b) plots the error variance for different node densities.

It shows, as expected, that the variance decreases when the node density increases. Beyond a node density threshold of less than 0.1, both the analytical and the non-parametric methods start to yield about the same variance as the empirical one. Furthermore, when N_a increases, more so at large enough values, the efficiency of the non-parametric method increases even at low node densities. Note that increasing the number of anchors N_a does not only result in a more accurate variance, but also in a more reliable localization [7].

VI. Proposed algorithm's implementation cost

As discussed in Section IV, the Proposed algorithm's implementation requires the $(i-N_a)$-th regular node to be able to compute its coordinates' initial guess $(\hat{x}_i, \hat{y}_i)$ as well as $\boldsymbol{\delta}_i$ which is used at the position correction step. As discussed above, since these quantities depend solely on the information locally available at the $(i-N_a)$-th regular node, their computation does not require any additional overhead or power cost. Furthermore, this node must perform matrix-inversion operations to the matrices $\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^T$ and $\boldsymbol{\Gamma}_i^T\mathbf{P}_i^{-1}\boldsymbol{\Gamma}_i$ in order to derive $(\hat{x}_i, \hat{y}_i)$ and $\boldsymbol{\delta}_i$, respectively. This kind of operations which is often highly computationally demanding may significatively increase the overall cost of the WSN. Nevertheless, since these matrices are 2-by-2 matrices, the entries of their inverses can be analytically and easily derived using the locally available information at the $(i-N_a)$-th regular. This proves that the computation of $(\hat{x}_i, \hat{y}_i)$ and $\boldsymbol{\delta}_i$ does not burden neither the implementation complexity of the proposed algorithm nor the overall cost of the WSN. Moreover, some iterations should be repeated, at most 5 times as shown in Fig. 4, to ensure the convergence of the proposed correction mechanism. Knowing that the required power to execute one instruction is in the range of 10^{-4} of the power consumed per transmitted bit, the power needed to execute this mechanism is then very negligible with respect to the overall power consumed by each node. On the other hand, as discussed in Sections IV and V, the proposed algorithm's implementation requires that each node transmits, upon reception a message from an anchor, its transmission capability and variance besides to the latter's coordinates and the distance between the two nodes. This results in additional few bits, with respect to the existent algorithms, thereby causing an additional power cost. We will shortly see below that this cost could be easily avoided.

Let p_i be the available power at the i-th node, b_i be the length in bits of the original packet sent when the existing algorithms are implemented (i.e., packet includes only the anchor's coordinates and its distance to the i-th node), and a_i be the cost in bits if Tc_i and ${\sigma_i}^2$ are added to the packet. If the power p_i allows the i-th node to transmit b_i bits over a Tc_i coverage distance, this power will also allow the latter node to transmit $b_i + a_i$ bits but over a coverage distance $\tilde{T}c_i < Tc_i$, where $\tilde{T}c_l$ is the new transmission capability of the i-th node. Since no matter are the transmission capabilities of the i-th node and the previous intermediate node, this node is always able to compute the EHP, the fact that Tc_i decreases to $\tilde{T}c_i$ does not affect the performance of the proposed localization algorithm. Therefore, the additional bits a_i could be broadcasted without any additional power cost. All the above discussion proves that the proposed localization algorithm can be implemented at a low cost. Furthermore, since it complies with the heterogeneous nature of WSNs and, further, is power efficient, it could easily find application in EH-WSNs where the power is considered as a scarce resource.

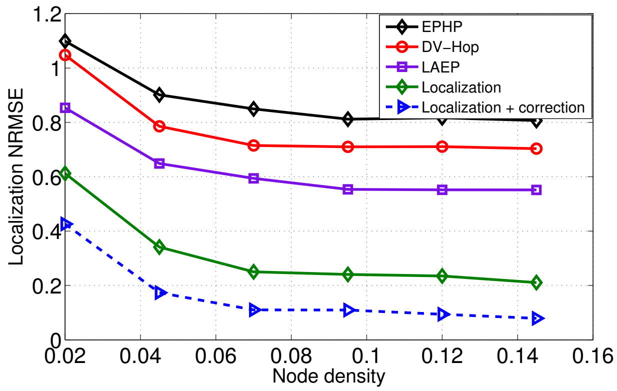
VII. Simulations results

In this section, we evaluate the performance of the proposed algorithm in terms of localization accuracy by simulations using Matlab. These simulations are conducted to compare, under the same network settings, the proposed algorithm with some of the best representative localization algorithms currently available in the literature, i.e., DV-Hop [7], LAEP [8] and EPHP [9]. All simulation results are obtained by averaging over 100 trials. In the simulations, nodes are uniformly deployed in a 2-D square area $S = 100 \times 100\, m^2$. We assume that $Tc_i \neq Tc_j$ if $i \neq j$ and that all transmission capabilities are set between 5 and 30 meters. We also assume that the number of anchors N_a is set to 20. As a performance metric, we propose to adopt the normalized root mean square error (NRMSE) which is defined as

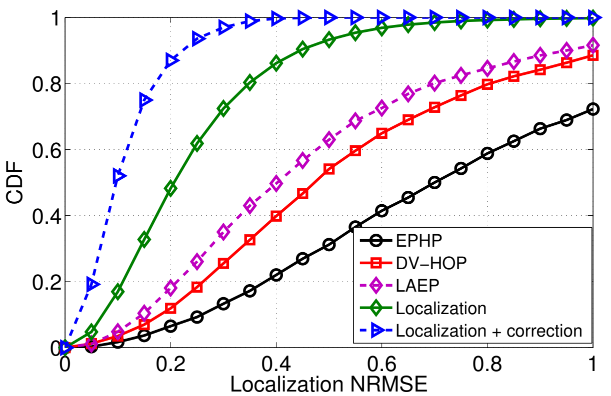
$$\text{NRMSE} = \frac{1}{N_u} \sum_{i=1}^{N_u} \frac{\sqrt{(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2}}{Tc_i}. \qquad (18)$$

Fig. 6(a) plots the localization NRMSE achieved by DV-Hop, EPHP, LAEP and the proposed algorithm for different node densities λ in HWSNs. From this figure, the proposed algorithm, with or without localization correction, always outperforms its counterparts. Indeed, our proposed algorithm turns out to be until about two, three and four times more accurate than LAEP, DV-Hop, and EPHP, respectively. Furthermore, as can be observed from Fig. 6(a), the NRMSE achieved by the proposed algorithm significantly decreases when the node density λ increases while those achieved by its counterparts slightly decreases then quickly saturate. This is expected since two conflicting phenomena arise when λ grows large. The first is that the approximation in (V-A) becomes more realistic and, hence, more accurate localization is performed. The second is the increase of the number of different transmission capabilities due to the heterogeneous nature of WSNs when the node density increases. Since more different are the transmission capabilities in the network, worse is the accuracy of the former algorithms. This explains why their performance quickly saturates when the node density increases. The proposed algorithm's accuracy, in contrast, increases with λ since it takes into account the difference between the transmission capabilities that is typical of HWSNs. This further proves the efficiency and suitability of the proposed localization algorithm to HWSNs.

Fig. 6(b) illustrates the localization NRMSE's CDF. Using the proposed algorithm, 90% of the regular nodes could estimate their position within almost the fifth of their transmission capabilities. In contrast, 20% of the nodes achieve the same accuracy with LAEP, about 14% with DV-Hop, and only 9%



(a) NRMSE vs. node density.



(b) NRMSE's CDF.

Fig. 6. Localization NRMSE.

with EPHP. This further proves the efficiency of the proposed localization algorithm.

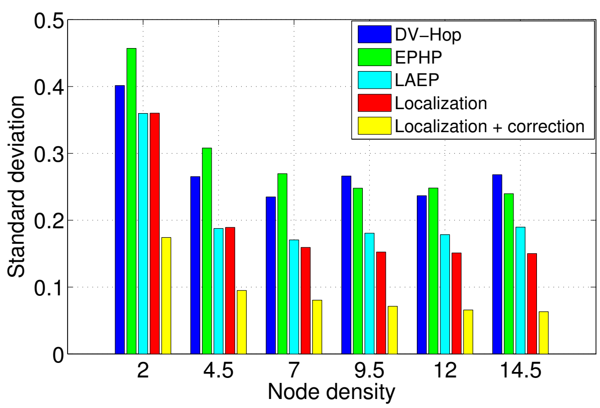


Fig. 7. Standard deviation vs. node density.

Fig. 7 plots the NRMSE's standard deviation achieved by all localization algorithms. As can be observed from this figure, the one achieved by the proposed algorithm substantially decreases when the node density increases while those achieved by the other algorithms slightly decrease. This is due once again to the fact that the proposed algorithm complies with the heterogeneous nature of the WSNs when the former algorithms do not. Furthermore, the NRMSE standard deviation achieved by the proposed algorithm approaches zero. This means that implementing our algorithm in HWSNs guarantees a very accurate localization for any given realization. This result is very interesting in terms of implementation strategy, since it proves that the result in Fig. 6(a) becomes more and more meaningful as λ grows large.

VIII. Conclusion

In this paper, a novel low-cost localization algorithm which accounts for the heterogeneous nature of WSNs was proposed. A novel approach is developed to accurately derive the EHP. Using the latter, the proposed algorithm is able to accurately locate the sensor nodes owing to a new low-cost implementation that avoids any additional power consumption. Furthermore, a correction mechanism which complies with the heterogeneous nature of WSNs was developed to further improve localization accuracy without incurring any additional costs. The proposed algorithm, whether applied with or without correction, is shown to outperform in accuracy the most representative WSN localization algorithms.

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