MIMO Relaying with Interference via the CMGF Transform

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***Abstract*—In this paper, we analyze the ergodic capacity of a two-hop multiple-antenna amplify and forward (AF) system, where the relay is subject to co-channel interference (CCI) while the destination is corrupted by additive white Gaussian noise (AWGN) only. A novel integral transform, called the complementary moment generating function transform (CMGF), is proposed as a unified tool to compute the ergodic capacity. When both the relay and destination perform maximum ratio combining (MRC), we derive a new analytical exact expression for the ergodic capacity. It is shown that the ergodic capacity is better improved by increasing the number of antennas at the relay N_r than that at destination N_d. Unfortunately, the system shows an incapability of canceling interference even if N_r and/or N_d grows large.**

***Index Terms*—Two-hop relaying, Co-channel interference, Ergodic capacity, Multiple antennas, Complementary moment generating function transform (CMGF).**

I. Introduction

The deployment of wireless relays has rekindled a wide interest from the wireless communication community as a means of achieving high throughput where traditional architectures are unsatisfactory, such as in cell-edge, indoor, etc.. Several relaying protocols have been introduced in the literature [1]. Of particular interest is the amplify-and-forward (AF) scheme due to its low complexity. In such a scheme, in fact, each relay mimics a simple repeater by forwarding a scaled version of the received signal to the destination node.

Nevertheless, deployed relays in future wireless systems generations will, inevitably, face a complex co-channel interference environment due to the highly aggressive frequency reuse. The latter actually causes a more severe performance degradation than thermal noise [2].

Aiming to understand the performance limitations of relaying systems in the presence of interference, significant contributions investigating the ergodic capacity in various practical scenarios have appeared. As far as the analysis of single antenna systems is concerned, some insightful results can be found in [3]-[4]. These studies have shed new insights into how the ergodic capacity is dominated by the interference power, especially at the relay. Recently, most research activity has been devoted to the analysis of multiple antenna (MIMO) systems, which have been shown to provide significant improvements to the achievable data rates. Some relevant contributions on the analysis of channel capacity for these systems are [5]-[6], where analytical bounds for the channel capacity over Rayleigh fading channels with various diversity-combining techniques are obtained.

This paper is a nontrivial and useful add-on of the framework proposed by [6], with the main objective and motivation of taking advantage of an MGF-based approach to obtain new closed-form expressions for the ergodic capacity of two-hop MIMO AF systems, an objective deemed impossible to acheive by the authors of [6].

II. System Model

Let us consider the two-hop MIMO network in Fig. 1, where both the relay R and destination D are equipped with N_r and N_d antennas, respectively, while the source S is equipped with a single antenna. We assume that the relay is subjected to M independently but not necessarily identically distributed co-channel interferers that dominate the noise effect, while the destination is corrupted by AWGN only. Interference-limited relay and noisy destination stems from cell-edge or frequency-division relaying [4], [7].

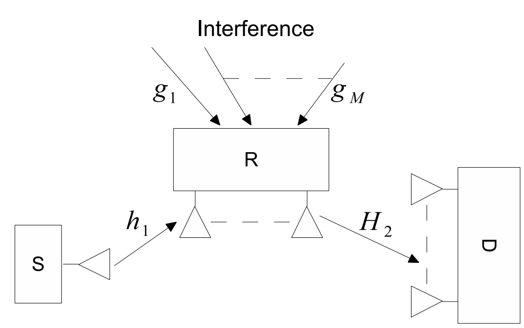

Fig. 1. System model.

Let the $N_r \times 1$ vectors $\{\mathbf{h}_1, \mathbf{g}_i\}$, $i = 1, \ldots, M$ denote the channels for the source-relay and i-th interference-relay links, respectively, with entries following identically independently distributed (i.i.d) complex circular Gaussian random variables $\mathcal{CN}(0,1)$. Let also x and s_{I_i} denote the source and the i-th interferer symbol satisfying $\mathrm{E}\{xx^*\} = P_s$ and $\mathrm{E}\{s_{I_i} s_{I_i}^*\} = P_{I_i}$, $i = 1, \ldots, M$. Then the received signal at

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Bold lower case letters denote vectors and lower case letters denote scalars. $\mathrm{E}\{x\}$ stands for the expectation of the random variable x, $*$ denotes the conjugate operator and, $\dagger$ denotes the conjugate transpose operator.

the interference-limited relay is given by

$$y_r = \mathbf{w}^\dagger \left[\mathbf{h}_1 x + \sum_{i=1}^{M} \mathbf{g}_i s_{I_i} \right], \quad (1)$$

where $\mathbf{w}$ is set to match the first hop, i.e., $\mathbf{w} = \mathbf{h}_1/\|\mathbf{h}_1\|$, also known as the MRC combiner. The relay node transmits a transformed version of the received signal to the destination such that

$$y_d = \mathbf{u}^\dagger \left[G\mathbf{H}_2 \mathbf{v} y_r + \mathbf{n} \right], \quad (2)$$

where $\mathbf{H}_2 = [h_{2_{i,j}}]_{i,j=1}^{N_r,N_d}$ is a $N_r \times N_d$ matrix and denotes the channel for the relay-destination link with entries following i.i.d $\mathcal{CN}(0,1)$, $\mathbf{n}$ is the $N_d \times 1$ AWGN vector at the destination node with $\mathrm{E}\{\mathbf{n}\mathbf{n}^*\} = N_0 \mathbf{I}$, where $\mathbf{I}$ is the identity matrix, $\mathbf{u}$ and $\mathbf{v}$ are the transmit precoding and receive filtering vectors at R and D, selected by using the channel matrix $\mathbf{H}_2$ as the first columns of $\mathbf{U}$ and $\mathbf{V}$, respectively, corresponding to the largest singular value of $\mathbf{H}_2$. Combining (1) and (2), the end-to-end signal-to-interference-plus-noise ratio (SINR) of the system can be expressed as

$$\gamma = \frac{P_s \Lambda G^2 |\mathbf{w}^\dagger \mathbf{h}_1|^2}{G^2 \Lambda \sum_{i=1}^{M} |\mathbf{w}^\dagger \mathbf{g}_i|^2 P_{I_i} + N_0}, \quad (3)$$

where Λ is the largest eigenvalue of the Wishart matrix $\mathbf{H}_2^\dagger \mathbf{H}_2$ and G is the power constraint factor given by

$$G^2 = \frac{P_r}{P_s \mathbf{h}_1^\dagger \mathbf{h}_1 + \sum_{i=1}^{M} |\mathbf{w}^\dagger \mathbf{g}_i|^2 P_{I_i}}. \quad (4)$$

By substituting (4) into (3), we obtain

$$\gamma = \frac{\Lambda |\mathbf{h}_1|^2 P_s}{\Lambda \frac{\sum_{i=1}^{M} |\mathbf{h_1}^\dagger \mathbf{g}_i|^2 P_{I_i}}{\|\mathbf{h_1}\|^2} + \frac{N_0}{P_r} \left(|\mathbf{h}_1|^2 P_s + \frac{\sum_{i=1}^{M} |\mathbf{h_1}^\dagger \mathbf{g}_i|^2 P_{I_i}}{\|\mathbf{h_1}\|^2} \right)}. \quad (5)$$

Finally, after noting $\rho_1 = P_s/N_0$, $\rho_2 = P_r/N_0$, and $\rho_{I_i} = P_{I_i}/N_0, i = 1, \ldots, M$, a more compact form of (5) is obtained, after some manipulations, as

$$\gamma = \frac{\gamma_1 \gamma_2}{\gamma_2 + \gamma_1 + 1}, \quad (6)$$

where $\gamma_1 = \frac{|\mathbf{h}_1|^2 \rho_1}{\chi}$, $\chi = \frac{\sum_{i=1}^{M} |\mathbf{h_1}^\dagger \mathbf{g}_i|^2 \rho_{I_i}}{\|\mathbf{h_1}\|^2}$, and $\gamma_2 = \Lambda \rho_2$.

III. ERGODIC CAPACITY ANALYSIS

The ergodic capacity is defined as the expected values of the instantaneous mutual information and is mathematically expressed as

$$C = \frac{1}{2} \mathrm{E} \left[\log_2 (1 + \gamma) \right], \quad (7)$$

in which γ stands for the end-to-end SINR and the factor $1/2$ accounts for the total number of time slots required for the transmission.

The singular value decomposition of $\mathbf{H}_2$ is given by $\mathbf{H}_2 = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^\dagger$, where $\mathbf{\Sigma}$ is the $N_d \times N_r$ matrix having the largest singular value $\sqrt{\Lambda}$ as the first element on the main diagonal. Further, $\mathbf{U}$ and $\mathbf{V}$, are unitary $N_d \times N_d$ and $N_r \times N_r$ matrices, respectively.

A. Novel MGF-Based Approach for Two-Hop Channel Capacity Computation

In this section, we propose a new integral transform for channel capacity computation by relying on the knowledge of the first hop complementary CDF (CCDF) and the second hop MGF.

Theorem 1: The ergodic capacity of two-hop AF relaying system can be computed as

$$C = \frac{1}{2\ln(2)} \left(\int_0^\infty e^{-s} \widehat{M}_{\gamma_1}(s) ds - \int_0^\infty e^{-s} \widehat{M}_{\gamma_1}(s) M_{\gamma_2}(s) ds \right) = \widehat{C}_1 - \widehat{C}_{12}, \quad (8)$$

where $M_X(\cdot)$ stands for the MGF of X and $\widehat{M}_X(\cdot)$ denotes the complementary MGF (CMGF) defined as

$$\widehat{M}_X(s) \triangleq \int_0^\infty e^{-sx} \widehat{F}_X(x) dx, \quad (9)$$

with $\widehat{F}_X(x)$ denoting the CCDF of X. In this paper, the integral in (8) is called CMGF transform, as it relies on a CMGF kernel function.

Proof: Combining (6) and (7), the ergodic capacity of the system can be computed by

$$C = \frac{1}{2} \mathrm{E} \left[\log_2 \left(\frac{(1+\gamma_1)(1+\gamma_2)}{1+\gamma_1+\gamma_2} \right) \right] = C_{\gamma_1} + C_{\gamma_2} - C_{\gamma_T}, \quad (10)$$

where $C_{\gamma_i} = \frac{1}{2\ln(2)} \mathrm{E} \Big[\ln(1+\gamma_i) \Big]_{i=1,2}$ and $C_{\gamma_T} = \frac{1}{2\ln(2)} \mathrm{E} \Big[\ln(1+\gamma_1+\gamma_2) \Big]$. Noticing that the quantities $C_{\gamma_X}, X \in \{1, 2, T\}$ can be expressed by means of the MGF-based approach in [8] as

$$C_{\gamma_X} = \frac{1}{2\ln(2)} \int_0^\infty \frac{e^{-s}}{s} \left(1 - M_{\gamma_X}(s)\right) ds, \quad (11)$$

and resorting to the key transformation

$$M_{\gamma_X}(s) = 1 - s \int_0^\infty e^{-sx} \widehat{F}_{\gamma_X}(x) dx, \quad (12)$$

then, pulling all together in (10), the ergodic capacity can be expressed as

$$C = \frac{1}{2\ln(2)} \left(\int_0^\infty e^{-s} \frac{1 - \left(1 - s\widehat{M}_{\gamma_1}(s)\right)}{s} ds + \int_0^\infty e^{-s} \frac{1 - M_{\gamma_2}(s)}{s} ds - \int_0^\infty e^{-s} \frac{1 - \left(1 - s\widehat{M}_{\gamma_1}(s)\right) M_{\gamma_2}(s)}{s} ds \right), \quad (13)$$

where $\widehat{M}$ is defined in (9). To this end, simplifying (13) yields the desired result.

We remark that the result in (8) offers a flexible and simple approach for the computation of the ergodic capacity that relies on the knowledge of the first-hop SIR CMGF and the second-hop SNR MGF. To the best of our knowledge, closed-form and exact expressions for these quantities do exist for most fading models. Moreover, in those scenarios where very complicated expressions of the CMGF/MGF of the per-hop SIR/SNR do not allow easy computation of the aforementioned integral in closed form, the result in (8) can efficiently and easily be obtained using standard computing environments, such as Mathematica. In fact, in contrast to [8], it is worth noting that the singularity of the $\frac{e^{-s}}{s}$ kernel function around zero is avoided by the integral simplification performed in (13) and that the evaluation of (8) does not face, in general, numerical problems.

B. Ergodic Capacity of Two-Hop MIMO AF Systems with Interference in Rayleigh Fading

Hereafter, we will restrict the scope of (8) to the yet challenging scenario described in section II.

Corollary 1: The ergodic capacity of two-hop MIMO AF systems with interference is obtained as

$$C = \frac{1}{2\ln(2)} \sum_{k=0}^{N_r-1} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \zeta_{i,j}(\mathbf{D}) \left(\frac{\Psi_{k,j}}{k+j} - \frac{1}{\Gamma(j)\Gamma(k+1)} \sum_{a=1}^{P} \sum_{b=Q}^{(P+Q)a-2a^2} \frac{\beta(a,b)}{\Gamma(b+1)} \Sigma_{k,j} \right), \quad (14)$$

where

$$\Psi_{k,j} = \begin{cases} {}_2\mathrm{F}_1\left(1, j, k+j+1; 1-\frac{\rho_{I_{<i>}}}{\rho_1}\right), & |1-\frac{\rho_{I_{<i>}}}{\rho_1}| < 1; \\ {}_2\mathrm{F}_1\left(k+1, 1, k+j+1; 1-\frac{\rho_1}{\rho_{I_{<i>}}}\right), & \frac{\rho_{I_{<i>}}}{\rho_1} > \frac{1}{2}, \end{cases} \quad (15)$$

and

$$\Sigma_{k,j} = \mathrm{G}^{1,1,1,1,2}_{1,[1,1],0,[1,2]} \left(\frac{\rho_2}{a}, \frac{\rho_1}{\rho_{I_{<i>}}} \,\middle|\, \begin{matrix} 0; 1+b; 1+k \\ \ldots; 0; 0, j-1 \end{matrix} \right), \quad (16)$$

wherein ${}_2\mathrm{F}_1(\cdot)$ and $\mathrm{G}^{p,q,k,r,l}_{A,[C,E],B,[D,F]}(\cdot,\cdot)$ denote the Gauss hypergeometric function [9, Eq(9.100)] and the generalized Meijer-G function [10], respectively. Moreover in (14), $\mathbf{D} = \mathrm{diag}(\rho_{I_1}, \rho_{I_2}, \ldots, \rho_{I_M})$, $\rho(\mathbf{D})$ is the number of distinct diagonal elements of $\mathbf{D}$, $\rho_{I_{<1>}} > \rho_{I_{<2>}} > \ldots > \rho_{I_{<M>}}$ are the distinct diagonal elements in decreasing order, $\tau_{\mathbf{i}}(\mathbf{D})$ is the multiplicity of $\rho_{I_{<i>}}$ and $\zeta_{i,j}(\mathbf{D})$ is the (i,j)-th characteristic coefficient of $\mathbf{D}$ [11]. For instance, when non-equal-power interferers are considered, we have $\tau_i(\mathbf{D}) = 1$ and $\zeta_{i,1}(\mathbf{D}) = \prod_{k=1,k\neq i}^{\rho(\mathbf{D})} 1/\left(1-\frac{\rho_{I_{<k>}}}{\rho_{I_{<i>}}}\right)$. In (14), we also note that the coefficients $\beta(a,b)$ are given by

$$\beta(a,b) = \frac{c_{a,b}\, b!}{a^{b+1}\prod_{l=1}^{P}(P-l)!(Q-l)!}, \quad (17)$$

where $P = \min(N_r, N_d)$ and $Q = \max(N_r, N_d)$.

Proof: According to the CMGF transform and the SINR expression in (6), the ergodic capacity of two-hop AF systems with multiple antennas at the relay-destination pair, interference at the relay and noise at the destination is obtained by the calculation of the two items $\widehat{C}_1$ and $\widehat{C}_{12}$.

$c_{a,b}$ is the coefficient of $e^{-ax}x^b$ in the expansion of $\frac{d}{dx}\det[S(x)]$, where S(x) is an $P \times P$ Hankel matrix with elements $\mathrm{S}_{i,j} = \gamma(Q-P+i+j-1, x)$, with $\gamma(\cdot)$ denoting the incomplete Gamma function [9, Eq (6.5.3)]. The coefficients $c_{a,b}$ can be readily determined using mathematical softwares such as Maple or Mathematica.

1) Calculation of $\widehat{C}_1$: In order to proceed, we need to find out the statistics of $\gamma_1 = \frac{|\mathbf{h}_1|^2\rho_1}{\chi}$ where $\chi = \frac{\sum_{i=1}^{M}|\mathbf{h_1}^{\dagger}\mathbf{g}_i|^2\rho_{I_i}}{\|\mathbf{h_1}\|^2}$. It is easy to observe that $|\mathbf{h}_1|^2$ is an exponential random variable with pdf

$$f_{|\mathbf{h}_1|^2}(x) = \frac{x^{N_r-1}}{(N_r-1)!}e^{-x}. \quad (18)$$

Also, according to [11], χ follows an hyper-exponential distribution with pdf

$$f_{\chi}(x) = \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \zeta_{i,j}(\mathbf{D}) \frac{\rho_{I_{<i>}}^{-j} x^{j-1}}{(j-1)!} e^{-x/\rho_{I_{<i>}}}. \quad (19)$$

Then invoking [9, Eq. (3.381.8)] and [9, Eq. (3.381.4)], the CCDF of γ_1 is obtained after some manipulations as

$$\widehat{F}_{\gamma_1}(x) = \sum_{k=0}^{N_r-1} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \zeta_{i,j}(\mathbf{D}) \frac{\Gamma(k+j)}{\Gamma(j)k!} \left(\frac{x\rho_{I_{<i>}}}{\rho_1}\right)^k \left(\frac{\rho_1}{\rho_{I_{<i>}}x+\rho_1}\right)^{k+j}. \quad (20)$$

The next step is to calculate $\widehat{M}_{\gamma_1}$. From (9), and using (20), along with some basic algebraic manipulations, we arrive at

$$\widehat{M}_{\gamma_1}(s) = \sum_{k=0}^{N_r-1} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \zeta_{i,j}(\mathbf{D}) \frac{\Gamma(k+j)}{\Gamma(j)} \frac{\rho_1}{\rho_{I_{<i>}}} \Psi\left(k+1, 2-j; \frac{\rho_1}{\rho_{I_{<i>}}}s\right), \quad (21)$$

where $\Psi(a;b;z)$ denotes the Triconomi confluent hypergeometric function [9, Eq. (9.211.1)]. Then, substituting (21) into $\widehat{C}_1$, the latter can be evaluated as

$$\widehat{C}_1 = \frac{1}{2\ln(2)} \sum_{k=0}^{N_r-1} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \zeta_{i,j}(\mathbf{D}) \frac{\Gamma(k+j)}{\Gamma(j)} \underbrace{\int_0^{\infty} e^{-\frac{\rho_{I_{<i>}}}{\rho_1}s}\Psi(k+1, 2-j, s)\, ds}_{I}. \quad (22)$$

Utilizing, [9, Eq. (7.621.6)], we obtain

$$I = \frac{\Gamma(j)}{\Gamma(k+j+1)}\Psi_{k,j}, \quad (23)$$

wherein $\Psi_{k,j}$ is given in (15). Finally, substituting (23) into (22) yields the desired result.

2) Calculation of $\widehat{C}_{12}$: Since $\widehat{M}_{\gamma_1}(s)$ is derived in (21), the remaining task is to figure out $M_{\gamma_2}(s)$. In order to derive the latter, one needs the closed-form statistics of Λ_2, the largest eigenvalues of the central Wishart matrix $\mathbf{H_2^{\dagger}H_2}$. According to [12], f_{γ_2} can be expressed as

$$f_{\gamma_2}(x) = \sum_{a=1}^{P} \sum_{b=Q}^{(P+Q)a-2a^2} \beta(a,b) \frac{a^{b+1}x^b}{\rho_2^{b+1}b!} e^{-\frac{ax}{\rho_2}}, \quad (24)$$

where $\beta(a,b)$ is defined in (17). Then, the MGF of γ_2 is obtained as

$$M_{\gamma_2}(s) = \sum_{a=1}^{P} \sum_{b=Q}^{(P+Q)a-2a^2} \frac{\beta(a,b)}{\left(\frac{\rho_2}{a}s+1\right)^{b+1}}. \quad (25)$$

To this end, substituting (21) and (25) into $\widehat{C}_{12}$, the latter can be evaluated as

$$\widehat{C}_{12} = \frac{1}{2\ln(2)} \sum_{k=0}^{N_r-1} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \zeta_{i,j}(\mathbf{D}) \frac{\Gamma(k+j)}{\Gamma(j)} \sum_{a=1}^{P} \sum_{b=Q}^{(P+Q)a-2a^2} \beta(a,b) \frac{\rho_1}{\rho_{I_{<i>}}} \underbrace{\int_0^\infty e^{-s} \frac{\Psi\left(k+1, 2-j, \frac{\rho_1}{\rho_{I_{<i>}}} s\right)}{\left(\frac{\rho_2}{a}s+1\right)^{b+1}} ds}_{\Phi}. \quad (26)$$

We now have to solve the integral Φ in order to derive $\widehat{C}_{12}$. To this end, noticing that $(1+\beta x)^{-\alpha} = \frac{1}{\Gamma(\alpha)} \mathrm{G}_{1,1}^{1,1}\left(\beta x \,\middle|\, \begin{matrix} 1-\alpha \\ 0 \end{matrix}\right)$ [9, Eq. (9.34.3)] and, $\Psi(a,b;z) = \frac{1}{\Gamma(a)\Gamma(a-b+1)} \mathrm{G}_{1,2}^{2,1}\left(z \,\middle|\, \begin{matrix} 1-a \\ 0, 1-b \end{matrix}\right)$ [9, Eq. (9.34.3)], the integral Φ is now given by

$$\Phi = A \int_0^\infty e^{-s} \mathrm{G}_{1,1}^{1,1}\left(\frac{\rho_2}{a} s \,\middle|\, \begin{matrix} -b \\ 0 \end{matrix}\right) \mathrm{G}_{1,2}^{2,1}\left(\frac{\rho_1}{\rho_{I_{<i>}}} s \,\middle|\, \begin{matrix} -k \\ 0, j-1 \end{matrix}\right) ds \quad (27)$$

where $A = 1/\Gamma(b+1)\Gamma(k+1)\Gamma(k+j)$. Integrals of this type can be evaluated by means of a G-function of two variables [10, Eq. (1.2)], as can be seen from a more general integral formula due to [10, Eq. (3.2)]. In what follows, let

$$G_{A,[C,E],B,[D,F]}^{p,q,k,r,l}\left(z_1, z_2 \,\middle|\, \begin{matrix} \alpha_1, ..., \alpha_A; \gamma_1, ..., \gamma_C; \epsilon_1, ..., \epsilon_E \\ \beta_1, ..., \beta_B; \delta_1, ..., \delta_D; \phi_1, ..., \phi_F \end{matrix}\right), \quad (28)$$

denote the generalization of Meijer's G-function to a function of two variables z_1, z_2. Then according to [10, Eq. (3.2)] and under consideration of the functional relation

$$\mathrm{G}_{C,D}^{r,q}\left(z_1 \,\middle|\, \begin{matrix} \gamma_1, ..., \gamma_C \\ \delta_1, ..., \delta_D \end{matrix}\right) \mathrm{G}_{E,F}^{l,k}\left(z_2 \,\middle|\, \begin{matrix} \epsilon_1, ..., \epsilon_E \\ \phi_1, ..., \phi_F \end{matrix}\right) = \quad (29)$$

$$\mathrm{G}_{0,[C,E],0,[D,F]}^{0,q,k,r,l}\left(z_1, z_2 \,\middle|\, \begin{matrix} ...; 1-\gamma_1, ..., 1-\gamma_C; 1-\epsilon_1, ..., 1-\epsilon_E \\ ....; \delta_1, ..., \delta_D; \phi_1, ..., \phi_F \end{matrix}\right),$$

we obtain

$$\Phi = \frac{\mathrm{G}_{1,[1,1],0,[1,2]}^{1,1,1,1,2}\left(\frac{\rho_2}{a}, \frac{\rho_1}{\rho_{I_{<i>}}} \,\middle|\, \begin{matrix} 0; 1+b; 1+k \\ ...; 0; 0, j-1 \end{matrix}\right)}{\Gamma(b+1)\Gamma(k+1)\Gamma(k+j)}. \quad (30)$$

Finally, substituting (30) into (26), $\widehat{C}_{12}$ can be expressed in a compact-form as

$$\widehat{C}_{12} = \frac{1}{2\ln(2)} \sum_{k=0}^{N_r-1} \sum_{i=1}^{\rho(\mathbf{D})} \sum_{j=1}^{\tau_i(\mathbf{D})} \frac{\zeta_{i,j}(\mathbf{D})}{\Gamma(j)\Gamma(k+1)} \sum_{a=1}^{P} \sum_{b=Q}^{(P+Q)a-2a^2} \frac{\beta(a,b)}{\Gamma(b+1)} \mathrm{G}_{1,[1,1],0,[1,2]}^{1,1,1,1,2}\left(\frac{\rho_2}{a}, \frac{\rho_1}{\rho_{I_{<i>}}} \,\middle|\, \begin{matrix} 0; 1+b; 1+k \\ ...; 0; 0, j-1 \end{matrix}\right). \quad (31)$$

Finally, pulling everything together yields the desired result.

IV. Numerical Results

Fig. 2 plots the ergodic capacity of two-hop multiple-antenna AF systems with different values of N_r and N_d under different interference power levels, i.e., weak interference $\rho_I = 0$ dB and strong interference $\rho_I = 25$ dB. As shown in the figure, the theoretical results match perfectly their empirical counterparts, confirming thereby the correctness of the analytical expressions. It is observed that the ergodic capacity is better improved by increasing the number of antennas at the relay N_r than that at the destination N_d. However, as the interference power grows large, the capacity gap between the different antenna configurations narrows down at low SNR. On the other hand, stronger interference has a detrimental effect on the capacity performance of the MRC scheme as shown by the considerable gap between the two interference power scenarios.

Fig. 3 investigates the interference reduction capability of two-hop MIMO AF systems. By letting N_r grow, we observe that the ergodic capacity increases at a rate that gradually becomes smaller without attaining the performance of an interference-free system. On the other hand increasing N_d is useless, more so when M is larger.

V. Conclusion

Named as CMGF, a novel integral transform was proposed as a unified tool to compute the ergodic of a two-hop multiple-antenna AF system, where the relay is subject to CCI, while the destination is only corrupted by AWGN. When both the relay and destination perform maximum ratio combining, we have been able to derive a new analytical exact expression for the ergodic capacity. The paper's findings have shed new lights on how the antenna number, the CCI numbers, and the interference power affect the performance of the system.

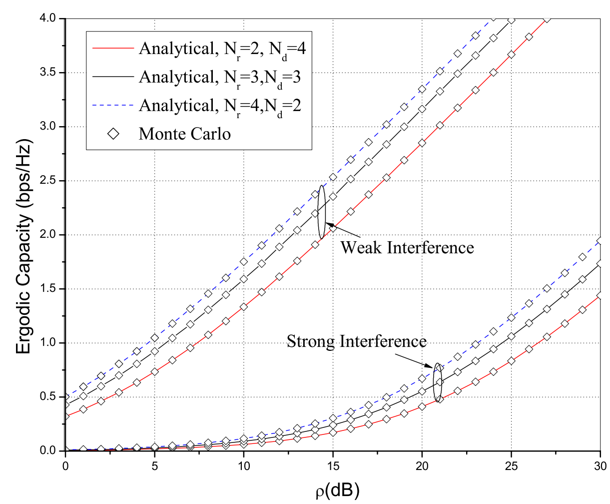

Fig. 2. The impact of the R-D MIMO link size (N_r, N_d) and the interference power ρ_I on the ergodic capacity of two-hop MIMO AF systems with $\rho_1 = \rho_2 = \rho$, and $M = 3$.

An accurate routine for the evaluation of the bivariate Meijer's G function can be found in [13].

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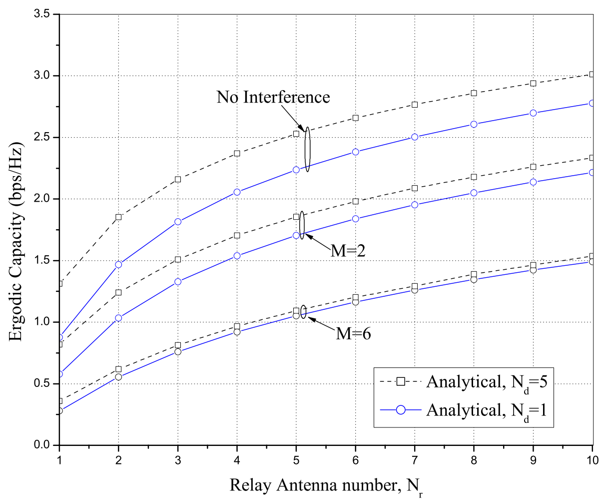

Fig. 3. Ergodic capacity versus the relay antenna number N_r with $\rho_1 = \rho_2 = 10$ dB, $\rho_I = [1, 5]$ dB for $M = 2$ and $\rho_I = [1, 2, 3, 4, 5, 6]$ dB for $M = 6$.

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