Distributed Beamforming for Spectrum-Sharing Relay Networks under Mutual Primary-Secondary Interference*

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***Abstract*—In this paper, we consider distributed beamforming for spectrum sharing networks comprising two secondary transceivers, multiple secondary relays and multiple primary transceivers. The aim of this work is to improve the secondary system performance. We assume that the relays that reliably decode the secondary signals participate in the beamforming process. We also assume the presence of mutual interference between the primary and secondary systems, while beamforming is used to suppress the interference inflicted on the primary system. However, the interference inflicted on the secondary system is not mitigated. To examine the impact of this interference on the performance of the secondary system, we derive closed-form expressions for the outage probability and bit error rate (BER) over independent and identically distributed Rayleigh fading channels. Numerical results demonstrate the efficacy of beamforming in making the secondary system performance resilient against the interference caused by the primary system.**

I. Introduction

Cognitive radio (CR) has been recently proposed as a smart and agile technology that allows non-licensed users to utilize licensed bands opportunistically. In this regard, secondary users (SUs) are allowed to get access to the spectrum with the primary users (PUs) while adhering to the interference limitations to the latter [1].

There have been considerable recent works on interference management in CR networks. Cooperative relaying in spectrum-sharing systems has proven to be effective in guaranteeing reliable transmission for the secondary systems [2]- [4]. In other words, cooperative diversity improves the secondary network performance without increasing the secondary transmit power, which imposes additional interference on the co-existing primary network. In [2], a closed-form expression for the outage performance is derived for a decode-and-forward (DF) CR network over Rayleigh fading channels, proving that full diversity is achieved. The relay selection strategies in multiple DF CR systems are studied in [3]- [4]. However, the aforementioned works, ignore the co-channel interference (CCI) from the primary transmitter to secondary receivers. In practical spectrum-sharing system, this generated interference is not negligible and should be considered in the performance analysis. In [5], the authors considered the performance of CR network with a single relay and multiple PUs under spectrum-sharing constraints. In this work, the outage performance is investigated for the interference-limited scenario. Authors in [6] extend the similar work of [5] with relay selection and imperfect channel state information (CSI) environment. One common observation of the aforementioned works is that they use power control techniques to manage the interference reflected on the primary receivers.

On the other hand, several interference mitigation techniques in CR systems have been over-viewed in [7], including transmit beamforming, precoding and spread spectrum. In [8]-[11], transmit beamforming is used to proactively mitigate the interference from secondary transmitters to the primary network. In [8], an opportunistic spectrum sharing approach is formulated that determines the optimal beamforming weights at the cognitive base-station in order to maximize the overall worst throughput of the CRs while guaranteeing a certain quality-of-service (QoS) for the PU. In [9], iterative optimization algorithms were developed to design the optimal weight beamformers aiming to maximize the worst signal-to-interference-plus-noise (SINR) ratio of multiple destinations. However, these algorithms and tools suffer from high computational complexity. Zero forcing beamforming (ZFB) is considered as a simple sub-optimal approach that can be practically implemented. In [10], a ZFB approach in a single relay with a collocated multi-antenna system was applied to improve the primary system performance in an overlay CR scenario. In our previous work [11], we investigate a dual-hop spectrum-sharing system with ZFB based considering only one PU and assuming negligible interference from this PU to the secondary receivers.

Motivated by the potential of combining cooperative diversity and beamforming, we adopt in this paper distributed ZFB in a dual-hop DF selective relaying CR network. Our main contributions can be summarized as follows:

- The ZFB weights are found via a linear optimization method aiming to maximize the received signal-to-noise (SNR) at the secondary destination.
- To examine the impact of the PUs' CCI on the secondary system performance, we derive closed-form expressions of the outage probability and bit error error (BER).
- To get more insights, we derive an asymptotic expression for the BER in order to get the diversity gain.

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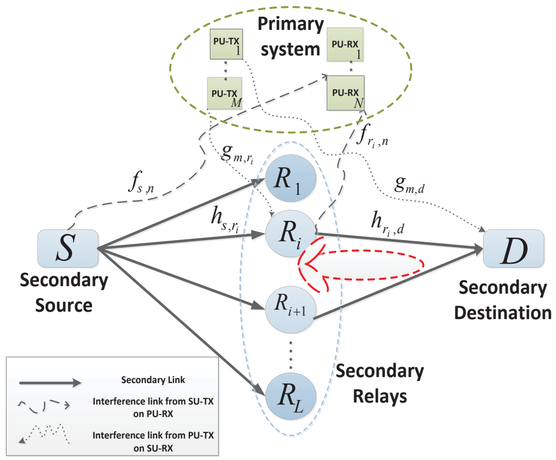

Fig. 1: Spectrum-Sharing System with dual-hop Relaying.

- We investigate the secondary system through many numerical examples and confirm the validity of the results by simulations.

As a result, the ZFB approach has the potential for improving the secondary performance and compensate the performance loss due to PUs' CCI in a simple practical manner compared to other complex approaches.

II. System and Channel Models

A. System model

We consider a dual-hop relaying system that is composed of a pair of secondary source S and destination D and a set of L DF secondary relays denoted by R_i for $i = 1, ..., L$ coexisting in the same spectrum band with a primary system which consists of a cluster of M primary transmitters (PU-TXs)- N primary receivers (PU-RXs) pairs as shown in Fig. 1. All nodes are equipped with one antenna. Due to the severe impairments such as multipath effects, there is no direct link between the source and destination and thus the source can only send messages via secondary relay nodes over two time-slot.

In the first time-slot, based on the interference channel state information (CSI) from S to the nth PU-RX, which suffers the most interference caused by S (the strongest interference channel), S adjusts its transmit power under predefined threshold Q and broadcasts its message to all relays. In the second time-slot, ZFB is applied to null the interference from the selected potential relays L_s (which decided to participate) to the PU-RXs so that the relays are always able to transmit without interfering with the PU-RXs. The ZFB processing vector, namely $\mathbf{w_{zf}}$, is optimized so as to maximize the received SNR at the secondary destination, while nulling the inflicted interference to the existing PU-RXs.

All channel coefficients are assumed to be independent Rayleigh flat fading such that $|h_{s,r_i}|^2, |f_{s,n}|^2, |f_{r_i,n}|^2, |g_{m,d}|^2$ and $|g_{m,r_i}|^2$ are exponential distributed random variables with parameters $\lambda_{sr_i}, \lambda_{sn}, \lambda_{r_in}, \lambda_{md}$ and λ_{mr_i} respectively. All the noises are assumed to be additive white complex Gaussian with zero-mean and variance σ^2. Let the ZFB vector $\mathbf{w_{zf}^T} = [w_1, w_2, ..., w_{L_s}]$ used to direct the signal to D. Let $\mathbf{h}_{r,d}^T = [h_{r_1,d}, ..., h_{r_{L_s},d}]$ be the channel vector between the potential relays and D. Let $\mathbf{F_{rp}^T} = [\mathbf{f_{r,p_1}}, ..., \mathbf{f_{r,p_N}}]$ be the channel matrix between the potential relays and all N PU-RXs where $\mathbf{f_{r,p_n}} = [f_{r_1,n}, ..., f_{r_{L_s},n}]$. It is assumed that S has perfect knowledge of the interference channel between itself and the PU-RX, i.e., $f_{s,n}$. We also assume that each selected relay R_i has perfect knowledge about its interference channels $f_{r_i,n}$ to the PU-RXs and then it shares this channel information with the D to design the ZFB process at the second-hop transmission. It is worth mentioning that the availability of these interference channel information can be acquired through a spectrum-band manager that mediates between the primary and secondary users [12]. It is also assumed that D has perfect knowledge of the channel coefficients between the selected relays and itself, which can be obtained via traditional channel estimation [13]. In our scheme, the ZFB weights associated with the selected relays are designed at the D by exploiting the aforementioned channel information. Then, each weight is sent back to the selected relay via a low date-rate feedback link, and that is applicable in slow fading environments [14].

B. Transmission protocol

In this dual-hop system, the communication process occurs over two time-slots. In the first time-slot, S broadcasts its signal to all L relays, then the received signal at the ith relay is given as

$$y_{r_i} = \sqrt{P} h_{s,r_i} x_s + \sum_{m=1}^{M} \sqrt{P_{int}}\, g_{m,r_i} \hat{x}_{im} + n_{i1}, \qquad (1)$$

where P is the source's transmit power, P_{int} is interference power from PU-TXs, x_s is the information symbol of S, $\hat{x}_{im}$ is the mth PU-TX interfering symbol at the ith relay and n_{i1} denotes the noise at the ith relay in the first time-slot. We assume that the transmitted symbols are equiprobable with unit energy.

In the second time-slot, the decoding set $\mathcal{C}$, which consists of the relays that can correctly decode x_s by using cyclic redundancy codes, transmits the decoded signals to the destination simultaneously by using beamforming. In particular, each relay weights the decoded signal and forwards it to the destination. Thus the received signal at D is given by

$$y_d = \sqrt{P_r} \mathbf{h}_{r,d}^H \hat{\mathbf{x}}_s + \sum_{m=1}^{M} \sqrt{P_{int}}\, g_{m,d} \hat{x}_{md} + n_2, \qquad (2)$$

where $\hat{\mathbf{x}}_s = \mathbf{w_{zf}} x_s$ is the weighted vector, $\hat{x}_{md}$ is the mth PU-TX interfering symbol at D and n_2 is the noise at D in the second time-slot. Therefore, the corresponding total received SINR at D given $\mathcal{C}$, denoted $\gamma_{d|\mathcal{C}}$, is given as

$$\gamma_{d|\mathcal{C}} = \frac{P_r |\mathbf{h}_{r,d}^H \mathbf{w_{zf}}|^2}{\sum_{m=1}^{M} P_{int}\, |g_{m,d}|^2 + \sigma^2}. \qquad (3)$$

C. Mathematical Model and Size of $\mathcal{C}$

In the underlay approach of this model, S can utilize the PU's spectrum as long as the interference it generates at the most affected PU-RX remains below the interference threshold Q. For that reason, P is constrained as $P = \min\left\{\frac{Q}{|f_{s,n}|^2}, P_s\right\}$ where P_s is the maximum transmission power of S [11]. So the received SINR, γ_{s,r_i} at the ith relay is given as

$$\gamma_{s,r_i} = \frac{\min\left\{\frac{Q}{|f_{s,n}|^2}, P_s\right\}|h_{s,r_i}|^2}{\sum_{m=1}^{M} P_{int}|g_{m,r_i}|^2 + \sigma^2}. \quad (4)$$

Next, we derive the CDF of $\gamma_{s,r_i} = \frac{U}{P_{int}V+\sigma^2}$, where $U = \min\left\{\frac{Q}{|f_{s,n}|^2}, P_s\right\}|h_{s,r_i}|^2$ and $V = \sum_{m=1}^{M}|g_{m,r_i}|^2$. By using the definition of CDF of γ_{s,r_i}, we find

$$F_{\gamma_{s,r_i}}(x) = \int_0^\infty \Pr(U < (P_{int}\, y + \sigma^2)x) f_V(y)dy. \quad (5)$$

Since V is the sum of M exponential random variables with parameter λ_{m,r_i}, it presents a chi-square random variable with $2M$ degrees of freedom and its PDF is given by

$$f_V(y) = \frac{\lambda_{m,r_i}^M y^{M-1} e^{-\lambda_{m,r_i} y}}{\Gamma(M)}. \quad (6)$$

The CDF of U is obtained with the help of [11, Eq.2] as,

$$F_U(x) = 1 + e^{-\frac{\lambda_{sr_i} x}{P_s}}\left(\frac{e^{-\frac{\lambda_{sn} Q}{P_s}}}{\frac{\lambda_{sn} Q}{\lambda_{sr_i} x} + 1} - 1\right). \quad (7)$$

Substituting (6) and (7) into (5), and after several algebraic manipulations, (5) is equivalently expressed as

$$\begin{aligned} F_{\gamma_{s,r_i}}(x) &= 1 + \psi e^{-\frac{\lambda_{sr_i}\sigma^2 x}{P_s}} \int_0^\infty y^{M-1} e^{-(\frac{\lambda_{sr_i}\sigma^2 x}{P_s} + \lambda_{mr_i})y} \\ &\times \left[(e^{-\frac{\lambda_{sn} Q}{P_s}} - 1) - \frac{\frac{\lambda_{sn} Q}{\lambda_{sr_i}} e^{-\frac{\lambda_{sn} Q}{P_s}}}{x P_{int}\, y + \sigma^2 x + \frac{\lambda_{sn} Q}{\lambda_{sr_i}}}\right] dy. \end{aligned} \quad (8)$$

where $\psi = \frac{\lambda_{mr_i}^M}{\Gamma(M)}$. After easy simplifications, and with the help of [15, Eqs. (3.351.3), (3.353.5)], the CDF of γ_{s,r_i} is derived as

$$\begin{aligned} F_{\gamma_{s,r_i}}(x) &= 1 + \psi e^{-\frac{\lambda_{sr_i}\sigma^2 x}{P_s}} \left[\frac{(e^{-\frac{\lambda_{sn} Q}{P_s}} - 1)\kappa!}{\frac{\lambda_{sr_i} P_{int} x}{P_s} + \lambda_{mr_i}}\right. \\ &- (\frac{\lambda_{sn} Q e^{-\frac{\lambda_{sn} Q}{P_s}}}{\lambda_{sr_i}}) \left((-1)^{\kappa-1}\beta^\kappa e^{\beta\mu} Ei[-\beta\mu]\right. \\ &+ \left.\left.\sum_{k=1}^{\kappa}\Gamma(k)(-\beta)^{\kappa-k}\mu^{-k}\right)\right], \end{aligned} \quad (9)$$

where $\kappa = M-1$, $\beta = \frac{\sigma^2}{P_{int}} + \frac{\lambda_{sn} Q}{\lambda_{sr_i} P_{int} x}$, $\mu = \lambda_{mr_i} + \frac{\lambda_{sr_i} P_{int} x}{P_s}$ and $Ei[.]$ is the exponential integral defined in [15].

As mentioned earlier, we define $\mathcal{C}$ to be the set of relays which perfectly decode the signals received in the first time-slot, which implies that there is no outage at these relays. This translates to the fact that the mutual information between S and each ith relay is above a specified target value. In this case, the potential ith relay is only required to meet the decoding constraint given as

$$\Pr[R_i \in \mathcal{C}] = \Pr\left[\frac{1}{2}\log_2(1 + \gamma_{s,r_i}) \geq R_{th}\right], \quad (10)$$

where $(1/2)$ is from the message transmission in two time-slot and R_{th} denotes the minimum target rate below which outage occurs. By using the the Binomial distribution, the probability $\Pr[|\mathcal{C}| = L_s]$ becomes

$$\Pr[|\mathcal{C}| = L_s] = \binom{L}{L_s} P_{\text{off}}^{L-L_s}(1 - P_{\text{off}})^{L_s}, \quad (11)$$

where P_{off} denotes the probability that the relay does not decode correctly the received signal and keeps silent in the second time-slot. Then P_{off} is computed as

$$P_{\text{off}} = F_{\gamma_{s,r_i}}(\gamma_{th}), \quad (12)$$

where $\gamma_{th} = 2^{2R_{min}} - 1$ is the SINR threshold.

III. Sub-optimal ZFB Weights Design

Our objective here is to maximize the received SNR at the secondary destination in order to enhance the performance of the secondary system while limiting the interference reflected on the PU-RXs. To be able to apply ZFB, the general assumption $L_s > N$ is considered. According to the ZFB principles, the transmit weight vector $\mathbf{w_{zf}}$, is chosen to lie in the orthogonal space of $\mathbf{F_{rp}}^H$ such that $|\mathbf{f}_{\mathbf{r,p_i}}^H \mathbf{w_{zf}}| = 0$ $\forall i = 1, ..., N$ and $|\mathbf{h}_{r,d}^H \mathbf{w_{zf}}|$ is maximized. So the problem formulation for finding the optimal weight vector is introduced as.

$$\begin{aligned} \max_{\mathbf{w_{zf}}} \quad & |\mathbf{h}_{r,d}^H \mathbf{w_{zf}}|^2 \\ \text{s.t.:} \quad & |\mathbf{f}_{\mathbf{r,p_n}}^H \mathbf{w_{zf}}| = 0, \quad \forall n = 1, ..., N \\ & \|\mathbf{w_{zf}}\| = 1. \end{aligned} \quad (13)$$

By applying a standard Lagrangian multiplier method, the weight vector that satisfies the above optimization method is given as

$$\mathbf{w_{zf}} = \frac{\mathbf{\Xi}^\perp \mathbf{h}_{r,d}}{\|\mathbf{\Xi}^\perp \mathbf{h}_{r,d}\|}, \quad (14)$$

where $\mathbf{\Xi}^\perp = \left(\mathbf{I} - \mathbf{F_{rp}}(\mathbf{F_{rp}}^H \mathbf{F_{rp}})^{-1}\mathbf{F_{rp}}^H\right)$ is the projection idempotent matrix with rank $(L_s - N)$ [11]. It is worth noting that the relays transmit only their own received signal and there is no data exchange among the relays. Thus, the algorithm works in a distributed manner.

IV. Performance Analysis

A. End-to-End SINR statistics

Now, after finding $\mathbf{w_{zf}}$, we substitute (14) into equation (3) to get

$$\gamma_{d|\mathcal{C}} = \frac{P_r||\mathbf{\Xi}^\perp \mathbf{h_{r,d}}||^2}{\sum_{m=1}^{M} P_{int}|g_{m,d}|^2 + \sigma^2}, \quad (15)$$

To analyze the system performance, we firstly need to obtain the CDF of $\gamma_{d|\mathcal{C}}$. Let $U_1 = P_r||\mathbf{\Xi}^{\perp}\mathbf{h_{r,d}}||^2$ and $V_1 = \sum_{m=1}^{M}|g_{m,d}|^2$, by following the same previous approach, we need the CDF of $U_1$and the PDF of V_1 which can obtained from [16, Eq.20] and (6), respectively. We find the conditional CDF of $\gamma_{d|\mathcal{C}}$

$$F_{\gamma_{d|\mathcal{C}}}(x) = \int_0^{\infty} \frac{\psi_1 \gamma\left(\tau, \frac{(P_{int}\, y+\sigma^2)x}{P_r}\right) y^{\kappa} e^{-\lambda_{md} y}}{\Gamma(L_s - N - 1)} dy, \quad (16)$$

where $\psi_1 = \frac{\lambda_{md}^M}{\Gamma(M)}$ and $\tau = L_s - N$. By representing the incomplete Gamma function into another form utilizing the identities [15, Eqs. (8.352.1), (1.11)], and after several mathematical manipulations, the integral in (16) is expressed as

$$F_{\gamma_{d|\mathcal{C}}}(x) = 1 - \psi_1 e^{-\frac{\sigma^2 x}{P_r}} \sum_{i=0}^{L_s-N-1} \sum_{j=0}^{i} \binom{j}{i} P_{int}^{j} (\sigma^2)^{i-j} x^i \times \int_0^{\infty} y^{M-1+j} e^{-(\lambda_{md}+\frac{P_{int}x}{P_r})y} dy. \quad (17)$$

With the help of [15, (3.3351.3)], the CDF of $\gamma_{d|\mathcal{C}}$ is given as

$$F_{\gamma_{d|\mathcal{C}}}(x) = 1 - \psi_1 \sum_{i=0}^{L_s-N-1} \sum_{j=0}^{i} \binom{j}{i} P_{int}^{j} (\sigma^2)^{i-j} \times e^{-\frac{\sigma^2 x}{P_r}} \frac{(M-1+j)! x^i}{(\lambda_{md} + \frac{P_{int}x}{P_r})^{M+j}}. \quad (18)$$

To compute the unconditional CDF denoted as $F_{\gamma_d}(\gamma)$, we use the total probability theorem to get

$$F_{\gamma_d}(x) = \sum_{L_s=0}^{N} \binom{L}{L_s} P_{\text{off}}^{L-L_s} (1-P_{\text{off}})^{L_s} + \sum_{L_s=N+1}^{L} \binom{L}{L_s} P_{\text{off}}^{L-L_s} (1-P_{\text{off}})^{L_s} F_{\gamma_{d|\mathcal{C}}}(x), \quad (19)$$

where $F_{\gamma_{d|\mathcal{C}}}(x)$ is substituted from (18). In the subsequent section, we make use of $F_{\gamma_d}(x)$ to derive closed-form expressions for the outage and end-to-end (E2E) BER performance.

B. Outage Probability

An outage event occurs when the total received SNR falls below a certain threshold γ_{th} and is expressed as $P_{out} = \Pr(\gamma_d < \gamma_{th})$. By using (19), the outage probability of the secondary system in a closed-form can be written as

$$P_{out} = F_{\gamma_d}(\gamma_{th}). \quad (20)$$

C. E2E BER

We analyze the BER performance due to errors occurring at D assuming that all participating relays have accurately decoded and regenerated the message. This probability could be evaluated using the following identity

$$P_e = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^{\infty} \frac{e^{-bx}}{\sqrt{x}} F_{\gamma_d}(x) dx \quad (21)$$

Since P_e depends on the modulation scheme, many expressions can be used. In this paper, we consider Binary Phase Shift Keying (BPSK) for which $(a, b) = (1, 1)$.

$$P_e = \frac{a\sqrt{b}}{2\sqrt{\pi}} \left(A \underbrace{\int_0^{\infty} \frac{e^{-bx}}{\sqrt{x}} dx}_{I_1} + B \underbrace{\int_0^{\infty} \frac{e^{-bx}}{\sqrt{x}} F_{\gamma_{d|\mathcal{C}}}(x) dx}_{I_2} \right) \quad (22)$$

where $A = \sum_{L_s=0}^{N} \binom{L}{L_s} P_{\text{off}}^{L-L_s} (1-P_{\text{off}})^{L_s}$ and $B = \sum_{L_s=N+1}^{L} \binom{L}{L_s} P_{\text{off}}^{L-L_s} (1-P_{\text{off}})^{L_s}$. I_1 is evaluated with the help of [15, (3.361.2)]. Next, to compute I_2, we represent the integrands of I_2 in terms of Meijer's G-functions using [17, Eq. 10, 11], which are given, respectively, as

$$\left(\lambda_{md} + \frac{P_{int}x}{P_r}\right)^{-\nu} = \eta x^{-\nu} G_{1,1}^{1,1}\left(\frac{P_{int}\, x}{\lambda_{md} P_r}\Big|_{\nu}^{1}\right), \quad (23)$$

where $\eta = \frac{(\frac{P_{int}}{P_r})^{-(M+j)}}{\Gamma(M+j)}$ and $\nu = (M+j)$.

$$e^{-(b+\frac{\sigma^2}{P_r})x} = G_{0,1}^{1,0}\left(\left(b + \frac{\sigma^2}{P_r}\right) x\Big|_{0}^{-}\right). \quad (24)$$

Thus, I_2 yields as

$$I_2 = 0.5 - \eta\,\Phi \int_0^{\infty} x^{-M-j+i-\frac{1}{2}} G_{1,1}^{1,1}\left(\frac{P_{int}\, x}{\lambda_{md} P_r}\Big|_{M+j}^{1}\right) \times G_{0,1}^{1,0}\left(\left(b + \frac{\sigma^2}{P_r}\right) x\Big|_{0}^{-}\right) dx, \quad (25)$$

where $\Phi = \frac{\lambda_{md}^M}{\Gamma(M)} \sum_{i=0}^{L_s-N-1} \sum_{j=0}^{i} \binom{j}{i} P_{int}^{j} (\sigma^2)^{i-j} (M-1+j)!$ Exploiting that the integral of the product of a power term and two Meijer's G-function [17, Eq. 21], I_2 results in

$$I_2 = 0.5 - \frac{\eta\,\Phi}{\Gamma(M+j)} \left(\frac{P_{int}}{P_r}\right)^{-(M+j)} \left(\frac{\sigma^2}{P_r} + b\right)^{-\alpha} \times G_{2,1}^{1,2}\left(\frac{P_{int}}{bP_r + \lambda_{md}\sigma^2}\Big|_{M+j}^{1,1-\alpha}\right), \quad (26)$$

where $\alpha = i - M - j + 0.5$. By incorporating the results of I_1 and I_2 into (22), a closed-form expression for the unconditional E2E BER at D is given as

$$P_e = 0.5A + B\left(0.5 - \frac{\eta\,\Phi}{\Gamma(M+j)} \left(\frac{P_{int}}{P_r}\right)^{-(M+j)} \times \left(\frac{\sigma^2}{P_r} + 1\right)^{-\alpha} G_{2,1}^{1,2}\left(\frac{P_{int}}{P_r + \lambda_{md}\sigma^2}\Big|_{M+j}^{1,1-\alpha}\right)\right). \quad (27)$$

It is worth noting that the Meijer's G-function is implemented in many mathematical softwares such as Matlab and Mathematica.

Diversity Gain: To gain some insight about the achieved diversity order and understand the impact of the interference threshold on the BER performance, we derive the asymptotic expression when a fixed power constraint is imposed. We consider the scenario where $Q \to \infty$ and $P_s \ll Q$. According to [18, Eqs. (5.1.7) and (5.1.20)], we have $\lim_{x\to\infty} e^x Ei(-x) = 0$.

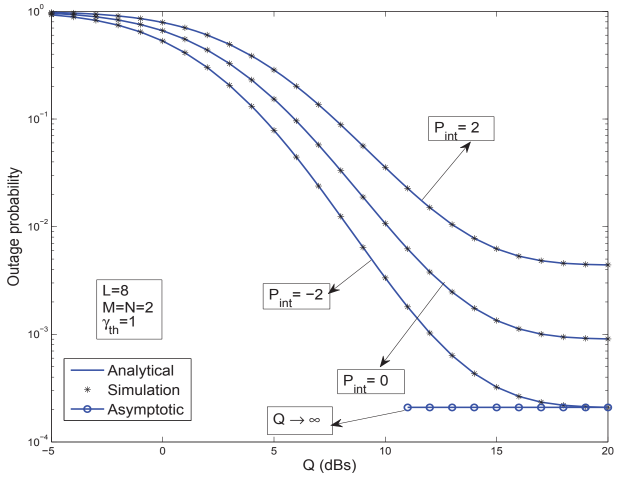

Fig. 2: Outage probability vs. Q (dB) for L=8 and $M = N = 2$.

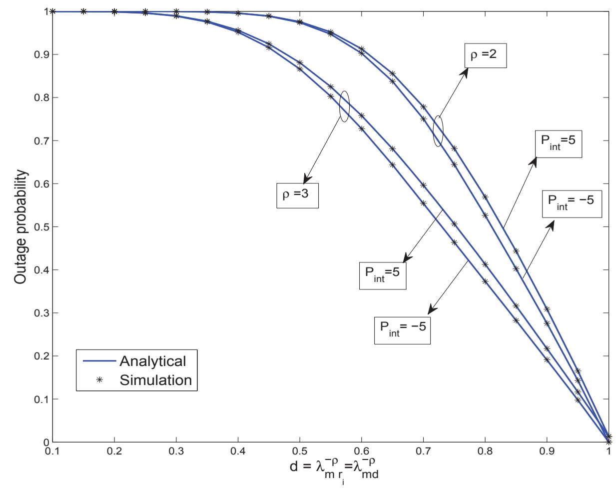

Fig. 3: Outage probability vs. d for L=10 and $M = N = 1$.

It is also known that $\lim\limits_{x\to\infty} xe^{-x} = 0$. Using these formulas, for $Q \to \infty$, the CDF in (18) is approximated as

$$\tilde{F}_{\gamma_{s,r_i}}(x) \approx 1 - \psi \frac{\kappa! e^{-\frac{\lambda_{sr_i}\sigma^2 x}{P_s}}}{\frac{\lambda_{sr_i} P_{int}\, x}{P_s} + \lambda_{mr_i}}. \quad (28)$$

Substituting (28) into (12), we get

$$\tilde{P}_{\text{off}} \approx \tilde{F}_{\gamma_{s,r_i}}(\gamma_{th}). \quad (29)$$

and then incorporating (29) into (27), we get an approximate expression for the asymptotic BER performance.

$$\tilde{P}_e \approx 0.5\tilde{A} + \tilde{B} I_2 \quad (30)$$

where $\tilde{A} = \sum_{L_s=0}^{N} \binom{L}{L_s} \tilde{P}_{\text{off}}^{L-L_s}(1-\tilde{P}_{\text{off}})^{L_s}$, $\tilde{B} = \sum_{L_s=N+1}^{L} \binom{L}{L_s} \tilde{P}_{\text{off}}^{L-L_s}(1-\tilde{P}_{\text{off}})^{L_s}$ and I_2 is the result of (26). It is interesting to observe from (30) that the approximate BER expression is independant of the interference threshold Q and results in a non-zero constant value. This means that $\tilde{P}_e$ saturates when the S transmit power exceeds the PU-RX threshold, leading to a diversity order of zero.

Mathematically, the asymptotic diversity order d with respect to Q is given by

$$\begin{aligned} d &= \lim_{Q\to\infty} -\frac{\log\ (\tilde{P}_e)}{\log\ (Q)} \\ &= \lim_{Q\to\infty} -\frac{\log(K)}{\log\ (Q)} = 0 \end{aligned} \quad (31)$$

where K is a non-zero constant. Noting that $P_s, P_{int}, P_r, L, M, \gamma_{th}, \lambda_{md}, \lambda_{sr_i}$ are all constants and using (30), we reach to (31). Hence, the diversity order is zero in this case.

V. Numerical Results and Discussion

In this section, we present numerical results to validate our derived expressions in Section IV. Without loss of generality, we assume that the relays are located on a straight line vertical to the distance between the source and destination. The distance between them equals one. We further assume that the primary system forms a cluster where PU-TXs are closely located to each others and as all PU-RXs. Unless otherwise stated, we also assume that $\lambda_{sn} = \lambda_{r_i n} = 1$, $\lambda_{sr_i} = \lambda_{r_i d} = 1, \forall i$ and $\lambda_{mr_i} = \lambda_{md} = 1, \forall i$. we fix the value of $\gamma_{th} = 1$ dB, $P_s = 20$ dB.

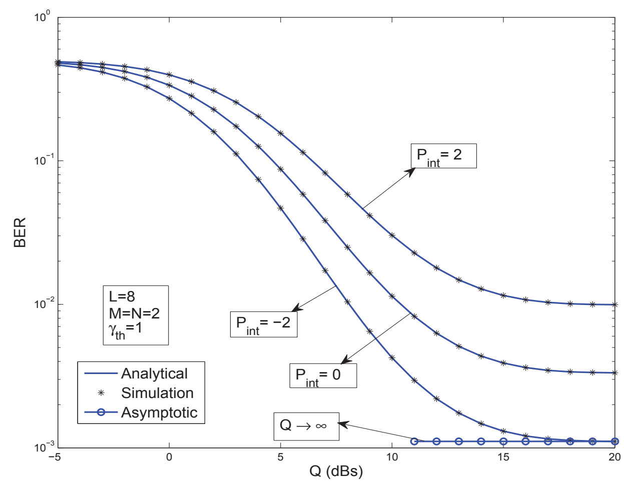

Fig. 4: BER vs. Q (dB) for L= 8 and $M = N = 2$.

Fig. 2 shows the outage performance versus Q for $L = 8$, $M = N = 2$ under different values of $P_{int} = -2, 0, 2$dB. As observed from the figure, as the value of Q increases, the outage performance improves substantially. Also, the figure shows the impact of co-channel interference on the outage performance. As P_{int} increases, it degrades the system performance significantly. However, a compensation of this loss in performance is gained by the use of beamforming process.

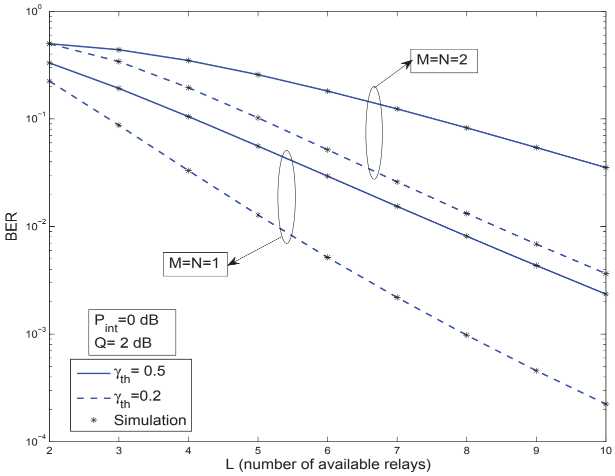

Fig. 5: BER vs. L for $M = N = 1, 2$.

A floor in the outage performance curve is noticed which is due to the interference level constraint.

Fig. 3 illustrates the outage probability for different locations of the PU-TX and different P_{int} values with $M = N = 1$, and $Q = 20$. It is assumed that $\lambda_{mr_i} = \lambda_{md} = d^{-\rho}$, where ρ is the path loss exponent and d is the distance between PU-TX and secondary receiver. It is observed that the outage performance is better for the lower PU-TX transmit power P_{int} and higher path loss exponent ρ. That happens because when PU-TX locates closer to the secondary receivers, the outage performance of the secondary system becomes worse.

Fig. 4 illustrates the BER performance versus Q for L= 8 and $M = N = 2$. Similar observation is obtained as in Fig. 2. It is seen that as Q increases, the BER performance improves. While the performance deteriorates when P_{int} increases. In addition, it is observed from the figure that BER curve saturates in high Q region and an error floor occurs which results in zero-diversity. This error flooring is due to the limitations on the secondary transmit powers and co-channel interferences.

Fig. 5 shows the BER performance versus the number of available relays L under different number of PU-TXs-RXs with $P_{int} = 0, Q = 2$ dB. It is obvious that the BER performance improves substantially as the number of relays increases. This is attributed to the combined cooperative diversity and beamforming which enhances the total received SINR at the secondary destination. Clearly, as the number of existing PU-TXs increases from one to two, the BER performance becomes worse as this will increase the sum of the CCIs that severely affects the received signals at secondary receivers. In addition, the figure shows that the BER performance is better when the SINR threshold γ_{th} is less strict.

VI. Conclusion

The impact of multiple primary transmitters co-channel interference on the secondary system performance was investigated for a cooperative dual-hop DF spectrum-sharing system. The proposed system limits the interference to the PU-RXs using a distributed ZFB approach and peak interference power constraints. The beamforming weights were optimized to maximize the received SNR at the secondary destination and to null the interference inflicted on the primary receivers. We evaluated the performance of the secondary system by deriving closed-form expressions for the outage probability and BER metrics. We also derived asymptotic expression for the BER to get insights on the error floor occurring due to interference constraints. Our numerical results showed that the combination of the distributed ZFB and the cooperative diversity enhances the secondary link performance by compensating the performance loss due to the CCIs.

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