On the Performance of Spectrum Sharing Two-Way Relay Networks with Distributed Beamforming*

Ali Afana[1], Ali Ghrayeb[1,2], Vahid Asghari[1,3] and Sofiéne Affes[3]
[1]ECE Department, Concordia University, Montréal, Québec, Canada.
[2] ECE Department, Texas A&M University, Doha, Qatar.
[3]INRS-EMT, University of Québec, Montréal, Québec, Canada.
Emails:{a_afa, aghrayeb}@ece.concordia.ca, {vahid, affes}@emt.inrs.ca.

***Abstract*—In this paper, we consider distributed beamforming for two-way spectrum-sharing system in an effort to enhance the performance of the secondary system and improve the bandwidth efficiency. In particular, we consider a cognitive radio network where a set of amplify-and-forward (AF) relays are employed to help a pair of secondary transceivers in the presence of multiple primary receivers. This set of the available relays participate in the beamforming process, where the beamformer weights are obtained via a convex optimization method. We analyze the performance of the proposed methods where closed-form expressions for the end-to-end outage and average error probabilities are derived for independent and identically distributed (i.i.d.) Rayleigh fading channels. Numerical results demonstrate the vitality of beamforming in enhancing the secondary system performance in addition to mitigating the interference to the primary users.**

I. Introduction

Cognitive Radio (CR) is a promising solution to enhance the utilization of the wireless spectrum. In this regard, spectrum sharing offers tremendous potential to allow secondary users (SUs) to get access to the spectrum with the primary users (PUs) while adhering to the interference limitations of the latter. On the other hand, applying the concept of cooperative relaying in the spectrum-sharing systems has received considerable interest due to its efficacy to guarantee reliable transmission for the secondary systems [1]- [3]. While the cooperative one-way relaying systems in cognitive radio networks (CRNs) and non-CRNs are heavily studied, the two-way relaying systems in spectrum-sharing environment are rarely investigated. Recently, in [2], the outage probability expression was derived for a cooperative two-way decode-and-forward (DF) relaying system where a SU helps two primary transceivers to communicate with each other while the outage performance of an amplify-and-forward (AF) two-way relaying system was investigated in [3]. However, in [2], [3], an overly spectrum-sharing strategy is assumed.

Beamforming is an alternative emerging technology to alleviate the inflicted interference in the spectrum-sharing systems [4], [5]. Recently, the problem of sum-rate maximization under constraints on interference on a primary receiver for multi-antenna cognitive two-way relay network has been investigated in [6]. In that paper, the authors have provided a structure of the optimal relay beamformer and proposed projection-based suboptimal beamforming schemes. The authors in [7] have obtained the optimal beamforming coefficients in a cognitive two-way relaying system using iterative semidefinite programming and bisection search methods with the objective of minimizing the interference at the PU with SUs signal-to-interference-plus-noise ratio constraints. This scheme suffers from high computational complexity and implementation difficulties. We remark that all previous works considered only one primary user that coexists with the secondary users. Zero forcing beamforming (ZFB) is an alternative sub-optimal approach that can be practically implemented.

Motivated by the great potential of combining two-way relaying and beamforming, we use in this paper collaborative distributed ZFB in two-way AF relaying in a spectrum sharing environment. In particular, we consider a CRN comprising two secondary sources communicating with each other in two consecutive time slots, a number of secondary AF relays and a number of PUs. The relays that receive the signals (from the sources) are used for relaying and beamforming process in the second time-slot. Specifically, those relays employ distributed ZFB to null the inflicted interference to the PUs in addition to improving the performance of the secondary system. We also limit the interference from the secondary source by imposing a peak constraint on the interference received at the PUs in the first time-slot. To analyze the performance, we derive closed-form expressions of the end-to-end outage and average error probabilities. As a result, the ZFB approach has the potential for improving the secondary performance and limiting the interference in a simple practical manner compared to other complex approaches.

II. System and Channel Models

We consider a two-way relaying system that is composed of two secondary transceivers S_j, $j = 1, 2$ and a set of L_s AF secondary relays denoted by R_i for $i = 1, ..., L_s$ coexisting in the same spectrum band with M primary receivers (PUs) as shown in Fig. 1. All nodes are equipped with one antenna. The two sources wish to communicate with each other in a half-duplex way. There is no direct link between the sources and thus they can only exchange messages via relay nodes over two time-slot (2-TS) protocol. The SUs are allowed to share the same frequency spectrum with the PUs as long as

* This paper was made possible by NPRP grant # 09-126-2-054 from the Qatar National Research Fund (a member of Qatar Foundation).

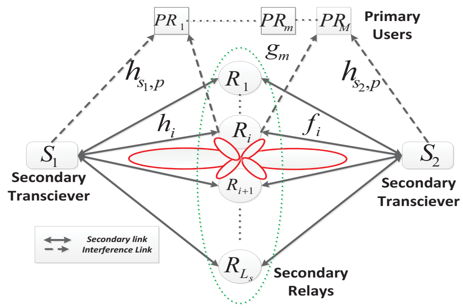

Fig. 1: Spectrum-Sharing System with Two-Way AF Relaying.

the interference to the PUs is limited to a predefined threshold. Both systems transmit simultaneously in an underlay manner. In the first time-slot (TS_1), based on the interference channel state information (CSI) from S_1 to the mth PU, which suffers the most interference caused by S_1, S_1 adjusts its transmit power under predefined threshold Q_1 and broadcasts its message to all relays. Simultaneously, in TS_1, based on the interference CSI from S_2 to the jth PU, which suffers the most interference caused by S_2 (the mth and jth PUs could be different or the same), S_2 adjusts its transmit power under predefined threshold Q_2 and broadcasts its message to all relays. In the second time-slot (TS_2), ZFB is applied to null the interference from the reliable relays L_s (that are allowed to participate) to the PUs so that the relays are always able to transmit without interfering with the PUs. The ZFB processing matrix, namely $\mathbf{W_{zf}}$, is optimized to maximize the received SNRs at both transceivers while nulling the inflicted interference to the existing PUs.

All channel coefficients are assumed to be independent Rayleigh flat fading and quasi-static, so that the channel gains remain unchanged during the transmission period. Let h_{s_1,r_i}, f_{s_2,r_i} denote the channel coefficients from the sources S_1 and S_2 to the ith relay, respectively, which are modeled as zero mean, circularly symmetric complex Gaussian (CSCG) random variables with variance λ_{s_1,r_i},λ_{s_2,r_i}. Denote $h_{s_1,p}$ and $h_{s_2,p}$ as the interference channel coefficients from S_1 and S_2 to the mth and jth PUs, and their channel power gains are $|h_{s_1,p}|^2$ and $|h_{s_2,p}|^2$, which are exponentially distributed with parameter $\lambda_{s_1,p}$ and $\lambda_{s_2,p}$, respectively. Let the ZFB vectors $\mathbf{w_{zf_1}^{\mathit{T}}} = [w_{11}, w_{12}, ..., w_{1L_s}]$ used to direct the signal to S_1 and $\mathbf{w_{zf_2}^{\mathit{T}}} = [w_{21}, w_{22}, ..., w_{2L_s}]$ used to direct the signal to S_2. Let $\mathbf{h}^T = [h_{r_1,s}, ..., h_{r_{L_s},s}]$ and $\mathbf{f}^T = [f_{r_1,s}, ..., f_{r_{L_s},s}]$ be the channel vectors between the relays and S_1 and S_2, respectively. Let $\mathbf{G_{rp}^{\mathit{T}}} = [\mathbf{g_{r,p_1}}, ..., \mathbf{g_{r,p_M}}]$ be the channel matrix between the relays and all M PUs where $\mathbf{g_{r,p_m}} = [g_{r_1,p_m}, ..., g_{r_{L_s},p_m}]$. It is assumed that S_1, S_2 and the relays have perfect knowledge of their interference channel power gains, which can be acquired through a spectrum-band manager that mediates between the primary and secondary users [1]- [3]. To be able to implement beamforming, global CSI is required at every relay. The ith relay needs to know the CSI for the links $(R_i - S_1)$ and $(R_i - S_2)$. Assume that the relays can share the CSI between each other to realize cooperative beamforming which can be obtained in practice through training. The interference from the primary transmitter is neglected and can be represented in terms of noise when its message is generated by random Gaussian codebooks [2].

As mentioned, the sources communicate with each other over two time-slots. In TS_1, S_1 and S_2 broadcast their signals to the relays simultaneously. The received signal at the ith relay is written as

$$y_{r_i}^1 = \sqrt{P_1} h_{s_1,r_i} x_{s_1} + \sqrt{P_2} f_{s_2,r_i} x_{s_2} + n_1, \tag{1}$$

where P_1 and P_2 are the S_1 and S_2 transmit powers, respectively, x_{s_1} and x_{s_2} are the information symbols of S_1 and S_2 with $\mathrm{E}[|x_{s_1}|^2]$=$\mathrm{E}[|x_{s_2}|^2] = 1$ and n_1 denotes the zero-mean CSCG noise at the ith relay with variance σ^2 in TS_1.

In TS_2, the relays that received the signal reliably weight the received signals and forward them to the two sources. The weighted transmitted signal in a vector form is

$$\mathbf{x_R} = \mathbf{W_{zf} y_r^1}, \tag{2}$$

where $\mathbf{W_{zf}}$ is the beamforming processing matrix and $\mathbf{y_r^1}$ is the relays received signals in a vector form. The received signal at S_2 is given as

$$\begin{aligned} y_{S_2}^2 \;&=\; \sqrt{P_1} A_r B_r \mathbf{f}^H \mathbf{W_{zf}} \mathbf{h} x_{s_1} + \sqrt{P_2} A_r B_r \mathbf{f}^H \mathbf{W_{zf}} \mathbf{f} x_{s_2} \\ &+\; A_r B_r \mathbf{f}^H \mathbf{W_{zf}} \mathbf{n_1} + n_2, \end{aligned} \tag{3}$$

where n_2 denotes the zero-mean CSCG noise at S_2 with variance σ^2, and A_r and B_r are normalization constants designed to ensure that the long-term total transmit power at the relays is constrained, and they are given as

$$A_r = \sqrt{\frac{1}{P_1 \|\mathbf{h}\|^2 + P_2 \|\mathbf{f}\|^2 + \sigma^2}}, B_r = \sqrt{\frac{P_r}{\mathrm{Tr}\left(\mathbf{W_{zf}} \mathbf{W_{zf}^{\mathit{H}}}\right)}}. \tag{4}$$

After removing the self-interference term from (3), the received signal at S_2 becomes

$$y_{S_2}^2 = \sqrt{P_1} A_r B_r \mathbf{f}^H \mathbf{W_{zf}} \mathbf{h} x_{s_1} + A_r B_r \mathbf{f}^H \mathbf{W_{zf}} \mathbf{n_1} + n_2. \tag{5}$$

The combined received signal-to-noise ratio (SNR) at S_2 is

$$\gamma_{eq}^{\text{2-TS}} = \frac{P_1 A_r^2 B_r^2 |\mathbf{f}^H \mathbf{W_{zf}} \mathbf{h}|^2}{A_r^2 B_r^2 |\mathbf{f}^H \mathbf{W_{zf}}|^2 \sigma^2 + \sigma^2}. \tag{6}$$

Similarly, the total received SNR at S_1 is obtained with the notations interchanged. Hereafter, since the analysis is the same for S_1 and S_2, we consider only S_2.

III. Sub-optimal ZFB Weights Design

Our objective here is to maximize the received SNRs at the two transceivers in order to enhance the performance of the secondary system while limiting the interference reflected on the PUs. To be able to apply ZFB, the general assumption $L_s > M$ is considered. According to the ZFB principles, the transmit weight vectors $\mathbf{w_{zf_1}}$, $\mathbf{w_{zf_2}}$ are chosen to lie in the orthogonal space of $\mathbf{G}_{\mathbf{rp}}^H$ such that $|\mathbf{g}_{\mathbf{r},\mathbf{p_i}}^H \mathbf{w_{zf_1}}| = 0$ and $|\mathbf{g}_{\mathbf{r},\mathbf{p_i}}^H \mathbf{w_{zf_2}}| = 0$, $\forall i = 1, ..., M$ and $|\mathbf{h}^H \mathbf{w_{zf_1}}|$, $|\mathbf{f}^H \mathbf{w_{zf_2}}|$ are maximized. So the problem formulation for finding the optimal weight vectors is divided into two parts as follows.

$$\begin{aligned} \max_{\mathbf{w_{zf_1}}} \quad & |\mathbf{h}^H \mathbf{w_{zf_1}}|^2 \\ \text{s.t.:} \quad & |\mathbf{g}_{\mathbf{r},\mathbf{p_i}}^H \mathbf{w_{zf_1}}| = 0, \quad \forall i = 1, ..., M \\ & \|\mathbf{w_{zf_1}}\| = 1. \end{aligned} \tag{7}$$

$$\begin{aligned} \max_{\mathbf{w_{zf_2}}} \quad & |\mathbf{f}^H \mathbf{w_{zf_2}}|^2 \\ \text{s.t.:} \quad & |\mathbf{g}_{\mathbf{r},\mathbf{p_i}}^H \mathbf{w_{zf_2}}| = 0, \quad \forall i = 1, ..., M \\ & \|\mathbf{w_{zf_2}}\| = 1. \end{aligned} \tag{8}$$

By applying a standard Lagrangian multiplier method, the weight vectors that satisfy the above optimization methods are given as

$$\mathbf{w_{zf_1}} = \frac{\mathbf{\Xi}^\perp \mathbf{h}}{\|\mathbf{\Xi}^\perp \mathbf{h}\|}, \ \mathbf{w_{zf_2}} = \frac{\mathbf{\Xi}^\perp \mathbf{f}}{\|\mathbf{\Xi}^\perp \mathbf{f}\|}, \tag{9}$$

where $\mathbf{\Xi}^\perp = \left(\mathbf{I} - \mathbf{G_{rp}}(\mathbf{G_{rp}}^H \mathbf{G_{rp}})^{-1} \mathbf{G_{rp}}^H\right)$ is the projection idempotent matrix with rank $(L_s - M)$ [1]. It can be observed from the rank of the matrix that the cooperative ZBF beamformer becomes effective only when $L_s > M$. Since each relay knows the CSI of the channels between itself and both secondary sources and between itself and the primary receivers, the ZFB matrix $\mathbf{W_{zf}}$ is made up by the diagonal of the product of the two ZFB vectors $\mathbf{w_{zf_1}}$ and $\mathbf{w_{zf_2}}$ which is represented as [6], [7] and references therein

$$\mathbf{W_{zf}} = (\mathbf{w_{zf_1}} \mathbf{w_{zf_2}}^T). \tag{10}$$

IV. Statistics of end-to-end $\gamma_{eq}^{\text{2-TS}}$

In the underlay approach of this model, the secondary source can utilize the PU's spectrum as long as the interference it generates at the PUs remains below the interference threshold Q_j, $\forall j = 1, 2$. For that reason, P_j is constrained as $P_j = \min\left\{\frac{Q_j}{|h_{s_j,p}|^2}, P_{s_j}\right\}$ where P_{s_j} is the maximum transmission power of S_j. We focus on the analysis of the case $\left(P_{s_j} \geq \frac{Q_j}{|h_{s_j,p}|^2}\right)$ as it determines the effect of the peak power constraint in the first time-slot on the performance of the secondary system. So the transmit powers P_1 and P_2 are constrained as $P_1 \leq \frac{Q_1}{|h_{s_1,p}|^2}$ and $P_2 \leq \frac{Q_2}{|h_{s_2,p}|^2}$.
Substituting (4), (10) and values of P_1, P_2 into (6), and after few manipulations, the equivalent SNR at S_2 can be written in the general form of $\gamma_{eq}^{\text{2-TS}} = \frac{\gamma_1 \ \gamma_3}{\gamma_1+\gamma_2+\gamma_3+1}$ as:

$$\gamma_{eq}^{\text{2-TS}} = \frac{\gamma_{q_1} \frac{\|\mathbf{\Xi}^\perp \mathbf{h}\|^2}{|h_{s_1,p}|^2} \gamma_r \left\|\mathbf{\Xi}^\perp \mathbf{f}\right\|^2}{\gamma_{q_1} \frac{\|\mathbf{\Xi}^\perp \mathbf{h}\|^2}{|h_{s_1,p}|^2} + \gamma_{q_2} \frac{\|\mathbf{\Xi}^\perp \mathbf{f}\|^2}{|h_{s_2,p}|^2} + \gamma_r \left\|\mathbf{\Xi}^\perp \mathbf{f}\right\|^2 + 1}, \tag{11}$$

where $\gamma_r = \frac{P_r}{\sigma^2}$, $\gamma_{q_1} = \frac{Q_1}{\sigma^2}$ and $\gamma_{q_2} = \frac{Q_2}{\sigma^2}$.
We first find the statistics of the new random variables defined above. Then, we compute the CDF and MGF of $\gamma_{eq}^{\text{2-TS}}$ which will be used for the derivation of the performance metrics. Let $\gamma_1 = \gamma_{q_1} \frac{\|\mathbf{\Xi}^\perp \mathbf{h}\|^2}{|h_{s_1,p}|^2}$, $\gamma_2 = \gamma_{q_2} \frac{\|\mathbf{\Xi}^\perp \mathbf{f}\|^2}{|h_{s_2,p}|^2}$ and $\gamma_3 = \gamma_r \left\|\mathbf{\Xi}^\perp \mathbf{f}\right\|^2$.

Lemma 1: (PDFs of γ_1 and γ_2 and CDF of γ_3): Let each entry of $\mathbf{h}$ and $\mathbf{f}$ be i.i.d. $\mathcal{CN} \sim (0,1)$, then $\left\|\mathbf{\Xi}^\perp \mathbf{h}\right\|^2$ and $\left\|\mathbf{\Xi}^\perp \mathbf{f}\right\|^2$ are chi-squared random variables with $2(L_s - M)$ degrees of freedom. Given that $|h_{s_1,p}|^2$ and $|h_{s_2,p}|^2$ are exponential random variables, The CDF of γ_3 and the PDFs of $f_{\gamma_i}(\gamma)$, $i = 1, 2$ are given respectively by [1]:

$$F_{\gamma_3}(\gamma) = 1 - \frac{\Gamma\left(L_s - M, \frac{\gamma}{\gamma_r}\right)}{(L_s - M - 1)!}, \ \gamma \geq 0. \tag{12}$$

$$f_{\gamma_i}(\gamma) = \frac{\lambda_{s_i,p}(L_s - M + 1)\gamma^{L_s - M}}{\gamma_{q_i}^{L_s - M}(\frac{\gamma}{\gamma_{q_i}} + \lambda_{s_i,p})^{L_s - M + 2}}, \ i = 1, 2. \tag{13}$$

In the subsequent sections, we consider the statistics of the random variable $\gamma_{eq}^{\text{2-TS}}$ defined by $\gamma_{eq}^{\text{2-TS}} = \frac{\gamma_1 \ \gamma_3}{\gamma_1+\gamma_2+\gamma_3}$ [1], which can be considered as a tractable tight upper bound to the actual equivalent SNR.

V. Outage Probability Analysis

An outage event occurs when $\gamma_{eq}^{\text{2-TS}}$ falls below a certain threshold γ_{th}, which can be characterized mathematically as

$$P_{out}^{\text{2-TS}} = \Pr(\gamma_{eq}^{\text{2-TS}} < \gamma_{th}) = F_{\gamma_{eq}^{\text{2-TS}}}(\gamma_{th}). \tag{14}$$

Theorem 1: A closed-form expression for the outage probability in the 2-TS protocol for a two-way AF relaying in spectrum-sharing system is given by

$$\begin{aligned} P_{out}^{\text{2-TS}} &= 1 - b \sum_{m=0}^{L_s - M - 1} \sum_{k=0}^{m} \sum_{r=0}^{k} \frac{(\gamma_{th})^{k-r}}{m!} \binom{m}{k} \binom{k}{r} a^{\frac{r}{2}} \\ &\times e^{-\frac{\gamma_{th}}{\gamma_r}} \frac{2^{\frac{-2L_s + 2M - 2 - r}{2}}}{\sqrt{2\pi}} \sqrt{\frac{a\gamma_{th}}{\gamma_r}} \left(\frac{\gamma_{th}}{\gamma_r}\right)^{m - \frac{3r}{2}} \\ &\times \sum_{p=0}^{L_s - M} \binom{L_s - M}{p} \gamma_{th}^{L_s - M - p} \\ &\times \sum_{s=0}^{N} \frac{1}{s!} \left(\frac{-\gamma_{th}^2}{\gamma_r} + \frac{\gamma_{th} a}{\gamma_r}\right)^s \frac{2^{L_s - M + 1}(\gamma_{th} + a)^\mu}{2\pi\Gamma(L_s - M + 2)} \\ &\times G_{6,4}^{2,6}\left(\frac{4\gamma_r^2(\gamma_{th} + a)^2}{(a\gamma_{th})^2} \middle|_{\Delta(2,(L_s+1)-\alpha_o),\Delta(1,1-a_r)}^{\Delta(2,1-\alpha_o),\Delta(1,1-b_r)}\right), \end{aligned} \tag{15}$$

where $a = \lambda_{s_2,p}\gamma_{q_2}$, $b = (L_s - M + 1)\Gamma(r + L_s - M + 1)$, $a_r = (\frac{1}{4} - \frac{1}{4}(-2L_s + 2M - 2 - r), \ \frac{3}{4} - \frac{1}{4}(-2L_s + 2M - 2 - r))$, $b_r = (\frac{1}{2} + \frac{1}{4}(1-r), \ \frac{1}{2} - \frac{1}{4}(1-r), \ \frac{1}{4}(1-r), \ \frac{-1}{4}(1-r))$, $\mu_o = p - k + \frac{3r}{2} - s - L_s + M - \frac{3}{2}, \alpha_o = \mu_o + L_s - M + 2, \Delta(i, a) = \frac{a}{i}, \frac{a+1}{i}, ..., \frac{a-i+1}{i}$ and $G_{:,:}^{:,:}(.|.)$ is the Meijer's G-function defined in [8].

Proof: To derive the outage probability of $\gamma_{eq}^{\text{2-TS}}$, conditioned on γ_1 and γ_2, we first express the CDF of $\gamma_{eq}^{\text{2-TS}}$ as

$$\begin{aligned} F_{\gamma_{eq}^{\text{2-TS}}}(\gamma_{th}) &= \int_0^\infty \Pr\left(\gamma_3 < \frac{\gamma_{th}(y+z)}{y - \gamma_{th}}\right) \\ &\times f_{\gamma_1}(y) f_{\gamma_2}(z) dy dz. \end{aligned} \tag{16}$$

Using variable change, $w = y - \gamma_{th}$, and after some algebraic manipulations, we have

$$F_{\gamma_{eq}^{\text{2-TS}}}(\gamma_{th}) = 1 - \int_0^\infty \Pr\left(\gamma_3 \geq \frac{\gamma_{th}(w+\gamma_{th}+z)}{w}\right) \times f_{\gamma_1}(w+\gamma_{th}) f_{\gamma_2}(z) dw dz. \quad (17)$$

Before proceeding in the derivation, the complementary of the CDF of γ_3, is expressed in another mathematical form using [8, eq. 8.352.2] and [8, eq. 1.111], then substituting it into (17) with some mathematical manipulations, we have

$$F_{\gamma_{eq}^{\text{2-TS}}}(\gamma_{th}) = 1 - \sum_{m=0}^{Ls-M-1} \sum_{k=0}^{m} \sum_{r=0}^{k} \frac{1}{m!} \left(\frac{\gamma_{th}}{\gamma_r}\right)^{m-k} \binom{m}{k} \times \binom{k}{r} e^{-\frac{\gamma_{th}}{\gamma_r}} \int_0^\infty \left(\frac{\gamma_{th}^2}{w\gamma_r}\right)^{k-r} f_{\gamma_1}(w+\gamma_{th}) \times \underbrace{\left(\int_0^\infty \left(\frac{\gamma_{th} z}{w\gamma_r}\right)^r e^{-\frac{z\gamma_{th}}{w\gamma_r}} f_{\gamma_2}(z) dz\right)}_{I_1} dw. \quad (18)$$

The inner integral I_1 can be solved using the variable change, $u = z + \lambda_{s_2,p}\gamma_{q_2}$, and using [8, eq. 3.383.4], leading to

$$I_1 = a^{\frac{r}{2}}\, b \left(\frac{\gamma_{th}}{w\gamma_r}\right)^{\frac{-r}{2}} e^{\frac{a\gamma_{th}}{2w\gamma_r}} W_{\frac{-2L_s+2M-2-r}{2}, \frac{1-r}{2}}\left(\frac{a\gamma_{th}}{w\gamma_r}\right), \quad (19)$$

where $W_{.,.}(.)$ is the Whittaker function [8].
Substituting the results in (19) and representing the exponential term using Taylor series representation [8, eq. 1.211.1], apply the binomial theorem [8, eq. 1.11.1] for the term $(w+\gamma_{th})^{Ls-M}$ and express the Whittaker function in terms of Meijer's G-function using [8, eq. 9.34.9] and [8, eq. 9.31.2], which after many manipulations results in

$$F_{\gamma_{eq}^{\text{2-TS}}}(\gamma_{th}) = 1 - \sum_{m=0}^{Ls-M-1} \sum_{k=0}^{m} \sum_{r=0}^{k} \frac{(\gamma_{th})^{k-r}}{m!} \binom{m}{k} \binom{k}{r} \times b\, a^{\frac{r+1}{2}} e^{-\frac{\gamma_{th}}{\gamma_r}} \frac{2^{\frac{-2Ls+2M-2-r}{2}}}{\sqrt{2\pi}} \left(\frac{\gamma_{th}}{\gamma_r}\right)^{m-\frac{3r+1}{2}} \times \left(\frac{-\gamma_{th}^2 + \gamma_{th} a}{\gamma_r}\right)^s \int_0^\infty \left(\frac{w^{p-k+\frac{3r}{2}-s-\frac{1}{2}}}{(w+\gamma_{th}+a)^\theta} \times \left(G_{4,2}^{0,4}\left(\frac{4\gamma_r^2 w^2}{(a\gamma_{th})^2}\Big|_{1-a_r}^{1-b_r}\right)\right)\right) dw. \quad (20)$$

where $\theta = L_s - M + 2$. The integral in (20) is solved using [10, eq. 2.24.2.4, vol. 3], then after few simplifications, *the outage probability* is expressed as in (15), thus completing the proof.

VI. Average error probability Analysis

Theorem 2: A closed-form expression for the average error probability in the 2-TS protocol for a two-way AF relaying based distributed ZFB in spectrum-sharing system is given by

$$P_e^{\text{2-TS}} = \frac{1}{2} - \frac{1}{2\sqrt{\pi}} \bar{\delta} \sum_{m=0}^{M-L_s-2} \frac{1}{c^m m!} \left(\frac{1}{A}\right)^{v+\frac{3}{4}} \times G_{4,4}^{4,1}\left(\frac{b^2}{A}\Big|_{L_s-M,0,0,-L_s+M}^{-v-\frac{1}{4},0,\frac{1}{2},-v+\frac{1}{4}}\right), \quad (21)$$

where $\bar{\delta} = \delta(M - L_s - 2)!, \delta = \frac{4(L_s-M+1)\gamma_r^{-(L_s-M)}}{c^{L_s-M+1}(\Gamma(L_s-M))^2}, v = 2L_s - 2M + m + \frac{3}{4}$, $A = 1$ for binary phase shift keying (BPSK) modulation scheme, $c = \lambda_{s_1,p}\gamma_{q_1}$ and $b = \frac{2}{\sqrt{\gamma_r}}$.

Proof: In order to obtain the average error probability for the secondary system, the MGF based approach will be used in this paper. Let $(\gamma_{eq}^{\text{2-TS}})^{-1} = \gamma_1^{-1} + \frac{\gamma_2}{\gamma_1\, \gamma_3} + \gamma_3^{-1} = X_1 + X_2 + X_3$ where $X_1 = \gamma_1^{-1}$, $X_2 = \frac{\gamma_2}{\gamma_1\, \gamma_3}$ and $X_3 = \gamma_3^{-1}$. As $(\gamma_{eq}^{\text{2-TS}})^{-1}$ is the sum of three independent random variables, the MGF of the $(\gamma_{eq}^{\text{2-TS}})^{-1}$, denoted by $M(s)$, results simply from the product of the three MGFs of X_1, X_2 and X_3, which is given as[1]

$$M(s) = \delta s^{2L_s-2M+1} e^{\frac{s}{c}} \Gamma\left(-L_s+M-1, \frac{s}{c}\right) \times \left(K_{L_s-M}\left(2\sqrt{\frac{s}{\gamma_r}}\right)\right)^2. \quad (22)$$

Despite seeming difficult, we use the following formula to compute the MGF of the $\gamma_{eq}^{\text{2-TS}}$, denoted by $\phi_{\gamma_{eq}^{\text{2-TS}}}(s)$, exploiting the MGF of $(\gamma_{eq}^{\text{2-TS}})^{-1}$ [9, Eq. 18]

$$\phi_{\gamma_{eq}^{\text{2-TS}}}(s) = 1 - 2\sqrt{s} \int_0^\infty J_1(2\beta\sqrt{s}) M(\beta^2) d\beta, \quad (23)$$

where $J_1(.)$ is the Bessel function of the first kind [8]. Utilizing the MGF-based form, the average error probability of coherent binary signaling is given by

$$P_e^{\text{2-TS}} = \frac{1}{\pi} \int_0^{\pi/2} \phi_{\gamma_{eq}^{\text{2-TS}}}\left(\frac{A}{\sin^2\varphi}\right) d\varphi, \quad (24)$$

where $A = 1$ for BPSK. Substituting (23) into (24) and after some manipulations, the formula of the error probability becomes

$$P_e^{\text{2-TS}} = \vartheta \int_0^\infty M(\beta^2) \int_0^{\frac{\pi}{2}} \sqrt{\frac{A}{\sin^2\varphi}} J_1\left(\sqrt{\frac{4\beta^2 A}{\sin^2\varphi}}\right) d\varphi d\beta, \quad (25)$$

Where $\vartheta = \frac{1}{2} - \frac{2}{\pi}$. The inner integral of (25) can be solved by using the variable change and equation [10, eq. 2.12.4.15] which results in the value $\frac{\sin(2\beta\sqrt{A})}{2\beta}$. So (25) simplifies to

$$P_e^{\text{2-TS}} = \frac{1}{2} - \frac{2}{\pi} \underbrace{\int_0^\infty M(\beta^2) \frac{\sin(2\beta\sqrt{A})}{2\beta} d\beta}_{I_2}, \quad (26)$$

To continue, we make use of the identity

$$\sin(2\beta\sqrt{A}) = \sqrt{\pi\beta\sqrt{A}}\; J_{\frac{1}{2}}(\sqrt{4\beta^2}\sqrt{A}). \quad (27)$$

[1] Note that many details in the proofs are omitted for space limitation.

Expressing Gamma function in (22) in another mathematical form using [8, eq. 8.352.2] and incorporating it with (27) into (26), I_2 becomes as

$$I_2 = \int_0^\infty \delta(M-L_s-2)! \frac{\sqrt{\pi\sqrt{A}}}{2} \sum_{m=0}^{M-L_s-2} \frac{1}{c^m m!} \times \beta^{2\varsigma} J_{\frac{1}{2}}(\sqrt{4\beta^2\sqrt{A}}) \left(K_{L_s-M}\left(2\sqrt{\frac{\beta^2}{\gamma_r}}\right)\right)^2 d\beta \quad (28)$$

where $\varsigma = (2L_s - 2M + m + \frac{3}{4})$. By solving I_2 in (28) using [11], we get (21). This completes the proof. ■

VII. Numerical Results and Discussion

In this section, we investigate the performance of some of the derived results through numerical examples and simulations. Unless otherwise stated, We assume that the relays are located on a straight line vertical to the distance between the two sources. The distance between the sources equals one. Furthermore, the path loss exponents is set to four. We also assume that $\lambda_{s_1,p} = \lambda_{s_2,p} = 1$ and fixed $\alpha = 0.5$.

Figs. 2 show the outage performance of S_2 versus Q_1 for $L_s = 6, 8, 10$, $M = 1, 2$ at $\gamma_{th} = 1$ dB, $\gamma_{q_2} = -2$ dB, and $\gamma_r = 10$ dB. As observed from the figures, as the value of Q_1 increases, the outage performance improves substantially. Moreover, by increasing the number of relays with ZFB, we observe significant improvements in the outage performance. This is attributed to the combined cooperative diversity and beamforming which enhances the total received SNR at the receiver. Clearly, as the number of existing PUs increases from one to two, the outage performance becomes worse because the secondary sources have to adapt their transmit powers according to the most affected PU.

Figs. 3 illustrate the average error probability performance versus $Q_1 = Q_2 = Q$ for L_s= 6, 8, 10 and $M = 1, 2, 3$, $\gamma_r = 5$ dB at $\gamma_{th} = 1$ dB. It is obvious that the average error probability performance improves substantially as the number of relays increases and Q becomes looser. With beamforming and increasing the number of relays, the gain becomes more. The larger the number of existing PUs, the worse the error probability, as expected.

VIII. Conclusion

We investigated a cooperative two-way AF relaying based distributed ZFB in a spectrum sharing environment. The proposed system limits the the interference to the primary users using a distributed ZFB approach and peak interference power constraints. We analyzed the performance of the secondary system by deriving the outage and error probabilities. Our numerical results showed that the combination of the distributed ZFB and the cooperative diversity enhances the system performance in addition to limiting interference to PUs.

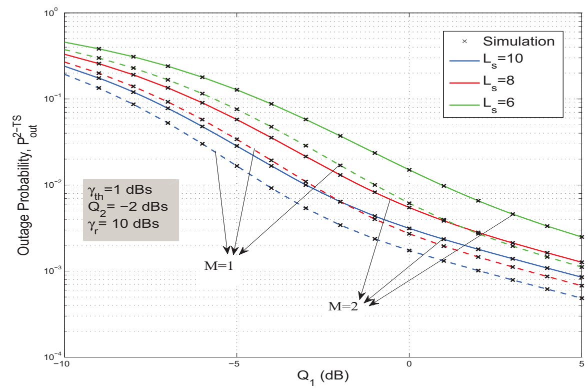

Fig. 2: Outage probability vs. Q_1 (dB) for L_s= 6, 8, 10 and M=1, 2

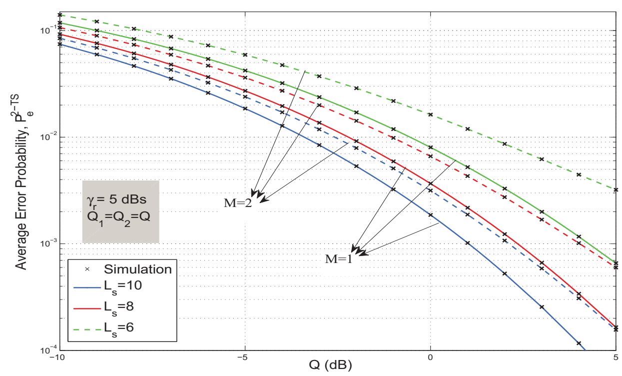

Fig. 3: Average error probability vs. Q (dB) for L_s=6, 8, 10 and M=1, 2.

References

[1] A. Afana, V. Asghari, A. Ghrayeb and S. Affes, "Enhancing the performance of spectrum-sharing systems via collaborative distributed beamforming and AF relaying," in *Proc. IEEE GLOBECOM*, Dec., 2012.

[2] Q. Li, S. H. Ting, A. Pandharipande, and Y. Han, "Cognitive spectrum sharing with two-way relaying systems," *IEEE Trans. Veh. Technol.,* vol. 60, no. 3, pp. 1233−1240, Mar. 2011.

[3] L. Yang, M. S. Alouini and K. Qaraqe, "On the performance of spectrum sharing systems with two-way relaying and multiuser diversity," *IEEE Commun. Lett.,* vol.16, no.8, pp.1240−1243, August 2012.

[4] K. Zarifi, S. Affes and A. Ghrayeb, "Joint source power control and relay beamforming in amplify-and-forward cognitive networks with multiple source-destination pairs," in *Proc. IEEE ICC,*pp.5−9 Jun. 2011.

[5] K. Zarifi, A. Ghrayeb and S. Affes, "Jointly optimal source power control and relay matrix design in multipoint-to-multipoint cooperative communication networks, *IEEE Trans. on Signal Process.,* vol. 59, no. 9, pp. 4313− 4330, Sept. 2011.

[6] K. Jitvanichphaibool, Y.-C. Liang, and R. Zhang, "Beamforming and power control for multi-antenna cognitive two-way relaying," in *Proc. IEEE Wireless Communications and Networking Conf. WCNC,* 2009.

[7] S.H. Safavi, M. Ardebilipour and S. Salari, "Relay beamforming in cognitive two-way networks with imperfect channel state information," *IEEE Wireless Commun. Lett.*, vol.1, no.4, pp.344−347, August 2012.

[8] I. S. Gradshteyn, I. M. Ryzhik, A. Jeffrey, and D. Zwillinger, *Table of integrals, series and products*, 7th ed. Elsevier, 2007.

[9] M. Di Renzo, F. Graziosi, and F. Santucci, "A unified framework for performance analysis of CSI-assisted cooperative communications over fading channels," *IEEE Trans. Commun.,* vol.57, no.9, Sep. 2009.

[10] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series, Volume 2,3: Special Functions,* Second Printing with corrections. Gordon and Breach Science Publishers, 1988.

[11] I. Trigui, S. Affes and A. Stephenne, "Closed-form error analysis of variable-gain multihop systems in Nakagami-m fading channels," *IEEE Trans. commun.*, vol.59, no.8, pp.2285−2295, August 2011.