Distributed Collaborative Beamforming with Minimum Overhead for Local Scattering Environments

(Invited Paper)

Slim Zaidi and Sofiène Affes
INRS-EMT, 800, de la Gauchetière West, Suite 6900, Montréal, H5A 1K6, Qc, Canada,
Email: {zaidi, affes}@emt.inrs.ca

***Abstract*—In this paper, transmit and receive collaborative beamforming (CB) techniques are considered to achieve a dual-hop communication from a source to a receiver, through a wireless network comprised of K independent terminals. Whereas the previous works assumed a model of plane wavefronts, here, a local scattering in the source or receiver vicinity is considered, thereby broadening the range of applications in real-world environments. Taking into account the local scattering, these CB techniques aim to maintain the beamforming response in the desired direction equal to unity. It is shown that the so-obtained collaborative beamformers are not suitable for a distributed implementation. We hence propose a novel beamformer solution that can be implemented in a distributed fashion and, further, well-approximates both transmit and receive collaborative beamformers. The performance of the proposed distributed CB (DCB) technique is analyzed and its advantages against the conventional DCB technique, which is designed without taking into account the presence of local scattering in the source or receiver vicinity, are analytically proved and further verified by simulations.**

***Index Terms*—distributed collaborative beamforming, local scattering, device/machine-2-device/machine (D2D/M2M) communications, wireless sensor networks (WSN).**

I. Introduction

The implementation of the collaborative beamforming (CB) technique can increase the transmission coverage, the link reliability, and the capacity of wireless networks [1]-[9]. Using this technique, a set of K independent terminals (mobile users, soldiers in battlefield, sensor nodes, relays, etc) play a central role in the signal transmission flow. Indeed, these terminals or devices (in D2D or M2M communications) multiply their received signals from the source with the complex conjugates of properly selected beamforming weights, and forward the resulting signals to the receiver. How to select these weights? This is an active subject of research. So far, several approaches have been proposed such as minimizing the total transmit power subject to the received quality of service constraint, maximizing the received signal-to-noise ratio (SNR) subject to two different types of power constraints, namely the total transmit power constraint and individual terminal power constraint, or fixing the beamforming response in the desired direction to unity. Due to its practical potential, the CB technique has garnered the attention of the research community. Assuming that the terminals are uniformly distributed, the conventional DCB technique has been presented in [1] and the characteristics of its resultant beampattern have been analyzed. Beampattern characteristics of the conventional DCB technique have been also evaluated in [2] when the terminals are Gaussian distributed. To enhance the beampattern proprieties, terminal selection algorithms aiming to narrow down the mainbeam and minimize the effect of sidelobes have been, respectively, presented in [3] and [4]. In [5] and [6], CB techniques that improve the energy efficiency have been proposed. A review on different CB techniques wherein properly selected weights achieve the design objective while satisfying the design constraints has been presented in [7]. The selected weights must often comply with the restrictions dictated by the network structure. For instance, when a CB technique is used in a wireless network that lacks a master terminal (MT) with a global knowledge of all network parameters, the terminals typically require to locally compute their weights based only on their limited knowledge about the network. This is also the case when the MT is available to compute all weights but the overhead associated with sending them to all terminals is prohibitive. This impediment motivates more investigation in this direction of research. Lending themselves to a distributed implementation, a variety of so-called distributed CB (DCB) techniques, wherein the selected weights solely depend on the information commonly available at every terminal and, hence, each is able to locally compute its own weight, have been proposed in [8] and [9].

Work supported by a Canada Research Chair in Wireless Communications and the Discovery Grants Program of NSERC.

In spite of their significant contributions, all the above works neglect the effect of the scattering and reflection and assume a simple model of plane wavefronts. Unfortunately, in practice, the propagation environments are often more complicated than this model. In fact, when the source or the receiver is scattered by a large number of scatterers within its vicinity, as in rural and suburban environments, several replicas of the transmit or receive signal are generated [10]- [13]. In such a case, the signal can be modeled as a superposition of independent, and identically distributed (i.i.d.) rays [10]. Commonly known as local scattering, the effect of

978-1-4577-1379-8/12/$26.00 ©2012 IEEE

this phenomenon on the CB technique was investigated in [11]. It has been shown that the performance of CB techniques, designed without taking into account the presence of local scattering, are deteriorated. Obviously, the performance of the CB techniques proposed in the above contributions may be acceptable in rural environments, but they become more and more unsatisfactory in suburban environments wherein local scattering is relatively important. Hence, the aim of this work is to go one significant step forward by pushing the frontier of the DCB real-world applicability to include both rural and suburban environments.

In this paper, we consider both transmit and receive CB schemes that aim to fix the beamforming response in the desired direction to unity. Depending on the scheme, the source S or the receiver is assumed to be scattered by a large number of scatterers within its vicinity that generate L i.i.d rays from the transmit or the receive signal. Taking into account this phenomenon, the beamforming vectors corresponding to both beamformers are derived. Unfortunately, it is shown that the so-obtained beamformers are not suitable for a distributed implementation. Using the fact that the number of terminals could be typically large in practice [8], [9], we propose a new distributed collaborative beamformer that not only can be implemented in a distributed fashion but also, well-approximates its transmit and receive collaborative beamformer counterparts. The performance of the proposed DCB technique is analyzed and its advantages against the conventional DCB technique, which is designed without taking into account the presence of local scattering in the source or receiver vicinity, is proved. It is shown that the maximum achievable SNR is performed by the proposed DCB technique even in suburban environments where the local scattering is relatively important, while the performance of the conventional DCB technique decreases in rural environments and becomes unsatisfactory in suburban environments. It is also proved that the proposed DCB technique is able to achieve until 3 dB of SNR gain against its conventional vis-a-vis.

The rest of this paper is organized as follows. The system model is described in Section II. The CB techniques in the presence of local scattering is described in Section III. A novel DCB solution is proposed in Section IV. Section V analyzes the performance of the proposed technique while Section VI verifies by computer simulations the theoretical results. Concluding remarks are given in Section VII.

Notation: Uppercase and lowercase bold letters denote matrices and vectors, respectively. $[\cdot]_{il}$ and $[\cdot]_i$ are the (i,l)-th entry of a matrix and i-th entry of a vector, respectively. $\mathbf{I}$ is the identity matrix and $\mathbf{e}_l$ is a vector with one in the l-th position and zeros elsewhere. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the Hermitian transpose, respectively. $\|\cdot\|$ is the 2-norm of a vector and $|\cdot|$ is the absolute value. $\mathrm{E}\{\cdot\}$ stands for the statistical expectation and ($\xrightarrow{ep1}$) $\xrightarrow{p1}$ denotes (element-wise) convergence with probability one. $J_1(\cdot)$ is the first order Bessel function of the first kind and $\odot$ is the element-wise product.

II. System model

As can be observed form Fig. 1, in this work, both receive and transmit CB schemes are of concern. As illustrated in Fig. 1.a, the system of interest in the receive configuration consists of a wireless network or subnetwork comprised of K uniformly and independently distributed terminals on $D(O,R)$, the disc with center at O and radius R, a receiver at O, and a source S located in the same plane containing $D(O,R)$ [1]- [8]. We assume that there is no direct link from the source to the receiver due to pathloss attenuation. Moreover, let (r_k,ψ_k) denote the polar coordinates of the k-th terminal and (A_s,ϕ_s) denote those of the source. The latter is assumed to be at $\phi_s=0$, without loss of the generality, and to be located in the far-field region, hence, $A_s \gg R$. Description of the transmit configuration in Fig. 1.b is straightforward from the previous, where only the source and receiver switch positions.

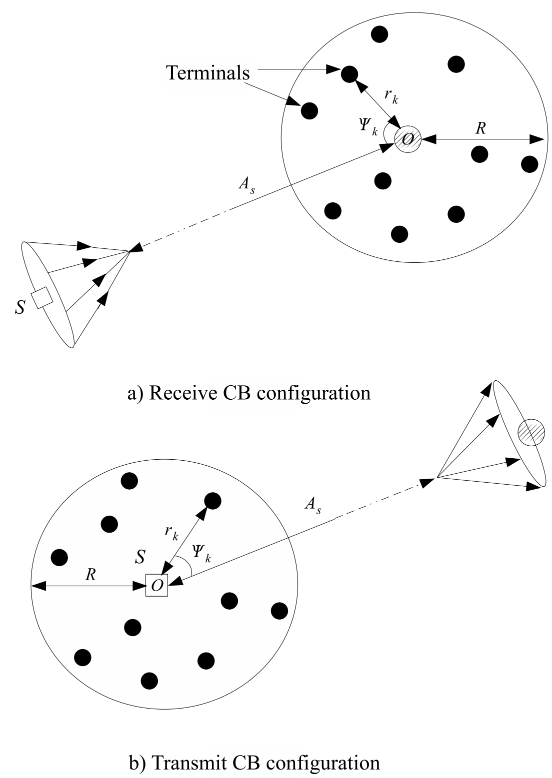

Fig. 1. Receive and transmit system configurations.

The following assumptions are further considered with respect to both configurations in Fig. 1.a and Fig. 1.b:

A1) The far-field source or receiver, respectively, is scattered by a large number of scatterers within its vicinity that generate L equal-power rays [10]-[13]. The l-th ray is characterized by its direction θ_l and its complex amplitude $\alpha_l = \rho_l e^{j\xi_l}$ where the amplitudes $\rho_l,\ l=1,\ldots,L$ and the phases $\xi_l,\ l=1,\ldots,L$ are i.i.d. random variables, and each phase is uniformly distributed over $[-\pi,\pi]$. The

θ_l, $l = 1, \ldots, L$ are also i.i.d. random variables distributed with variance σ_θ^2 and probability density functions (pdf) $p(\theta)$. All θ_ls, ξ_ls, and ρ_ls are mutually independent [10]-[13].

A2) The channel gain $[\mathbf{f}]_k$ between the k-th terminal and the receiver or the source, respectively, is a zero-mean unit-variance circular Gaussian random variable [8].

A3) The source signal s is a zero-mean random variable with power p_s while noises at terminals and the receiver are zero-mean Gaussian random variables with variances ${\sigma_v}^2$ and ${\sigma_n}^2$, respectively. The source signal, noises, and the terminals forward channel gains are mutually independent.

A4) The k-th terminal is aware of its own coordinates (r_k, ψ_k), its forward channel $[\mathbf{f}]_k$, the directions of the source ϕ_s, K, and σ_θ^2 while being oblivious to the locations and the forward channels of *all* other terminals in the network.

Using A1 and the fact that $A_s \gg R$, the channel gain between the k-th terminal and the source or the receiver, respectively, can be represented as

$$[\mathbf{g}]_k = \sum_{l=1}^{L} \alpha_l e^{-j\frac{2\pi}{\lambda} r_k \cos(\theta_l - \psi_k)} \tag{1}$$

where λ is the wavelength.

III. CB Techniques in the Presence of Local Scattering

A. Receive CB Configuration

In this scheme, a dual-hop communication is established from the source S to the receiver. In the first time slot, the source sends its signal s to the wireless network. Let $\mathbf{y}$ denotes the received signal vector at the terminals given by

$$\mathbf{y} = \mathbf{g}s + \mathbf{v}, \tag{2}$$

where $\mathbf{v}$ is the terminals' noise vector. In the second time slot, the k-th terminal multiplies its received signal with the complex conjugate of the beamforming weight w_k and forwards the resulting signal to the receiver. It follows from (2) that the received signal at O is

$$\begin{aligned} r &= \mathbf{f}^T(\mathbf{w}^* \odot \mathbf{y}) + n = \mathbf{w}^H(\mathbf{f} \odot \mathbf{y}) + n \\ &= \mathbf{w}^H(\mathbf{f} \odot \mathbf{g}s + \mathbf{f} \odot \mathbf{v}) + n \\ &= s\mathbf{w}^H\mathbf{h} + \mathbf{w}^H(\mathbf{f} \odot \mathbf{v}) + n, \end{aligned} \tag{3}$$

where $\mathbf{w} \triangleq [w_1 \ldots w_K]$ is the beamforming vector, $\mathbf{h} \triangleq \mathbf{f} \odot \mathbf{g}$, $\mathbf{f} \triangleq [[\mathbf{f}]_1 \ldots [\mathbf{f}]_K]^T$, and n is the receiver noise.

As mentioned above, several approaches can be adopted to properly select the beamforming weights. In this paper, we are only concerned with the technique that aims to maintain the beamforming response in the source direction equal to unity. In what follows, we design a receive CB technique and discuss its implementability in a distributed fashion. Mathematically, we have to solve the following problem:

$$\mathbf{w}_{\mathrm{r}}^H \mathbf{h} = 1, \tag{4}$$

where $\mathbf{w}_{\mathrm{r}}$ is the beamforming vector associated with the receive CB technique. Since the noises at terminals are zero-mean Gaussian random variables, $\mathbf{w}_{\mathrm{r}} = \mathbf{w}_{\mathrm{o}}$ the optimum beamforming vector which satisfies

$$\mathbf{w}_{\mathrm{o}} = \arg\min P_{\mathbf{w},n}^{\mathrm{r}} \qquad \text{s.t.} \qquad \mathbf{w}^H\mathbf{h} = 1, \tag{5}$$

where $P_{\mathbf{w},n}^{\mathrm{r}}$ is the aggregate noise power due to the thermal noise at the receiver and the forwarded noises from the terminals given by

$$P_{\mathbf{w},n}^{\mathrm{r}} = \mathbf{w}^H\mathbf{\Lambda}\mathbf{w} + \sigma_n^2, \tag{6}$$

where $\mathbf{\Lambda} \triangleq \sigma_v^2 \mathrm{diag}\{|[\mathbf{f}]_1|^2 \ldots |[\mathbf{f}]_K|^2\}$. Using (6) in (5) we obtain the following optimization problem

$$\mathbf{w}_{\mathrm{o}} = \arg\min \mathbf{w}^H\mathbf{\Lambda}\mathbf{w} \qquad \text{s.t.} \qquad \mathbf{w}^H\mathbf{h} = 1. \tag{7}$$

Obliviously, since the channel is the sum of random variables, there is no practical purpose from claiming that the optimal solution of (7) is simply $\mathbf{h}$ due to prohibitive overhead required for its instantaneous estimation. Using the fact that $\mathbf{w}^H\mathbf{h} = 1$, one can rewrite (7) as

$$\mathbf{w}_{\mathrm{o}} = \arg\max \frac{\mathbf{w}^H \mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}\mathbf{w}}{\mathbf{w}^H\mathbf{\Lambda}\mathbf{w}} \;\; \text{s.t.} \;\; \mathbf{w}^H \mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}\mathbf{w} = 1 \tag{8}$$

where the expectation is taken with respect to the rays' directions θ_ls and their complex amplitudes α_ls. It can be readily shown that $\mathbf{w}_{\mathrm{o}}$ is the principal eigenvector of the matrix $\mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}$ scaled to satisfy the constraint [14]. However, it can be observed from (8) that $\mathbf{w}_{\mathrm{o}}$ can not be directly derived using the actual form of the latter matrix. Therefore, for the sake of analytical tractability, a useful approximation may be developed. This requires a more in-depth analytical study of the matrix $\mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}$. Using the assumption A1, one can deduce the following property:

$$\mathrm{E}\left\{\alpha_l^*\alpha_m\right\} = \begin{cases} 0 & l \neq m \\ \frac{1}{L} & l = m \end{cases}. \tag{9}$$

Consequently, $\mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\}$ is reduced to

$$\mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\} = \int p(\theta)\mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta, \tag{10}$$

where $\mathbf{a}(\theta) \triangleq [[\mathbf{a}(\theta)]_1 \ldots [\mathbf{a}(\theta)]_K]^T$ with $[\mathbf{a}(\theta)]_k = [\mathbf{f}]_k e^{-j\frac{2\pi}{\lambda} r_k \cos(\theta - \psi_k)}$.

Nevertheless, if σ_θ is relatively small as in typical suburban environments, the relationship between $\mathbf{a}(\theta)$ and θ can be accurately described by the first three non-zero terms of the Taylor series of $\mathbf{a}(\theta)$ at 0 and, hence,

$$\mathbf{a}(\theta) = \mathbf{a} + \mathbf{a}'\theta + \mathbf{a}''\theta^2, \tag{11}$$

where $\mathbf{a} = \mathbf{a}(0)$, and $\mathbf{a}'$ and $\mathbf{a}''$ are, respectively, the first and the second derivatives of $\mathbf{a}(\theta)$ at 0. Therefore,

$$\begin{aligned} \mathrm{E}\left\{\mathbf{h}\mathbf{h}^H\right\} &\approx \mathbf{a}\mathbf{a}^H + \frac{\sigma_\theta^2}{2}\left(\mathbf{a}\mathbf{a}''^H + \mathbf{a}''\mathbf{a}^H + 2\mathbf{a}'\mathbf{a}'^H\right) \\ &\approx \frac{1}{2}\left(\mathbf{a}(\sigma_\theta)\mathbf{a}(\sigma_\theta)^H + \mathbf{a}(-\sigma_\theta)\mathbf{a}(-\sigma_\theta)^H\right). \end{aligned} \tag{12}$$

It is noteworthy that the approximation in (12), previously exploited differently in angular spread and direction of arrival estimation of scattered sources [12], [13], is independent of the pdf $p(\theta)$. Using (12), the optimization problem in (8) can be written as

$$\mathbf{w}_{\rm o} \approx \arg\max \frac{\mathbf{w}^H \boldsymbol{\Xi} \mathbf{w}}{\mathbf{w}^H \boldsymbol{\Lambda} \mathbf{w}} \qquad \text{s.t} \quad \mathbf{w}^H \boldsymbol{\Xi} \mathbf{w} = 2 \tag{13}$$

where $\boldsymbol{\Xi} = \left(\mathbf{a}(\sigma_\theta)\mathbf{a}(\sigma_\theta)^H + \mathbf{a}(-\sigma_\theta)\mathbf{a}(-\sigma_\theta)^H\right)$. It can be shown that $\mathbf{w}_{\rm o} = \mu \boldsymbol{\Lambda}^{-1} \rho_{\max}(\boldsymbol{\Xi})$ where $\rho_{\max}(\boldsymbol{\Xi})$ is the principal eigenvector of the matrix $\boldsymbol{\Xi}$ and μ is the factor chosen such that the constraint in (13) is satisfied. In the sequel, the expression of $\rho_{\max}(\boldsymbol{\Xi})$ is derived.

It is easy to note that the rank of $\boldsymbol{\Xi}$ is inferior or equal to two, which means that this matrix has at most two eigenvectors. In addition, it can be seen that

$$\begin{aligned}\boldsymbol{\Xi}\left(\mathbf{a}(\sigma_\theta)+\mathbf{a}(-\sigma_\theta)\right) = {} & \mathbf{a}(\sigma_\theta)\left(\|\mathbf{a}(\sigma_\theta)\|^2+\mathbf{a}(\sigma_\theta)^H\mathbf{a}(-\sigma_\theta)\right) + \\ & \mathbf{a}(-\sigma_\theta)\left(\|\mathbf{a}(-\sigma_\theta)\|^2+\mathbf{a}(-\sigma_\theta)^H\mathbf{a}(\sigma_\theta)\right),\end{aligned} \tag{14}$$

and

$$\begin{aligned}\boldsymbol{\Xi}\left(\mathbf{a}(\sigma_\theta)-\mathbf{a}(-\sigma_\theta)\right) = {} & \mathbf{a}(\sigma_\theta)\left(\|\mathbf{a}(\sigma_\theta)\|^2-\mathbf{a}(\sigma_\theta)^H\mathbf{a}(-\sigma_\theta)\right) - \\ & \mathbf{a}(-\sigma_\theta)\left(\|\mathbf{a}(-\sigma_\theta)\|^2-\mathbf{a}(-\sigma_\theta)^H\mathbf{a}(\sigma_\theta)\right).\end{aligned} \tag{15}$$

It is direct to show from the definition of $\mathbf{a}(\phi)$ that $\|\mathbf{a}(\sigma_\theta)\| = \|\mathbf{a}(-\sigma_\theta)\|$ and, further, $\mathbf{a}(-\sigma_\theta)^H\mathbf{a}(\sigma_\theta) \approx \mathbf{a}(\sigma_\theta)^H\mathbf{a}(-\sigma_\theta)$ for small σ_θ. Therefore, from (14) and (15), $\mathbf{a}(\sigma_\theta)+\mathbf{a}(-\sigma_\theta)$ and $\mathbf{a}(\sigma_\theta)-\mathbf{a}(-\sigma_\theta)$ are both eigenvectors of $\boldsymbol{\Xi}$ and, in addition, $\rho_{\max}(\boldsymbol{\Xi}) \approx \mathbf{a}(\sigma_\theta)+\mathbf{a}(-\sigma_\theta)$, when σ_θ is relatively small. Consequently, $\mathbf{w}_{\rm o}$ is given by

$$\mathbf{w}_{\rm o} = \frac{\mu}{K}\boldsymbol{\Lambda}^{-1}\left(\mathbf{a}(\sigma_\theta)+\mathbf{a}(-\sigma_\theta)\right) \tag{16}$$

where

$$\mu = \left(\frac{1}{\sigma_v^2} + \frac{\mathbf{a}(\sigma_\theta)^H \boldsymbol{\Lambda}^{-1}\mathbf{a}(-\sigma_\theta)}{K}\right)^{-1}. \tag{17}$$

Note that $\mathbf{w}_{\rm o}$ is valid for any given pdf $p(\theta)$.

Nevertheless, since the terminals are independent entities, the receive CB technique is implementable only if the k-th terminal can locally compute its corresponding beamforming weight $[\mathbf{w}_{\rm o}]_k$ that depends on μ and the k-th entry of $\boldsymbol{\Lambda}^{-1}\left(\mathbf{a}(\sigma_\theta)+\mathbf{a}(-\sigma_\theta)\right)/K$. According to A4, the latter depends solely on the information locally available at the kth terminal while μ is function of all terminals' locations and forward channels and, hence, cannot be computed at each terminal. Therefore, $\mathbf{w}_{\rm o}$ cannot be implemented in a distributed fashion.

B. Transmit CB configuration

In this scheme, a dual-hop communication is also considered from the source S to the receiver. In the first time slot, the source sends its signal s to the terminals while, in the second time slot, the k-th terminal multiplies its received signal with the complex conjugate of the beamforming weight w_k and forwards the resulting signal to the far-field receiver. In order to select w_k for $k = 1 \dots K$, the same criterion as above is used and, hence, we have to solve

$$\mathbf{w}_{\rm t} = \arg\min P_{\mathbf{w},n}^{\rm t} \qquad \text{s.t.} \qquad \mathbf{w}^H\mathbf{h} = 1 \tag{18}$$

where $\mathbf{w}_{\rm t}$ is the beamforming vector associated with the transmit CB technique and $P_{\mathbf{w},n}^{\rm t}$ is the aggregate noise power given by

$$P_{\mathbf{w},n}^{\rm t} = \mathbf{w}^H \mathrm{E}\left\{(\mathbf{h}\odot\mathbf{v})\left(\mathbf{h}^H\odot\mathbf{v}^H\right)\right\}\mathbf{w} + \sigma_n^2. \tag{19}$$

The expectation in (19) is taken with respect to the rays' directions θ_ls, their complex amplitudes α_ls and the forwarded noises from the terminals $\mathbf{v}$. Using the property in (9), it is straightforward to prove that $P_{\mathbf{w},n}^{\rm t} = P_{\mathbf{w},n}^{\rm r}$ and, therefore, $\mathbf{w}_{\rm t} = \mathbf{w}_{\rm o} = \mathbf{w}_{\rm r}$. Thus, both transmit and receive CB techniques are not suitable for a distributed implementation.

IV. Proposed distributed beamformer

In order to get around the problem underlined above, one can substitute μ with a quantity that can be computed at each individual terminal and, in addition, well-approximates its original counterpart. Using the fact that the number of terminals K could be typically large in many practical cases [8], [9], μ can be substituted with $\mu_p = \lim_{K\to\infty}\mu$ in (16). Although μ_p is a good approximation of μ, it must also solely depend on the information commonly available at all the terminals. This will be proved in the following.

It is direct to show, from (17), that

$$\mu_p = \left(\frac{1}{\sigma_v^2} + \lim_{K\to\infty}\frac{\mathbf{a}(\sigma_\theta)^H\boldsymbol{\Lambda}^{-1}\mathbf{a}(-\sigma_\theta)}{K}\right)^{-1}. \tag{20}$$

It follows from the definition of $\mathbf{a}(\phi)$ that

$$\frac{\mathbf{a}(-\sigma_\theta)^H\boldsymbol{\Lambda}^{-1}\mathbf{a}(\sigma_\theta)}{K} = \frac{\sum_{k=1}^K e^{j\frac{2\pi}{\lambda}r_k(\cos(\psi_k+\sigma_\theta)-\cos(\psi_k-\sigma_\theta))}}{K\sigma_v^2}. \tag{21}$$

Using the strong law of large numbers and the fact that r_k, ψ_k and $[\mathbf{f}]_k$ are all mutually statistically independent, we obtain

$$\lim_{K\to\infty}\frac{\mathbf{a}(-\sigma_\theta)^H\boldsymbol{\Lambda}^{-1}\mathbf{a}(\sigma_\theta)}{K} \xrightarrow{p1} \frac{1}{\sigma_v^2}\mathrm{E}\left\{e^{j\frac{2\pi}{\lambda}r_k(\cos(\psi_k+\sigma_\theta)-\cos(\psi_k-\sigma_\theta))}\right\}. \tag{22}$$

Since the terminals are uniformly distributed on $D(O,R)$, it can be shown that [1]

$$\mathrm{E}\left\{e^{j\frac{2\pi}{\lambda}r_k(\cos(\psi_k+\sigma_\theta)-\cos(\psi_k-\sigma_\theta))}\right\} = 2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}, \tag{23}$$

where $\alpha(\phi) \triangleq (4\pi R/\lambda)\sin(\phi/2)$. By substituting (23) in (22), we have

$$\lim_{K\to\infty}\frac{\mathbf{a}(-\sigma_\theta)^H\boldsymbol{\Lambda}^{-1}\mathbf{a}(\sigma_\theta)}{K} \xrightarrow{p1} \frac{2}{\sigma_v^2}\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}. \tag{24}$$

Therefore, it follows from (20)-(24) that

$$\mu_{\rm p} \xrightarrow{p1} \sigma_v^2\left(1+2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}\right)^{-1}, \tag{25}$$

when the number of terminals K is large enough. As it can be observed from (25), $\mu_{\rm p}$ does not depend on the locations and the forward channels of any terminals and, therefore, it is locally computable at each individual terminal. Substituting μ by $\mu_{\rm p}$ in (16), we introduce a new DCB technique whose beamforming vector

$$\mathbf{w}_{\rm p} = \frac{\mu_{\rm p}}{K}\mathbf{\Lambda}^{-1}\left(\mathbf{a}\left(\sigma_\theta\right) + \mathbf{a}\left(-\sigma_\theta\right)\right) \tag{26}$$

not only can be implemented in a distributed fashion, but also well-approximates its counterpart $\mathbf{w}_{\rm o}$, when K is large enough[1]. Moreover, it is valid for any given pdf $p(\theta)$. Note that in the conventional scenario, when there is no local scattering phenomenon (i.e. plane-wave propagation), $\sigma_\theta \longrightarrow 0$ and hence (26) is reduced to

$$\mathbf{w}_{\rm c} = \frac{1}{K}\mathbf{a}, \tag{27}$$

the beamforming vector associated with the well-known conventional DCB technique [1], [2].

V. Performance analysis of the proposed distributed beamformer

As discussed above, the proposed distributed collaborative beamformer well-approximates its transmit and receive CB counterparts and achieves the same performances in both considered configurations. Thus, for the sake of simplicity, in what follows, we only focus on the receive CB scheme. One way to prove the efficiency of the proposed DCB technique is undoubtedly comparing its achieved SNR with the SNR performed when the conventional DCB technique, which is designed without taking into account the presence of local scattering in the source vicinity, is used. To this end, we introduce the following performance measure:

$$\Upsilon(\sigma_\theta) = \frac{\xi_{\mathbf{w}_{\rm c}}}{\xi_{\mathbf{w}_{\rm p}}}, \tag{28}$$

where

$$\xi_{\mathbf{w}} = \frac{P_{\mathbf{w}}(\phi_s)}{P^{\rm r}_{\mathbf{w},n}}, \tag{29}$$

is the achieved SNR when the beamforming vector $\mathbf{w}$ is used. In (29), commonly known as the beampattern, $P_{\mathbf{w}}(\phi_\star) = p_\star\left|\mathbf{w}^H\sum_{l=1}^{L}\alpha_l\mathbf{a}(\phi_\star+\theta_l)\right|^2$ is the received power from a transmitter at direction $\phi_\star$ with power $p_\star$. It is noteworthy that Υ is an excessively complex function of the random variables r_k, ψ_k, $[\mathbf{f}]_k$ for $k = 1,\ldots,K$ and α_l, θ_l for $l = 1,\ldots,L$ and, hence, a random quantity of its own. Therefore, it is practically more appealing to investigate the behavior and the properties of $\tilde{\Upsilon}$ given by [8]

$$\tilde{\Upsilon}(\sigma_\theta) = \frac{\tilde{\xi}_{\mathbf{w}_{\rm c}}}{\tilde{\xi}_{\mathbf{w}_{\rm p}}}, \tag{30}$$

where $\tilde{\xi}_{\mathbf{w}} = \tilde{P}_{\mathbf{w}}(\phi_s)/\tilde{P}^{\rm r}_{\mathbf{w},n}$ is the achieved average-signal-to-average-noise ratio (ASANR) when $\mathbf{w}$ is implemented with $\tilde{P}_{\mathbf{w}}(\phi_\star) = \mathrm{E}\left\{P_{\mathbf{w}}(\phi_\star)\right\}$, called the average beampattern, and $\tilde{P}^{\rm r}_{\mathbf{w},n} = \mathrm{E}\left\{P^{\rm r}_{\mathbf{w},n}\right\}$ is the average noise power. Thus, using the proposed DCB technique, it can be shown that

$$\tilde{P}^{\rm r}_{\mathbf{w}_{\rm p},n} = \frac{2\sigma_v^2}{K}\left(1+2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}\right)^{-1} + {\sigma_n}^2, \tag{31}$$

and

$$\tilde{P}_{\mathbf{w}_{\rm p}}\left(\phi_\star\right) = \frac{2p_\star}{K\left(1+2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}\right)}\left(1+\frac{2(K-1)\Omega\left(\phi_\star\right)}{\left(1+2\frac{J_1(\alpha(2\sigma_\theta))}{\alpha(2\sigma_\theta)}\right)}\right), \tag{32}$$

with

$$\Omega(\phi) = \int p(\theta)\left(\frac{J_1(\alpha(\phi+\theta+\sigma_\theta))}{\alpha(\phi+\theta+\sigma_\theta)} + \frac{J_1(\alpha(\phi+\theta-\sigma_\theta))}{\alpha(\phi+\theta-\sigma_\theta)}\right)^2 d\theta. \tag{33}$$

When the conventional DCB technique is implemented, it can be proved that

$$\tilde{P}^{\rm r}_{\mathbf{w}_{\rm c},n} = \frac{\sigma_v^2}{K} + {\sigma_n}^2, \tag{34}$$

and

$$\tilde{P}_{\mathbf{w}_{\rm c}}(\phi_\star) = \frac{p_\star}{K}\left(1+(K-1)\Gamma\left(\phi_\star\right)\right), \tag{35}$$

with

$$\Gamma(\phi) = \int p(\theta)\left(2\frac{J_1(\alpha(\phi+\theta))}{\alpha(\phi+\theta)}\right)^2 d\theta. \tag{36}$$

Note that, from (31)-(36), $\tilde{\Upsilon}(\sigma_\theta)$ is independent of the pdf $p(\theta)$. It is also noteworthy that the integrals in (33) and (36) can be computed numerically with any desired accuracy by using the most popular mathematical software packages such as Matlab and Mathematica, after properly choosing the pdf $p(\theta)$. In fact, several statistical distributions for θ_l have been proposed so far such as the Laplace, Gaussian or Uniform distribution [10]-[13]. Moreover, from (31)-(36), when K is large enough, it holds for small σ_θ that

$$\tilde{\Upsilon}(\sigma_\theta) \approx \frac{1}{4}\left(1 + {}_0F_1\left(;2;-4\pi^2\left(\frac{R}{\lambda}\sigma_\theta\right)^2\right)\right)^2. \tag{37}$$

Since the hypergeometric function ${}_0F_1\left(;2;-4\pi^2x^2\right)$ has a maximum peak value of 1 at $x = 0$, the above expression indicates that regardless of the value of R/λ, $\tilde{\xi}_{\mathbf{w}_{\rm p}} \approx \tilde{\xi}_{\mathbf{w}_{\rm c}}$, when there is no local scattering in the source vicinity. This is expected since $\mathbf{w}_{\rm p}$ boils down to $\mathbf{w}_{\rm c}$ in such a case. In addition, for small x, ${}_0F_1\left(;2;-4\pi^2x^2\right)$ decreases inversely proportional to x while for large x, it has an oscillatory tail but converges to 0 as x increases. Thus, it can be observed from (37) that, as σ_θ increases, the ASANR gain achieved using $\mathbf{w}_{\rm p}$ instead of $\mathbf{w}_{\rm c}$ increases and can reach as much as 3 dB. Therefore, the proposed DCB technique is much more efficient in terms of achieved ASANR compared to the conventional DCB technique, which is designed without taking into account the presence of local scattering, and this holds for any given pdf $p(\theta)$.

Further, it can be shown that

$$\lim_{K\to\infty}\tilde{\Upsilon}(\sigma_\theta) = \lim_{K\to\infty}\Upsilon(\sigma_\theta), \tag{38}$$

[1] We will see in Section VI that K in the range of 20 is already sufficient to perfectly fit the asymptotic solution while K in the range of 10 readily offers an acceptable approximation within a dB fraction.

and, hence, $\tilde{\Upsilon}(\sigma_\theta)$ is a meaningful performance measure that asymptotically converges to Υ when K is large enough. Consequently, the proposed DCB technique is also much more efficient in enhancing the achieved SNR for any given realization of r_k, ψ_k and $[\mathbf{f}]_k$ for $k = 1, \ldots, K$, when K is large enough, and this holds for any distribution of θ_ls.

VI. Simulation Results

Computer simulations are provided to support the theoretical results. All the simulation results are obtained by averaging over 10^6 random realizations of r_k, ψ_k, $[\mathbf{f}]_k$ for $k = 1, \ldots, K$ and α_l, θ_l for $l = 1, \ldots, L$. In all examples, we assume that the noises' powers σ_n^2 and σ_v^2 are 10 (dB) below the source transmit power p_s and $K = 20$. It is also assumed that the number of rays is $L = 6$ and that their phases are uniformly distributed. Figs. 2, 3 and 4 display $\tilde{P}_{\mathbf{w}_\mathrm{p}}(\phi_\star)$, $\tilde{P}_{\mathbf{w}_\mathrm{o}}(\phi_\star)$ and $\tilde{P}_{\mathbf{w}_\mathrm{c}}(\phi_\star)$ for different values of R and σ_θ. As can be observed from these figures, when the conventional DCB technique is used, the beamforming response in the desired direction decreases if R or σ_θ increases while it remains equal to unity when the proposed DCB technique is implemented. Therefore, the proposed technique is much more efficient than its conventional vis-a-vis in keeping the beamforming response in the desired direction equal to unity. Fig. 5 plots $\tilde{\xi}_{\mathbf{w}_\mathrm{p}}$ and $\tilde{\xi}_{\mathbf{w}_\mathrm{o}}$ versus σ_θ for $R = 1$ and different values of K. From this figure, the performance of the proposed DCB technique fits perfectly with that of the transmit and receive CB techniques when K is in the range of 20 while it looses only a fraction of a dB when K is in the range of 10. Fig. 6 displays $\tilde{\xi}_{\mathbf{w}_\mathrm{p}}$, $\tilde{\xi}_{\mathbf{w}_\mathrm{o}}$ and $\tilde{\xi}_{\mathbf{w}_\mathrm{c}}$ versus σ_θ for $R = 1$. It can be verified from Fig. 6 that the proposed DCB technique is able to perform the maximum achievable SNR even in suburban environments where σ_θ is in the range of 20 degrees, while the SNR performed by its conventional vis-a-vis decreases by 0.5 dB for $\sigma_\theta = 7$ degrees and becomes unsatisfactory in suburban environments. In such environments, the proposed technique achieves until 3 dB of SNR gain. Fig. 7 shows $\tilde{\Upsilon}(\sigma_\theta)$ for different values of R. It can be seen from this figure that for small σ_θ, $\tilde{\Upsilon}(\sigma_\theta)$ is a decreasing function of σ_θ and R and, hence, the SNR gain achieved when the proposed DCB technique is implemented increases with σ_θ and R. This corroborates our discussions in Section V.

VII. Conclusion

We consider both transmit and receive CB techniques to achieve a dual-hop communication from a source to a receiver, through a wireless network comprised of K uniformly and independently distributed terminals. Here, local scattering in the source or receiver vicinity is assumed when in previous works the effect of scattering and reflection was neglected. Taking into account this phenomenon, the beamforming vectors corresponding to both beamformers are derived. It is shown that the so-obtained beamformers are not suitable for a distributed implementation. Using the fact that the number of terminals could be typically large in practice, we proposed a novel distributed collaborative beamformer that can

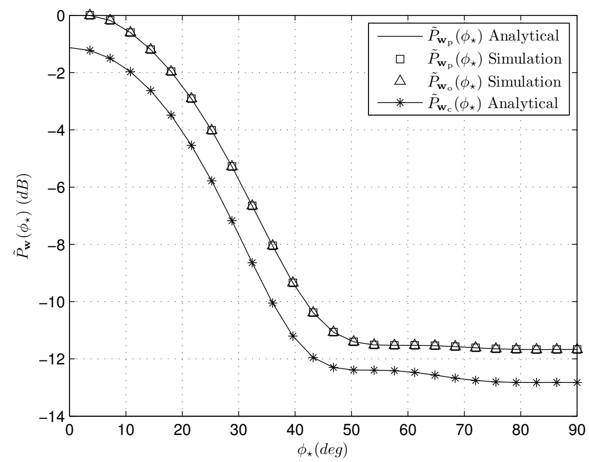

Fig. 2. $\tilde{P}_{\mathbf{w}_\mathrm{p}}(\phi_\star)$, $\tilde{P}_{\mathbf{w}_\mathrm{c}}(\phi_\star)$ and $\tilde{P}_{\mathbf{w}_\mathrm{o}}(\phi_\star)$ for $\sigma_\theta = 10$ (deg) and $R = 1$.

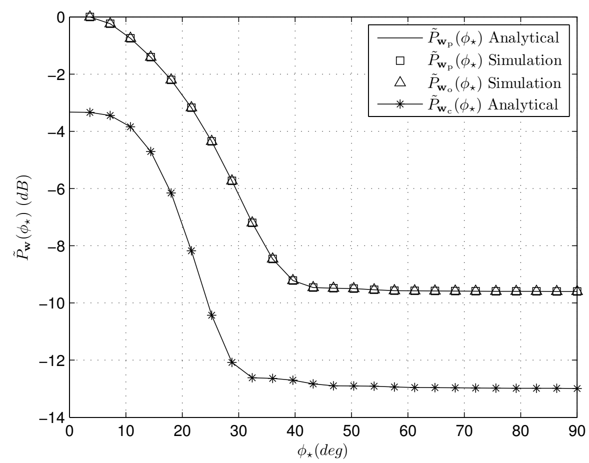

Fig. 3. $\tilde{P}_{\mathbf{w}_\mathrm{p}}(\phi_\star)$, $\tilde{P}_{\mathbf{w}_\mathrm{c}}(\phi_\star)$ and $\tilde{P}_{\mathbf{w}_\mathrm{o}}(\phi_\star)$ for $\sigma_\theta = 10$ (deg) and $R = 2$.

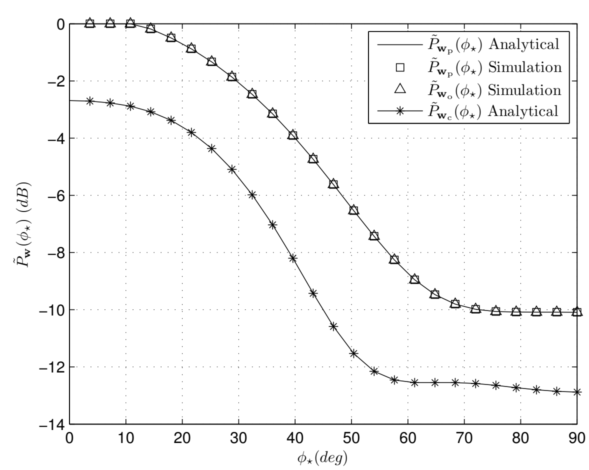

Fig. 4. $\tilde{P}_{\mathbf{w}_\mathrm{p}}(\phi_\star)$, $\tilde{P}_{\mathbf{w}_\mathrm{c}}(\phi_\star)$ and $\tilde{P}_{\mathbf{w}_\mathrm{o}}(\phi_\star)$ for $\sigma_\theta = 17$ (deg) and $R = 1$.

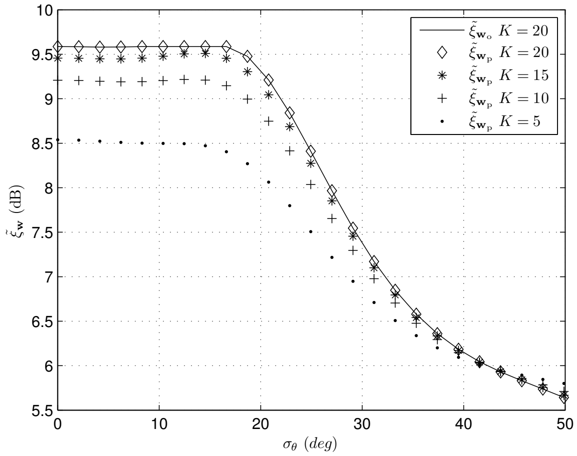

Fig. 5. $\tilde{\xi}_{\mathbf{w}_\mathrm{p}}$ and $\tilde{\xi}_{\mathbf{w}_\mathrm{o}}$ versus σ_θ for $R = 1$ and $K = 5, 10, 15, 20$.

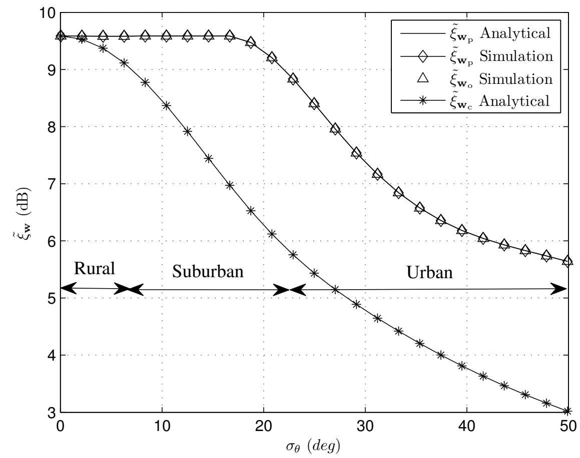

Fig. 6. $\tilde{\xi}_{\mathbf{w}_\mathrm{p}}$, $\tilde{\xi}_{\mathbf{w}_\mathrm{c}}$ and $\tilde{\xi}_{\mathbf{w}_\mathrm{o}}$ versus σ_θ for $R = 1$.

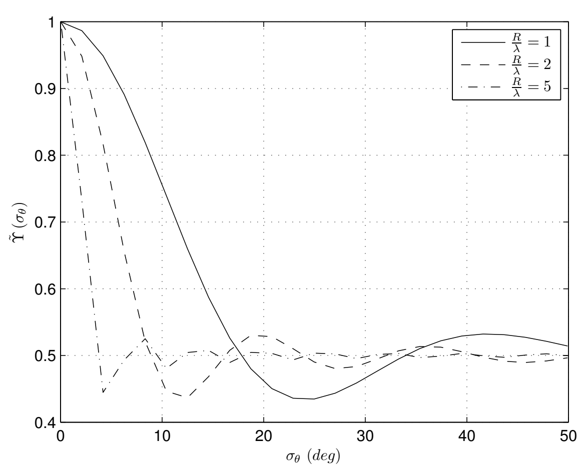

Fig. 7. $\tilde{\Upsilon}(\sigma_\theta)$ for $R = 1, 2, 5$.

be implemented in a distributed fashion and, further, well-approximates its transmit and receive CB counterparts. The performance of the proposed DCB technique is analyzed and its advantages against the conventional DCB technique, which is designed without taking into account the presence of local scattering in the source or receiver vicinity, was proved. It was shown that the maximum achievable SNR is performed by the proposed DCB technique even in suburban environments where local scattering is important, while the performance of the conventional DCB technique decreases in rural environments and becomes unsatisfactory in suburban environments. It was also proved that the proposed DCB technique is able to achieve until 3 dB of SNR gain against its conventional vis-a-vis. Moreover, the proposed technique exactly achieves the asymptotic performance of the theoretical non-distributed transmit and receive CB techniques when K is in the range of 20 while it approaches it within a fraction of a dB only when K is in the range of 10.

References

[1] H. Ochiai, P. Mitran, H. V. Poor, and V. Tarokh, "Collaborative beamforming for distributed wireless ad hoc sensor networks," *IEEE Trans. Signal Process.,* vol. 53, pp. 4110-4124, Nov. 2005.

[2] M. F. A. Ahmed and S. A. Vorobyov, "Collaborative beamforming for wireless sensor networks with Gaussian distributed sensor nodes," *IEEE Trans. Wireless Commun.,* vol. 8, pp. 638-643, Feb. 2009.

[3] K. Zarifi, A. Ghrayeb, and S. Affes , "Distributed beamforming for wireless sensor networks with improved graph connectivity and energy efficiency," *IEEE Trans. Signal Process.,* vol. 58, pp. 1904-1921, Mar. 2010.

[4] M. F. A. Ahmed and S. A. Vorobyov, "Sidelobe control in collaborative beamforming via node selection," *IEEE Trans. Signal Process.,* vol. 58, pp. 6168-6180, Dec. 2010.

[5] M. Pun, D. R. Brown III, and H. V. Poor, "Opportunistic collaborative beamforming with one-bit feedback," *IEEE Trans. Wireless Commun.,* vol. 8, pp. 2629-2641, May 2009.

[6] Z. Han and H. V. Poor, "Lifetime improvement in wireless sensor networks via collaborative beamforming and cooperative transmission," *IET Microw. Antennas Propag.,* vol. 1, pp. 1103-1110, Dec 2007.

[7] L. C. Godara, "Application of antenna arrays to mobile communications, Part II: Beam-forming and direction-of-arrival considerations," *Proc. IEEE,* vol. 85, pp. 1195-1245, Aug. 1997

[8] K. Zarifi, S. Zaidi, S. Affes, and A. Ghrayeb, "A distributed amplify-and-forward beamforming technique in wireless sensor networks," *IEEE Trans. Signal Process.,* vol. 59, pp. 3657-3674, Aug. 2011.

[9] K. Zarifi, S. Affes, and A. Ghrayeb, "Collaborative null-steering beamforming for uniformly distrubuted wireless sensor networks," *IEEE Trans. Signal Process.,* vol. 58, pp. 1889-1903, Mar. 2010.

[10] D. Astly and B. Ottersten, "The effects of local scattering on direction of arrival estimation with MUSIC," *IEEE Trans. Signal Process.,* vol. 47, pp. 3220-3234, Dec. 1999.

[11] A. Amar, "The effect of local scattering on the gain and beamwidth of a collaborative beampattern for wireless sensor networks," *IEEE Trans. Wireless Commun.,* vol. 9, pp. 2730-2736, Sep. 2010.

[12] M. Souden, S. Affes, and J. Benesty, "A two-stage approach to estimate the angles of arrival and the angular spreads of locally scattered sources," *IEEE Trans. Signal Process.,* vol. 56, pp. 1968-1983, May 2008.

[13] M. Bengtsson and B. Ottersten, "Low-complexity estimators for distributed sources," *IEEE Trans. Signal Process.,* vol. 48, pp. 2185-2194, Aug. 2000.

[14] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z.-Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Trans. Signal Process.,* vol. 56, pp. 4306-4316, Sep. 2008.