This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2011 proceedings.

Joint Estimation of the Ricean K-factor and the SNR for SIMO Systems Using Higher Order Statistics

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***Abstract*—In this paper, we propose a joint estimator of the Ricean K-factor and the signal-to-noise ratio (SNR) for single-input multiple-output (SIMO) systems. The second-order moment and the fourth-order cross-moment matrix of the received signal envelope are used to estimate the desired parameters. The K-factor is estimated using the kurtosis of the Ricean channel gain while the SNR is obtained by separately estimating the powers of the useful signal and the additive noise. Two versions are developed depending on the value of the kurtosis of the transmitted data. Unlike the autocorrelation-based K-factor and SNR joint estimator developed in [1] that is tailored for unmodulated signals, the proposed method applies to any digital modulation and does not request the knowledge of the maximum Doppler spread. Simulation results are included to illustrate the performance behavior of the new estimator.**

Index Terms : Ricean K-factor, SNR, joint estimation, kurtosis, higher order statistics.

I. Introduction

In wireless communications, the Ricean channel model is often used to simulate and study real-life scenarios. The Ricean distribution is mainly characterized by the K-factor which is defined as the ratio of the signal power in the direct component over the scattered power. In general, the value of the Ricean K-factor is a measure of the fading severity. Therefore, the estimation of the Ricean K-factor is a good indicator of the channel quality and is important in the link budget calculations [2]. Moreover, many systems require the SNR estimate. Indeed, the latter is needed when it comes to control the transmitting power, carry handoffs or dynamically allocate the resources [3] [4].

Many K-factor estimators have been developed in the two last decades. In [5], the Kolmogorov-Smirnov statistic was used to measure the maximum deviation between the theoretical Ricean distribution and its empirical counterpart. In [6], the ratio between the first moment and the zero crossing rate of the received signal instantaneous frequency is used to estimate the K-factor. In [2] and [7], a general class of moment-based and an In-phase and Quadrature-phase (I/Q) based estimators were proposed. The first moment and the root mean square fluctuation of the power gain of the received signal are used to estimate the K-factor in [8]. The Ricean channel kurtosis and the ratio of the first-order and the squared second-order moments are used to estimate the K-factor in [9]. For the SNR estimation, in [10] a new moment-based estimator for SIMO configurations has been recently proposed in which the fourth-order moments matrix is considered to separately estimate the useful signal and the noise powers. In [11], a maximum likelihood (ML) SNR estimator for non-coherent M-ary frequency shift keying over fading channels and white Gaussian noise is developed.

Concerning the K-factor estimation, all the aforementioned methods consider non-modulated signals or perform its estimation from the estimates of the channel coefficients and not directly from the received data. Solely in [1], a joint K-factor and local average SNR estimation based on the autocorrelation function of a modulated signal is presented. Both data-aided (DA) and non-data-aided (NDA) estimations are discussed. In the DA case, the transmitted data symbols are assumed to be perfectly known *a priori*. In the NDA case, the transmitted symbols are completely unknown but, the proposed approach in [1] is only valid when the autocorrelation function of the transmitted data is non-*zero*. Moreover, this autocorrelation-based approach assumes the perfect knowledge of the Doppler spread.

In this paper, we propose a new NDA estimator that is valid for any type of digital modulation (M-QAM, M-PAM, M-PSK, etc...) and applicable in SIMO systems. The proposed approach estimates jointly the Ricean K-factor and the SNR without the *a priori* knowledge of the transmitted symbols (NDA) or the Doppler spread of the line-of-sight (LOS) component. The K-factor is estimated from the kurtosis of the Ricean channel. The powers of the useful signal and the additive noise are estimated using the fourth-order cross-moments (across the multiple receiving antenna branches) and the second-order moments of the received signal envelope. The SNR per-antenna is then easily deduced.

The remainder of this paper is organized as follows. In section II, the signal model is described. A new joint K-factor and SNR estimator based on higher-order statistics (HOS) of the received signal envelope is presented in section III where different cases are discussed, depending on the value of the kurtosis of the considered digital modulation. In section IV, we assess the performance of the proposed estimator, through computer simulations, and compare it with the autocorrelation-based approach of [1]. Section V concludes the paper.

II. SIGNAL MODEL

We consider the uplink transmission in a SIMO system. The base station have a two-dimensional arbitrary array of N_a receiving antenna elements. Denoting the total number of received samples by N (over each antenna branch), the expression of the baseband received signal at the i^{th} antenna element and at discrete time index, $\{t\}_{t=1}^{N}$, is given by:

$$y_i(t) = h_i(t)a(t) + \omega_i(t), \quad (1)$$

where $\{h_i(t)\}_{i=1}^{N_a}$ are the Ricean channel coefficients at instant t, $a(t)$ is the transmitted signal and $\omega_i(t)$ is an additive white Gaussian noise (AWGN) of zero mean. The noise components at all antenna elements, $\omega_i(t)$, are assumed temporally and spatially uncorrelated with equal average power $E[|\omega_i(t)|^2] = N_0$, where $E[.]$ stands for the statistical expectation. We assume that the same value of the K-factor is observed at the array antenna elements. Without loss of generality, the energy of the transmitted signal $a(t)$ is normalized to one, i.e., $E[|a(t)|^2] = 1$. The Ricean channel coefficients $h_i(t)$ can be modeled as the sum of a non line-of-sight (NLOS) component, $\tilde{h}_i(t)$, and a LOS component [2] as follows:

$$h_i(t) = \sqrt{\frac{P_i K}{K+1}} \exp\{j(\omega_D \cos(\theta_0)t + \phi_0)\} + \sqrt{\frac{P_i}{K+1}} \tilde{h}_i(t), \quad (2)$$

where $P_i = E[|h_i(t)|^2]$ is the received signal power at the i^{th} antenna element, ω_D is the maximum Doppler spread, θ_0 and ϕ_0 are, respectively, the angle of arrival and the phase of the LOS component.

III. NEW JOINT K-FACTOR AND SNR ESTIMATION

In this paper, we propose a new joint K-factor and SNR estimator for any type of digital modulation. To that end, the fourth-order cross-moment matrix, M_4, is considered (the temporal index, t, is dropped for ease of notation) and its entries are defined as:

$$M_4(i,j) = E\left[|y_i|^2 |y_j|^2\right]. \quad (3)$$

The channel coefficients, h_i, the transmitted signal, a, and the zero-mean noise, ω_i, are assumed to be mutually independent. The received signals at different antenna elements are considered uncorrelated as well and hence arbitrary antenna configurations can be used. Then exploiting these assumptions, the fourth-order cross-moment in (3) reduces simply to:

$$M_4(i,j) = K_a P_i P_j + P_i N_0 + P_j N_0 + N_0^2; \text{ for } i \neq j \quad (4)$$

where $K_a = \frac{E\left[|a|^4\right]}{E[|a|^2]^2}$ is the kurtosis of the transmitted signal which can be easily determined off-line. For instance, for a M-QAM, $K_a = 1 + \frac{2}{5}(1 - \frac{3}{M-1})$ and for M-PSK, $k_a = 1$. Similarly, the diagonal elements of the fourth-order cross-moment matrix, M_4, are given by:

$$\begin{aligned} M_4(i,i) &= E\left[|h_i|^4\right] E\left[|a|^4\right] + E\left[|\omega_i|^4\right] \\ &\quad + 4E\left[|h_i|^2\right] E\left[|a|^2\right] E\left[|\omega_i|^2\right], \\ &= K_R K_a P_i^2 + 4P_i N_0 + K_\omega N_0^2, \end{aligned} \quad (5)$$

where $K_R = \frac{E\left[|h_i|^4\right]}{E[|h_i|^2]^2}$ and $K_\omega = \frac{E\left[|\omega_i|^4\right]}{E[|\omega_i|^2]^2}$ are, respectively, the Ricean channel and the noise kurtosises. The kurtosis of a complex AWGN is $K_\omega = 2$ (see [12]) while the kurtosis of the Ricean channel depends only on the K-factor [9] as follows:

$$K_R = 2 - \left(\frac{K}{K+1}\right)^2. \quad (6)$$

We mention here that the channel kurtosis is an important metric to measure the peakedness of a distribution. For instance, $K_R = 2$ and $K_R = 1$ represent the most severe fading (Rayleigh channel) and no fading situations, respectively.

We also use the second-order moment of the received signal at each antenna element that is given by:

$$\begin{aligned} M_2(i) &= E\left[|y_i|^2\right], \\ &= P_i + N_0. \end{aligned} \quad (7)$$

In the following, we develop two algorithms depending on the value of the transmitted signal's kurtosis, K_a.

A. Case 1: modulations with kurtosis $K_a \neq 1$

In this subsection, we consider modulation schemes with $K_a \neq 1$ such as M-PAM, M-QAM etc... In this case, using (4) and (7), we obtain:

$$P_i P_j = \frac{M_4(i,j) - M_2(i) M_2(j)}{K_a - 1}, \quad (8)$$

and

$$P_i - P_j = M_2(i) - M_2(j). \quad (9)$$

Denote $\hat{M}_2(i) = \frac{1}{N}\sum_{t=1}^{N} |y_i(t)|^2$ and $\hat{M}_4(i,j) = \frac{1}{N}\sum_{t=1}^{N} |y_i(t)|^2 |y_j(t)|^2$ as the sample estimates of the second-order moment and the fourth-order cross-moment, respectively. Then, resolving for P_i and P_j from (8) and (9), an estimate, $\hat{P}_j(i)$, for the received signal power at the j^{th} antenna branch, P_j, is obtained using the statistic extracted from each antenna pair (i,j) as follows:

$$\begin{aligned} \hat{P}_j(i) &= \frac{\hat{M}_2(j) - \hat{M}_2(i)}{2} \\ &+ \frac{\sqrt{\left(\hat{M}_2(j) - \hat{M}_2(i)\right)^2 + 4\frac{\hat{M}_4(i,j) - \hat{M}_2(j)\hat{M}_2(i)}{K_a - 1}}}{2}. \end{aligned} \quad (10)$$

Then, the final estimate $\hat{P}_j$ can be refined by averaging over all the antenna pairs (i,j) with $i \neq j$:

$$\hat{P}_j = \frac{1}{N_a - 1}\sum_{i \neq j} \hat{P}_j(i). \quad (11)$$

The noise power is also obtained as an average of all the estimates obtained on each antenna element from (7) as follows:

$$\hat{N}_0 = \frac{1}{N_a}\sum_{j=1}^{N_a} \left(\hat{M}_2(j) - \hat{P}_j\right). \quad (12)$$

Finally, the estimated SNR over the j^{th} antenna is simply the ratio:

$$\hat{\rho}_j = \frac{\hat{P}_j}{\hat{N}_0}. \tag{13}$$

The Ricean kurtosis is estimated from the diagonal elements of M_4, i.e., $M_4(j,j)$ as follows:

$$\hat{K}_R = \frac{1}{K_a N_a} \sum_{j=1}^{N_a} \left(2 + \frac{\hat{M}_4(j,j) - 2(\hat{P}_j + \hat{N}_0)^2 \hat{P}_j^2}{\hat{P}_j^2} \right). \tag{14}$$

The K-factor is then deduced from (6):

$$\hat{K} = \frac{\sqrt{2 - \hat{K}_R}}{1 - \sqrt{2 - \hat{K}_R}}. \tag{15}$$

B. Case 2: modulations with kurtosis $K_a = 1$

This is for example the case of all the constant-envelope modulations such as M-PSK. In this case, using (5) and (7), we obtain the following system of equations:

$$P_i - P_j = M_2(i) - M_2(j), \tag{16}$$

and

$$\frac{P_i^2}{P_j^2} = \frac{M_4(i,i) - 2M_2^2(i)}{M_4(j,j) - 2M_2^2(j)}. \tag{17}$$

The received signal power at the j^{th} antenna element can be estimated using all the cross-moments over the antenna pairs (i,j) as follows:

$$\hat{P}_j = \frac{1}{N_a - 1} \sum_{i \neq j} \frac{\hat{M}_2(i) - \hat{M}_2(j)}{\sqrt{\frac{\hat{M}_4(i,i) - 2\hat{M}_2^2(i)}{\hat{M}_4(j,j) - 2\hat{M}_2^2(j)}} - 1}. \tag{18}$$

The Ricean kurtosis is then easily deduced as follows:

$$\hat{K}_R = \frac{1}{N_a} \sum_{j=1}^{N_a} \left(\frac{\hat{M}_4(j,j) - 2\hat{M}_2^2(j)}{\hat{P}_j^2} + 2 \right). \tag{19}$$

The noise power and the K-factor are, respectively, estimated using (12) and (15). The SNR is estimated using (13).

Note that unlike the estimators presented in [1], the new NDA approach does not require the *a priori* estimation of the Doppler frequency. Additionally, it does not assume the knowledge of the cosine of the angle of arrival of the LOS component. Moreover, the new HOS-based estimator is applicable to all possible modulation types. More so, in contrast to the classical autocorrelation-based method, its performance holds irrespectively of the autocorrelation lag. According to simulation results disclosed in [1], an inappropriate lag value would affect seriously the Doppler spread estimation and as a result the K-factor and the SNR estimates will be inaccurate.

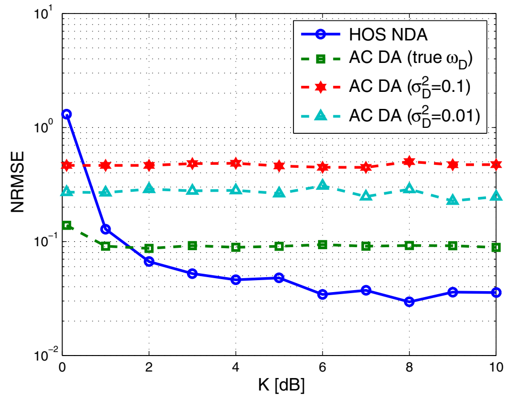
IV. Simulation results

In this section, we assess and compare the performance of the new estimator against the DA and NDA estimators developed in [1]. Given the fact that we do not exploit the correlation of the channel coefficients, there is no constraint on the antenna array geometry. A uniform linear array with five antenna elements spaced by a half wavelength is used as an example. Each antenna element exhibits a different SNR, i.e., $\boldsymbol{\rho} = [\rho_1 = 10,\ \rho_2 = 9,\ \rho_3 = 8,\ \rho_4 = 7,\ \rho_5 = 6]$ dB when we plot the performance in terms of the Ricean K-factor. The carrier frequency is 1.9 GHz, the speed of the mobile is set to 100 Km/h and the received signal is sampled at a symbol rate of 15 Kb/s. The distortion phase and the angle of arrival of the LOS component are, respectively, set to $\phi_0 = 0$ and $\theta_0 = \frac{\pi}{6}$ (note here that our new estimator is resilient to phase distortion and therefore we simply choose $\phi_0 = 0$ without loss of generality). In [1], the product of the Doppler frequency, the cosine of θ_0 and the time sampling period T_s is estimated using the imaginary parts of the autocorrelation function associated to two different lags. However in our simulations, for the autocorrelation-based estimator we first assume the product of the maximum Doppler spread and the time sampling period as perfectly known (in order to simulate the best operation conditions for that estimator). Then, to consider the effect of the estimation accuracy of $\omega_D T_s$ on the autocorrelation-based estimator performance, we consider a normalized Doppler spread estimate ($\hat{\omega}_D T_s$) with a Gaussian error estimation, or equivalently:

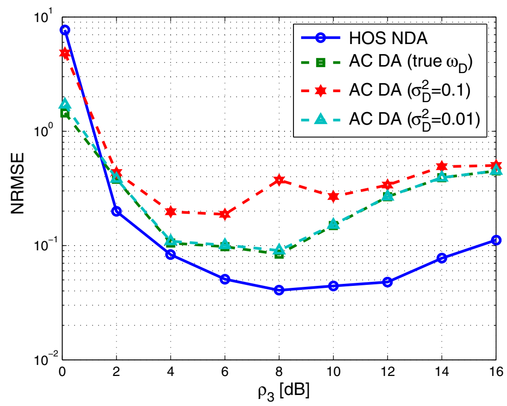
$$\hat{\omega}_D T_s = \omega_D T_s + \epsilon, \tag{20}$$

where ϵ is normally distributed with zero mean and variance σ_D^2, i.e., $N\left(0, \sigma_D^2\right)$. In our simulations, we consider three different values for σ_D^2 ($\sigma_D^2 = 0,\ 0.1$ and 0.01). As previously mentioned, the new K-factor and SNR joint estimator does not require however any *a priori* knowledge of the transmitted data or the Doppler spread value.

We first consider a binary phase-shift-keying (BPSK) modulation $\left(a(t) \in \{-1, 1\},\ K_a = 1\right)$. In this case, the DA approach in [1] is a more appropriate benchmark for the performance evaluation of our new estimator. In [1], it is shown that the performance of the DA and NDA methods depends on the autocorrelation lag value. In our simulations, we consider a lag equal to 20 for the K-factor estimation and a lag equal to one for the SNR estimation. The selected lags yield the best overall performance for the DA approach of [1]. As shown in Fig. 1 (a) and Fig. 1 (b), the performance of the autocorrelation-based estimator (referred to as AC) depends significantly on the accuracy of estimating $\omega_D T_s$, especially for the K-factor estimation. Indeed, even for a small error ϵ, ($\sigma_D^2 = 0.01$), the performance of the DA approach degrades noticeably. Yet, the performance in terms of SNR estimation does not change for a small value of σ_D^2 ($\sigma_D^2 = 0.01$). The new estimator referred to as HOS exhibits the lowest normalized root mean square error (NRMSE) for both the K-factor and the SNR estimates. Note that when K tends to zero, the ratio $\frac{\hat{M}_4(i,i) - 2\hat{M}_2^2(i)}{\hat{M}_4(j,j) - 2\hat{M}_2^2(j)}$ in (17) gets closer to zero. As a result, the

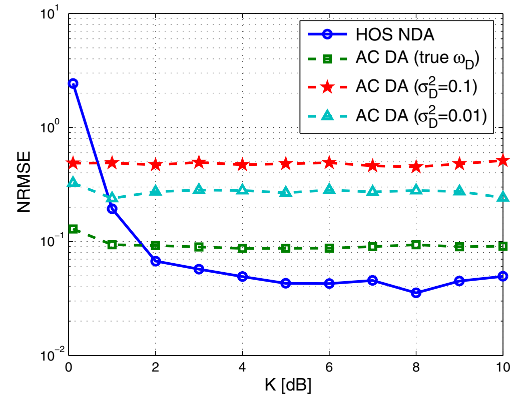


(a) NRMSE of the K-factor estimates



(b) NRMSE of the SNR estimates on the third antenna element for $K = 10$ dB

Fig. 1. NRMSE of the new estimator against the DA estimator of [1], BPSK modulation.



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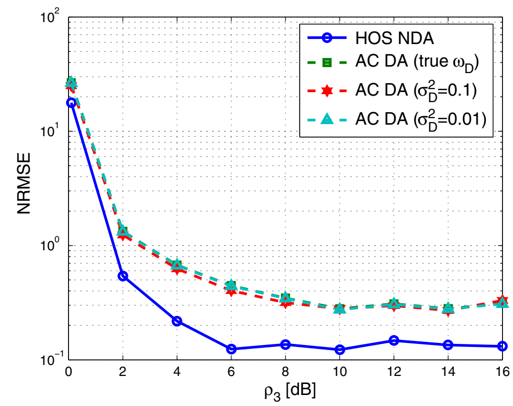


(b) NRMSE of the SNR estimates on the third antenna element for $K = 10$ dB

Fig. 2. NRMSE of the new estimator against the DA estimator of [1], 16-QAM modulation.

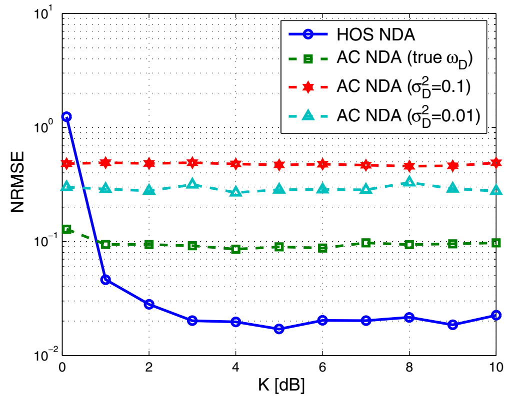
estimate of the received signal power is less accurate. Thus, the new estimator exhibits higher NRMSE for both the K-factor and the SNR estimates when K is small. Note also that, for SNR estimation, only the NRMSE of the third antenna-element (central branch) is shown in Fig. 1 (b). The performance in terms of SNR estimation over the other antenna elements, not included due to lack of space, exhibits the same behavior.

Now, we consider an M-QAM modulated signal ($M = 16$, $K_a = 1.32$). The transmitted symbols are independent and identically distributed. We compare the new method to the DA approach in [1] for which the transmitted symbols are assumed known. The same autocorrelation lag values are used as in the previous case. As shown in Fig. 2 (a), the same results for the K-factor estimation are observed as in the case of the BPSK modulation. Indeed, the estimation error ϵ affects the performance of the DA approach. However, the SNR estimation is insensitive to the value of the variance σ_D^2. The new estimator presents lower NRMSE for the entire SNR range and almost for all the K-factor values. As for the BPSK modulation, when the K-factor is small, the new estimator shows higher NRMSE. Indeed, in this case, the ratio $\frac{\hat{M}_4(j,j)-2(\hat{P}_j+\hat{N}_0)^2}{\hat{P}_j^2} = \left(\frac{K}{K+1}\right)^2$ in (6) tends to zero which yields erroneous K-factor estimates.

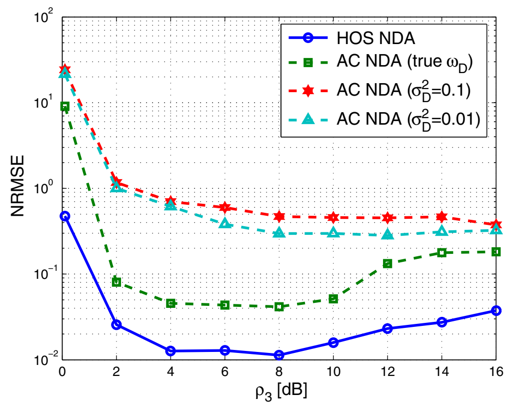
Finally, we apply the NDA approach derived in [1] to the On-Off Keying (OOK) modulation $\big(a(t) \in \{0,1\},\ K_a = 2\big)$. The same correlation lags are used. In this case, the autocorrelation-based estimator exhibits the same performance behavior for the K-factor estimation, as it can be seen from Fig. 3 (a). However, the SNR estimation using the NDA approach exhibits higher NRMSE when $\hat{\omega}_D T_s$ is not perfectly known. When the true value of the Doppler spread is used the NDA approach NRMSE decreases. As in the previous cases, the new K-factor and SNR joint estimator shows the lowest NRMSE, as shown in Fig. 3 (a) and Fig. 3 (b).

V. Conclusion

In this paper, we proposed a new approach to jointly estimate the Ricean K-factor and the SNR for SIMO systems based on the HOS of the received signal. In contrast to the autocorrelation-based method developed in [1], the new HOS-based estimator is valid for any digital modulation and does not require the estimation of the Doppler spread. The new estimator has two versions depending on the value of the



(a) NRMSE of the K-factor estimates



(b) NRMSE of the SNR estimates on the third antenna element for $K = 10$ dB

Fig. 3. NRMSE of the new estimator against the NDA estimator of [1], OOK modulation.

transmitted data kurtosis being equal or not to one. For both versions, simulation results show noticeable low estimation errors for wide ranges of the SNR and the K-factor values.

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