# Joint Power Control and Relay Design in Underlay Cognitive Networks with Multiple Transmitter-Receiver Pairs

*(Invited Paper)*


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***Abstract*—We consider an underlay cognitive network with $L$ transmitter-receiver pairs and $K$ relays in a two-hop half-duplex cooperative scheme. While the transmitters use a shared channel to transmit to the relays, the relays use either a shared channel or $L$ orthogonal channels to communicate with the $L$ receivers. For both second-hop-channel cases, we jointly determine the transmitters' powers and the relaying weights that maximize the worst signal-to-interference-plus-noise ratio subject to the transmitters' total and individual and the relays' total power constraints while guaranteeing that the interference powers on $M$ existing primary users are below some required thresholds.**



***Index Terms*—Cognitive network, cooperative system, multiple source-destination system, power control, beamforming.**


## I. Introduction

There is a growing research interest in the applications of multi-point to multi-point wireless cooperative networks wherein multiple sources communicate with their dedicated destinations through a set of relays [1]- [8]. It is likely that such networks operate in a frequency band that is pre-occupied by an existing wireless cellular system. This forces the network to perform as a cognitive (secondary) network whose inter-terminal communications should satisfy maximum interference power admissible to the receivers in the existing (primary) system. In this paper, we consider a cognitive dual-hop multi-point to multi-point wireless network with $L$ single-antenna source-destination pairs with $K$ relaying antennas that operate in the presence of $M$ primary receivers. In the first phase, $L$ sources transmit and the relay antennas receive different faded and noisy mixtures of the sources' signals. In the second phase, we consider two different relaying schemes: 1) Common relay-destination channel (CRDC) case. In this case, the relays multiply their received signals with properly-selected beamforming weights and retransmit them to the destinations through a shared channel; and 2) Orthognal relay-destination channel (ORDC) case wherein the relays transmit to each destination using a dedicated channel. In both cases, the goal is to find the set of jointly optimal sources' transmit powers and relays' beamforming weights that maximize the minimum signal-to-interference-plus-noise ratio (SINR) among all destinations while concurrently satisfying constraints on sources' and relays' total transmit powers, sources' individual transmit powers, and the maximum interference power that is admissible to the $M$ primary users. We derive a joint-optimality condition and use it to develop an efficient algorithm to iteratively optimize the sources' transmit powers and the relays' beamforming weights. Simulation results show that our joint optimization technique for both CRDC and ORDC cases converge very fast and outperform the conventional algorithms that try to optimize the sources and relays separately.

The rest of this paper is organized as follows. The system model and the problem formulation are presented in Section II. The sources' transmit power and relays' beamforming weights are jointly optimized in Section III. Simulation results are presented in Section IV and concluding remarks are drawn in Section V.

*Notation:* Uppercase and lowercase bold letters denote matrices and vectors, respectively. $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote the transpose, the conjugate, and the Hermitian transpose, respectively. $|\cdot|$ is the absolute value, $\|\cdot\|$ is the 2-norm of a vector, and $\mathrm{tr}\,(\cdot)$ is the trace of a matrix. $\mathrm{E}\{\cdot\}$ stands for the statistical expectation and $\mathbf{I}_K$ is the $K \times K$ identity matrix. $\mathbf{1}$ is the vector with all entries equal to 1 and $\mathbf{e}_i$ is the vector with 1 at the $i$-th position and zeros elsewhere. $[\cdot]_l$ and $[\cdot]_{nl}$ stand for the $l$th entry of a vector and the entry at the $n$th row and the $l$th column of a matrix, respectively. $\lambda_{\max}(\cdot)$ is the maximum-modulus eigenvalue and $\Omega(\cdot)$ is the eigenvector associated with the maximum-modulus eigenvalue normalized such that its last entry is 1. $\mathbf{D}(\mathbf{a})$ is a diagonal matrix whose diagonal elements are the entries of $\mathbf{a}$. $\mathbf{a}_{\bar{l}}$ is the $L-1 \times 1$ vector obtained by removing the $l$-th entry of the $L \times 1$ vector $\mathbf{a}$ and $\mathbf{A}_{\bullet\bar{l}}$ is the $N \times (L-1)$ matrix obtained by removing the $l$th column of the $N \times L$ matrix $\mathbf{A}$. $\mathrm{vec}(\mathbf{A})$ is the $NL \times 1$ vector obtained by stacking the columns of $\mathbf{A}$ on top of one another. $\otimes$ is the Kronecker product.

## II. System model and problem representation

### A. CRDC case

Consider the dual-hop multipoint-to-multipoint cognitive network shown in Fig. 1 with $L$ source-destination pairs $(S_l, D_l),\ l = 1, \ldots, L$ and $K$ relays all operating in a frequency band allocated to $M$ primary users $U_m,\ m = 1, \ldots, L$. All terminals involved are single-antenna and there is no

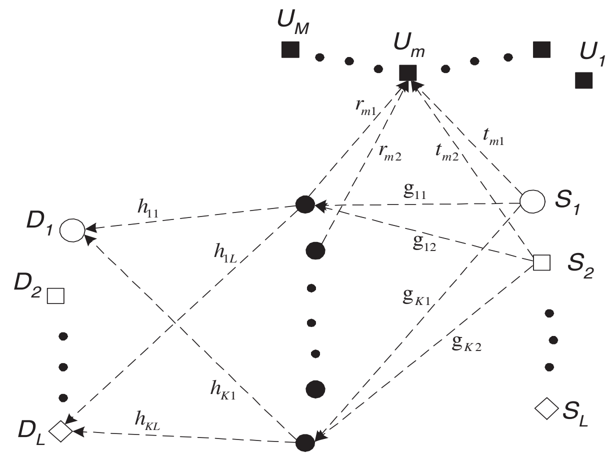


Fig. 1. Dual-hop multipoint-to-multipoint network in the shared RD channel case.

direct link from any source to any destination and all required $S_l - D_l,\ l = 1, \ldots, L$ channel links are established through the use of the following dual-hop cooperative scheme: In the first phase, $S_l,\ l = 1, \ldots, L$ transmit and the $K$ relays receive a faded and noisy mixtures of the transmitted signals. In the second phase, each relay multiplies its received signal with a properly-selected beamforming weight and forwards the result towards the destinations $D_l,\ l = 1, \ldots, L$.

Let $s_l$ denote the transmitted signal from $S_l$ where we must have $\mathrm{E}\left\{|s_l|^2\right\} = p_l \leq P_l$ for $l = 1, \ldots, L$. Further, due to regulation it should also hold that $\sum_{l=1}^{L} p_l \leq P_{L+1}$ where $P_{L+1} < \sum_{l=1}^{L} P_l$. The above source power constraints can be represented in a unified manner as

$$\mathbf{u}_l^T \mathbf{p} \leq P_l \qquad l = 1, \ldots, L+1 \tag{1}$$

where $\mathbf{u}_l \triangleq \mathbf{e}_l,\ l = 1, \ldots, L,\ \mathbf{u}_{L+1} \triangleq \mathbf{1}$, and $\mathbf{p} \triangleq [p_1 \ \ldots \ p_L]^T$. Denote the channel gain from $S_l$ to the $m$-th primary user $U_m$ as $t_{ml}$. Then, the interference power inflicted from $S_l,\ l = 1, \ldots, L$ on $U_m$ is $\mathbf{u}_{L+1+m}^T \mathbf{p}$ where $\mathbf{u}_{L+1+m} \triangleq \left[|t_{m1}|^2 \ \ldots, |t_{mL}|^2\right]^T$. Let $P_{L+1+m}$ be the maximum admissible interference power caused by the cognitive sources $S_l,\ l = 1, \ldots, L$ to $U_m$. Then, $\mathbf{p}$ should also satisfy

$$\mathbf{u}_l^T \mathbf{p} \leq P_l \qquad l = L+2, \ldots, L+M+1. \tag{2}$$

Assume that $g_{kl}$ denote the channel gain from $S_l$ to $k$-th relay and introduce $\mathbf{g}_l \triangleq [g_{1l} \ \ldots \ g_{Kl}]^T$ and $\mathbf{G} \triangleq [\mathbf{g}_1 \ \ldots \ \mathbf{g}_L]$. Then, the $K \times 1$ received signal vector at the relays is

$$\mathbf{y} = \mathbf{G}\mathbf{s} + \mathbf{v} \tag{3}$$

where $\mathbf{s} \triangleq [s_1 \ \ldots \ s_L]^T$ and $\mathbf{v} \triangleq [v_1 \ \ldots \ v_K]^T$ is the relays' noise vector. Let $w_k^*$ denote the beamforming weight at the $k$-th relay and introduce the beamforming weigh vector $\mathbf{w} \triangleq [w_1 \ \ldots w_K]^T$ and $\mathbf{W} \triangleq \mathbf{D}(\mathbf{w})$. Then, the transmitted signal vector from the $K$ relays in the second phase is

$$\mathbf{x} \triangleq [x_1 \ \ldots \ x_K]^T = \mathbf{W}^* \mathbf{y}. \tag{4}$$

From (3) and (4) it can be shown that the relays' total transmit power is

$$\begin{aligned} \mathrm{E}\left\{\mathbf{x}^H \mathbf{x}\right\} &= \mathrm{tr}\left(\mathbf{W}^* \left(\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \mathbf{\Sigma}_v\right) \mathbf{W}^T\right) \\ &= \mathbf{w}^H \mathbf{\Lambda}_1(\mathbf{p}) \mathbf{w} \end{aligned} \tag{5}$$

where $\mathbf{\Lambda}_1(\mathbf{p}) \triangleq \mathbf{D}\left(\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \mathbf{\Sigma}_v\right)$ with $\mathbf{\Sigma}_v \triangleq \mathrm{E}\left\{\mathbf{v}\mathbf{v}^H\right\}$. Due to regulations, the relays' total transmit power is also constrained by $\bar{P}_1$, that is, we must have

$$\mathbf{w}^H \mathbf{\Lambda}_1(\mathbf{p}) \mathbf{w} \leq \bar{P}_1. \tag{6}$$

Let $r_{mk}$ stand for the channel gain from the $k$-th relay to $U_m$ and introduce $r_m \triangleq \{r_{m1} \ \ldots \ r_{mK}\}^T$. Then, the interference power afflicted by the relays on $U_m$ can be shown to be equal to

$$\mathrm{E}\left\{\left|\mathbf{r}_m^T \mathbf{x}\right|^2\right\} = \mathbf{w}^H \mathbf{\Lambda}_{m+1}(\mathbf{p}) \mathbf{w} \tag{7}$$

where $\mathbf{\Lambda}_{m+1}(\mathbf{p}) \triangleq \mathbf{D}(\mathbf{r}_m)\left(\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \mathbf{\Sigma}_v\right)\mathbf{D}(\mathbf{r}_m)^H$. Assuming that the acceptable threshold of the latter interference power is $\bar{P}_{m+1}$, it should hold that

$$\mathbf{w}^H \mathbf{\Lambda}_m(\mathbf{p}) \mathbf{w} \leq \bar{P}_m \qquad m = 2, \ldots, M+1. \tag{8}$$

Let $h_{kl}$ be the channel gain from the $k$-th relay to $D_l$ and introduce $\mathbf{h}_l \triangleq [h_{1l} \ldots h_{Kl}]^T$. The received signal at $D_l$ is

$$z_l = \mathbf{h}_l^T \mathbf{x} + n_l \qquad l = 1, \ldots, L \tag{9}$$

where $n_l$ is the noise at $D_l$ with $\mathrm{E}\left\{|n_l|^2\right\} = \sigma_{n_l}^2$. Using (3) and (4) in (9), it can be readily shown for $l = 1, \ldots, L$ that

$$z_l = \mathbf{w}^H \mathbf{f}_l^{(l)} s_l + \sum_{k=1, k\neq l}^{L} \mathbf{w}^H \mathbf{f}_k^{(l)} s_k + \mathbf{w}^H \left(\mathbf{h}_l \odot \mathbf{v}\right) + n_l \tag{10}$$

where $\mathbf{f}_k^{(l)} \triangleq \mathbf{h}_l \odot \mathbf{g}_k$. Using (10), the SINR at $D_l$ is

$$\eta_l\left(\mathbf{w}, \mathbf{p}\right) = \frac{p_l \left|\mathbf{w}^H \mathbf{f}_l^{(l)}\right|^2}{\sum_{k=1, k\neq l}^{L} p_k \left|\mathbf{w}^H \mathbf{f}_k^{(l)}\right|^2 + \mathbf{w}^H \mathbf{\Gamma}_v^{(l)} \mathbf{w} + \sigma_{n_l}^2} \tag{11}$$

where $\mathbf{\Gamma}_v^{(l)} \triangleq \mathbf{D}(\mathbf{h}_l)\mathbf{\Sigma}_v \mathbf{D}(\mathbf{h}_l)^H$. We use $\bar{\eta}_l\left(\mathbf{w}, \mathbf{p}\right) \triangleq \eta_l\left(\mathbf{w}, \mathbf{p}\right)/\gamma_l$ as the $S_l - D_l$ communication performance metric where $\gamma_l$ is a normalization factor that can be selected proportional to the target SINR at $D_l$. We aim to derive the jointly-optimal power vector $\mathbf{p}$ and the beamforming vector $\mathbf{w}$ that maximize the worst normalized SINR $\bar{\eta}_l\left(\mathbf{w}, \mathbf{p}\right)$ subject to the $L + 2M + 2$ constraints imposed by (1), (2), (6), and (8). The problem can be mathematically represented as:

$$\max_{\mathbf{w}, \mathbf{p}} \quad \min_{1 \leq l \leq L} \bar{\eta}_l\left(\mathbf{w}, \mathbf{p}\right) \tag{12a}$$

$$\text{s.t.} \quad (1), (2), (6), \text{and } (8). \tag{12b}$$

## B. ORDC case

For the sake of notational simplicity in the ORDC case, we consider a $K$-relay antenna (compare with the $K$ single-antenna relays in the CRDC case). Moreover, we ignore the presence of the primary users. Including the primary users' effect in the ORDC case is straightforward and is discussed in [8]. In the second phase of the ORDC scheme, the relay-destinations transmissions are carried out in $L$ dedicated channels. In particular, the relay multiplies its $K$-dimensional received signal vector by a relaying matrix $\mathbf{W}^{(l)^*}$ and forwards the resulting signal vector to the $l$th destination in the $l$th dedicated channel for $l = 1, \dots, L$. The signal transmitted from the relay in the $l$-th dedicated channel can then be expressed as

$$\mathbf{x}^{(l)} = \mathbf{W}^{(l)^*}\mathbf{y}. \tag{13}$$

From (3) and (13) it follows that the relay's transmit power in the $l$-th channel is

$$\begin{aligned} p_r^{(l)} &= \mathrm{E}\left\{\mathbf{x}^{(l)^H}\mathbf{x}^{(l)}\right\} \\ &= \mathrm{tr}\left(\mathbf{W}^{(l)^*}\left(\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \boldsymbol{\Sigma}_v\right)\mathbf{W}^{(l)^T}\right) \\ &= \mathbf{w}^{(l)^H}\boldsymbol{\Lambda}(\mathbf{p})\mathbf{w}^{(l)} \end{aligned} \tag{14}$$

where $\boldsymbol{\Lambda}(\mathbf{p}) \triangleq \left(\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \boldsymbol{\Sigma}_v\right) \otimes \mathbf{I}_K$, and $\mathbf{w}^{(l)} \triangleq \mathrm{vec}\left(\mathbf{W}^{(l)}\right)$. The following power constraint must be upheld:

$$\mathbf{w}^{(l)^H}\boldsymbol{\Lambda}(\mathbf{p})\mathbf{w}^{(l)} \leq P^{(l)} \qquad l = 1, \dots, L \tag{15}$$

where $P^{(l)}$ is the upper-bound on the relay's transmit power in the $l$th channel. Introducing $\mathbf{h}^{(l)} \triangleq \left[h_1^{(l)} \;\dots\; h_K^{(l)}\right]^T$, the received signal at the $l$th destination in its dedicated channel is

$$z_l = \mathbf{h}^{(l)^T}\mathbf{x}^{(l)} + n_l \qquad l = 1, \dots, L. \tag{16}$$

Using (3) and (13) in (16), it can be readily shown for $l = 1, \dots, L$ that

$$z_l = \mathbf{w}^{(l)^H}\mathbf{f}_l^{(l)}s_l + \sum_{\substack{k=1,\\ k\neq l}}^{L}\mathbf{w}^{(l)^H}\mathbf{f}_k^{(l)}s_k + \mathbf{w}^{(l)^H}\left(\mathbf{v}\otimes\mathbf{h}^{(l)}\right) + n_l \tag{17}$$

where $\mathbf{f}_k^{(l)} \triangleq \mathbf{g}_k \otimes \mathbf{h}^{(l)}$. From (17), it is direct to obtain the SINR at the $l$th destination as

$$\eta_l\left(\mathbf{w}^{(l)}, \mathbf{p}\right) = \frac{p_l\left|\mathbf{w}^{(l)^H}\mathbf{f}_l^{(l)}\right|^2}{\sum\limits_{\substack{k=1\\ k\neq l}}^{L} p_k\left|\mathbf{w}^{(l)^H}\mathbf{f}_k^{(l)}\right|^2 + \mathbf{w}^{(l)^H}\boldsymbol{\Gamma}_v^{(l)}\mathbf{w}^{(l)} + \sigma_{n_l}^2} \tag{18}$$

where $\boldsymbol{\Gamma}_v^{(l)} \triangleq \boldsymbol{\Sigma}_v \otimes \mathbf{h}^{(l)}\mathbf{h}^{(l)^H}$. Introducing $\bar{\eta}_l\left(\mathbf{w}^{(l)}, \mathbf{p}\right) \triangleq \eta_l\left(\mathbf{w}^{(l)}, \mathbf{p}\right)/\gamma_l$, our problem of interest in the ORDC case can be formally expressed as

$$\max_{\mathbf{W},\mathbf{p}} \min_{1\leq l\leq L} \bar{\eta}_l\left(\mathbf{w}^{(l)}, \mathbf{p}\right) \tag{19a}$$

$$\text{subject to (1) and (15)} \tag{19b}$$

where $\mathbf{W} \triangleq [\mathbf{W}^{(1)} \dots \mathbf{W}^{(L)}]$.

## III. Jointly-optimal sources' transmit power and relays' beamforming weights

### A. CRDC case

In the first subsection, we derive a necessary and sufficient condition for the jointly-optimal sources' transmit power vector $\mathbf{p}_o$ and relays' beamforming vector $\mathbf{w}_o$. In the next two subsections, we assume the knowledge of $\mathbf{w}_o$ ($\mathbf{p}_o$) and use the necessary and sufficient joint-optimality condition to obtain $\mathbf{p}_o$ ($\mathbf{w}_o$). Our results will then be used in the last subsection to introduce an efficient iterative algorithm to jointly optimize the sources' transmit power and relays' beamforming vectors. The proofs of all results are outlined in [13].

*1) Necessary and sufficient joint-optimality condition: Theorem 1:* $(\mathbf{w}_o, \mathbf{p}_o)$ are the jointly-optimal solutions to (12) if and only if **C1**, **C2**, and **C3** are satisfied.

**C1:** $(\mathbf{w}_o, \mathbf{p}_o)$ balances all normalized SINRs, that is,

$$\bar{\eta} = \bar{\eta}_l(\mathbf{w}_o, \mathbf{p}_o) \qquad l = 1, \dots, L. \tag{20}$$

**C2:** $\mathbf{w}_o$ is the solution to

$$\max_{\mathbf{w}} \min_{1\leq l\leq L} \bar{\eta}_l(\mathbf{w}, \mathbf{p}_o) \tag{21a}$$

$$\text{s.t. } \mathbf{w}^H\boldsymbol{\Lambda}_m(\mathbf{p}_o)\mathbf{w} \leq \bar{P}_m \;\; m = 1, \dots, M+1. \tag{21b}$$

**C3:** There is a constraint in (1) and (2) that is satisfied with equality, that is,

$$\mathbf{u}_{l^\star}^T\mathbf{p}_o = P_{l^\star} \;\; \text{for an} \;\; l^\star \in \{1, \dots, L+M+1\}. \tag{22}$$

**C1** is the necessary SINR-balancing property [9]- [12]. If **C2** does not hold, then $(\mathbf{w}_o, \mathbf{p}_o)$ results in a smaller objective function in (12) than $(\tilde{\mathbf{w}}, \mathbf{p}_o)$ where $\tilde{\mathbf{w}}$ is the solution to (21). This is in contradiction with the optimality of $(\mathbf{w}_o, \mathbf{p}_o)$. Note that **C2** implies that at least one of the constraints in (21b) is satisfied with equality. **C3** is proved in [13].

*2) Deriving $\mathbf{p}_o$ for the given $\mathbf{w}_o$:* Assuming the knowledge of $\mathbf{w}_o$, $\mathbf{p}_o$ can be obtained using **C1** and **C3**. Let the $L\times L$ non-negative matrix $\boldsymbol{\Psi}(\mathbf{w})$ such that $[\boldsymbol{\Psi}(\mathbf{w})]_{lk} \triangleq \left|\mathbf{w}^H\mathbf{f}_k^{(l)}\right|^2$ for $l \neq k$ and $[\boldsymbol{\Psi}(\mathbf{w})]_{ll} \triangleq 0$, the $L \times L$ positive diagonal matrix $\boldsymbol{\Omega}(\mathbf{w})$ such that $[\boldsymbol{\Omega}(\mathbf{w})]_{ll} \triangleq \gamma_l / \left|\mathbf{w}^H\mathbf{f}_l^{(l)}\right|^2$, and the $L \times 1$ positive vector $\boldsymbol{\sigma}(\mathbf{w})$ such that $[\boldsymbol{\sigma}(\mathbf{w})]_l \triangleq \mathbf{w}^H\boldsymbol{\Gamma}_v^{(l)}\mathbf{w} + \sigma_{n_l}^2$. Then, it is straightforward to show from (11) and **C1** that

$$\boldsymbol{\Omega}(\mathbf{w}_o)\boldsymbol{\Psi}(\mathbf{w}_o)\mathbf{p}_o + \boldsymbol{\Omega}(\mathbf{w}_o)\boldsymbol{\sigma}(\mathbf{w}_o) = \frac{1}{\bar{\eta}}\mathbf{p}_o. \tag{23}$$

Using (22) in (23), it also follows that

$$\frac{1}{P_l^\star}\mathbf{u}_{l^\star}^T\boldsymbol{\Omega}(\mathbf{w}_o)\boldsymbol{\Psi}(\mathbf{w}_o)\mathbf{p}_o + \frac{1}{P_l^\star}\mathbf{u}_{l^\star}^T\boldsymbol{\Omega}(\mathbf{w}_o)\boldsymbol{\sigma}(\mathbf{w}_o) = \frac{1}{\bar{\eta}}. \tag{24}$$

Introducing

$$\boldsymbol{\Theta}_l(\mathbf{w}) \triangleq \begin{bmatrix} \boldsymbol{\Omega}(\mathbf{w})\boldsymbol{\Psi}(\mathbf{w}) & \boldsymbol{\Omega}(\mathbf{w})\boldsymbol{\sigma}(\mathbf{w}) \\ \frac{1}{P_l}\mathbf{u}_l^T\boldsymbol{\Omega}(\mathbf{w})\boldsymbol{\Psi}(\mathbf{w}) & \frac{1}{P_l}\mathbf{u}_l^T\boldsymbol{\Omega}(\mathbf{w})\boldsymbol{\sigma}(\mathbf{w}) \end{bmatrix}, \tag{25}$$

it holds from (23) and (24) that

$$\boldsymbol{\Theta}_{l^\star}(\mathbf{w}_o)\begin{bmatrix}\mathbf{p}_o \\ 1\end{bmatrix} = \frac{1}{\bar{\eta}}\begin{bmatrix}\mathbf{p}_o \\ 1\end{bmatrix}. \tag{26}$$

It follows from Eq. (26) that $1/\bar{\eta}$ and $[\mathbf{p}_o^T\ 1]^T$ are a jointly-positive eigenpair of $\mathbf{\Theta}_{l^\star}(\mathbf{w}_o)$. As $\mathbf{\Theta}_l(\mathbf{w})$ are nonnegative primitive matrices, it can be shown that [14, Ch. 8]

$$\begin{bmatrix}\mathbf{p}_o \\ 1\end{bmatrix} = \Omega\left(\mathbf{\Theta}_{l^\star}(\mathbf{w}_o)\right). \tag{27}$$

It remains to obtain $l^\star$. $\mathbf{p}_o$ is required to satisfy all inequalities in (1) and (2). This requirement is met if and only if [8], [13]

$$l^\star = \underset{1\leq l\leq L+M+1}{\operatorname{argmax}} \ \lambda_{\max}\left(\mathbf{\Theta}_l\left(\mathbf{w}_o\right)\right). \tag{28}$$

*3) Deriving* $\mathbf{w}_o$ *for the given* $\mathbf{p}_o$*:* When $\mathbf{p}_o$ is known, $\mathbf{w}_o$ may be obtained using **C2**. Let $\mathbf{F} \triangleq \left[\mathbf{f}_1^{(l)}\ldots\mathbf{f}_L^{(l)}\right]$ and $\mathbf{\Xi}_l(\mathbf{p}) \triangleq \mathbf{F}_{\bullet\bar{l}}\mathbf{p}_{\bar{l}}\mathbf{F}_{\bullet\bar{l}}^H + \mathbf{\Gamma}_v^{(l)}$. Then, (11) can be used to equivalently represent (21) as

$$\max_{\mathbf{w}} \ z \tag{29a}$$

$$\text{s.t. } \mathbf{w}^H\mathbf{\Theta}_m(\mathbf{p}_o)\mathbf{w} \leq \bar{P}_m \ \ m{=}1,\ldots,M+1 \tag{29b}$$

$$z\left(\mathbf{w}^H\mathbf{\Xi}_l(\mathbf{p}_o)\mathbf{w} + \sigma_{n_l}^2\right) \leq [\mathbf{p}_o]_l\left|\mathbf{w}^H\mathbf{f}_l^{(l)}\right|^2 \ \ l{=}1,\ldots,L \tag{29c}$$

Problem (29) is NP-hard. We can find (an approximate) solution to (29) in a polynomial time using a semi-definite relaxation (SDR) approach [3], [15], [16] along with a bisection method as follows. Let $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$ and $\mathbf{F}_l \triangleq \mathbf{f}_l^{(l)}\mathbf{f}_l^{(l)H}$. Then, the SDR version of (29) can be rewritten as

$$\max_{\mathbf{X}} \ z \tag{30a}$$

$$\text{s.t. } \operatorname{tr}\left(\mathbf{X}\mathbf{\Theta}_m(\mathbf{p}_o)\right) \leq \bar{P}_m \ \ m{=}1,\ldots,M+1 \tag{30b}$$

$$z\left(\operatorname{tr}\left(\mathbf{X}\mathbf{\Xi}_l(\mathbf{p}_o)\right) + \sigma_{n_l}^2\right) \leq [\mathbf{p}_o]_l\operatorname{tr}\left(\mathbf{X}\mathbf{F}_l\right) \ \ l{=}1,\ldots,L \tag{30c}$$

$$\mathbf{X} \succeq \mathbf{0}. \tag{30d}$$

For a fixed $z = z_q$, the associated feasibility problem of (30)

$$\text{Find } \mathbf{X} \tag{31a}$$

$$\text{s.t. } \operatorname{tr}\left(\mathbf{X}\mathbf{\Theta}_m(\mathbf{p}_o)\right) \leq \bar{P}_m \ \ m{=}1,\ldots,M+1 \tag{31b}$$

$$z_q\left(\operatorname{tr}\left(\mathbf{X}\mathbf{\Xi}_l(\mathbf{p}_o)\right) + \sigma_{n_l}^2\right) \leq [\mathbf{p}_o]_l\operatorname{tr}\left(\mathbf{X}\mathbf{F}_l\right) \ \ l{=}1,\ldots,L \tag{31c}$$

$$\mathbf{X} \succeq \mathbf{0} \tag{31d}$$

is comprised only of convex constraints and, hence, can be efficiently solved using standard convex optimization tools. Let $\mathbf{X}_o$ and $z_o$ be the optimal variable and the optimal value of the objective function in (30), respectively. Then, (31) is feasible if and only if $z_q \leq z_o$. This property can be used to obtain $\mathbf{X}_o$ with an arbitrary accuracy as follows. First, find $z_{\min}$ and $z_{\max}$ such that $z_{\min} \leq z_o \leq z_{\max}$. Then, solve (31) with $z_q = (z_{\min} + z_{\max})/2$ for $q = 1$. If (31) is feasible, set $z_{\min} = z_1$, if not, set $z_{\max} = z_1$ and solve (31) with $z_q = (z_{\min} + z_{\max})/2$ for $q = 2$. Continue this procedure until $z_{\max} - z_{\min} \leq \epsilon$ where $\epsilon$ is an arbitrarily-selected accuracy measure. The last feasible $\mathbf{X}$ is an approximation to $\mathbf{X}_o$, the solution to (30). $\mathbf{X}$ can become arbitrarily close to $\mathbf{X}_o$ by selecting smaller thresholds $\epsilon$. If the so-obtained $\mathbf{X}_o$ turns out to be rank-one, then $\mathbf{w}_o$ is a properly-normalized principal eigenvector of $\mathbf{X}_o$ [13]. Otherwise, an efficient approximation of $\mathbf{w}_o$ can be obtained by applying standard randomization techniques [15] on $\mathbf{X}_o$.

*4) Jointly optimizing* $\mathbf{p}$ *and* $\mathbf{w}$*:* Based on our results in Sections III-A2 and III-A3, we propose Algorithm I to optimize $\mathbf{p}$ and $\mathbf{w}$.

**Algorithm I: Joint optimization of the sources' power and the relays' beamforming vectors in the CRDC case**

1: `Initialize:` $n = 0$; $\mathbf{p}_{[0]} \geq \mathbf{0}$
2: $n = n + 1$

%% Finding $\mathbf{w}_{[n]}$ given $\mathbf{p}_{[n-1]}$:
3: `Substitute` $\mathbf{p}_{[n-1]}$ `in lieu of` $\mathbf{p}_o$ `in (30) and obtain the solution` $\mathbf{X}_{[n]}$
4A: $\mathbf{X}_{[n]}$ `rank-one: Normalize the principal eigenvector of` $\mathbf{X}_{[n]}$ `to obtain` $\mathbf{w}_{[n]}$
4B: $\mathbf{X}_{[n]}$ `higher rank: Approximate` $\mathbf{w}_{[n]}$ `using a randomization technique`

%% Finding $\mathbf{p}_{[n]}$ given $\mathbf{w}_{[n]}$:
5: $l_{[n]}^\star = \underset{1\leq l\leq L+M+1}{\operatorname{argmax}} \ \lambda_{\max}\left(\mathbf{\Theta}_l\left(\mathbf{w}_{[n]}\right)\right)$.
6: $\begin{bmatrix}\mathbf{p}_{[n]} \\ 1\end{bmatrix} = \Omega\left(\mathbf{\Theta}_{l_{[n]}^\star}\left(\mathbf{w}_{[n]}\right)\right)$

7: `Go back to step 2 until` $\left\|\mathbf{p}_{[n]} - \mathbf{p}_{[n-1]}\right\| \leq \delta$ `where` $\delta$ `is a small number or` $n = n_{\max}$

Simulations show that when $\mathbf{X}_{[n]}$ is rank-one in all iterations, Algorithm I converges very rapidly. However, if $\mathbf{X}_{[n]}$ is not rank-one, $\mathbf{p}_{[n]}$ and $\mathbf{w}_{[n]}$ may not converge in Algorithm I and one may have to stop the algorithm after the number of iterations reaches $n_{\max}$. In the latter case, $\mathbf{p}_{[n_{\max}]}$ and $\mathbf{w}_{[n_{\max}]}$ are only approximates of the optimal pair of $\mathbf{p}_o$ and $\mathbf{w}_o$

### *B. ORDC case*

Similar steps as in the CRDC case can be followed to jointly optimize $\mathbf{W}$ and $\mathbf{p}$. However, unlike in the CRDC case, when $\mathbf{p}_o$ is known, $\mathbf{W}_o$ can be found in closed form as [8]

$$\mathbf{w}_o^{(l)} = \zeta_l(\mathbf{p}_o)\mathbf{R}_l(\mathbf{p}_o)^{-1}\mathbf{f}_l^{(l)} \qquad l = 1,\ldots,L \tag{32a}$$

where $\zeta_l(\mathbf{p}) \triangleq \sqrt{P^{(l)}}\left(\mathbf{f}_l^{(l)H}\mathbf{R}_l(\mathbf{p})^{-1}\mathbf{\Lambda}(\mathbf{p})\mathbf{R}_l(\mathbf{p})^{-1}\mathbf{f}_l^{(l)}\right)^{-1/2}$ and $\mathbf{R}_l(\mathbf{p}) \triangleq P^{(l)}\left(\mathbf{F}_{\bullet\bar{l}}^{(l)}\mathbf{D}(\mathbf{p}_{\bar{l}})\mathbf{F}_{\bullet\bar{l}}^{(l)H} + \mathbf{\Gamma}_v^{(l)}\right) + \sigma_{n_l}^2\mathbf{\Lambda}(\mathbf{p})$ with $\mathbf{F}^{(l)} \triangleq \left[\mathbf{f}_1^{(l)}\cdots\mathbf{f}_L^{(l)}\right]$. The above result along with our discussion in Section III-A2 give rise to Algorithm II that iteratively finds $\mathbf{W}_o$ and $\mathbf{p}_o$. As $\mathbf{W}_o$ and $\mathbf{p}_o$ are closed-form functions of one another, the convergence of $\mathbf{W}_{[n]}$ and $\mathbf{p}_{[n]}$ to $\mathbf{W}_o$ and $\mathbf{p}_o$ is guaranteed in Algorithm II [8]. Note that in Algorithm II we have

$$\mathbf{\Theta}_l(\mathbf{W}) \triangleq \begin{bmatrix}\mathbf{\Omega}(\mathbf{W})\mathbf{\Psi}(\mathbf{W}) & \mathbf{\Omega}(\mathbf{W})\boldsymbol{\sigma}(\mathbf{W}) \\ \frac{1}{P_l}\mathbf{u}_l^T\mathbf{\Omega}(\mathbf{W})\mathbf{\Psi}(\mathbf{W}) & \frac{1}{P_l}\mathbf{u}_l^T\mathbf{\Omega}(\mathbf{W})\boldsymbol{\sigma}(\mathbf{W})\end{bmatrix} \tag{33}$$

where the $L \times L$ non-negative matrix $\mathbf{\Psi}(\mathbf{W})$ is such that $[\mathbf{\Psi}(\mathbf{W})]_{lk} \triangleq \left|\mathbf{w}^{(l)H}\mathbf{f}_k^{(l)}\right|^2$ for $l \neq k$ and $[\mathbf{\Psi}(\mathbf{W})]_{ll} \triangleq 0$, the $L \times L$ positive diagonal matrix $\mathbf{\Omega}(\mathbf{W})$ is such that

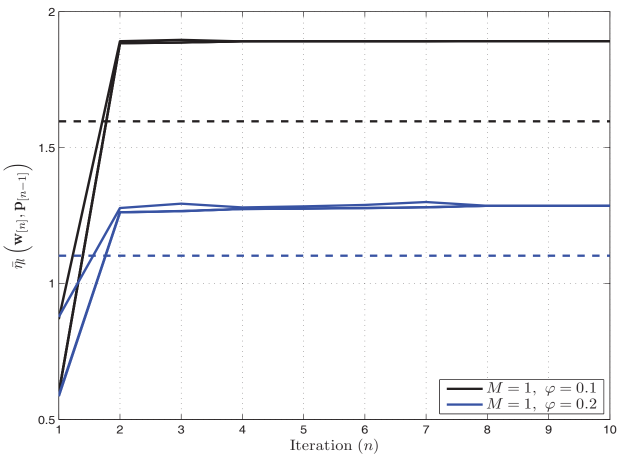


Fig. 2. $\bar{\eta}_l\left(\mathbf{w}_{[n]},\mathbf{p}_{[n-1]}\right)$ v.s. the iteration index $n$ for $M=1$ in the CRDC case.

$[\boldsymbol{\Omega}(\mathbf{W})]_{ll} \triangleq \gamma_l / \left|{\mathbf{w}^{(l)}}^H \mathbf{f}_l^{(l)}\right|^2$, and the $L \times 1$ positive vector $\boldsymbol{\sigma}(\mathbf{W})$ is such that $[\boldsymbol{\sigma}(\mathbf{W})]_l \triangleq {\mathbf{w}^{(l)}}^H \boldsymbol{\Gamma}_v^{(l)} \mathbf{w}^{(l)} + \sigma_{n_l}^2$.

**Algorithm II: Joint optimization of the sources' power and the relays' beamforming vectors in the ORDC case**

1: `Initialize:` $n=0$; $\mathbf{p}_{[0]} \geq \mathbf{0}$
2: `Repeat`
3: $n=n+1$
4: $\mathbf{w}_{[n]}^{(l)} = \zeta_l\left(\mathbf{p}_{[n-1]}\right)\mathbf{R}_l\left(\mathbf{p}_{[n-1]}\right)^{-1}\mathbf{f}_l^{(l)} \qquad l=1,\ldots,L$
5: $\mathbf{W}_{[n]} = \left[\mathbf{W}_{[n]}^{(1)} \cdots \mathbf{W}_{[n]}^{(L)}\right]$
6: $l_{[n]}^\star = \underset{1\le l\le L+1}{\operatorname{argmax}} \lambda_{\max}\left(\boldsymbol{\Theta}_l\left(\mathbf{W}_{[n]}\right)\right)$
7: $\begin{bmatrix}\mathbf{p}_{[n]}\\ 1\end{bmatrix} = \Omega\left(\boldsymbol{\Theta}_{l_{[n]}^\star}\left(\mathbf{W}_{[n]}\right)\right)$
8: `until` $\left\|\mathbf{p}_{[n+1]}-\mathbf{p}_{[n]}\right\| \le \epsilon$ `where` $\epsilon$ `is a small number.`

## IV. Simulations

### A. CRDC case

A numerical experiment is conducted to investigate the performance of Algorithm I. In this example, $L=4$, $K=20$, $\sigma_{n_l}^2=\sigma^2$ for $l=1,\ldots,L$, and $\boldsymbol{\Sigma}_v=\sigma^2\mathbf{I}_K$. The individual sources' transmit power constraints $P_l$, $l=1,\ldots,L$ are such that $10\log_{10}(P_l/\sigma^2)=15$. In turn, the sources' total transmit power constraint is $P_{L+1}=0.9\sum_{l=1}^{L}P_l$. Maximum admissible interference power levels of primary users $P_l$, $l=L+2,\ldots,L+M+1$ and $\bar{P}_m$, $m=2,\ldots,M+1$ are all equal and $10\log_{10}(\bar{P}_2/\sigma^2)=5$. Finally, the relays' total transmit power constraint $\bar{P}_1=LP_1$. The target SINRs are set to $\gamma_l=15$ (dB) for $l=1,\ldots,L$. All source-relay, relay-destination, source-primary user, and relay-primary user channel gains are randomly and independently drawn from zero-mean circular complex Gaussian distributions with $\mathrm{E}\left\{|g_{k1}|^2\right\}=16\sigma^2$, $\mathrm{E}\left\{|g_{kl}|^2\right\}=\sigma^2$, $l=2,\ldots,L$, $\mathrm{E}\left\{|h_{kl}|^2\right\}=\sigma^2$, $l=1,\ldots,L$, $\mathrm{E}\left\{|t_{ml}|^2\right\}=\mathrm{E}\left\{|r_{ml}|^2\right\}=\varphi\sigma^2$, $l=1,\ldots,L$.

The following two-step technique to optimize the sources' transmit power and relays' beamforming vectors is also examined and is compared to the proposed technique: 1) Let all sources transmit with a common power $P_{\mathrm{c}}$. Maximize $P_{\mathrm{c}}$ such that the sources' transmit power vector $\mathbf{p}_{\mathrm{c}}=P_{\mathrm{c}}\mathbf{1}$ satisfies all $L+M+1$ constraints in (1) and (2); 2) Find $\mathbf{w}_{\mathrm{c}}$ as the solution to[1]

$$\max_{\mathbf{w}} \min_{1\le l\le L} \bar{\eta}_l(\mathbf{w},\mathbf{p}_{\mathrm{c}}) \tag{34a}$$

$$\text{s.t. } \mathbf{w}^H\boldsymbol{\Lambda}_m(\mathbf{p}_{\mathrm{c}})\mathbf{w} \le \bar{P}_m \;\; m=1,\ldots,M+1. \tag{34b}$$

Note that obtaining (an approximate of) $\mathbf{w}_{\mathrm{c}}$ also requires using a SDR technique to solve an NP-hard problem. The above algorithm can be viewed as a "separated" optimization technique of $\mathbf{p}$ and $\mathbf{w}$ in contrast to our proposed technique to jointly optimize $\mathbf{p}$ and $\mathbf{w}$.

In Fig. 2, Algorithm I is used to plot $\bar{\eta}_l\left(\mathbf{w}_{[n]},\mathbf{p}_{[n-1]}\right)$, $l=1,\ldots,L$ versus the iteration index $n$ for $M=1$ and two different $\varphi$. As can be observed from the figure, all normalized SINRs converge to a common value in a few iterations. The dashed lines show $\min_{1\le l\le L}\bar{\eta}_l(\mathbf{w}_{\mathrm{c}},\mathbf{p}_{\mathrm{c}})$. For both examined $\varphi$, the proposed algorithm offers a considerably larger minimum normalized SINR than the above-described separated optimization technique.

### B. ORDC case

In the ORDC case, we consider $K=8$ relay antennas and $L=(K/2)+1=5$ source-destinations pairs and assume that the noise power at all relay antennas and all destinations is $\sigma^2$ and set $P^{(l)}=\gamma_l=10\sigma^2$ for $l=1,\ldots,L$.

Fig. 3 shows $\bar{\eta}_l\left(\mathbf{w}_{[n]}^{(l)},\mathbf{p}_{[n-1]}\right)$ from Algorithm II for $l=1,\ldots,L$ versus the iteration index $n$ in the case that the upper-bounds of the source powers are equal and are given by $P_l=1.1\cdot\gamma_l=11\sigma^2$, $l=1,\ldots,L$ and the upper-bound on the sources total transmit power is $P_{L+1}=L\gamma_l=50\sigma^2$. As can be observed from Fig. 3, all normalized SINRs converge to the same value in $n=4$ iterations. This verifies the efficiency of Algorithm I to obtain $\mathbf{W}_o$ and $\mathbf{p}_o$.

In the next numerical example, it is assumed that $P_l=\varphi\cdot p$ for $l=1,\ldots,L$ and $P_{L+1}=Lp=5p$. Then, Algorithm II is used to obtain the optimal balanced normalized SINRs $\bar{\eta}_l\left(\mathbf{w}_o^{(l)},\mathbf{p}_o\right)$. Fig. 4 shows $\bar{\eta}_l\left(\mathbf{w}_o^{(l)},\mathbf{p}_o\right)$ versus $p/\sigma^2$ for several $\varphi$. For the sake of comparison, we have also shown with the dashed line the minimum of the normalized SINRs when only the relaying matrices are optimized and the transmit powers of all sources are equal to $p$. Note that in the latter case the sources total transmit power is equal to $P_{L+1}$, and, hence, is always larger than or equal to the sources total transmit power in the case when the sources' powers and the relaying matrices are jointly optimized. Despite the above fact, Fig. 4 shows that our joint optimization approach always performs better than the case that the sources transmit with equal powers.

[1] See, for instance, [15] for a similar approach to optimize the beamforming vector.

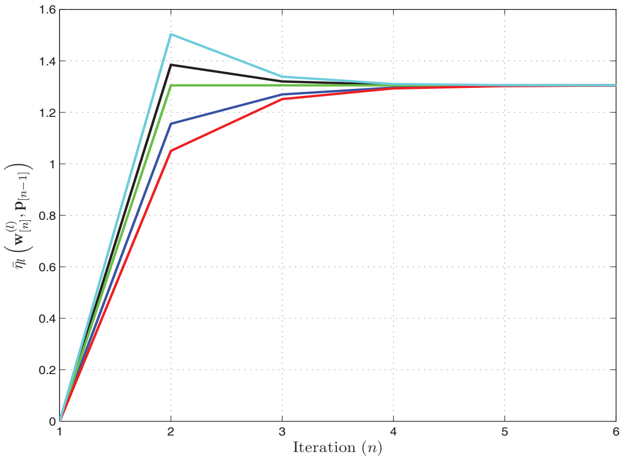


Fig. 3. $\bar{\eta}_l\left(\mathbf{w}_{[n]}^{(l)}, \mathbf{p}_{[n-1]}\right)$ v.s. the iteration index $n$ for $K = 8$ and $L = 5$ in the ORDC case.

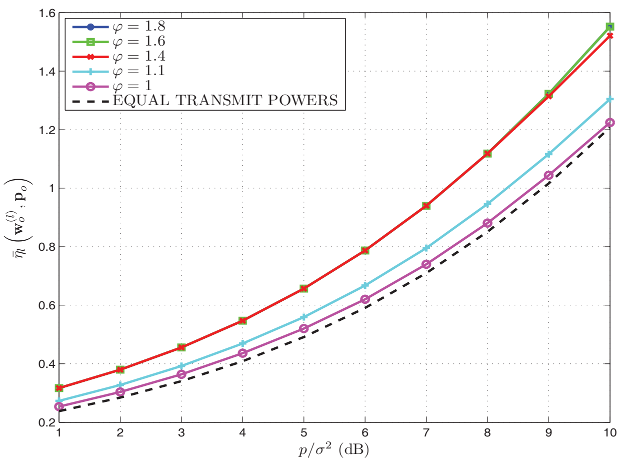


Fig. 4. $\bar{\eta}_l\left(\mathbf{w}_o^{(l)}, \mathbf{p}_o\right)$ v.s. $p/\sigma^2$ for several $\varphi$ for $K = 8$ and $L = 5$ in the ORDC case.

## V. Conclusions

We considered a cognitive cooperative network wherein $K$ relaying antennas receive faded and noisy mixtures of the transmitted signals from $L$ sources, multiply them with a properly-selected beamforming weights, and forward the results towards $L$ destinations. Both common relay-destination channel (CRDC) and orthogonal relay-destination channel (ORDC) cases were studied. Aiming to maximize the minimum signal-to-interference-plus-noise ratio among all $L$ destinations, we jointly optimized the sources' transmit powers and the relays' beamforming weights while concurrently satisfying all the following constraints: 1) The sources and the relays total transmit power constraints; 2) The sources individual transmit power constraints; and 3) The maximum interference power that the cognitive sources and relays are allowed to inflict on the $M$ existing primary users. In both CRDC and ORDC cases, the sources' transmit powers and the relays' beamforming weights are optimized using efficient iterative algorithms. While unlike in the ORDC case the proposed algorithm for the CRDC case does not guarantee the globally optimal solution, simulation results verified in both cases that the performance of the proposed joint optimization techniques are still superior to the technique that separately optimizes the sources' transmit powers and the relays' beamforming weights.

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