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Joint source power control and relay beamforming in amplify-and-forward cognitive networks with multiple source-destination pairs

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***Abstract*— We consider a multipoint-to-multipoint cognitive network wherein the communications from L sources to their designated destinations are carried out through the use of K relays in a dual-hop amplify-and-forward cooperative scheme. Aiming to maximize the worst signal-to-interference-plus-noise ratio of the L destinations, we develop a technique that jointly optimizes the sources' transmit powers and the relays' beamforming weights while satisfying the sources and the relays total transmit power constraints as well as the sources individual power constraints and further guaranteeing that the interference powers inflicted from the cognitive network on the M existing primary users are below acceptable thresholds.**

I. Introduction

There is a growing research interest in the applications of multi-point to multi-point wireless networks wherein multiple sources communicate with their dedicated destinations [1]-[8]. When all sources concurrently transmit in the same frequency band, destinations experience significant interfering powers from undesired sources. Proper beamforming techniques at sources and/or destinations may be used to alleviate the interference effect and substantially improve the signal-to-interference-plus-noise ratio (SINR) at the destinations. However, if the network terminals are single antenna, transmit or receive beamforming techniques are infeasible. In such cases, the beamforming gain may alternatively be achieved by means of using multiple relaying terminals in the form of a virtual antenna array that receives the signals from the sources and retransmits their properly-processed versions towards the destinations [3], [5], [7]-[9].

It is likely that such a dual-hop multi-point to multi-point wireless network operates in a frequency band that is pre-occupied by, for instance, an existing wireless cellular system. This obliges the network to act as a cognitive (secondary) network whose inter-terminal communications are constrained by the maximum interference power admissible to the receivers in the existing (primary) system. In this paper, we consider a cognitive dual-hop multi-point to multi-point wireless network with L single-antenna source-destination pairs and K single-antenna relays that operate in the presence of M primary receivers. In the first phase, L sources transmit and the K relays receive different faded and noisy mixtures of the sources' signals. In the second phase, the relays multiply their received signals with properly-selected beamforming weights and retransmit them to the destinations. The goal is to find the set of jointly optimal sources' transmit powers and relays' beamforming weights that maximize the minimum SINR among all destinations while concurrently satisfying constraints on sources' and relays' total transmit powers, sources' individual transmit powers, and the maximum interference power that is admissible to the M primary users.

We derive a joint-optimality condition and use it to develop an efficient algorithm to iteratively optimize the sources' transmit powers and the relays' beamforming weights. Each of the iterations uses a semi-definite relaxation (SDR) technique to obtain (an approximate) solution to an NP-hard problem. The algorithm converges to the optimal values of the sources' transmit powers and the relays' beamforming weights if the SDR technique has a rank-one solution matrix at every iteration. when the SDR technique does not have a rank-one solution, simulation results show that the proposed joint optimization technique still outperforms the technique that separately optimizes the sources' transmit powers and the relays' beamforming weights.

The rest of this paper is organized as follows. The system model and the problem formulation is presented in Section II. A necessary and sufficient condition for the joint optimality of the sources' transmit power and relays' beamforming vectors is derived and an iterative algorithm to obtain the latter vectors is proposed in Section III. Simulation results are presented in Section IV and concluding remarks are drawn in Section VI.

Notation: Uppercase and lowercase bold letters denote matrices and vectors, respectively. $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote the transpose, the conjugate, and the Hermitian transpose, respectively. $|\cdot|$ is the absolute value, $\|\cdot\|$ is the 2-norm of a vector, and $\mathrm{tr}(\cdot)$ is the trace of a matrix. $\mathrm{E}\{\cdot\}$ stands for the statistical expectation and $\mathbf{I}_K$ is the $K \times K$ identity matrix. $\mathbf{1}$ is the vector with all entries equal to 1 and $\mathbf{e}_i$ is the vector with 1 at the i-th position and zeros elsewhere. $[\cdot]_l$ and $[\cdot]_{nl}$ stand for the lth entry of a vector and the entry at the nth row and the lth column of a matrix, respectively. $\lambda_{\max}(\cdot)$ is the maximum-modulus eigenvalue and $\Omega(\cdot)$ is the eigenvector associated with the maximum-modulus eigenvalue normalized such that its last entry is 1. $\mathbf{D}(\mathbf{a})$ is a diagonal matrix whose diagonal elements are the entries of $\mathbf{a}$. $\mathbf{a}_{\bar{l}}$ is the $L-1 \times 1$ vector obtained by removing the l-th entry of the $L \times 1$ vector

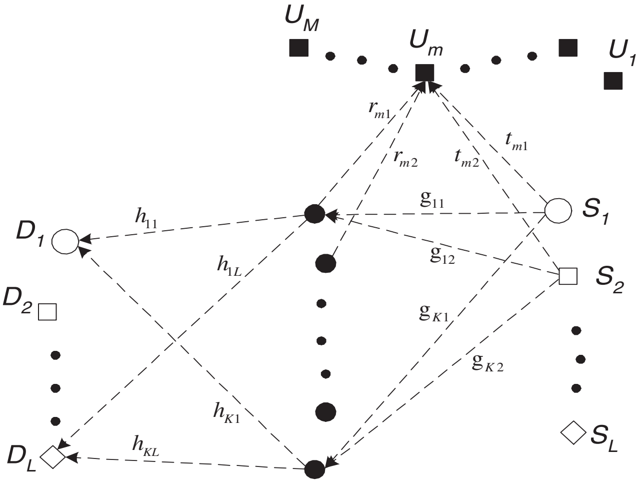

Fig. 1. Dual-hop multipoint-to-multipoint cognitive network

$\mathbf{a}$ and $\mathbf{A}_{\bullet\bar{l}}$ is the $N \times (L-1)$ matrix obtained by removing the lth column of the $N \times L$ matrix $\mathbf{A}$. $\mathrm{rank}(\cdot)$ is the rank of a matrix.

II. System model and problem representation

Consider the dual-hop multipoint-to-multipoint cognitive network shown in Fig. 1 with L source-destination pairs (S_l, D_l), $l = 1, \dots, L$ and K relays all operating in a frequency band allocated to M primary users U_m, $m = 1, \dots, L$. All terminals involved are single-antenna. For the sake of simplicity, it is assumed that there is no direct link from S_l to D_l' for $l, l' \in \{1, \dots, L\}$ and all required $S_l - D_l$, $l = 1, \dots, L$ channel links are established through the use of the following dual-hop cooperative scheme: In the first phase, S_l, $l = 1, \dots, L$ transmit and the K relays receive a faded and noisy mixtures of the transmitted signals. In the second phase, each relay multiplies its received signal with a properly-selected beamforming weight and forwards the result towards the destinations D_l, $l = 1, \dots, L$.

Let s_l be the transmitted signal from S_l where we must have $\mathrm{E}\left\{|s_l|^2\right\} = p_l \le P_l$ for $l = 1, \dots, L$. Further, due to regulation it should also hold that $\sum_{l=1}^{L} p_l \le P_{L+1}$ where $P_{L+1} < \sum_{l=1}^{L} P_l$ as, otherwise, the total power constraint is trivially satisfied. The above source power constraints can be represented in a more compact form as

$$\mathbf{u}_l^T \mathbf{p} \le P_l \qquad l = 1, \dots, L+1 \tag{1}$$

where $\mathbf{u}_l \triangleq \mathbf{e}_l$, $l = 1, \dots, L$, $\mathbf{u}_{L+1} \triangleq \mathbf{1}$, and $\mathbf{p} \triangleq [p_1 \ \dots \ p_L]^T$. Denoting the channel gain from S_l to the m-th primary user U_m as t_{ml}, the interference power inflicted from S_l, $l = 1, \dots, L$ on U_m is $\mathbf{u}_{L+1+m}^T \mathbf{p}$ where $\mathbf{u}_{L+1+m} \triangleq \left[|t_{m1}|^2 \ \dots, |t_{mL}|^2\right]^T$. Assume that P_{L+1+m} is the maximum admissible interference power caused by the cognitive sources S_l, $l = 1, \dots, L$ to U_m. Then, $\mathbf{p}$ should also satisfy

$$\mathbf{u}_l^T \mathbf{p} \le P_l \qquad l = L+2, \dots, L+M+1. \tag{2}$$

Let g_{kl} denote the channel gain from S_l to k-th relay and introduce $\mathbf{g}_l \triangleq [g_{1l} \ \dots \ g_{Kl}]^T$ and $\mathbf{G} \triangleq [\mathbf{g}_1 \ \dots \ \mathbf{g}_L]$. Then, the $K \times 1$ received signal vector at the relays is given by

$$\mathbf{y} = \mathbf{G}\mathbf{s} + \mathbf{v} \tag{3}$$

where $\mathbf{s} \triangleq [s_1 \ \dots \ s_L]^T$ and $\mathbf{v} \triangleq [v_1 \ \dots \ v_K]^T$ is the relays' noise vector. Let w_k^* denote the beamforming weight at the k-th relay and introduce the beamforming weigh vector $\mathbf{w} \triangleq [w_1 \ \dots w_K]^T$ and $\mathbf{W} \triangleq \mathbf{D}(\mathbf{w})$. Then, the transmitted signal vector from the K relays in the second phase is

$$\mathbf{x} \triangleq [x_1 \ \dots \ x_K]^T = \mathbf{W}^* \mathbf{y}. \tag{4}$$

From (3) and (4) it is direct to show that the relays' total transmit power is given by

$$\begin{aligned} \mathrm{E}\left\{\mathbf{x}^H \mathbf{x}\right\} &= \mathrm{tr}\left(\mathbf{W}^* \left(\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \mathbf{\Sigma}_v\right) \mathbf{W}^T\right) \\ &= \mathbf{w}^H \mathbf{\Lambda}_1(\mathbf{p}) \mathbf{w} \end{aligned} \tag{5}$$

where $\mathbf{\Lambda}_1(\mathbf{p}) \triangleq \mathbf{D}\left(\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \mathbf{\Sigma}_v\right)$ with $\mathbf{\Sigma}_v \triangleq \mathrm{E}\left\{\mathbf{v}\mathbf{v}^H\right\}$. Due to regulations, the relays' total transmit power is constrained by $\bar{P}_1$, that is, we must have

$$\mathbf{w}^H \mathbf{\Lambda}_1(\mathbf{p}) \mathbf{w} \le \bar{P}_1. \tag{6}$$

Let r_{mk} stand for the channel gain from the k-th relay to U_m and introduce $r_m \triangleq \{r_{m1} \ \dots \ r_{mK}\}^T$. Then, the interference power afflicted by the relays on U_m can be shown to be equal to

$$\mathrm{E}\left\{\left|\mathbf{r}_m^T \mathbf{x}\right|^2\right\} = \mathbf{w}^H \mathbf{\Lambda}_{m+1}(\mathbf{p}) \mathbf{w} \tag{7}$$

where $\mathbf{\Lambda}_{m+1}(\mathbf{p}) \triangleq \mathbf{D}(\mathbf{r}_m)\left(\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \mathbf{\Sigma}_v\right)\mathbf{D}(\mathbf{r}_m)^H$. Assuming that the acceptable threshold of the latter interference power is $\bar{P}_{m+1}$, it should hold that

$$\mathbf{w}^H \mathbf{\Lambda}_m(\mathbf{p}) \mathbf{w} \le \bar{P}_m \qquad m = 2, \dots, M+1. \tag{8}$$

Let h_{kl} represent the channel gain from the k-th relay to D_l and introduce $\mathbf{h}_l \triangleq [h_{1l} \dots h_{Kl}]^T$. The received signal at D_l is then given by

$$z_l = \mathbf{h}_l^T \mathbf{x} + n_l \qquad l = 1, \dots, L \tag{9}$$

where n_l is the noise at D_l with $\mathrm{E}\left\{|n_l|^2\right\} = \sigma_{n_l}^2$. Using (3) and (4) in (9), it can be readily shown for $l = 1, \dots, L$ that

$$z_l = \mathbf{w}^H \mathbf{f}_l^{(l)} s_l + \sum_{k=1, k \ne l}^{L} \mathbf{w}^H \mathbf{f}_k^{(l)} s_k + \mathbf{w}^H \left(\mathbf{h}_l \odot \mathbf{v}\right) + n_l \tag{10}$$

where $\mathbf{f}_k^{(l)} \triangleq \mathbf{h}_l \odot \mathbf{g}_k$. Using (10), it is direct to show that the SINR at D_l is

$$\eta_l(\mathbf{w}, \mathbf{p}) = \frac{p_l \left|\mathbf{w}^H \mathbf{f}_l^{(l)}\right|^2}{\sum_{k=1, k \ne l}^{L} p_k \left|\mathbf{w}^H \mathbf{f}_k^{(l)}\right|^2 + \mathbf{w}^H \mathbf{\Gamma}_v^{(l)} \mathbf{w} + \sigma_{n_l}^2} \tag{11}$$

where $\mathbf{\Gamma}_v^{(l)} \triangleq \mathbf{D}(\mathbf{h}_l)\mathbf{\Sigma}_v\mathbf{D}(\mathbf{h}_l)^H$. In this paper, we use $\bar{\eta}_l(\mathbf{w}, \mathbf{p}) \triangleq \eta_l(\mathbf{w}, \mathbf{p})/\gamma_l$ as the $S_l - D_l$ communication performance metric where γ_l is a normalization factor that can be selected proportional to the target SINR at D_l. We aim to derive the jointly-optimal power vector $\mathbf{p}$ and the beamforming vector $\mathbf{w}$ that maximize the worst normalized SINR $\bar{\eta}_l(\mathbf{w}, \mathbf{p})$

subject to the $L+2M+2$ constraints imposed by (1), (2), (6), and (8). Formally, our problem of interest can be expressed as follows:

$$\max_{\mathbf{w},\mathbf{p}} \quad \min_{1\leq l\leq L} \bar{\eta}_l(\mathbf{w},\mathbf{p}) \tag{12a}$$

$$\text{s.t.} \quad (1),(2),(6),\text{and } (8). \tag{12b}$$

As will be shown in Section III-A, all normalized SINRs become equal at the optimum point of the objective function. Therefore, our approach to maximize the minimum of the normalized SINRs aims to preserve fairness among all source-destination pairs. The max-min SINR optimization problem has been studied before in the literature and a similar SINR-balancing property has been observed [10]- [13]. However, most studied problems in this context can be cast as maximizing the minimum of the users' (normalized) SINRs subject to a *single* constraint on the sources' total transmit power similar to the last constraint in (1). The works in [14] and [7] are among the exceptions to above. In [14], the authors propose a technique to jointly optimize the sources' transmit powers and some beamforming vectors in the case that the sources' transmit powers are constrained by several linear inequalities as in (1) and (2). Unfortunately, the technique introduced in [14] cannot be used to solve (12) as (6) and (8) pose $M+1$ additional constraints that jointly depend on $\mathbf{w}$ and $\mathbf{p}$ and, further, are nonconvex with respect to the total design parameters $(\mathbf{w},\mathbf{p})$. Assuming that the transmissions from the relays to the destinations are carried out in dedicated orthogonal channels, we have developed in [7] (also [8]) a technique that jointly optimizes the sources' transmit powers and relays' beamforming vectors subject to a group of convex and nonconvex constraints. Yet, our technique in [7] cannot be adopted to solve (12) due to, in part, the following fundamental difference between (12) and the problem considered in [7]: Fixing the sources' transmit powers in [7], the optimal beamforming vectors can be obtained in a closed form while, as will be discussed in Section III-C, fixing $\mathbf{p}$ in (12), the optimal $\mathbf{w}$ is the solution to an NP-hard problem.

While different constraints require developing different techniques to obtain the jointly-optimal sources' transmit powers and beamforming vectors, most these techniques share a similar iterative structure: 1) Obtain the optimal sources' transmit powers for the given beamforming vector(s); 2) Derive the optimal beamforming vector(s) for the given sources' transmit powers; and 3) Continue steps 1 and 2 until the algorithm converges. Our proposed technique follows similar steps.

III. Jointly-optimal sources' transmit power and relays' beamforming vectors

In the first subsection, we derive a necessary and sufficient condition for the jointly-optimal sources' transmit power vector $\mathbf{p}_o$ and relays' beamforming vector $\mathbf{w}_o$. In the next two subsections, we assume the knowledge of $\mathbf{w}_o$ ($\mathbf{p}_o$) and use the necessary and sufficient joint-optimality condition to obtain $\mathbf{p}_o$ ($\mathbf{w}_o$). Our results will then be used in the last subsection to introduce an efficient iterative algorithm to jointly optimize the sources' transmit power and relays' beamforming vectors. While the proofs of all results are presented in [15], in some cases, they are also outlined in this paper.

A. Necessary and sufficient joint-optimality condition

Theorem 1: $(\mathbf{w}_o,\mathbf{p}_o)$ are the jointly-optimal solutions to (12) if and only if **C1**, **C2**, and **C3** are satisfied.

C1: $(\mathbf{w}_o,\mathbf{p}_o)$ balances all normalized SINRs, that is,

$$\bar{\eta} = \bar{\eta}_l(\mathbf{w}_o,\mathbf{p}_o) \qquad l=1,\ldots,L. \tag{13}$$

C2: $\mathbf{w}_o$ is the solution to

$$\max_{\mathbf{w}} \min_{1\leq l\leq L} \bar{\eta}_l(\mathbf{w},\mathbf{p}_o) \tag{14a}$$

$$\text{s.t. } \mathbf{w}^H\mathbf{\Lambda}_m(\mathbf{p}_o)\mathbf{w} \leq \bar{P}_m \;\; m=1,\ldots,M+1. \tag{14b}$$

C3: There is a constraint in (1) and (2) that is satisfied with the equality, that is,

$$\mathbf{u}_{l^\star}^T\mathbf{p}_o = P_{l^\star} \;\; \text{for an} \;\; l^\star\in\{1,\ldots,L+M+1\}. \tag{15}$$

C1 is due to the fact that if a normalized SINR, say, $\bar{\eta}_{\tilde{l}}(\mathbf{w}_o,\mathbf{p}_o)$ is larger than the others, a decrease in $[\mathbf{p}_o]_{\tilde{l}}$ increases all $\bar{\eta}_l(\mathbf{w}_o,\mathbf{p}_o)$ for $l=1,\ldots,L,\; l\neq\tilde{l}$, and, therefore, the objective function without violating any of (1), (2), (6), and (8). This contradicts the optimality of $(\mathbf{w}_o,\mathbf{p}_o)$. If **C2** does not hold, then $(\mathbf{w}_o,\mathbf{p}_o)$ results in a smaller objective function in (12) than $(\tilde{\mathbf{w}},\mathbf{p}_o)$ where $\tilde{\mathbf{w}}$ is the solution to (14). This is also in contradiction with the optimality of $(\mathbf{w}_o,\mathbf{p}_o)$. Note that **C2** implies that at least one of the constraints in (14b) is satisfied with the equality. If **C3** does not hold, then $\left(\sqrt{\beta}\mathbf{w}_o,\alpha\mathbf{p}_o\right)$ with

$$\alpha \triangleq \min_{1\leq l\leq L+M+1}\frac{P_l}{\mathbf{u}_l^T\mathbf{p}_o} > 1 \tag{16}$$

$$\beta \triangleq \min_{1\leq m\leq M+1}\beta_m \tag{17}$$

where $\beta_1 \triangleq \bar{P}_1\left(\alpha\mathbf{w}_o^H\mathbf{\Lambda}_1(\mathbf{p}_o)\mathbf{w}_o-(\alpha-1)\mathbf{w}_o^H\mathbf{\Sigma}_v\mathbf{w}_o\right)^{-1}$ and $\beta_m \triangleq \bar{P}_m\big(\alpha\mathbf{w}_o^H\mathbf{\Lambda}_m(\mathbf{p}_o)\mathbf{w}_o-(\alpha-1)\mathbf{w}_o^H\mathbf{D}(\mathbf{r}_{m-1})\mathbf{\Sigma}_v\mathbf{D}(\mathbf{r}_{m-1})^H\mathbf{w}_o\big)^{-1}$, $m=2,\ldots,M$ satisfies (1), (2), (6), and (8). Moreover, $\bar{\eta}_l\left(\sqrt{\beta}\mathbf{w}_o,\alpha\mathbf{p}_o\right) > \bar{\eta}_l(\mathbf{w}_o,\mathbf{p}_o)$ for $l=1,\ldots,L$. This also contradicts the optimality of $(\mathbf{w}_o,\mathbf{p}_o)$.

B. Deriving $\mathbf{p}_o$ for the given $\mathbf{w}_o$

Assuming the knowledge of $\mathbf{w}_o$, $\mathbf{p}_o$ can be obtained using **C1** and **C3**. Let the $L\times L$ non-negative matrix $\mathbf{\Psi}(\mathbf{w})$ such that $[\mathbf{\Psi}(\mathbf{w})]_{lk} \triangleq \left|\mathbf{w}^H\mathbf{f}_k^{(l)}\right|^2$ for $l\neq k$ and $[\mathbf{\Psi}(\mathbf{w})]_{ll}\triangleq 0$, the $L\times L$ positive diagonal matrix $\mathbf{\Omega}(\mathbf{w})$ such that $[\mathbf{\Omega}(\mathbf{w})]_{ll} \triangleq \gamma_l/\left|\mathbf{w}^H\mathbf{f}_l^{(l)}\right|^2$, and the $L\times 1$ positive vector $\boldsymbol{\sigma}(\mathbf{w})$ such that $[\boldsymbol{\sigma}(\mathbf{w})]_l \triangleq \mathbf{w}^H\mathbf{\Gamma}_v^{(l)}\mathbf{w}+\sigma_{n_l}^2$. Then, it is straightforward to show from (11) and **C1** that

$$\mathbf{\Omega}(\mathbf{w}_o)\mathbf{\Psi}(\mathbf{w}_o)\mathbf{p}_o+\mathbf{\Omega}(\mathbf{w}_o)\boldsymbol{\sigma}(\mathbf{w}_o) = \frac{1}{\bar{\eta}}\mathbf{p}_o. \tag{18}$$

Using (15) in (18), it also follows that

$$\frac{1}{P_l^\star}\mathbf{u}_{l^\star}^T\mathbf{\Omega}(\mathbf{w}_o)\mathbf{\Psi}(\mathbf{w}_o)\mathbf{p}_o+\frac{1}{P_l^\star}\mathbf{u}_{l^\star}^T\mathbf{\Omega}(\mathbf{w}_o)\boldsymbol{\sigma}(\mathbf{w}_o) = \frac{1}{\bar{\eta}}. \tag{19}$$

Introducing

$$\boldsymbol{\Lambda}_l(\mathbf{w}) \triangleq \begin{bmatrix} \boldsymbol{\Omega}(\mathbf{w})\boldsymbol{\Psi}(\mathbf{w}) & \boldsymbol{\Omega}(\mathbf{w})\boldsymbol{\sigma}(\mathbf{w}) \\ \frac{1}{P_l}\mathbf{u}_l^T\boldsymbol{\Omega}(\mathbf{w})\boldsymbol{\Psi}(\mathbf{w}) & \frac{1}{P_l}\mathbf{u}_l^T\boldsymbol{\Omega}(\mathbf{w})\boldsymbol{\sigma}(\mathbf{w}) \end{bmatrix}, \quad (20)$$

it holds from (18) and (19) that

$$\boldsymbol{\Lambda}_{l^\star}(\mathbf{w}_o)\begin{bmatrix}\mathbf{p}_o \\ 1\end{bmatrix} = \frac{1}{\bar{\eta}}\begin{bmatrix}\mathbf{p}_o \\ 1\end{bmatrix}. \quad (21)$$

It follows from Eq. (21) that $1/\bar{\eta}$ and $[\mathbf{p}_o^T\ 1]^T$ are a jointly-positive eigenpair of $\boldsymbol{\Lambda}_{l^\star}(\mathbf{w}_o)$. It can be shown that $\boldsymbol{\Lambda}_l(\mathbf{w})$ are nonnegative primitive matrices for $l = 1,\ldots,2L+1$ [8]. Therefore, $\lambda_{\max}(\boldsymbol{\Lambda}_l(\mathbf{w}_o))$ and its associated eigenvector $\Omega(\boldsymbol{\Lambda}_l(\mathbf{w}_o))$ are the unique positive eigenpair of $\boldsymbol{\Lambda}_l(\mathbf{w}_o)$ [16, Ch. 8]. As such, we have

$$\begin{bmatrix}\mathbf{p}_o \\ 1\end{bmatrix} = \Omega(\boldsymbol{\Lambda}_{l^\star}(\mathbf{w}_o)). \quad (22)$$

It remains to obtain $l^\star$. $\mathbf{p}_o$ is required to satisfy all inequalities in (1) and (2). This requirement is met if and only if [8], [15]

$$l^\star = \underset{1\le l\le L+M+1}{\operatorname{argmax}} \lambda_{\max}(\boldsymbol{\Lambda}_l(\mathbf{w}_o)). \quad (23)$$

Further, even if $l^\star$ is not unique, $\Omega(\boldsymbol{\Lambda}_{l^\star}(\mathbf{w}_o))$, and, consequently, $\mathbf{p}_o$ is unique. The above discussion shows that, given $\mathbf{w}_o$, $\mathbf{p}_o$ can be obtained as follows: 1) Compute $\boldsymbol{\Lambda}_l(\mathbf{w}_o)$ from (20) for $l = 1,\ldots,L+M+1$; 2) Compute $\lambda_{\max}(\boldsymbol{\Lambda}_l(\mathbf{w}_o))$ for $l = 1,\ldots,L+M+1$ and find $l^\star$ from (23); 3) Obtain $\mathbf{p}_o$ from (22).

C. *Deriving $\mathbf{w}_o$ for the given $\mathbf{p}_o$*

When $\mathbf{p}_o$ is known, $\mathbf{w}_o$ may be obtained using **C2**. Let $\mathbf{F} \triangleq \left[\mathbf{f}_1^{(l)}\ldots\mathbf{f}_L^{(l)}\right]$ and $\boldsymbol{\Xi}_l(\mathbf{p}) \triangleq \mathbf{F}_{\bullet\bar{l}}\mathbf{p}_{\bar{l}}\mathbf{F}_{\bullet\bar{l}}^H + \boldsymbol{\Gamma}_v^{(l)}$. Then, (11) can be used to equivalently represent (14) as

$$\max_{\mathbf{w}}\ z \quad (24a)$$
$$\text{s.t. } \mathbf{w}^H\boldsymbol{\Lambda}_m(\mathbf{p}_o)\mathbf{w} \le \bar{P}_m \ \ m=1,\ldots,M+1 \quad (24b)$$
$$z\left(\mathbf{w}^H\boldsymbol{\Xi}_l(\mathbf{p}_o)\mathbf{w} + \sigma_{n_l}^2\right) \le [\mathbf{p}_o]_l\left|\mathbf{w}^H\mathbf{f}_l^{(l)}\right|^2 \ \ l=1,\ldots,L \quad (24c)$$

A special case of (24) simplifies to the NP-hard max-min fair beamforming problem discussed in [17]. Therefore, (24) is an NP-hard problem in general. One can find (an approximate) solution to (24) in a polynomial time using a semi-definite relaxation (SDR) approach [3], [17], [18] along with a bisection method as follows. First, let $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$ and $\mathbf{F}_l \triangleq \mathbf{f}_l^{(l)}\mathbf{f}_l^{(l)H}$. Then, (24) can be rewritten as

$$\max_{\mathbf{X}}\ z \quad (25a)$$
$$\text{s.t. } \operatorname{tr}(\mathbf{X}\boldsymbol{\Lambda}_m(\mathbf{p}_o)) \le \bar{P}_m \ \ m=1,\ldots,M+1 \quad (25b)$$
$$z\left(\operatorname{tr}(\mathbf{X}\boldsymbol{\Xi}_l(\mathbf{p}_o)) + \sigma_{n_l}^2\right) \le [\mathbf{p}_o]_l\operatorname{tr}(\mathbf{X}\mathbf{F}_l) \ \ l=1,\ldots,L \quad (25c)$$
$$\mathbf{X} \succeq \mathbf{0} \quad (25d)$$
$$\operatorname{rank}(\mathbf{X}) = 1. \quad (25e)$$

Following the standard SDR technique, we drop the non-convex rank-one constraint (25e) and simplify (25) to

$$\max_{\mathbf{X}}\ z \quad (26a)$$
$$\text{s.t. } \operatorname{tr}(\mathbf{X}\boldsymbol{\Lambda}_m(\mathbf{p}_o)) \le \bar{P}_m \ \ m=1,\ldots,M+1 \quad (26b)$$
$$z\left(\operatorname{tr}(\mathbf{X}\boldsymbol{\Xi}_l(\mathbf{p}_o)) + \sigma_{n_l}^2\right) \le [\mathbf{p}_o]_l\operatorname{tr}(\mathbf{X}\mathbf{F}_l) \ \ l=1,\ldots,L \quad (26c)$$
$$\mathbf{X} \succeq \mathbf{0}. \quad (26d)$$

For a fixed $z = z_q$, the associated feasibility problem of (26)

$$\text{Find } \mathbf{X} \quad (27a)$$
$$\text{s.t. } \operatorname{tr}(\mathbf{X}\boldsymbol{\Lambda}_m(\mathbf{p}_o)) \le \bar{P}_m \ \ m=1,\ldots,M+1 \quad (27b)$$
$$z_q\left(\operatorname{tr}(\mathbf{X}\boldsymbol{\Xi}_l(\mathbf{p}_o)) + \sigma_{n_l}^2\right) \le [\mathbf{p}_o]_l\operatorname{tr}(\mathbf{X}\mathbf{F}_l) \ \ l=1,\ldots,L \quad (27c)$$
$$\mathbf{X} \succeq \mathbf{0} \quad (27d)$$

is comprised only of convex constraints and, hence, can be efficiently solved using standard convex optimization tools. Let $\mathbf{X}_o$ and z_o be the optimal variable and the optimal value of the objective function in (26), respectively. It is straightforward to show that (27) is feasible if and only if $z_q \le z_o$. This property can be used to obtain $\mathbf{X}_o$ with an arbitrary accuracy as follows. First, find $z_{\min}$ and $z_{\max}$ such that $z_{\min} \le z_o \le z_{\max}$. For instance, $z_{\min} = 0$ and $z_{\max} = \max_{\mathbf{w}} p_l|\mathbf{w}^H\mathbf{f}_l^{(l)}|^2/\left(\mathbf{w}^H\boldsymbol{\Xi}_l(\mathbf{p}_o)\mathbf{w}\right) = \mathbf{f}_l^{(l)H}\boldsymbol{\Xi}_l(\mathbf{p}_o)^{-1}\mathbf{f}_l^{(l)}$ may be selected. Then, solve (27) with $z_q = (z_{\min} + z_{\max})/2$ for $q = 1$. If (27) is feasible, set $z_{\min} = z_1$, if not, set $z_{\max} = z_1$ and solve (27) with $z_q = (z_{\min} + z_{\max})/2$ for $q = 2$. Continue this procedure until $z_{\max} - z_{\min} \le \epsilon$ where ϵ is an arbitrarily-selected accuracy measure. The last feasible $\mathbf{X}$ is an approximation to $\mathbf{X}_o$, the solution to (26). $\mathbf{X}$ can become arbitrarily close to $\mathbf{X}_o$ by keep selecting smaller thresholds ϵ. In this paper, CVX [19], [20], a free package for specifying and solving convex problems, is used to solve (27). If the so-obtained $\mathbf{X}_o$ turns out to be rank-one, then $\mathbf{w}_o$ is a properly-normalized principal eigenvector of $\mathbf{X}_o$ [15]. Otherwise, an efficient approximation of $\mathbf{w}_o$ can be obtained by applying standard randomization techniques [17], [18], [21] on $\mathbf{X}_o$.

D. *Jointly optimizing $\mathbf{p}$ and $\mathbf{w}$*

Based on our results in Sections III-B and III-C, Algorithm I is used to optimize $\mathbf{p}$ and $\mathbf{w}$. Assume that $\mathbf{X}_{[n]}$ is rank-one in all iterations of Algorithm I. Then, $\mathbf{p}_{[n]}$, $\mathbf{w}_{[n]}$, and $\mathbf{X}_{[n]} = \xi_{[n]}\mathbf{w}_{[n]}\mathbf{w}_{[n]}^H$ respectively converge to $\mathbf{p}_\infty$, $\mathbf{w}_\infty$, and $\mathbf{X}_\infty = \xi_\infty\mathbf{w}_\infty\mathbf{w}_\infty^H$ where $\xi_{[n]}$ and ξ_∞ are scaling factors. Since $\mathbf{X}_\infty$ is rank-one, it follows from Step 3 of Algorithm I along with our discussion in Section III-C that $\mathbf{w}_\infty$ is the solution to

$$\max_{\mathbf{w}}\ z \quad (28a)$$
$$\text{s.t. } \mathbf{w}^H\boldsymbol{\Lambda}_m(\mathbf{p}_\infty)\mathbf{w} \le \bar{P}_m \ \ m=1,\ldots,M+1 \quad (28b)$$
$$z\left(\mathbf{w}^H\boldsymbol{\Xi}_l(\mathbf{p}_\infty)\mathbf{w} + \sigma_{n_l}^2\right) \le [\mathbf{p}_\infty]_l\left|\mathbf{w}^H\mathbf{f}_l^{(l)}\right|^2 \ \ l=1,\ldots,L \quad (28c)$$

and, hence, to

$$\max_{\mathbf{w}}\ \min_{1\le l\le L}\ \bar{\eta}_l(\mathbf{w},\mathbf{p}_\infty) \quad (29a)$$
$$\text{s.t. } \mathbf{w}^H\boldsymbol{\Lambda}_m(\mathbf{p}_\infty)\mathbf{w} \le \bar{P}_m \ \ m=1,\ldots,M+1. \quad (29b)$$

Algorithm I: Joint optimization of the sources' power and the relays' beamforming vectors

1: `Initialize:` $n = 0$; $\mathbf{p}_{[0]} \geq \mathbf{0}$
2: $n = n + 1$

%% Finding $\mathbf{w}_{[n]}$ given $\mathbf{p}_{[n-1]}$:
3: `Substitute` $\mathbf{p}_{[n-1]}$ `in lieu of` $\mathbf{p}_o$ `in (26) and obtain the solution` $\mathbf{X}_{[n]}$
4A: $\mathbf{X}_{[n]}$ `rank-one: Normalize the principal eigenvector of` $\mathbf{X}_{[n]}$ `to obtain` $\mathbf{w}_{[n]}$
4B: $\mathbf{X}_{[n]}$ `higher rank: Approximate` $\mathbf{w}_{[n]}$ `using a randomization technique`

%% Finding $\mathbf{p}_{[n]}$ given $\mathbf{w}_{[n]}$:
5: $l^\star_{[n]} = \underset{1 \leq l \leq L+M+1}{\operatorname{argmax}} \; \lambda_{\max}\left(\mathbf{\Lambda}_l\left(\mathbf{w}_{[n]}\right)\right).$
6: $\begin{bmatrix} \mathbf{p}_{[n]} \\ 1 \end{bmatrix} = \Omega\left(\mathbf{\Lambda}_{l^\star_{[n]}}\left(\mathbf{w}_{[n]}\right)\right)$

7: `Go back to step 2 until` $\left\|\mathbf{p}_{[n]} - \mathbf{p}_{[n-1]}\right\| \leq \delta$ `where` δ `is a small number or` $n = n_{\max}$

In turn, Steps 5 and 6 of Algorithm I establish the fact that

$$\begin{bmatrix} \mathbf{p}_\infty \\ 1 \end{bmatrix} = \Omega\left(\mathbf{\Lambda}_{l^\star_\infty}(\mathbf{w}_\infty)\right) \tag{30}$$

where

$$l^\star_\infty = \underset{1 \leq l \leq L+M+1}{\operatorname{argmax}} \; \lambda_{\max}\left(\mathbf{\Lambda}_l\left(\mathbf{w}_\infty\right)\right). \tag{31}$$

Recalling our discussion in Section III-B, Eqs. (30) and (31) immediately prove that $\mathbf{w}_\infty$ and $\mathbf{p}_\infty$ jointly balance all SINRs, that is,

$$\bar{\eta}_\infty = \bar{\eta}_l\left(\mathbf{w}_\infty, \mathbf{p}_\infty\right) \qquad l = 1, \ldots, L \tag{32}$$

and, moreover,

$$\mathbf{u}^T_{l^\star_\infty}\mathbf{p}_\infty = P_{l^\star_\infty}. \tag{33}$$

It follows from Eqs. (29), (32), and (33) that **C1**, **C2**, and **C3** are satisfied by $(\mathbf{w}_\infty, \mathbf{p}_\infty)$ and, hence, $(\mathbf{w}_\infty, \mathbf{p}_\infty) = (\mathbf{w}_o, \mathbf{p}_o)$ and $\bar{\eta}_\infty = \bar{\eta}$.

Simulations show that when $\mathbf{X}_{[n]}$ is rank-one in all iterations, Algorithm I converges very rapidly. However, if $\mathbf{X}_{[n]}$ is not rank-one, $\mathbf{p}_{[n]}$ and $\mathbf{w}_{[n]}$ may not converge in Algorithm I and one may have to stop the algorithm after the number of iterations reaches $n_{\max}$. Numerical results show that high SINR performance can be achieved by using $\mathbf{p}_{[n_{\max}]}$ and $\mathbf{w}_{[n_{\max}]}$ in the latter case.

IV. Simulations

Two numerical experiments are conducted to investigate the performance of the proposed algorithm. In both examples, $L = 4$, $K = 20$, $\sigma^2_{n_l} = \sigma^2$ for $l = 1, \ldots, L$, and $\mathbf{\Sigma}_v = \sigma^2\mathbf{I}_K$. The individual sources' transmit power constraints P_l, $l = 1, \ldots, L$ are such that $10\log_{10}(P_l/\sigma^2) = 15$. In turn, the sources' total transmit power constraint is $P_{L+1} = 0.9\sum_{l=1}^{L} P_l$. Maximum admissible interference power levels of primary users P_l, $l = L+2, \ldots, L+M+1$ and $\bar{P}_m$, $m = 2, \ldots, M+1$ are all equal and $10\log_{10}(\bar{P}_2/\sigma^2) = 5$. Finally, the relays' total transmit power constraint $\bar{P}_1 = LP_1$. The target SINRs are set to $\gamma_l = 15$ (dB) for $l = 1, \ldots, L$. All source-relay, relay-destination, source-primary user, and relay-primary user channel gains are randomly and independently drawn from zero-mean circular complex Gaussian distributions with $\mathrm{E}\left\{|g_{k1}|^2\right\} = 16\sigma^2$, $\mathrm{E}\left\{|g_{kl}|^2\right\} = \sigma^2$, $l = 2, \ldots, L$, $\mathrm{E}\left\{|h_{kl}|^2\right\} = \sigma^2$, $l = 1, \ldots, L$, $\mathrm{E}\left\{|t_{ml}|^2\right\} = \mathrm{E}\left\{|r_{ml}|^2\right\} = \varphi\sigma^2$, $l = 1, \ldots, L$.

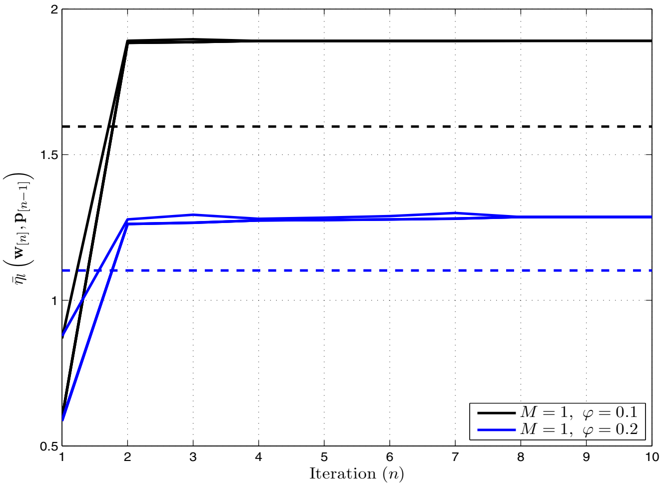

Fig. 2. $\bar{\eta}_l\left(\mathbf{w}_{[n]}, \mathbf{p}_{[n-1]}\right)$ v.s. the iteration index n for $M = 1$.

The following two-step technique to optimize the sources' transmit power and relays' beamforming vectors is also examined and is compared to the proposed technique: 1) Let all sources transmit with a common power P_{c}. Maximize P_{c} such that the sources' transmit power vector $\mathbf{p}_{\mathrm{c}} = P_{\mathrm{c}}\mathbf{1}$ satisfies all $L+M+1$ constraints in (1) and (2); 2) Find $\mathbf{w}_{\mathrm{c}}$ as the solution to[1]

$$\max_{\mathbf{w}} \min_{1 \leq l \leq L} \bar{\eta}_l\left(\mathbf{w}, \mathbf{p}_{\mathrm{c}}\right) \tag{34a}$$

$$\text{s.t. } \mathbf{w}^H\mathbf{\Lambda}_m(\mathbf{p}_{\mathrm{c}})\mathbf{w} \leq \bar{P}_m \;\; m = 1, \ldots, M+1. \tag{34b}$$

Note that obtaining (an approximate of) $\mathbf{w}_{\mathrm{c}}$ also requires using a SDR technique to solve an NP-hard problem. The above algorithm can be viewed as a "separated" optimization technique of $\mathbf{p}$ and $\mathbf{w}$ in contrast to our proposed technique to jointly optimize $\mathbf{p}$ and $\mathbf{w}$.

In Fig. 2, Algorithm I is used to plot $\bar{\eta}_l\left(\mathbf{w}_{[n]}, \mathbf{p}_{[n-1]}\right)$, $l = 1, \ldots, L$ versus the iteration index n for $M = 1$ and two different φ. As can be observed from the figure, all normalized SINRs converge to a common value in a few iterations. The dashed lines show $\min_{1 \leq l \leq L} \bar{\eta}_l\left(\mathbf{w}_{\mathrm{c}}, \mathbf{p}_{\mathrm{c}}\right)$. For both examined φ, the proposed algorithm offers a considerably larger minimum normalized SINR than the above-described separated optimization technique.

Fig. 3 shows the same curves as in Fig. 2 for $M = 2$. While all normalized SINR values are reduced due to the presence of an additional primary user, still $\bar{\eta}_l\left(\mathbf{w}_{[n]}, \mathbf{p}_{[n-1]}\right)$, $l =$

[1] See, for instance, [17] for a similar approach to optimize the beamforming vector.

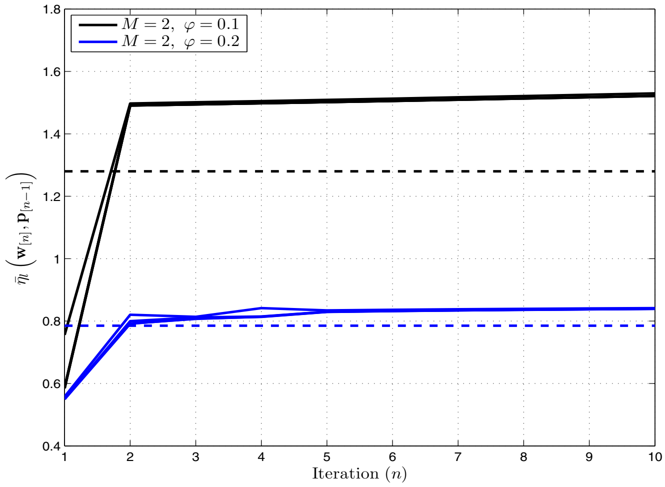

Fig. 3. $\bar{\eta}_l\left(\mathbf{w}_{[n]}, \mathbf{p}_{[n-1]}\right)$ v.s. the iteration index n for $M = 2$.

$1, \ldots, L$ rapidly converge and are significantly larger than the corresponding $\min_{1 \le l \le L} \bar{\eta}_l\left(\mathbf{w}_{\rm c}, \mathbf{p}_{\rm c}\right)$.

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VI. Conclusions

We considered a cognitive cooperative network wherein each of the K single antenna relays receives a faded and noisy mixture of the transmitted signals from L sources, multiplies it with a properly-selected beamforming weight, and forwards the result towards L destinations in a common channel. Aiming to maximize the minimum signal-to-interference-plus-noise ratio among all L destinations, we developed a technique to jointly optimize the sources' transmit powers and the relays' beamforming weights while concurrently satisfying all the following constraints: 1) The sources and the relays total transmit power constraints; 2) The sources individual transmit power constraints; and 3) The maximum interference power that the cognitive sources and relays are allowed to inflict on the M existing primary users. The sources' transmit powers and the relays' beamforming weights are optimized using an efficient iterative algorithm each of whose iterations involves a semidefinite relaxation (SDR) technique to solve an NP-hard problem. The algorithm converges to the optimal values if the SDR technique has a rank-one solution matrix. In the case that the SDR technique does not have a rank-one solution, simulation results verified that the performance of the proposed joint optimization technique is still superior to the technique that separately optimizes the sources' transmit powers and the relays' beamforming weights.

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