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# Topology-assisted techniques to relay selection for homogeneously distributed wireless sensor networks

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***Abstract*—We consider a multi-relay amplify-and-forward cooperative communication scheme in wireless sensor networks with uniformly distributed nodes. Fixing the average total transmission power from the network and preserving fairness among the selected relays by constraining them to transmit with equal average powers, we aim to improve the signal reception quality at the far-field receiver by means of a proper choice of the relays. Assuming that the nodes' forward channels are not known, the following three relay selection schemes are proposed and their performances are analyzed. 1) Optimal relay selection scheme that maximizes the average SNR at the receiver by exploiting $K$ nodes with the highest SNRs as relays; 2) geometry-based relay selection scheme that is energy-efficient and achieves a close-to-optimal average SNR performance at the receiver by using $K$ closest nodes to the source as relays; and 3) random relay selection scheme that is energy-efficient and further guarantees a fair usage of all nodes by randomly selecting $K$ relays from a specific area around the source. By minimizing an outage probability, a strategy to determine this area is also proposed. Finally, it is shown for all relay selection schemes that the SNR variance at the receiver converges to zero as $K$ increases.**

## I. Introduction and Motivation

Exploiting multiple transmit antennas in communication networks can substantially improve the communication performance by providing the spatial diversity and/or increasing the data rate and the multiplexing gain. However, in many systems such as wireless sensor networks (WSNs), it is impractical to implement multiple antennas at the transmitting terminals. As an alternative approach to communication with multiple transmit antennas, cooperative communication techniques in wireless sensor networks can provide diversity and multiplexing gain by means of using the existing idle nodes as a virtual antenna array that relay the source transmitted signal [1], [2]. How to select the relays from the pool of available nodes can have a major effect on the cooperative communication efficiency. This has motivated extensive research efforts on proposing a variety of relay selection techniques for such networks. In [3], it is proposed to select the relay with the best contribution in signal-to-noise ratio (SNR) at the receiver. In [4], the closest node to the receiver is selected to cooperate with the source. Assuming that the source-relay and the relay-receiver channels are known in all nodes, the best harmonic-mean single relay selection technique is proposed in [5]. It can be shown that selecting the optimal relay among multiple available nodes provides the full diversity order as if all nodes act as relays [5]. However, this strategy results in a rapid power depletion of the selected relay. In contrast, using multiple relays not only can provide a full diversity order under fairly general assumptions, but also makes it possible to decrease the transmission power from each individual relay proportionally to the number of selected relays under a total transmit power constraint. This, in turn, can dramatically increase the network lifetime. For a review on some techniques to multiple relay selection refer to [6].

In spite of substantial works on efficient relay selection techniques, there is only a limited research on developing relay selection strategies that take explicitly into account the network topology. We intend to contribute in this thrust of research by introducing energy-fair relay selection techniques in WSNs where the nodes are uniformly distributed according to a two-dimensional homogeneous Poisson process [7], [8]. We consider a two-phase amplify-and-forward (AF) cooperative scheme where in the first phase the source broadcasts its signal and in the second phase $K$ nodes act as relays by amplifying their received signals and resending them to the receiver through orthogonal channels. Aiming to prolong the network lifetime by avoiding overloading some of the selected relays, we choose an energy-fair approach wherein all selected relays transmit with equal average powers. We then develop the following three relay selection techniques for the pragmatic scenario where the nodes are unaware of their forward channels to the receiver:

1) Optimal relay selection technique where the $K$ nodes with the highest SNRs are selected as relays. Among all those techniques that do not use any information of the nodes' forward channels to select the proper set of relays, this approach can provide the highest SNR at the receiver. However, it entails a large signaling overhead as the set of $K$ relays may have to be updated at the beginning of every transmission cycle. In fact, the nodes' backward channel links can change quite dramatically in a relatively short period of time. Therefore, when this technique is used, it may not be possible to mark some nodes as inappropriate relay candidates and let them go to the sleeping mode and save energy. As such, all nodes have to alternately switch between the listening and transmitting modes. This results in a significant waste of energy.

2) Geometry-based relay selection technique that is based on

the fact that the variations in the network topology are typically slower than those of the channel links. This technique selects the $K$ closest nodes to the source as relays for a predetermined number of transmission cycles and leaves all other nodes in the sleeping mode to save energy. After a number of cycles passed, all nodes go back to the active mode, the new set of $K$ closest nodes start to act as relays, and all other nodes return to the sleeping mode. Although being energy-efficient, this technique may overexploit the nodes that continuously remain close to the source in networks with a static topology.

3) Random relay selection technique where $K$ relays are randomly selected from the nodes within an $R$-distance from the source. All other nodes both within and outside this neighborhood remain in the sleeping mode. The relay list is updated after a predetermined number of transmission cycles to avoid overexploiting a specific set of nodes. At the cost of a possibly noticeable drop in the SNR performance, this technique is both energy-efficient and fair toward the nodes.

For all the aforementioned techniques, we derive the average SNRs at the $K$ selected relays and at the receiver and analyze their properties. We then present a variance analysis for SNR at the receiver and show that, regardless of the used relay selection technique, the SNR variance goes to zero as $K$ grows large. We also define an outage probability, by minimizing which, the optimal $R$ in the random relay selection technique can be obtained.

The rest of the paper is organized as follows. The system description, the transmission protocol, and the signal model are presented in Sec. II. The relay selection techniques are formally proposed and the probabilistic properties of the SNRs at the relays are analyzed in Sec. III. The average and the variance of the SNR at the receiver are studied in Sec. IV. Simulation results are presented and are compared to the analytical results in Sec. V. Concluding remarks are given in Section VI.

## II. System Model

### A. System description and Transmission Protocol

Consider the WSN in Fig. 1 whose nodes are uniformly distributed with density $\rho$ in a 2-dimensional plane. The source $s$ is one of the nodes in the network and aims to send information to the receiver in the far-field. In order to provide diversity, $K$ other nodes in the network are exploited as relays in a two-phase AF cooperative scheme. In the first phase, the source broadcasts its signal. In the second phase, $K$ nodes act as relays by amplifying their received signals and resending them to the receiver through orthogonal channels.

### B. Signal Model

The received signal at the $k$th relay in the first phase is

$$y_k = \sqrt{p_s} h_{s,k} D_{s,k}^{-\nu/2} x + n_k \qquad k = 1, \ldots, K \tag{1}$$

where $n_k \sim \mathcal{CN}(0, \sigma^2)$ is noise, $p_s$ is the transmission power from the source and $x$ is the normalized transmitted signal. $D_{s,k}$ is the distance between the source and the $k$th selected relay and $\nu \geq 2$ is the path-loss factor and varies for different

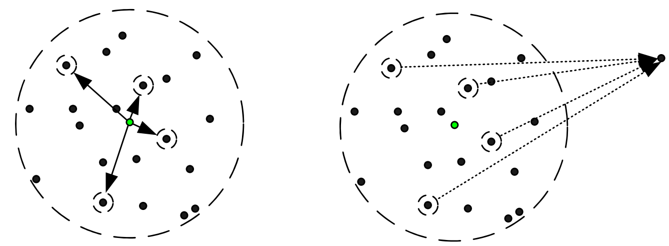


Fig. 1: Network model and the transmission protocol.

channel environments. $h_{s,k}$ is the Rayleigh fading channel between the source and the $k$th selected relay with the variance $1/2$ per real dimension. From (1), the SNR at the $k$th relay is

$$\gamma_k = \beta |h_{s,k}|^2 D_{s,k}^{-\nu}, \tag{2}$$

where $\beta = p_s/\sigma^2$. In the second phase, the $k$th relay sends $z_k = \sqrt{\beta_k} y_k$ where

$$\beta_k = \frac{1}{K} \cdot \frac{P_T}{\mathrm{E}\left\{|y_k|^2\right\}} = \frac{1}{K} \cdot \frac{P_T}{\sigma^2(\bar{\gamma}_k + 1)} \qquad k = 1, \ldots, K \tag{3}$$

is the normalization factor at the $k$-th relay [9]. In (3), $\bar{\gamma}_k$ represents the average received SNR at the $k$th relay. Regardless of the technique used to select the relays, it can be observed from (3) that the average transmit power from each relay is $P_T/K$. Therefore, the average of the total transmission power from the whole network during the relaying phase is $P_T$. This property guarantees an equitable power dissipation from the selected relays and further makes it possible to fairly compare the performances of different relay selection schemes. The signal at the receiver due to the $k$th relay is

$$y_d^{[k]} = \sqrt{\beta_k} h_{k,d} D^{-\nu/2} y_k + n_d^{[k]} \qquad k = 1, \ldots, K \tag{4}$$

where $n_d^{[k]}$ is the receiver noise, $h_{k,d}$ is the Rayleigh fading channel between the $k$th relay and the receiver with the variance $1/2$ per real dimension, and $D$ is the distance of the far-field receiver from the nodes.

Assume that the receiver uses the optimal maximum ratio combining (MRC) to estimate $x$ from the set of signals received from the relays. Using (1) and (4) and after some manipulation, it can be shown that the soft symbol estimate at the MRC receiver output is equal to

$$y_d = \sum_{k=1}^{K} \frac{\sqrt{p_s \alpha_k} h_{s,k}^* h_{k,d}^* D_{s,k}^{-\nu/2}}{\sigma^2 \left(1 + \alpha_k |h_{k,d}|^2\right)} \cdot y_d^{[k]} \tag{5}$$

where

$$\alpha_k \triangleq \beta_k D^{-\nu} = \frac{1}{K} \cdot \frac{P_T}{\sigma^2 D^\nu} \cdot \frac{1}{\bar{\gamma}_k + 1} \tag{6}$$

and $^*$ is the complex conjugate. It can be concluded from (4) and (5) that SNR at the receiver is given by

$$\gamma_d = \sum_{k=1}^{K} \theta_k, \tag{7}$$

where

$$\theta_k \triangleq \frac{\alpha_k \left|h_{k,d}\right|^2}{1+\alpha_k \left|h_{k,d}\right|^2} \cdot \gamma_k = \frac{\alpha_k \left|h_{k,d}\right|^2}{1+\alpha_k \left|h_{k,d}\right|^2} \cdot \frac{\beta |h_{s,k}|^2}{D_{s,k}^{\nu}}. \quad (8)$$

## III. Relay Selection Schemes and SNR analysis

SNR at the $k$th relay depends not only on the channel $h_{s,k}$, but also on the distance of the selected relay from the source, and, therefore, on the network topology. In this section, three different relay selection schemes are introduced and SNRs of all schemes are analyzed. In the following we use the superscripts $(o)$, $(g)$ and $(r)$ in order to refer to the optimal, the geometry-based, and the random relay selection schemes, respectively.

**Optimal Relay Selection–** Aiming to maximize $\gamma_d$, it immediately follows from (7) that the best relaying set is the set of $K$ nodes with the $K$ largest $\theta_k$. $\theta_k$ depends on $h_{k,d}$ which is unknown at node $k$ and therefore cannot be directly adopted as a measure to select the relays. However, as can be observed from (8), $\theta_k$ is an increasing function of $\gamma_k$, which is linearly proportional to $|h_{s,k}|^2 D_{s,k}^{-\nu}$ that is known at the node $k$. The above discussion suggests that, being unaware of $h_{k,d}$, the optimal relaying set is the set of $K$ nodes with the largest $\gamma_k$. We require the following theorem to analyze the performance of the optimal relay selection scheme.

**Theorem 1.** *Consider a large WSN wherein the nodes are uniformly distributed with density $\rho$ and the channels between the source and relays are Rayleigh fading with variance $1/2$ per real dimension. Then, the cumulative probability distribution (CDF) of the $k$-th largest SNR at the relays is given by*[1]

$$F_{\gamma_{(k)}^{(o)}}(\gamma) = G(u(\gamma), k) = e^{-u(\gamma)} \sum_{j=0}^{k-1} \frac{u(\gamma)^j}{j!}, \quad (9)$$

*where*

$$u(\gamma) = \frac{2\rho\pi}{\nu} \int_1^{\infty} z^{\frac{2}{\nu}-1} e^{-\frac{\gamma z}{\beta}} dz. \quad (10)$$

*Proof:* See [10]. ∎

The integral in (10) is bounded and has a closed form solution for all feasible values of $\nu$. Using this theorem, we can find the probability distribution function (PDF) of the SNR as $f_{\gamma_{(k)}^{(o)}}(\gamma) = \partial F_{\gamma_{(k)}^{(o)}}(\gamma)/\partial\gamma$ and obtain the average SNR, $\bar{\gamma}_k^{(o)}$. Note that $\bar{\gamma}_k^{(o)}$ depends on $k$. Therefore, when the optimal relay selection scheme is used, each relay should know its index in the relaying queue. Then $\beta_k^{(o)}$ can be determined from (3) and be used as the normalization factor at the $k$th relay. Among the relay selection techniques that do not have access to any information regarding the nodes' forward channels, the presented scheme provides the best possible SNR at the receiver. However, as discussed in Section I, the above optimal relay selection scheme requires all the nodes to continuously propel between the transmission and listening modes and, thus, can entail a significant waste in the network energy.

**Geometry-based Relay Selection–** In many practical scenarios, the changes in the nodes positions are much slower than the variations in their channel links. This, along with the fact that $\theta_k$ in (8) in inversely proportional to $D_{s,k}^{\nu}$, motivates a relay selection technique that selects $K$ closest nodes to the source as relays for a predetermined number of transmission cycles while leaving the rest of the nodes in the network in the sleeping mode. Compared to the optimal relay selection technique, the above geometry-based relay selection technique is much more energy-efficient at the cost of a potential decrease in the signal reception quality.

Note that $h_{s,k}$ is independent from $D_{s,k}$ in this scheme and we have

$$\bar{\gamma}_k^{(g)} = \beta \mathrm{E}\left\{|h_{s,k}|^2\right\} \mathrm{E}\left\{D_{s,k}^{-\nu}\right\} = \beta \mathrm{E}\left\{D_{s,k}^{-\nu}\right\}. \quad (11)$$

Using a similar technique as in [8], the PDF of $D_{s,k}$ can be derived as

$$f_{D_{s,k}}^{(g)}(r) = \frac{2(\rho\pi)^k}{(k-1)!} r(r^2-1)^{k-1} e^{-\rho\pi(r^2-1)} \quad r > 1. \quad (12)$$

It follows from (11) and (12) that

$$\bar{\gamma}_k^{(g)} = \frac{2\beta(\rho\pi)^k}{(k-1)!} \int_1^{\infty} r^{1-\nu}(r^2-1)^{k-1} e^{-\rho\pi(r^2-1)} dr. \quad (13)$$

After some simple manipulations, we obtain

$$\begin{aligned}\bar{\gamma}_k^{(g)} &= \frac{\beta(\rho\pi)^k}{(k-1)!} \sum_{i=0}^{\infty} \binom{-\frac{\nu}{2}}{i} \int_0^{\infty} u^{i+k-1} e^{-\rho\pi u} du \\ &= \frac{\beta(\rho\pi)^k}{(k-1)!} \sum_{i=0}^{\infty} \binom{-\frac{\nu}{2}}{i} \frac{(i+k-1)!}{(\rho\pi)^{i+k}}, \end{aligned} \quad (14)$$

where $\binom{\alpha}{k} = \alpha(\alpha-1)\cdots(\alpha-k+1)/k!$. This along with (3) indicates that the normalization factor $\beta_k^{(g)}$ also depends on $k$.

The geometry-based relay selection technique increases the network energy-efficiency and reduces its signaling complexity compared to the optimal relay selection scheme. However, in networks with a more static topology, the geometry-based relay selection technique tends to overexploit the rarely-changing set of $K$ closest nodes to the source. This can eventually result in the nodes battery depletion and a network disconnectivity.

**Random Relay Selection–** One approach to avoid the above problem is to randomly select $K$ relays from $O(s,R)$, the disc of radius $R$ centered at $s$, and leave all other nodes both inside and outside the disc in the sleeping mode. After a predetermined number of transmission cycles, a new set of $K$ nodes is selected from $O(s,R)$ to avoid overexploiting a fixed set of nodes. When the nodes are randomly selected using to the above technique, the PDF of $D_{s,k}$ is given by

$$f_{D_{s,k}}^{(r)}(r) = \frac{2r}{R^2-1} \qquad r > 1. \quad (15)$$

[1] In all our derivations, we have assumed a relay-free disc of unit radius around the source.

Exploiting the independency of the distance $D_{s,k}$ and the channel $h_{s,k}$ in this relay selection scheme, (11) and (15) can be used to obtain

$$\bar{\gamma}_k^{(r)} = \beta \int_1^R \frac{2t^{1-\nu}}{R^2-1} dt = \begin{cases} \beta \cdot \frac{2\ln(R)}{R^2-1} & \nu = 2 \\ \beta \cdot \frac{2(R^{2-\nu}-1)}{(R^2-1)(2-\nu)} & \nu \neq 2 \end{cases}. \tag{16}$$

Note from (16) that $\bar{\gamma}_k^{(r)}$, and, hence, $\beta_k^{(r)}$ are independent from $k$. Therefore, when the random relay selection scheme is used, it is not required that the relays are aware of their position in the relaying queue.

It can also be observed from (16) that the choice of $R$ has a significant effect on the performance of the random relay selection scheme. While $R$ should not be selected very small so that there are not enough relays on $O(s,R)$, it should also not be selected very large so that the selected relays suffer from a long distance from $s$ and a weak SNR problem. In what follows, we use the above discussion to propose a systematic approach to select $R$. First, let us define the following events:

$$\begin{aligned} \mathcal{A} &= \{\text{There are at least } K \text{ relays on } O(s,R)\} \\ \bar{\mathcal{A}} &= \{\text{There are at most } K-1 \text{ relays on } O(s,R)\} \\ \mathcal{B} &= \left\{\gamma_k^{(r)} \geq \gamma^\star \;\; \text{for } k = 1,2,...,K\right\} \end{aligned} \tag{17}$$

where $\gamma^\star$ is the minimum acceptable SNR at the selected relays and can be chosen such that a minimum communication rate between the source and the relays is guaranteed. Then, we can define the outage probability as the probability that there are at most $K-1$ relays on $O(s,R)$ *or* at least one of the $K$ selected relays has a SNR less than $\gamma^\star$. More formally,

$$\begin{aligned} \mathrm{P_{out}}(R) &= 1 - \Pr(\mathcal{A},\mathcal{B}) \\ &= 1 - \Pr(\mathcal{B}|\mathcal{A})\Pr(\mathcal{A}) \\ &= 1 - \left(1 - F_{\gamma_k^{(r)}}(\gamma^\star)\right)^K (1 - \Pr(\bar{\mathcal{A}})), \end{aligned} \tag{18}$$

where $F_{\gamma_k^{(r)}}(.)$ is the CDF of the SNR when the relays are randomly selected from $O(s,R)$ and is given by [10]

$$F_{\gamma_k^{(r)}}(\gamma) = \frac{2}{\nu(R^2-1)} \int_1^{R^\nu} t^{\frac{2}{\nu}-1} e^{-\frac{\gamma t}{\beta}} dt. \tag{19}$$

Therefore,

$$\mathrm{P_{out}}(R) = 1 - \left(\frac{2\int_1^{R^\nu} t^{\frac{2}{\nu}-1} e^{-\frac{\gamma^\star t}{\beta}} dt}{\nu(R^2-1)}\right)^K \times \left(1 - e^{-\rho\pi(R^2-1)} \sum_{l=0}^{K-1} \frac{(\rho\pi(R^2-1))^l}{l!}\right) \tag{20}$$

where the expression inside the second parentheses at the right-hand side of (20) is due to the fact that if nodes are uniformly distributed on a 2-dimensional plane with the density $\rho$, the number of nodes inside an area $A$ on the plane is a Poisson r.v. with the parameter $\rho\mu(A)$ where $\mu(A)$ is the standard Lebesgue measure of $A$ [8], [11]. The optimal $R$ can then be selected as

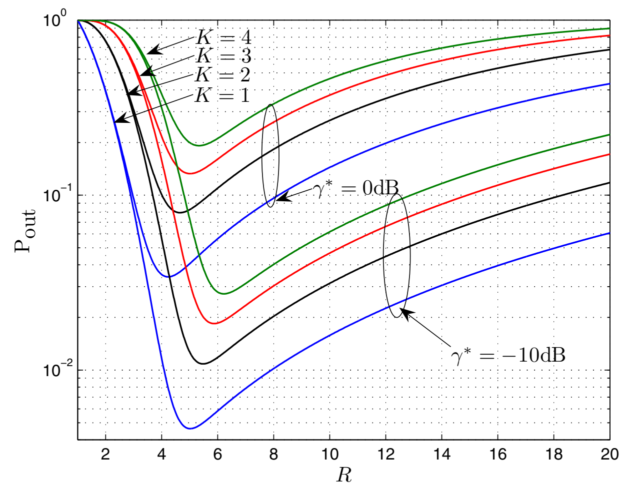


Fig. 2: $\mathrm{P_{out}}(R)$ as a function of $R$ for different $K$ and $\gamma^\star$.

$$R^\star = \arg\min_{\mathbf{R}} \mathrm{P_{out}}(R). \tag{21}$$

Fig. 2 shows $\mathrm{P_{out}}(R)$ versus $R$ for different $K$ and $\nu = 2$, $\rho = 0.1$, $\gamma^\star = 0$ and $-10$ (dB), and $\beta = 25$ (dB). As can be observed from Fig. 2, each outage probability curve has a minimum point that corresponds to $R^\star$.

## IV. SNR ANALYSIS AT THE RECEIVER

When MRC is used at the receiver, it is straightforward to show that the average SNR at this terminal is given by

$$\bar{\gamma}_d = \sum_{k=1}^K \underbrace{\mathrm{E}\left\{\frac{\alpha_k |h_{k,d}|^2}{1+\alpha_k|h_{k,d}|^2}\right\}}_{\phi_k} \bar{\gamma}_k. \tag{22}$$

As $h_{k,d}$ is a zero-mean Gaussian r.v. with the variance of $1/2$ per real dimension, $|h_{k,d}|^2$ is a unit-mean exponentially distributed r.v. and, therefore,

$$\phi_k = \int_0^\infty \frac{\alpha_k x}{1+\alpha_k x} \cdot e^{-x} dx = 1 - \frac{e^{\frac{1}{\alpha_k}}}{\alpha_k} \mathrm{E}_1\left(\frac{1}{\alpha_k}\right) \tag{23}$$

where $\mathrm{E}_1(z) = \int_z^\infty e^{-t}/t \; dt$ is the exponential integral. It is of practical value to study the variations of $\gamma_d$ around $\bar{\gamma}_d$. Using (7) and (8) along with the facts that $\alpha_k$ is inversely proportional to $K$ and $h_{s,k}$ and $h_{k,d}$ are independent r.v.s, it can be readily shown that

$$\mathrm{var}\left(\gamma_d^{(\bullet)}\right) = \frac{\zeta_1^{(\bullet)}}{K} + \zeta_2^{(\bullet)} F_K^{(\bullet)}, \tag{24}$$

where $\bullet$ is either "$o$", "$g$" or "$r$" and $\zeta_1^{(\bullet)}$ and $\zeta_2^{(\bullet)}$ are two scalars independent from $K$. In the following, $F_K^{(\bullet)}$ in the random, the geometry-based, and the optimal relay selection schemes are analyzed and the behavior of $\mathrm{var}\left(\gamma_d^{(\bullet)}\right)$ when $K$ grows large are investigated.

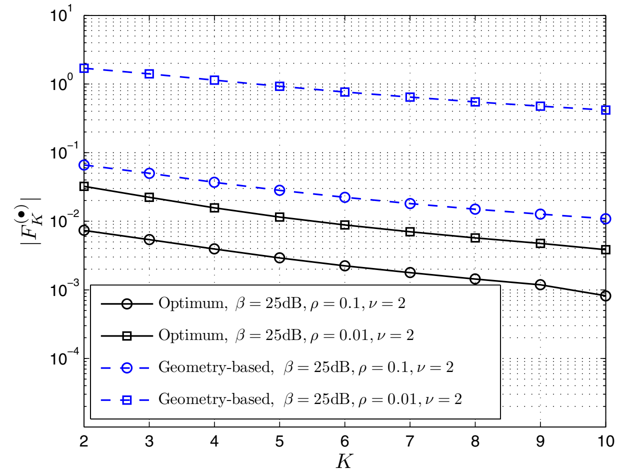


Fig. 3: $\left|F_K^{(o)}\right|$ and $\left|F_K^{(g)}\right|$ versus $K$ for $\rho = 0.1$ and $\rho = 0.01$.

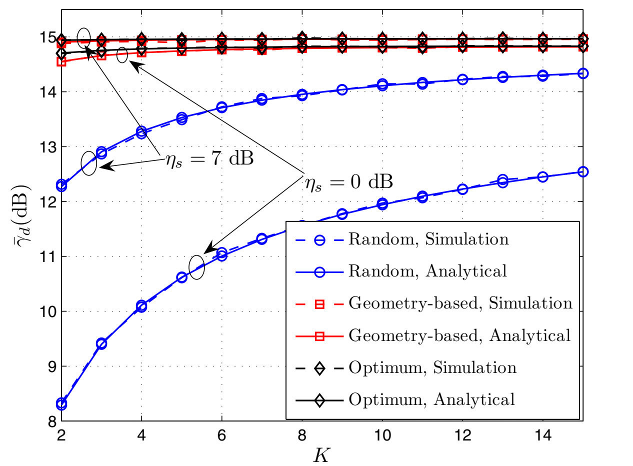


Fig. 4: $\bar{\gamma}_d$ versus $K$ for $\rho = 1$, $\eta_d = 15$ (dB), and $\eta_s = 0$, 7 (dB).

**Random Relay Selection–** When the random relay selection scheme is used, the relays' distances from the source are independent r.v.s and it can be shown that $F_K^{(r)} = 0$ [10]. Therefore, var $\left(\gamma_d^{(r)}\right)$ converges to zero with the rate $\mathcal{O}(1/K)$.

**Geometry-based Relay Selection–** In the geometry-based relay selection scheme, the relays' distances from the source are not independent r.v.s. and we have [10]

$$F_K^{(g)} = \frac{1}{K^2} \sum_{m=1}^{K} \sum_{l=1}^{m-1} \mathrm{E}\left\{D_{s,l}^{-\nu} D_{s,m}^{-\nu}\right\} - \mathrm{E}\left\{D_{s,l}^{-\nu}\right\} \mathrm{E}\left\{D_{s,m}^{-\nu}\right\}. \tag{25}$$

The analysis of var $\left(\gamma_d^{(g)}\right)$ requires the knowledge of $f_{D_{s,l},D_{s,m}}^{(g)}(\cdot,\cdot)$, the joint PDF of $D_{s,l}$ and $D_{s,m}$. The following theorem holds.

**Theorem 2.** *For $m > l$, the joint PDF of $D_{s,l}$ and $D_{s,m}$ in the geometry-based relay selection scheme is given by (26).*

*Proof:* See [10]. ■

Using (12) and (26), analytical expressions of $\mathrm{E}\left\{D_{s,l}^{-\nu}\right\}$ and $\mathrm{E}\left\{D_{s,l}^{-\nu} D_{s,m}^{-\nu}\right\}$ can be obtained and then be used to derive $F_K^{(g)}$. Although $F_K^{(g)}$ is a rather complicated function (not brought here), simulation results shown in Fig. 3 verify that $\left|F_K^{(g)}\right|$ is a decreasing function of $K$ for the tested values of $\nu$ and $\rho$. The latter observation along with (24) suggests that var $\left(\gamma_d^{(g)}\right)$ should converge to zero as the number of relays increases.

**Optimal Relay Selection–** When the optimal relay selection technique is used, we can define

$$F_K^{(o)} = \frac{1}{\beta^2 K^2} \sum_{m=1}^{K} \sum_{l=1}^{m-1} \mathrm{E}\left\{\gamma_l \gamma_m\right\} - \mathrm{E}\left\{\gamma_l\right\} \mathrm{E}\left\{\gamma_m\right\}. \tag{27}$$

Note that the normalization factor $\beta^2$ in (27) is only used to make $F_K^{(o)}$ and $F_K^{(g)}$ comparable. To analyze var $\left(\gamma_d^{(o)}\right)$, we require $f_{\gamma_m,\gamma_l}^{(o)}(.,.)$, the joint PDF of $\gamma_l$ and $\gamma_m$. The following theorem derives the latter function.

**Theorem 3.** *For $m > l$, the joint PDF of $\gamma_l$ and $\gamma_m$, the SNR of the $l$th and the $m$th relays with the highest SNRs, is given by (28).*

*Proof:* See [10]. ■

Again, simulation results shown in Fig. 3 verify the decreasing behavior of $\left|F_K^{(o)}\right|$ as $K$ grows. This observation shows that var $\left(\gamma_d^{(o)}\right)$ should also converge to zero as the number of relays increases.

Convergence of var $(\gamma_d)$ to zero implies that if $K$ is large enough, then, for any arbitrary set of realizations of $D_{s,k}$, $h_{s,k}$, and $h_{k,d}$, $\gamma_d$ should be close to $\bar{\gamma}_d$. This confirms that $\bar{\gamma}_d$ is a sensible performance measure for the considered cooperative WSN and the derived properties of the average SNR at the receiver also approximately hold for the instantaneous SNR at this terminal. Moreover, the above result verifies that the proposed relay selection schemes effectively decrease the signal power fluctuations at the receiver. This is an expected effect due to the spatial diversity provided by the $K$ independent channel paths.

## V. Simulations

In this section, further numerical simulations are used to validate the analytical results. Fig. 4 shows the analytical and the numerical $\bar{\gamma}_d^{(r)}$, $\bar{\gamma}_d^{(g)}$, and $\bar{\gamma}_d^{(o)}$ versus $K$ for $D = 1000$, $R = 20$, $\nu = 2$, $\sigma^2 = 1$, $\rho = 1$, $\eta_d = P_T/(\sigma^2 D^\nu) = 15$ (dB), and two different $\eta_s \triangleq p_s/(\sigma^2 R^\nu) = 0$ (dB) and 7 (dB). Note that $\eta_s$ is the average SNR on the boundary of $O(s, R)$, and, therefore, $\gamma_k^{(r)} \geq \eta_s$ for $k = 1, 2, ..., K$. The figure shows the results of averaging over $10^5$ random realizations of the channel links and source-relay distances. Fig. 4 further verifies that the derived average SNR expressions accurately predict their numerical counterparts.

$$f^{(g)}_{D_{s,l},D_{s,m}}(r,t) = \begin{cases} 4(\rho\pi)^m \cdot \dfrac{t(t^2-1)^{(m-l-1)}}{(m-l-1)!} \cdot \dfrac{r(r^2-1)^{(l-1)}}{(l-1)!} \cdot \left(\dfrac{t^2-r^2}{t^2-1}\right)^{m-l-1} \cdot e^{-\rho\pi(t^2-1)} & t \geq r \\ 0 & t < r \end{cases} \quad (26)$$

$$f^{(o)}_{\gamma_m,\gamma_l}(x,y) = \begin{cases} \dfrac{\partial}{\partial x} G\left(u(x)-u(y), m-l\right) \cdot \dfrac{\partial}{\partial y} G\left(u(y), l\right) & y \geq x \\ 0 & y < x \end{cases} \quad (28)$$

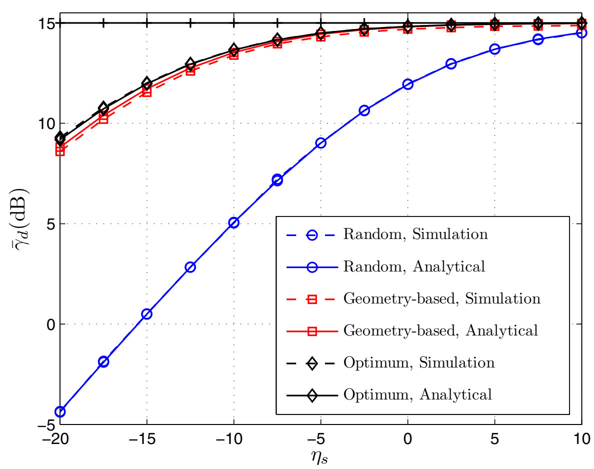


Fig. 5: $\bar{\gamma}_d$ versus $\eta_s$ for $\rho = 1$, $K = 10$, and $\eta_d = 15$ (dB).

Fig. 5 shows the analytical and the numerical $\bar{\gamma}_d^{(r)}$, $\bar{\gamma}_d^{(g)}$, and $\bar{\gamma}_d^{(o)}$ versus $\eta_s$ for $\rho = 1$ and $K = 10$. In Fig. 5, $\eta_d = 15$ (dB) is also plotted as a reference. As can be observed from the figure, the numerical results very closely follow their analytical counterparts. Fig. 5 also shows that when $\eta_s$ is very small, $\bar{\gamma}_d^{(o)}$ and $\bar{\gamma}_d^{(g)}$ are considerably higher than $\bar{\gamma}_d^{(r)}$. However, increasing $\eta_s$, all $\bar{\gamma}_d^{(r)}$, $\bar{\gamma}_d^{(g)}$, and $\bar{\gamma}_d^{(o)}$ rapidly increase and converge to $\eta_d$. This indicates that if the average SNRs at all relays are large enough, the average SNR at the receiver approaches $\eta_d$ irrespective to the selected relaying set. This figure also shows that $\bar{\gamma}_d^{(g)}$ is very close to $\bar{\gamma}_d^{(o)}$ for an extended range of $\eta_s$ starting as low as $-20$ (dB). This observation verifies the close-to-optimal performance of the geometry-based relay selection scheme even at a very low SNR regime. Note also that $\bar{\gamma}_d^{(r)}$ enters the one-dB vicinity of $\eta_d$ at a moderate $\eta_s \approx 5$ (dB).

## VI. Conclusions

We have presented the following three multiple relay selection schemes in wireless sensor networks with uniformly distributed nodes in the case that the forward channels' information is not available at the relays: 1) The optimal relay selection scheme where $K$ nodes with the highest instantaneous SNRs are selected for relaying. Among all relay selection techniques that do not use any information regarding the nodes' forward channels, this technique achieves the highest possible SNR at the receiver at the cost of energy-inefficiency and a considerable signaling overhead; 2) the geometry-based relay selection scheme that ignores the instantaneous channel variations and chooses $K$ closest nodes to the source as relays. While achieving a close-to-optimal average SNR at the receiver, it can be substantially more energy-efficient than its optimal counterpart. The main disadvantage of this scheme is the possibility of overexploiting the group of nodes that stay close to the source in networks with a more static topology; and 3) the random relay selection scheme using which $K$ nodes are randomly selected from an $R$-neighborhood around the source. Defining a suitable outage probability, a method to select $R$ in this scheme is proposed.

The SNR performance at the relays and the receivers for all above relay selection schemes have been analyzed. The SNR variance at the receiver has also been studied and it has been shown that it converges to zero as the number of relays grows. Various numerical simulations have also been used to verify the analytical derivations.

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