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A dual-hop amplify-and-forward MIMO cooperative beamformer in distributed wireless sensor networks

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***Abstract*—In this paper, we propose a cooperative beamforming (CB) technique for dual-hop amplify-and-forward communication in WSNs with one source, $N-1$ interferences, and K relay nodes. First, an optimal beamformer is obtained that maximizes the signal-to-interference-plus-noise ratio at the intended receiver in the far-field subject to the following constraints: 1) The relay nodes total transmit power is bounded; and 2) the received power from the N transmitters at $L-1$ unintended receivers in the far-field is zero. It is shown that the optimal beamformer can only be implemented if each relay node knows the locations and the backward channels of all other relay nodes in the network. As this knowledge is not typically available at nodes in WSNs, we use a technique to approximate the optimal beamforming coefficients with quantities that depend only on the locally-available information at each individual relay node. The average beampattern expression of the proposed CB technique is then derived and its properties are analyzed. In particular, it is shown that the average gain of the beamformer linearly increases with K in the direction of the intended receiver while remaining fixed in the directions of the unintended receivers.**

***Index Terms*—Amplify-and-forward relaying, beampattern, cooperative beamforming, multiple-input multiple-output communication, wireless sensor networks.**

I. Introduction

Under a total transmission power constraint, an efficient beamforming technique can increase the impinging power to the intended receiver proportionally to the number of transmitting antennas. This property is particularly beneficial when a beamforming technique is adopted to establish a communication link from small battery-powered sensor nodes in wireless sensor networks (WSNs) to a receiver in the far-field: Increasing the number of cooperative nodes K that imitate the transmitting antennas, the received power at the destination is proportionally increased with K while the dissipating power from each cooperative node can be proportionally decreased with K. The latter effect facilitates a substantial increase in the network's lifetime. Usually referred to as cooperative beamforming (CB) techniques, the beamforming techniques tailored for WSNs are of increasing interest to the research community [1]-[7].

The theory of antenna arrays is used to analyze a phased-array CB technique in [1]. Assuming that the second-order statistics of the channel coefficients are known, several CB designs are developed in [2] for a single-source single-destination system. A null-steering CB technique is proposed in [3] and the properties of its associated average beampattern are analyzed. A cross-layer approach to the CB design is introduced in [4] that facilitates a higher throughput than those of the conventional CB techniques.

This paper proposes a CB technique for a dual-hop communication scheme in ad-hoc WSNs wherein a source node uses K neighboring relay nodes that are uniformly distributed in the network to transmit its message to an intended receiver in the far-field outside the network. Due to the network's ad-hoc structure, the source is also neighbored by $N-1$ interfering transmitters and, further, it is assumed that the intended receiver is surrounded by $L-1$ unintended receivers. In the first time slot, all transmitters broadcast their signals while in the second time slot the overheard signals at the relay nodes are multiplied by proper beamforming weights and are forwarded to the receivers. The optimal beamforming weights are computed that maximize signal-to-interference-plus-noise ratio (SINR) at the intended receiver subject to two constraints: One constraint upper-bounds the relay nodes' total transmit power and the other enforces the received power from all N transmitters at the unintended receivers to zero.

It turns out that the optimal beamforming weights are complicated functions of the coordinates and the channel gains of all relay nodes in the WSN. As such, a local computation of the optimal weights requires each node to have a global knowledge on the network. The decentralized and unsupervised nature of WSNs renders such a requirement impractical in most scenarios. To get around this problem and facilitate the implementation of the CB technique, a technique is proposed that efficiently approximates the so-obtained optimal beamforming weights based on the locally-available information at each individual relay node. The average beampattern expression associated with the latter beamformer is derived and its properties are analyzed. In particular, it is shown that the average gain of the proposed beamformer is linearly proportional to the number of relay nodes K in the direction of the intended receiver while is independent from K in the directions of the unintended receivers.

The rest of the paper is organized as follows. The signal model is introduced in Section II. The optimal beamforming and the proposed CB techniques are presented in Section III. The average beampattern expression is obtained and its properties are analyzed in Section IV. Simulation results are presented in Section V and the concluding remarks are given in Section VI.

Notation: Lowercase boldface letters indicate vectors, while uppercase boldface letters present matrices. $\odot$ and $\otimes$ are the

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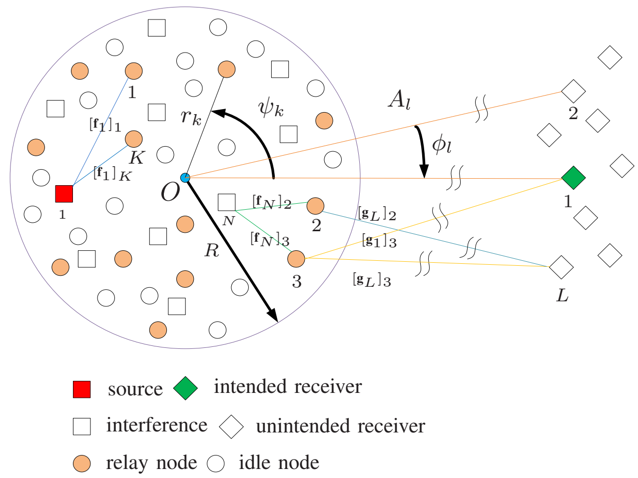

Fig. 1: System model.

element-wise product and the Kronecker product, respectively. $\mathbf{I}$ is the identity matrix. $\mathbf{e}_l$ is a vector with one in the lth position and zeros elsewhere. $[\mathbf{A}]_{kj}$ is the entry at the kth row and the jth column of $\mathbf{A}$, $[\mathbf{a}]_j$ is the jth entry of $\mathbf{a}$, and $\mathrm{diag}\{\mathbf{a}\}$ is the diagonal matrix whose diagonal entries are the entries of $\mathbf{a}$. $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote the transpose, the conjugate, and the conjugate transpose. $\mathrm{E}\{\cdot\}$ stands for the statistical expectation, $|\cdot|$ is the absolute value, and $J_1(\cdot)$ is the first-order Bessel function of the first kind.

II. System Model

Fig. 1 shows the system of our interest that is comprised of one source and $N-1$ interferences in the WSN, K WSN relay nodes that along with some idle nodes are uniformly distributed on $D(O,R)$, the disc centered at O with radius R, and L receivers in the far-field. The first receiver is the intended receiver and the rest are unintended ones. It is assumed that the direct link from each transmitter to any far-field receiver is negligible. The location of the kth relay node is denoted in polar coordinates by (r_k,ψ_k). The channel coefficients from the N transmitters to the K relay nodes are shown by $[\mathbf{f}_i]_j$, $i=1,2,\ldots,N$, $j=1,2,\ldots,K$ and the forward channel coefficients from the relay nodes to the receivers are denoted by $[\mathbf{g}_l]_j$, $l=1,2,\ldots,L$, $j=1,2,\ldots,K$. The intended receiver is located at (A_1,ϕ_1) and $L-1$ unintended receivers coordinates are (A_l,ϕ_l), $l=2,3,\ldots,L$. For the sake of simplicity, it is assumed that $\phi_1=0$ and all the receivers have the same distance from O, that is, $A_l=A$, $l=1,2,\ldots,L$. Extending our results to the case of unequal A_l, $l=1,2,\ldots,L$ is straightforward. The following assumptions are used throughout the paper.

- **A1:** The effects of scattering or signal reflection from the relay nodes to the receivers are negligible.
- **A2:** The $N\times 1$ transmitted signal vector $\mathbf{s}(t)$ and the $K\times 1$ relays noise vector $\mathbf{v}(t)$ are zero-mean with the covariance matrices $\mathbf{I}$ and $\sigma_v^2\mathbf{I}$, respectively, and $n_\bullet(t)$, the noise at an arbitrary far-filed point $F_\bullet(A_\bullet,\phi_\bullet)$, is zero-mean with variance σ_n^2. $\mathbf{s}(t)$, $\mathbf{v}(t)$, and $n_\bullet(t)$ are mutually statistically independent.
- **A3:** The communication links from the transmitters to the relays are statistically independent and follow the Rayleigh fading channel model, that is, $[\mathbf{f}_i]_j$, $i=1,2,\ldots,N$, $j=1,2,\ldots,K$ are independent zero-mean complex circular Gaussian random variables.
- **A4:** Each relay node is unaware of the locations and the backward channels of all other relay nodes in WSN.

A1-**A3** are common in the literature of array processing for planar wave and **A4** is due to the decentralized structure of WSNs. Note that **A1** is valid when the WSN size is small compared to the distance between the relay nodes and the receivers. In this case, the large scale fading effects are dominant [4]. The received signal at the kth relay node in the first time-slot is given by

$$[\mathbf{x}(t)]_k=\sum_{i=1}^{N}[\mathbf{f}_i]_k[\mathbf{s}(t)]_i+[\mathbf{v}(t)]_k. \tag{1}$$

In the second time slot, the relay nodes multiply their received signals with the beamforming weights and forward them to the receivers. The relays' transmitted signals can be represented as

$$\mathbf{y}(t)=\mathbf{W}^*\mathbf{x}(t)e^{j2\pi ft} \tag{2}$$

where $\mathbf{W}\triangleq\mathrm{diag}\{\mathbf{w}\}=\mathrm{diag}\left\{[w_1\ \ldots\ w_K]^T\right\}$ is the diagonal matrix of the beamforming weights and f is the carrier frequency. Using (1) and (2), it can be shown that the total transmit power from the relay nodes is

$$P_T=\sum_{k=1}^{K}\mathrm{E}\left\{|[\mathbf{y}(t)]_k|^2\right\}=\mathbf{w}^H\mathbf{D}\mathbf{w} \tag{3}$$

where $\mathbf{D}\triangleq\mathrm{diag}\left\{\left[\mathbf{F}\mathbf{F}^H\right]_{11}\ \ldots\ \left[\mathbf{F}\mathbf{F}^H\right]_{KK}\right\}+\sigma_v^2\mathbf{I}$ with $\mathbf{F}\triangleq[\mathbf{f}_1\cdots\mathbf{f}_N]$.

Due to **A1**, the received signal at an arbitrary point $F_\bullet(A_\bullet,\phi_\bullet)$ in the far-field is given by

$$z_\bullet(t)=\eta_{F_\bullet}e^{j2\pi ft}\left[\mathbf{w}^H\mathbf{H}^{(\bullet)}\mathbf{s}\left(t-\frac{A_\bullet}{c}\right)\right.$$
$$\left.+\,\mathbf{g}_\bullet^T\mathbf{W}^H\mathbf{v}\left(t-\frac{A_\bullet}{c}\right)\right]+n_\bullet(t) \tag{4}$$

where $\eta_{F_\bullet}\triangleq A_\bullet^{-\gamma/2}e^{-j(2\pi/\lambda)A_\bullet}$ with γ denoting the path-loss exponent and λ indicating the signal wavelength, $\mathbf{g}_\bullet\triangleq\left[e^{j(2\pi/\lambda)r_1\cos(\phi_\bullet-\psi_1)}\ \ldots\ e^{j(2\pi/\lambda)r_K\cos(\phi_\bullet-\psi_K)}\right]^T$ represents the forward channel vector from the relay nodes to $F_\bullet(A_\bullet,\phi_\bullet)$, $\mathbf{H}^{(\bullet)}\triangleq\mathbf{g}_\bullet\odot\mathbf{F}$, and c is the speed of an electromagnetic wave.

Let P_S, P_I, and P_N denote the signal power, the total interferences power, and the compound noise power sensed at the intended receiver, respectively. It is direct to show from (4) that

$$\begin{aligned}P_S&=\mathbf{w}^H\mathbf{R}\mathbf{w}\\P_I&=\mathbf{w}^H\mathbf{M}\mathbf{w}\\P_N&=\mathbf{w}^H\mathbf{Q}\mathbf{w}+\sigma_n^2\end{aligned} \tag{5}$$

where $\mathbf{R} \triangleq A^{-\gamma}\mathbf{h}_1\mathbf{h}_1^H$ with $\mathbf{h}_1 \triangleq \mathbf{g}_1 \odot \mathbf{f}_1$, $\mathbf{M} \triangleq A^{-\gamma}\mathbf{H}_{\bar{1}}^{(1)}\mathbf{H}_{\bar{1}}^{(1)^H}$ with $\mathbf{H}_{\bar{1}}^{(1)} \triangleq [\mathbf{g}_1 \odot \mathbf{f}_2 \ \ldots \ \mathbf{g}_1 \odot \mathbf{f}_N]$, and $\mathbf{Q} \triangleq A^{-\gamma}\sigma_v^2\mathbf{I}$.

III. The optimal and the cooperative beamforming techniques

In this section, the optimal beamforming vector is derived. The result is then used to introduce a CB technique that can be implemented in the decentralized WSN of our interest. Ideally, the goal is to maximize SINR at the intended receiver subject to the two following constraints:

- **C1:** The total transmit power from the relay nodes is upper-bounded by a certain threshold p_{th}.
- **C2:** The received power from all N transmitters at unintended receivers is zero.

Using (3) and (5), the above goal can be formally presented as

$$\max_{\mathbf{w}} \frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\mathbf{w}^H\left(\mathbf{M}+\mathbf{Q}\right)\mathbf{w}+\sigma_n^2} \quad \text{subject to} \quad \begin{array}{ll} \mathbf{w}^H\mathbf{D}\mathbf{w} & \leq \ p_{\mathrm{th}} \\ \mathbf{w}^H\mathbf{T} & = 0 \end{array} \tag{6}$$

where $\mathbf{T} \triangleq \left[\mathbf{H}^{(2)} \ \ldots \ \mathbf{H}^{(L)}\right]$. The solution to (6) is [8]

$$\mathbf{w}_{\mathrm{opt}} = \frac{\alpha A^{\gamma}}{\sqrt{Kp_{\mathrm{th}}}}\left(\left(\mathbf{\Lambda}+\mathbf{H}_{\bar{1}}^{(1)}\mathbf{H}_{\bar{1}}^{(1)^H}\right)^{-1}\mathbf{h}_1 - \mathbf{D}^{-1}\mathbf{T}\left(\mathbf{T}^H\mathbf{D}^{-1}\mathbf{T}\right)^{-1}\mathbf{T}^H\left(\mathbf{\Lambda}+\mathbf{H}_{\bar{1}}^{(1)}\mathbf{H}_{\bar{1}}^{(1)^H}\right)^{-1}\mathbf{h}_1\right) \tag{7}$$

where $\mathbf{\Lambda} \triangleq \left(A^{\gamma}\sigma_n^2/p_{\mathrm{th}}\right)\cdot\mathbf{D}+\sigma_v^2\mathbf{I}$ and α is a factor chosen such that $\mathbf{w}^H\mathbf{D}\mathbf{w} = p_{\mathrm{th}}$. The matrix inversion lemma can be used to equivalently represent (7) as

$$\mathbf{w}_{\mathrm{opt}} = \frac{\alpha A^{\gamma}}{\sqrt{Kp_{th}}}\left(\mathbf{\Lambda}^{-1}\mathbf{h}_1 - \mathbf{\Lambda}^{-1}\mathbf{H}_{\bar{1}}^{(1)}\mathbf{m} - \mathbf{D}^{-1}\mathbf{T}\mathbf{q}\right) \tag{8}$$

where

$$\mathbf{m} \triangleq \left(\frac{1}{K}\cdot\mathbf{I}+\frac{1}{K}\mathbf{H}_{\bar{1}}^{(1)^H}\mathbf{\Lambda}^{-1}\mathbf{H}_{\bar{1}}^{(1)}\right)^{-1}\frac{1}{K}\mathbf{H}_{\bar{1}}^{(1)^H}\mathbf{\Lambda}^{-1}\mathbf{h}_1 \tag{9}$$

$$\mathbf{q} \triangleq \left(\mathbf{T}^H\mathbf{D}^{-1}\mathbf{T}\right)^{-1}\mathbf{T}^H\left(\mathbf{\Lambda}^{-1}\mathbf{h}_1 - \mathbf{\Lambda}^{-1}\mathbf{H}_{\bar{1}}^{(1)}\mathbf{m}\right). \tag{10}$$

It is direct to show from (8) that $[\mathbf{w}_{\mathrm{opt}}]_k$, the optimal beamforming weight at the kth relay node, depends on the locations and the backward channels of all relay nodes. According to **A4**, this information is not available at the kth relay, and, hence, $\mathbf{w}_{\mathrm{opt}}$ cannot be implemented in the decentralized WSN of our interest.

In what follows, we derive a vector whose kth entry ($k = 1,\ldots,K$) efficiently approximates $[\mathbf{w}_{\mathrm{opt}}]_k$ and does not depend on the location and the backward channel of any relay node but the kth one. First, note that when the total transmit power from the K relays is fixed, the transmit power from each relay is inversely proportional to K while the SINR linearly increases with K [7]. Therefore, to improve the relay nodes' lifetime and the received signal quality, it is beneficial to have a large K by recruiting as many idle nodes as possible. Using **A3** and the strong law of large numbers along with the Squeeze Theorem, we have $\lim_{K\to\infty}(1/K)\cdot\mathbf{H}_{\bar{1}}^{(1)^H}\mathbf{\Lambda}^{-1}\mathbf{h}_1 = \mathbf{0}$ [8]. The latter result can be used in (9) and (10) to show that

$$\lim_{K\to\infty}\mathbf{m} = \mathbf{0}$$

$$\lim_{K\to\infty}\mathbf{q} = \left(\mathbf{T}^H\mathbf{D}^{-1}\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{\Lambda}^{-1}\mathbf{h}_1. \tag{11}$$

As such, when K is large enough, $\mathbf{w}_{\mathrm{opt}}$ in (8) can be well-approximated by

$$\check{\mathbf{w}} = \frac{\alpha A^{\gamma}}{\sqrt{Kp_{th}}}\left(\mathbf{\Lambda}^{-1}\mathbf{h}_1 - \mathbf{D}^{-1}\mathbf{T}\mathbf{E}^{-1}\mathbf{u}\right) \tag{12}$$

where $\mathbf{E} \triangleq \mathbf{T}^H\mathbf{D}^{-1}\mathbf{T}$ and $\mathbf{u} \triangleq \mathbf{T}^H\mathbf{\Lambda}^{-1}\mathbf{h}_1$. It follows from (12) that

$$[\check{\mathbf{w}}]_k = \frac{\alpha A^{\gamma}}{\sqrt{Kp_{\mathrm{th}}}}\left(\left[\mathbf{\Lambda}^{-1}\mathbf{h}_1\right]_k - \sum_{p=1}^{(L-1)N}\left[\mathbf{D}^{-1}\mathbf{T}\right]_{kp}\left[\mathbf{E}^{-1}\mathbf{u}\right]_p\right). \tag{13}$$

It can be readily verified that $\left[\mathbf{\Lambda}^{-1}\mathbf{h}_1\right]_k$ and $\left[\mathbf{D}^{-1}\mathbf{T}\right]_{kp}$, $p = 1,\ldots,(L-1)N$ do not depend on the location and the backward channels of any relay node other than the kth one, and, therefore, are computable at the kth relay node. However, α and the entries of $\mathbf{E}$ and $\mathbf{u}$ depend on the locations and the backward channels of all relay nodes and need to be substituted by proper approximating quantities. Using the fact that the nodes are uniformly distributed on $D(O,R)$, it can be proved that [8]

$$\lim_{K\to\infty}\mathbf{E} = \bar{\mathbf{E}} \triangleq \mathrm{I}_{\mathbf{E}}\cdot\bar{\mathbf{E}}_{\phi}\otimes\mathbf{I} \tag{14}$$

$$\lim_{K\to\infty}\mathbf{u} = \bar{\mathbf{u}} \triangleq \mathrm{I}_{\mathbf{u}}\cdot\bar{\mathbf{u}}_0\otimes\mathbf{e}_1 \tag{15}$$

$$\lim_{K\to\infty}\alpha = \bar{\alpha} \triangleq \frac{p_{\mathrm{th}}}{A^{\gamma}\sqrt{\mu - \bar{\mathbf{u}}^T\bar{\mathbf{E}}^{-1}\bar{\mathbf{u}}}} \tag{16}$$

where μ, $\mathrm{I}_{\mathbf{E}}$, and $\mathrm{I}_{\mathbf{u}}$ are constants that do not depend on the location and the backward channels of any relay node and the $L-1\times L-1$ matrix $\bar{\mathbf{E}}_{\phi}$ and the $L-1\times 1$ vector $\bar{\mathbf{u}}_0$ are such that

$$\left[\bar{\mathbf{E}}_{\phi}\right]_{mn} = \begin{cases} \frac{2J_1(\alpha(\phi_{m+1}-\phi_{n+1}))}{\alpha(\phi_{m+1}-\phi_{n+1})} & m \neq n \\ 1 & m = n \end{cases}$$

$$\left[\bar{\mathbf{u}}_0\right]_m = \frac{2J_1\left(\alpha(\phi_{m+1})\right)}{\alpha(\phi_{m+1})} \tag{17}$$

where $\alpha(\phi) \triangleq (4\pi/\lambda)\sin(\phi/2)$. It follows from (14), (15), and (17) that $\bar{\mathbf{E}}$ and $\bar{\mathbf{u}}$ are also computable at every relay node as they are not functions of the nodes' locations and backward channels and solely depend on the directions of the unintended receivers. Substituting α, $\mathbf{E}$, and $\mathbf{u}$ in (12) by $\bar{\alpha}$, $\bar{\mathbf{E}}$, and $\bar{\mathbf{u}}$, respectively, we obtain the following large-K approximation of $\mathbf{w}_{\mathrm{opt}}$:

$$\tilde{\mathbf{w}} = \frac{\sqrt{p_{\mathrm{th}}}}{\sqrt{K\left(\mu - \bar{\mathbf{u}}^T\bar{\mathbf{E}}^{-1}\bar{\mathbf{u}}\right)}}\left(\mathbf{\Lambda}^{-1}\mathbf{h}_1 - \mathbf{D}^{-1}\mathbf{T}\bar{\mathbf{E}}^{-1}\bar{\mathbf{u}}\right). \tag{18}$$

As discussed above, $[\tilde{\mathbf{w}}]_k$ only depends on the location and the backward channel of the kth node, and, hence, $\tilde{\mathbf{w}}$ can be

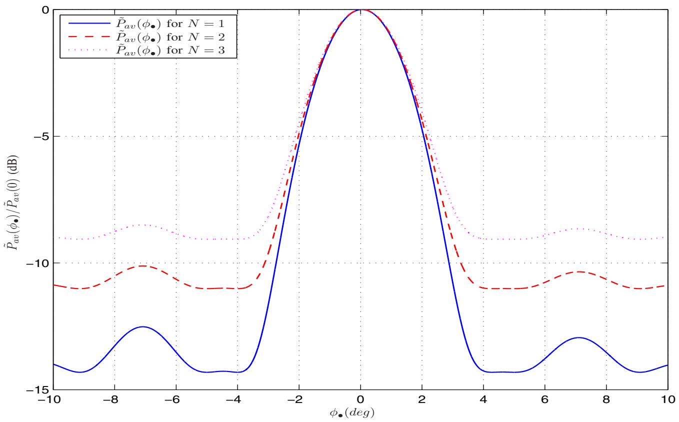

Fig. 2: $\tilde{P}_{av}(\phi_\bullet)/\tilde{P}_{av}(0)$ (dB) versus $\phi_\bullet$ (deg) for $K = 30$ and $N = 1, 2$, and 3.

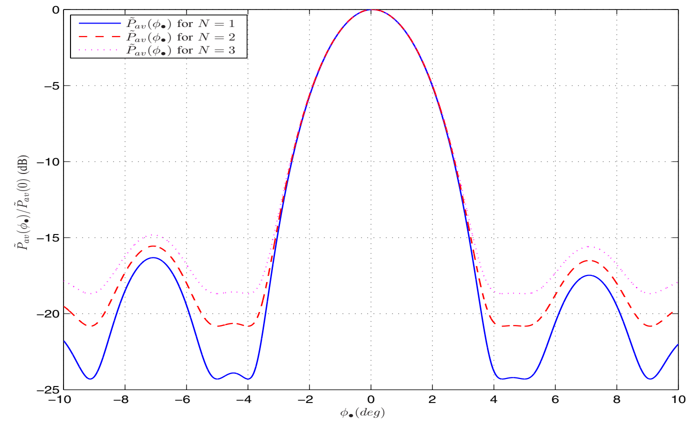

Fig. 3: $\tilde{P}_{av}(\phi_\bullet)/\tilde{P}_{av}(0)$ (dB) versus $\phi_\bullet$ (deg) for $K = 300$ and $N = 1, 2$, and 3.

implemented in the decentralized WSN of our interest. Note also that as $\tilde{\mathbf{w}}$ is a large-K approximation of the optimal beamforming vector $\mathbf{w}_{\text{opt}}$, its SINR performance is close to that of $\mathbf{w}_{\text{opt}}$ when K is large enough.

IV. Average beampattern and its properties

We define the average beampattern $\tilde{P}_{av}(\phi_\bullet)$ at an arbitrary far-field point $F_\bullet(A_\bullet, \phi_\bullet)$ as the average received power at $F_\bullet(A_\bullet, \phi_\bullet)$ where averaging is carried out over the relay nodes' locations [1], [3], [4]. Using (18) in (4) and taking into account the fact that the relay nodes are uniformly distributed on $D(O, R)$, we have [8]

$$\tilde{P}_{av}(\phi_\bullet) = a_1 + a_2(N-1) + a_3(K-1)\xi(\phi_\bullet) \qquad (19)$$

where a_1, a_2, and a_3 are constants and

$$\xi(\phi_\bullet) \triangleq \left(\frac{2J_1(\alpha(\phi_\bullet))}{\alpha(\phi_\bullet)} - \bar{\mathbf{u}}_0^T \bar{\mathbf{E}}_\phi^{-1} \bar{\mathbf{u}}_{\phi_\bullet}\right)^2 \qquad (20)$$

with $\bar{\mathbf{u}}_{\phi_\bullet}$ being an $L-1 \times 1$ vector such that

$$[\bar{\mathbf{u}}_{\phi_\bullet}]_m = \frac{2J_1(\alpha(\phi_{m+1} - \phi_\bullet))}{\alpha(\phi_{m+1} - \phi_\bullet)}. \qquad (21)$$

Note that $\xi(\phi_\bullet)$ is the only term in (19) that depends on $\phi_\bullet$. This term essentially determines the spatial distribution of the received power as a function of $\phi_\bullet$. The critical directions are $\phi_1 = 0$, the direction of the intended receiver, and ϕ_l, $l = 2, \ldots, L$, those of the unintended receivers. Using the fact that $\lim_{\phi_\bullet \to 0} 2J_1(\alpha(\phi_\bullet))/\alpha(\phi_\bullet) = 1$, it can be shown that

$$\xi(0) = \left(1 - \bar{\mathbf{u}}_0^T \bar{\mathbf{E}}_\phi^{-1} \bar{\mathbf{u}}_0\right)^2. \qquad (22)$$

It follows from (19) and (22) that the received power at the direction of the intended receiver linearly increases with K. To obtain $\xi(\phi_l)$, first note from (17) and (21) that $\bar{\mathbf{u}}_{\phi_l} = \bar{\mathbf{E}}_\phi \mathbf{e}_l$. Using the latter result in (20) and appealing to the definition of $\bar{\mathbf{u}}_0$ in (17), it can be readily shown that

$$\xi(\phi_l) = 0 \qquad l = 2, \ldots, L. \qquad (23)$$

It can be observed from (19) and (23) that, given N, the received power at the directions of unintended receivers ϕ_l, $l = 2, \ldots, L$ is a fixed value that is independent from ϕ_l and, further, it does not increase with K. Our results in this section verify that the proposed cooperative beamformer can efficiently suppress the transmitters effect in the directions of the unintended receivers while substantially increasing the received power in the direction of the intended receiver.

V. Simulations

Computer simulations are used to validate the analytical results. In all examples, $R = 10$, $\lambda = 1$, and three unintended receivers are assumed at $[\phi_2, \phi_3, \phi_4] = [5, -4, -5]$ (deg). Figs. 2 and 3 plot $\tilde{P}_{av}(\phi_\bullet)/\tilde{P}_{av}(0)$ (dB) versus $\phi_\bullet$ (deg) for $K = 30$ and $K = 300$, respectively. In these figures, the solid line, the dashed line, and the dotted line correspond to $N = 1$, $N = 2$, and $N = 3$, respectively. All curves in Figs. 2 and 3 have a maximum at the direction of the intended receiver and minimums very close to the directions of the unintended receivers. Figs. 2 and 3 also show that the gap between the mainlobe and sidelobes of the beampatterns grows as K increases. For instance, the largest sidelobe peak is 12.52 (dB) less than the mainlobe peak for $N = 1$ in Fig. 2, while the largest sidelobe peak is 16.32 (dB) lower than the mainlobe peak for $N = 1$ in Fig. 3.

Fig. 4 displays $\tilde{P}_{av}(0)$ and $\tilde{P}_{av}(\phi_l)$, $l = 2, 3, 4$ versus K for $N = 3$. As can be observed from Fig. 4, the curves corresponding to $\tilde{P}_{av}(\phi_l)$, $l = 2, 3, 4$ are indistinguishable from one another and, moreover, $\tilde{P}_{av}(\phi_l)$ does not increase as K grows. This corroborates the results obtained in Section IV. Moreover, as predicted in Section IV, $\tilde{P}_{av}(0)$ linearly increases with K (note that $\tilde{P}_{av}(0)$ is plotted in a logarithmic scale).

VI. Conclusion

A cooperative beamforming (CB) technique was proposed for a relay-assisted multiple-input multiple-output communication in wireless sensor networks with uniformly distributed

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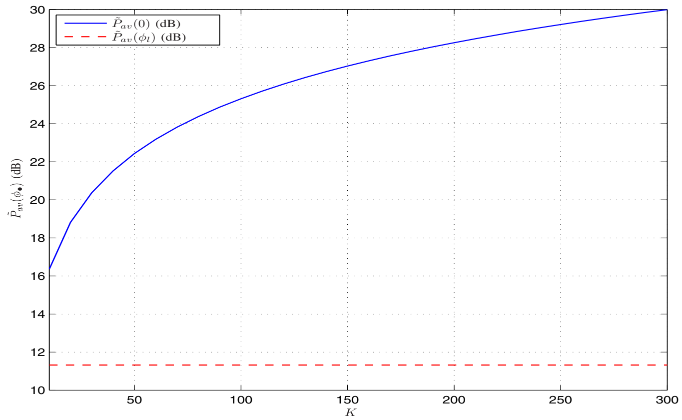

Fig. 4: $\tilde{P}_{av}(0)$ and $\tilde{P}_{av}(\phi_l)$, $l = 2, 3$, and 4 versus K for $N = 3$.

nodes. First, an optimal beamforming expression was derived that maximizes the signal-to-interference-plus-noise ratio (SINR) at the intended receiver while enforcing the transmitters' signals at all unintended receivers to zero. The so-obtained SINR-optimal beamformer does not lend itself to a decentralized implementation. Therefore, an efficient approximation of the latter beamformer was also proposed that can be implemented based only on the locally-available information at the relay nodes. The average beampattern expression of the proposed CB technique was then derived and its properties were analyzed and were validated by numerical simulations.

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