

# Joint power control and relay matrix design for cooperative communication networks with multiple source-destination pairs

Keyvan Zarifi<sup>\*†</sup>, Ali Ghrayeb<sup>†</sup>, and Sofiène Affes<sup>\*</sup>

<sup>\*</sup>INRS-EMT, Université du Québec, 800, de la Gauchetière west, Montreal, QC, H5A 1K6, Canada  
 {zarifi,affes}@emt.inrs.ca

<sup>†</sup>Concordia University, 1455 de Maisonneuve west, Montreal, QC, H3G 1M8, Canada  
 aghrayeb@ece.concordia.ca

**Abstract**— A network with  $L$  single-antenna source-destination pairs is considered that uses a half-duplex  $K$ -antenna relay to establish the end-to-end links in a dual-hop cooperative communication scheme. The sources share a common channel to concurrently transmit their signals and the relay multiplies its received signal vector with  $L$  relaying matrices and then forwards the resulting signal vectors to the destinations in dedicated channels. Under the relay's transmit power constraint at every channel as well as both sources' individual and total power constraints, the jointly optimal sources' powers and the relaying matrices are obtained that maximize the minimum of the normalized signal-to-interference-plus-noise ratios at the destinations. Numerical simulations are then used to verify the analytical results.

## I. INTRODUCTION

Cooperative communication has garnered significant interest as a means to improve the reliability, the coverage, and the capacity of ad-hoc wireless networks wherein multiple sources aim to communicate with their intended destinations [1]-[4]. When sources use a common channel, the effect of co-channel interference makes the task of achieving the desired quality of services at every source-destination pair more challenging. In such cases, the conventional cooperative schemes that opt for optimizing the communication network only at the relaying layer or use separate designs at the transmission and the relaying stages may show inadequate performances.

We present a joint source-relay design technique for a dual-hop cooperative network wherein a multiple-antenna relay is used to establish the links between the source-destination pairs. The proposed technique jointly optimizes the transmission powers at the sources and the linear processing at the relay. Despite the significant contributions to the literature of multipoint-to-multipoint cooperative communication networks, to the best of our knowledge, none of the existing works has investigated the problem of joint power control at the sources and the processing of the signal at the multiple-antenna relay. In the network under our investigation,  $L$  sources transmit their signals in a common channel in the first phase and the relay multiplies its received signal vector with  $L$  relaying matrices and forwards the resulting vectors in

dedicated channels to the destinations in the second phase.<sup>1</sup> We obtain the jointly-optimal sources' powers and the set of relaying matrices that maximize the minimum of the signal-to-interference-plus-noise ratios (SINRs) at the destinations subject to individual and total transmit power constraints at the sources as well as the relay's transmit power constraint at every relaying channel. We show that the jointly-optimal sources' powers and the relaying matrices are the solution to a multi-constraint optimization problem both whose objective function and set of constraints are non-convex with respect to the design parameters. The solution to this optimization problem is then derived in the form of two sets of equations one of which presents the optimal relaying matrices as explicit functions of the optimal source power vector and the other expresses the optimal source power vector as an explicit function of the optimal relaying matrices. An efficient iterative solution that is guaranteed to converge to the jointly-optimal relaying matrices and the source power vector is then presented.

This paper is organized as follows. The system model and the problem formulation are presented in Section II and the optimal source power vector for a fixed set of relaying matrices is derived in Section III. The jointly-optimal sources' powers and the relaying matrices are obtained in Section IV. The simulation results are presented in Section V and the concluding remarks are given in Section VI.

*Notation:* Uppercase and lowercase bold letters denote matrices and vectors, respectively.  $(\cdot)^T$ ,  $(\cdot)^*$ , and  $(\cdot)^H$  denote the transpose, the conjugate, and the Hermitian transpose, respectively.  $|\cdot|$  is the absolute value,  $\|\cdot\|$  is the 2-norm of a vector, and  $\text{tr}(\cdot)$  is the trace of a matrix.  $\mathbb{E}\{\cdot\}$  stands for the statistical expectation and  $\mathbf{I}_K$  is the  $K \times K$  identity matrix.  $\mathbf{1}$  is the vector with all entries equal to 1 and  $\mathbf{e}_i$  is the vector with 1 at the  $i$ -th position and zeros elsewhere.  $[\cdot]_l$  and  $[\cdot]_{nl}$  stand for the  $l$ th entry of a vector and the entry at the  $n$ th row and the  $l$ th column of a matrix, respectively.  $\lambda_{\max}(\cdot)$  is the maximum-modulus eigenvalue and  $\Omega(\cdot)$  is the eigenvector associated with the maximum-modulus eigenvalue normalized such that its last entry is 1.  $\mathbf{D}(\mathbf{a})$  is a diagonal matrix whose diagonal elements are the entries of  $\mathbf{a}$ .  $\mathbf{a}_{\bar{l}}$  is the  $L - 1 \times 1$

<sup>1</sup>The results concerning the case that the relay does not use  $L$  dedicated channels and, instead, forwards a processed version of its received signal in a single channel will be presented elsewhere.

vector obtained by removing the  $l$ -th entry of the  $L \times 1$  vector  $\mathbf{a}$ .  $\mathbf{A}_{\bullet \bar{l}}$  is the  $N \times L - 1$  matrix obtained by removing the  $l$ th column of the  $N \times L$  matrix  $\mathbf{A}$  and  $\text{vec}(\mathbf{A})$  is the  $NL \times 1$  vector obtained by stacking the columns of  $\mathbf{A}$  on top of one another.  $\otimes$  is the Kronecker product.  $\in$ ,  $\text{Card}(\cdot)$ ,  $\cup$ , and  $\subseteq$  denote the membership in a set, the cardinality of a set, the union of two sets, and the subset, respectively.

## II. SYSTEM MODEL AND PROBLEM REPRESENTATION

Consider a cooperative communication network with  $L$  single-antenna source-destination pairs where each source aims to transmit its signal to its intended destination. Due to, for instance, large distances between the sources and the destinations all direct source-destination links are negligibly weak and the links between the source-destination pairs are established using a half-duplex  $K$ -antenna relay and through the following two-phase communication protocol: In the first phase, sources share a common channel to transmit their signals. In the second phase, the relay-destinations transmissions are carried out in  $L$  dedicated channels.<sup>2</sup> In particular, the relay multiplies its  $K$ -dimensional received signal vector by a relaying matrix  $\mathbf{W}^{(l)*}$  and forwards the resulting signal vector to the  $l$ th destination in the  $l$ th dedicated channel for  $l = 1, \dots, L$ .

Let  $s_l$  denote the transmitted signal from the  $l$ th source with  $\text{E}\{|s_l|^2\} = p_l$  and assume that we must have  $p_l \leq P_l$ ,  $l = 1, \dots, L$  and, further, following regulations, it should also hold that  $\sum_{l=1}^L p_l \leq P_{L+1}$ . Note that  $P_{L+1} < \sum_{l=1}^L P_l$ , as otherwise the total power constraint is trivially satisfied. A more compact form of the above constraints is given by

$$\mathbf{u}_l^T \mathbf{p} \leq P_l \quad l = 1, \dots, L + 1 \quad (1)$$

where  $\mathbf{u}_l \triangleq \mathbf{e}_l$ ,  $l = 1, \dots, L$  and  $\mathbf{u}_{L+1} \triangleq \mathbf{1}$ . Let  $g_{kl}$  denote the channel gain from the  $l$ th source to  $k$ th relay antenna. Introducing  $\mathbf{s} \triangleq [s_1 \dots s_L]^T$ ,  $\mathbf{g}_l \triangleq [g_{1l} \dots g_{Kl}]^T$ , and  $\mathbf{G} \triangleq [\mathbf{g}_1 \dots \mathbf{g}_L]$ , the received signal vector at the relay can be represented as

$$\mathbf{y} = \mathbf{G}\mathbf{s} + \mathbf{v} \quad (2)$$

where  $\mathbf{v} \triangleq [v_1 \dots v_K]^T$  is the relay's noise vector. The signal transmitted from the relay in the  $l$ -th dedicated channel can then be expressed as

$$\mathbf{x}^{(l)} = \mathbf{W}^{(l)*} \mathbf{y}. \quad (3)$$

From (2) and (3) it follows that the relay's transmit power in the  $l$ -th channel is

$$\begin{aligned} p_r^{(l)} &= \text{E} \left\{ \mathbf{x}^{(l)H} \mathbf{x}^{(l)} \right\} \\ &= \text{tr} \left( \mathbf{W}^{(l)*} (\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \mathbf{\Sigma}_v) \mathbf{W}^{(l)T} \right) \\ &= \mathbf{w}^{(l)H} \mathbf{\Lambda}(\mathbf{p}) \mathbf{w}^{(l)} \end{aligned} \quad (4)$$

<sup>2</sup>Depending on the duplexing mode, dedicated relay-destination channels may be devised in time or in frequency. All results in this paper hold for both cases. See also [4]-[7] for other cooperative schemes with dedicated relay channels.

where  $\mathbf{p} \triangleq [p_1 \dots p_L]^T$ ,  $\mathbf{\Lambda}(\mathbf{p}) \triangleq (\mathbf{G}\mathbf{D}(\mathbf{p})\mathbf{G}^H + \mathbf{\Sigma}_v) \otimes \mathbf{I}_K$ , and  $\mathbf{w}^{(l)} \triangleq \text{vec}(\mathbf{W}^{(l)})$ . Due to regulations, the following power constraint must also be upheld:

$$\mathbf{w}^{(l)H} \mathbf{\Lambda}(\mathbf{p}) \mathbf{w}^{(l)} \leq P^{(l)} \quad l = 1, \dots, L \quad (5)$$

where  $P^{(l)}$  is the upper-bound on the relay's transmit power in the  $l$ th channel. Introducing  $P_{l+L+1}(\mathbf{W}^{(l)}) \triangleq P^{(l)} - \text{tr}(\mathbf{W}^{(l)*} \mathbf{\Sigma}_v \mathbf{W}^{(l)T})$  and  $\mathbf{u}_{l+L+1}(\mathbf{W}^{(l)})$  such that  $[\mathbf{u}_{l+L+1}(\mathbf{W}^{(l)})]_l \triangleq [\mathbf{G}^H \mathbf{W}^{(l)T} \mathbf{W}^{(l)*} \mathbf{G}]_{ll}$ , (5) can be equivalently represented as [8]

$$\mathbf{u}_l \left( \mathbf{W}^{(l-L-1)} \right)^T \mathbf{p} \leq P_l \left( \mathbf{W}^{(l-L-1)} \right) \quad (6)$$

where  $l = L + 2, \dots, 2L + 1$ . The similar structures of (1) and (6) makes it possible to express all power constraints in a unified form. Let  $h_k^{(l)}$  represent the channel gain from the  $k$ -th relay antenna to the  $l$ th destination in the  $l$ -th orthogonal channel. Introducing  $\mathbf{h}^{(l)} \triangleq [h_1^{(l)} \dots h_K^{(l)}]^T$ , the received signal at the  $l$ th destination in its dedicated channel is

$$z_l = \mathbf{h}^{(l)T} \mathbf{x}^{(l)} + n_l \quad l = 1, \dots, L \quad (7)$$

where  $n_l$  is the  $l$ th destination noise. Using (2) and (3) in (7), it can be readily shown for  $l = 1, \dots, L$  that

$$z_l = \mathbf{w}^{(l)H} \mathbf{f}_l^{(l)} s_l + \sum_{\substack{k=1, \\ k \neq l}}^L \mathbf{w}^{(l)H} \mathbf{f}_k^{(l)} s_k + \mathbf{w}^{(l)H} (\mathbf{v} \otimes \mathbf{h}^{(l)}) + n_l \quad (8)$$

where  $\mathbf{f}_k^{(l)} \triangleq \mathbf{g}_k \otimes \mathbf{h}^{(l)}$ . Let  $\sigma_{n_l}^2 = \text{E}\{|n_l|^2\}$  and  $\mathbf{\Sigma}_v \triangleq \text{E}\{\mathbf{v}\mathbf{v}^H\} = \mathbf{D}([\sigma_{v_1}^2 \dots \sigma_{v_K}^2]^T)$ . From (8), it is direct to obtain the SINR at the  $l$ th destination as

$$\eta_l(\mathbf{w}^{(l)}, \mathbf{p}) = \frac{p_l \left| \mathbf{w}^{(l)H} \mathbf{f}_l^{(l)} \right|^2}{\sum_{\substack{k=1 \\ k \neq l}}^L p_k \left| \mathbf{w}^{(l)H} \mathbf{f}_k^{(l)} \right|^2 + \mathbf{w}^{(l)H} \mathbf{\Gamma}_v^{(l)} \mathbf{w}^{(l)} + \sigma_{n_l}^2} \quad (9)$$

where  $\mathbf{\Gamma}_v^{(l)} \triangleq \mathbf{\Sigma}_v \otimes \mathbf{h}^{(l)} \mathbf{h}^{(l)H}$ . In this work,  $\bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}) \triangleq \eta_l(\mathbf{w}^{(l)}, \mathbf{p}) / \gamma_l$  is used as the  $l$ th pair communication performance measure where  $\gamma_l$  is a normalization factor that can be selected proportional to the target SINR at the  $l$ th destination. We aim to derive the jointly-optimal power vector  $\mathbf{p}_o$  and relaying matrices  $\mathbf{W}_o^{(l)}$ ,  $l = 1, \dots, L$  that maximize the minimum  $\bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p})$  subject to the  $2L + 1$  constraints in (1) and (6). A max-min (normalized) SINR optimization strategy is used when the goal is to preserve fairness among multiple communication links [9], [10]. The problem of interest can be formally expressed as

$$\max_{\mathbf{W}, \mathbf{p}} \min_{1 \leq l \leq L} \bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}) \quad (10a)$$

$$\text{subject to (1) and (6)} \quad (10b)$$

where  $\mathbf{W} \triangleq [\mathbf{W}^{(1)} \dots \mathbf{W}^{(L)}]$ .

There is a rich literature on the joint optimization of transmit powers and receivers in which the studied problems

can be typically cast as maximizing the minimum of the users' (normalized) SINRs subject to a total transmit power constraint similar to the last constraint in (1) [9], [11], [12]. In contrast, (10) is a problem with  $2L + 1$  constraints on  $\mathbf{p}$  and  $\mathbf{w}^{(l)}$ ,  $l = 1, \dots, L$ . Moreover, note from (6) that  $L$  constraints in (10) are non-convex with respect to the total design parameters  $(\mathbf{w}^{(l)}, \mathbf{p})$ . As such, the conventional joint power control and receiver design techniques cannot be directly adopted to solve (10). However, a two-step approach similar to that of the conventional techniques can still be used to solve our problem: The first step determines the optimal power vector for a fixed set of relaying matrices and the second step obtains the jointly-optimal power vector and relaying matrices. These two steps are respectively presented in the next two sections.

### III. THE OPTIMAL POWER VECTOR FOR A GIVEN SET OF RELAYING MATRICES

Consider a fixed set of relaying matrices  $\mathbf{W}^{(1)} \dots \mathbf{W}^{(L)}$ . Due to technical reasons, we assume that

$$\left| \mathbf{w}^{(l)H} \mathbf{f}_k^{(l)} \right| > 0 \quad l, k \in \{1, \dots, K\}. \quad (11)$$

The following reasons support (11) in practice: 1)  $\mathbf{f}_k^{(l)}$  is a stochastic vector and the probability that  $\mathbf{w}^{(l)H} \mathbf{f}_k^{(l)} = 0$  is zero for an arbitrarily-selected  $\mathbf{w}^{(l)}$ . 2) As the noise power in (9) is larger than zero, the zero-forcing receiver is not SINR-optimal and, hence, the optimal  $\mathbf{w}^{(l)}$  is not orthogonal to  $\mathbf{f}_k^{(l)}$ ,  $k \neq l$ . Given  $\mathbf{W} \triangleq [\mathbf{W}^{(1)} \dots \mathbf{W}^{(L)}]$ , (10) reduces to

$$\max_{\mathbf{p}} \min_{1 \leq l \leq L} \bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}) \quad (12a)$$

$$\text{subject to } \mathbf{u}_l^T \mathbf{p} \leq P_l \quad l = 1, \dots, 2L + 1 \quad (12b)$$

where, for the ease of representation, we have dropped the arguments of  $\mathbf{u}_l(\mathbf{W}^{(l-L-1)})$  and  $P_l(\mathbf{W}^{(l-L-1)})$  in (6) and combined the resulting inequalities with (1) to obtain (12b). The following observations are essential in deriving  $\mathbf{p}_o(\mathbf{W})$ ; the solution to (12):

*Observation 1:* When  $\mathbf{p} = \mathbf{p}_o(\mathbf{W})$ , at least one of the constraints in (12b) holds with equality. Otherwise one can multiply  $\mathbf{p}_o(\mathbf{W})$  with an  $\alpha_1 > 1$  and increase all  $\bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}_o(\mathbf{W}))$ . This contradicts the optimality of  $\mathbf{p}_o(\mathbf{W})$ .

When  $\mathbf{p} = \mathbf{p}_o(\mathbf{W})$ ,  $\mathcal{E}_o(\mathbf{W})$  shows the set of constraints indices in (12b) that hold with equality, that is,  $\mathbf{u}_l^T \mathbf{p}_o(\mathbf{W}) = P_l$  if and only if  $l \in \mathcal{E}_o(\mathbf{W})$ .

*Observation 2:*  $\mathbf{p}_o(\mathbf{W})$  balances all normalized SINRs, that is,

$$\bar{\eta}(\mathbf{W}) = \bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}_o(\mathbf{W})) \quad l = 1, \dots, L. \quad (13)$$

Eq. (13) is a result of (11) which implies that  $\bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p})$  is a strictly decreasing function of all sources' powers except  $p_l$ . Therefore, if  $\bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}_o(\mathbf{W}))$  is larger than the others, a decrease in  $[\mathbf{p}_o(\mathbf{W})]_l$  increases all other normalized SINRs, and, therefore, the objective function. This contradicts the optimality of  $\mathbf{p}_o(\mathbf{W})$ . Note that the SINR balancing property is an essential part of most multiuser power control techniques.

Let us introduce the  $L \times L$  non-negative matrix  $\Psi(\mathbf{W})$  such that  $[\Psi(\mathbf{W})]_{lk} \triangleq \left| \mathbf{w}^{(l)H} \mathbf{f}_k^{(l)} \right|^2$  for  $l \neq k$  and  $[\Psi(\mathbf{W})]_{ll} \triangleq 0$ , the  $L \times L$  positive diagonal matrix  $\Omega(\mathbf{W})$  such that  $[\Omega(\mathbf{W})]_{ll} \triangleq \gamma_1 / \left| \mathbf{w}^{(1)H} \mathbf{f}_1^{(1)} \right|^2$ , and the  $L \times 1$  positive vector  $\sigma(\mathbf{W})$  such that  $[\sigma(\mathbf{W})]_l \triangleq \mathbf{w}^{(l)H} \Gamma_v^{(l)} \mathbf{w}^{(l)} + \sigma_{n_l}^2$ . Then, it is straightforward to show from (9) that  $\mathbf{p}_o(\mathbf{W})$  in (13) satisfies

$$\Omega(\mathbf{W}) \Psi(\mathbf{W}) \mathbf{p} + \Omega(\mathbf{W}) \sigma(\mathbf{W}) = \frac{1}{\bar{\eta}(\mathbf{W})} \mathbf{p}. \quad (14)$$

Using (14) along with Observation 1, we have

$$\frac{1}{P_l} \mathbf{u}_l^T \Omega(\mathbf{W}) \Psi(\mathbf{W}) \mathbf{p} + \frac{1}{P_l} \mathbf{u}_l^T \Omega(\mathbf{W}) \sigma(\mathbf{W}) = \frac{1}{\bar{\eta}(\mathbf{W})} \quad (15)$$

for  $l \in \mathcal{E}_o(\mathbf{W})$ . Let

$$\Lambda_l(\mathbf{W}) \triangleq \begin{bmatrix} \Omega(\mathbf{W}) \Psi(\mathbf{W}) & \Omega(\mathbf{W}) \sigma(\mathbf{W}) \\ \frac{1}{P_l} \mathbf{u}_l^T \Omega(\mathbf{W}) \Psi(\mathbf{W}) & \frac{1}{P_l} \mathbf{u}_l^T \Omega(\mathbf{W}) \sigma(\mathbf{W}) \end{bmatrix} \quad (16)$$

for  $l = 1, \dots, 2L + 1$ . We have from (14) and (15) that  $[\mathbf{p}_o(\mathbf{W})^T \mathbf{1}]^T$  satisfies

$$\Lambda_l(\mathbf{W}) \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} = \frac{1}{\bar{\eta}(\mathbf{W})} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} \quad l \in \mathcal{E}_o(\mathbf{W}). \quad (17)$$

Eq. (17) shows that  $1/\bar{\eta}(\mathbf{W})$  and  $[\mathbf{p}_o(\mathbf{W})^T \mathbf{1}]^T$  are a jointly-positive eigenpair of  $\Lambda_l(\mathbf{W})$  for all  $l \in \mathcal{E}_o(\mathbf{W})$ . It can be proved that [8]  $\Lambda_l(\mathbf{W})$  is a nonnegative primitive matrix<sup>3</sup> for  $l = 1, \dots, 2L + 1$ , and, therefore,  $\lambda_{\max}(\Lambda_l(\mathbf{W}))$  and its associated eigenvector are the unique positive eigenpair of  $\Lambda_l(\mathbf{W})$  [13, Ch. 8]. Let  $\Omega(\Lambda_l(\mathbf{W})) \triangleq [\mathbf{p}_l(\mathbf{W})^T \mathbf{1}]^T$  for  $l = 1, \dots, 2L + 1$ . It follows from the above discussion that

$$\bar{\eta}(\mathbf{W}) = \frac{1}{\lambda_{\max}(\Lambda_l(\mathbf{W}))} \quad l \in \mathcal{E}_o(\mathbf{W}) \quad (18a)$$

$$\begin{bmatrix} \mathbf{p}_o(\mathbf{W}) \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{p}_l(\mathbf{W}) \\ 1 \end{bmatrix} = \Omega(\Lambda_l(\mathbf{W})) \quad l \in \mathcal{E}_o(\mathbf{W}). \quad (18b)$$

When  $\text{Card}(\mathcal{E}_o(\mathbf{W})) > 1$ , from Eq. (18a) it follows that we must have  $\lambda_{\max}(\Lambda_m(\mathbf{W})) = \lambda_{\max}(\Lambda_n(\mathbf{W}))$  for  $m, n \in \mathcal{E}_o(\mathbf{W})$ . Moreover, (18b) suggests that  $\mathbf{p}_o(\mathbf{W})$  is not unique unless  $\mathbf{p}_m(\mathbf{W}) = \mathbf{p}_n(\mathbf{W})$  for  $m, n \in \mathcal{E}_o(\mathbf{W})$ . The following theorem obtains  $\mathcal{E}_o(\mathbf{W})$  and proves that  $\lambda_{\max}(\Lambda_m(\mathbf{W})) = \lambda_{\max}(\Lambda_n(\mathbf{W}))$  and  $\mathbf{p}_m(\mathbf{W}) = \mathbf{p}_n(\mathbf{W})$  for  $m, n \in \mathcal{E}_o(\mathbf{W})$ . The proof of the theorem is given in [8].

*Theorem 1:* Let  $\{l_1, \dots, l_M\} \subseteq \{1, \dots, 2L + 1\}$ . The following Eqs. (19)-(22) are equivalent:

$$\mathcal{E}_o(\mathbf{W}) = \{l_1, \dots, l_M\} \quad (19)$$

$$\lambda_{\max}(\Lambda_{l_1}(\mathbf{W})) = \dots = \lambda_{\max}(\Lambda_{l_M}(\mathbf{W})) > \lambda_{\max}(\Lambda_l(\mathbf{W})) \quad (20)$$

$$\frac{1}{\bar{\eta}(\mathbf{W})} = \lambda_{\max}(\Lambda_{l_1}(\mathbf{W})) = \dots = \lambda_{\max}(\Lambda_{l_M}(\mathbf{W})) \neq \lambda_{\max}(\Lambda_l(\mathbf{W})) \quad (21)$$

<sup>3</sup>A nonnegative matrix is primitive if it is irreducible and only has one eigenvalue of maximum-modulus [13].

$$\mathbf{p}_o(\mathbf{W}) = \mathbf{p}_{l_1}(\mathbf{W}) = \dots = \mathbf{p}_{l_M}(\mathbf{W}) \neq \mathbf{p}_l(\mathbf{W}) \quad (22)$$

where  $l$  in Eqs. (20)-(22) is any index in  $\{1, \dots, 2L+1\} - \{l_1, \dots, l_M\}$ .

Using Theorem 1, (12) can be solved for a given  $\mathbf{W} = [\mathbf{W}^{(1)} \dots \mathbf{W}^{(L)}]$  as follows:

- 1) Form  $\Lambda_l(\mathbf{W})$  in (16) and compute  $\lambda_{\max}(\Lambda_l(\mathbf{W}))$  for  $l = 1, \dots, 2L+1$ .
- 2) Sort the set of  $\lambda_{\max}(\Lambda_l(\mathbf{W}))$ ,  $l = 1, \dots, 2L+1$  in a non-increasing order. There may be  $M \geq 1$  eigenvalues that are equal to one another but larger than all others. Label the indices of the corresponding matrices as  $l_1, \dots, l_M$  and find  $\bar{\eta}(\mathbf{W})$  from (21).
- 3) Form  $\mathcal{E}_o(\mathbf{W})$  as in (19). Choose an arbitrary  $l_m \in \mathcal{E}_o(\mathbf{W})$  and compute  $\Omega(\Lambda_{l_m}(\mathbf{W})) = [\mathbf{p}_{l_m}(\mathbf{W})^T \ 1]^T$ . Obtain the unique  $\mathbf{p}_o(\mathbf{W})$  from (22).

Note also from Theorem 1 that when  $\mathbf{p} = \mathbf{p}_o(\mathbf{W})$ , the  $l_1$ th, the  $l_2$ th,  $\dots$ , the  $l_M$ th constraints in (12b) hold with equality and all other constraints hold with a strict inequality.

One may conjecture that it is unlikely to have  $M = \text{Card}(\mathcal{E}_o(\mathbf{W})) > 1$ . However, as will be shown in Section IV, when the optimal set of relaying matrices  $\mathbf{W}_o \triangleq [\mathbf{W}_o^{(1)} \dots \mathbf{W}_o^{(L)}]$  is used,  $M_o \triangleq \text{Card}(\mathcal{E}_o(\mathbf{W}_o)) \geq L+1$ .

#### IV. JOINTLY-OPTIMAL POWER VECTOR AND RELAYING MATRICES

It follows from Theorem 1 that  $\mathbf{p}_o$  is uniquely given by  $[\mathbf{p}_o^T \ 1]^T = \Omega(\Lambda_l(\mathbf{W}_o))$ ,  $l \in \mathcal{E}_o(\mathbf{W}_o)$ . To characterize  $\mathbf{W}_o$ , note also from Theorem 1 that when a set of relaying matrices  $\mathbf{W}$  is used, we have  $\bar{\eta}(\mathbf{W}) = 1/\lambda_{\max}(\Lambda_l(\mathbf{W}))$ ,  $l \in \mathcal{E}_o(\mathbf{W})$ . Therefore, it should hold that  $\lambda_{\max}(\Lambda_l(\mathbf{W}_o)) \leq \lambda_{\max}(\Lambda_l(\mathbf{W}))$  where  $l \in \mathcal{E}_o(\mathbf{W}_o)$  and  $\mathbf{W}$  is any arbitrary set of relaying matrices. In other words,  $\mathbf{W}_o$  must minimize  $\lambda_{\max}(\Lambda_l(\mathbf{W}))$  for  $l \in \mathcal{E}_o(\mathbf{W}_o)$ . However, the latter minimization should be performed over the set of feasible relaying matrices that satisfy (6). It follows from the above discussion that  $\mathbf{W}_o$  and  $\mathbf{p}_o$  are jointly optimal if and only if

$$\mathbf{W}_o = \underset{\mathbf{W}}{\text{argmin}} \lambda_{\max}(\Lambda_l(\mathbf{W})) \quad l \in \mathcal{E}_o(\mathbf{W}_o) \quad (23a)$$

$$\text{subject to } \mathbf{u}_l(\mathbf{W}^{(l-L-1)})^T \mathbf{p}_o \leq P_l(\mathbf{W}^{(l-L-1)}) \quad l = L+2, \dots, 2L+1 \quad (23b)$$

where

$$\begin{bmatrix} \mathbf{p}_o \\ 1 \end{bmatrix} = \Omega(\Lambda_l(\mathbf{W}_o)) \quad l \in \mathcal{E}_o(\mathbf{W}_o). \quad (23c)$$

The optimization problem (23) has a complicated structure. The aim in (23a) is to jointly minimize the common maximum-modulus eigenvalue of multiple non-symmetric matrices  $\lambda_{\max}(\Lambda_l(\mathbf{W}))$ ,  $l \in \mathcal{E}_o(\mathbf{W}_o)$ . To the best of our knowledge, there is no systematic method to solve such a problem in general. Note also that the above minimization should be carried out over a feasible set that satisfies (23b). The feasible set in (23b) changes with  $\mathbf{p}_o$  which, due to (23c), itself is a function of the to-be-determined  $\mathbf{W}_o$ . It should also be mentioned that as  $\mathbf{W}_o$  is unknown, so is  $\mathcal{E}_o(\mathbf{W}_o)$  in (23a) and

(23c) and, further,  $\mathcal{E}_o(\mathbf{W}_o)$  may have a nonempty intersection with the set of constraints indices  $\{L+2, \dots, 2L+1\}$  in (23b). Therefore, some  $\mathbf{u}_l(\mathbf{W}^{(l-L-1)})$  and  $P_l(\mathbf{W}^{(l-L-1)})$  that determine the feasible set of the relaying matrices in (23b) may also have an indirect effect on this feasible set through  $\mathbf{p}_o$  in (23c). The above discussion underlines our need to represent (23) in an equivalent but a simpler form. The following two observations are instrumental in achieving this task.

*Observation 3:*  $\mathbf{W}_o$  and  $\mathbf{p}_o$  satisfy all constraints in (6) with equality [8], that is,

$$\mathbf{u}_l(\mathbf{W}_o^{(l-L-1)})^T \mathbf{p}_o = P_l(\mathbf{W}_o^{(l-L-1)}) \quad (24)$$

for  $l = L+2, \dots, 2L+1$ .

*Observation 4:* When  $\mathbf{W}_o$  and  $\mathbf{p}_o$  are jointly used, there is a constraint in (1) that holds with equality [8], that is,

$$\mathbf{u}_l^T \mathbf{p}_o = P_l \quad (25)$$

for at least one  $l \in \{1, \dots, L+1\}$ .

When  $\mathbf{W}_o$  and  $\mathbf{p}_o$  are jointly used, let  $\mathcal{A}_o(\mathbf{W}_o) \triangleq \{l \in \{1, \dots, L+1\} | \mathbf{u}_l^T \mathbf{p}_o = P_l\}$ . Then, it follows from the above observations that  $\mathcal{E}_o(\mathbf{W}_o) = \mathcal{A}_o(\mathbf{W}_o) \cup \{L+2, \dots, 2L+1\}$  and (23) may be equivalently expressed as

$$\mathbf{W}_o = \underset{\mathbf{W}}{\text{argmin}} \lambda_{\max}(\Lambda_l(\mathbf{W})) \quad l \in \mathcal{E}_o(\mathbf{W}_o) \quad (26a)$$

$$\text{subject to } \mathbf{u}_l(\mathbf{W}^{(l-L-1)})^T \mathbf{p}_o = P_l(\mathbf{W}^{(l-L-1)}) \quad l = L+2, \dots, 2L+1 \quad (26b)$$

where

$$\begin{bmatrix} \mathbf{p}_o \\ 1 \end{bmatrix} = \Omega(\Lambda_l(\mathbf{W}_o)) \quad l \in \mathcal{A}_o(\mathbf{W}_o) \cup \{L+2, \dots, 2L+1\}. \quad (26c)$$

As the equality constraints in (26b) hold for  $\mathbf{W} = \mathbf{W}_o$ , (26b) implies that  $\{L+2, \dots, 2L+1\} \subseteq \mathcal{E}_o(\mathbf{W}_o)$ . It follows from the equivalence of (19) and (22) that  $\{L+2, \dots, 2L+1\} \subseteq \mathcal{E}_o(\mathbf{W}_o)$  is an alternative representation of  $[\mathbf{p}_o^T \ 1]^T = \Omega(\Lambda_{L+2}(\mathbf{W}_o)) = \dots = \Omega(\Lambda_{2L+1}(\mathbf{W}_o))$ . Therefore, (26c) is partially enforced by (26b). We can remove this redundancy by replacing  $l \in \mathcal{A}_o(\mathbf{W}_o) \cup \{L+2, \dots, 2L+1\}$  in (26c) by  $l \in \mathcal{A}_o(\mathbf{W}_o)$ . Moreover, the equivalence of (19) and (20) establishes the fact that  $l^* \in \mathcal{A}_o(\mathbf{W}_o)$  if and only if  $l^* = \arg \max_{1 \leq l \leq L+1} \lambda_{\max}(\Lambda_l(\mathbf{W}_o))$ . The optimization problem (26) can now be equivalently represented as

$$\mathbf{W}_o = \underset{\mathbf{W}}{\text{argmin}} \lambda_{\max}(\Lambda_l(\mathbf{W})) \quad l \in \mathcal{E}_o(\mathbf{W}_o) \quad (27a)$$

$$\text{subject to } \mathbf{u}_l(\mathbf{W}^{(l-L-1)})^T \mathbf{p}_o = P_l(\mathbf{W}^{(l-L-1)}) \quad l = L+2, \dots, 2L+1 \quad (27b)$$

where

$$\begin{bmatrix} \mathbf{p}_o \\ 1 \end{bmatrix} = \Omega(\Lambda_{l^*}(\mathbf{W}_o)) \quad l^* = \arg \max_{1 \leq l \leq L+1} \lambda_{\max}(\Lambda_l(\mathbf{W}_o)). \quad (27c)$$

The advantage of (27) to (26) is that  $l$  and  $l^*$  in (27b) and (27c) belong to the disjoint sets of  $\{L+2, \dots, 2L+1\}$  and  $\{1, \dots, L+1\}$ , respectively. Moreover, the set of  $\Lambda_l(\mathbf{W}_o)$  in (27c) depends on those pairs of  $\mathbf{u}_l$  and  $P_l$  that are independent from the relaying matrices. These facts are useful in developing a simple iterative algorithm that alternately optimizes  $\mathbf{p}$  and  $\mathbf{W}$  to obtain  $\mathbf{p}_o$  and  $\mathbf{W}_o$ . The structural properties of  $\Lambda_l(\mathbf{W})$  can be used to represent (27) in a form that lends itself to the iterative solution. The following theorem whose proof is given in [8] holds.

*Theorem 2:* The optimization problem (27) can be equivalently expressed as

$$\mathbf{w}_o^{(l)} = \underset{\mathbf{w}^{(l)}}{\operatorname{argmax}} \bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}_o) \quad l = 1, \dots, L \quad (28a)$$

$$\text{subject to } \mathbf{u}_l(\mathbf{W}^{(l-L-1)})^T \mathbf{p}_o = P_l(\mathbf{W}^{(l-L-1)}) \quad l = L+2, \dots, 2L+1 \quad (28b)$$

where

$$\begin{bmatrix} \mathbf{p}_o \\ 1 \end{bmatrix} = \Omega(\Lambda_{l^*}(\mathbf{W}_o)) \quad l^* = \underset{1 \leq l \leq L+1}{\operatorname{argmax}} \lambda_{\max}(\Lambda_l(\mathbf{W}_o)). \quad (28c)$$

It should be mentioned that similar techniques as in the proof of Theorem 2 have been used before in [9] to solve the conventional problem of joint power control and receiver design with a single constraint on sources total transmit power. Note that  $\bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}_o)$  depends only on  $\mathbf{w}^{(l)}$  and  $\mathbf{w}^{(l)}$  appears only in the  $l$ -th constraint in (28b). Therefore, when  $\mathbf{p}_o$  is known, the subproblem (28a)-(28b) may be solved by independently maximizing  $\bar{\eta}_l(\mathbf{w}^{(l)}, \mathbf{p}_o)$  subject to the  $l$ -th constraint in (28b) for  $l = 1, \dots, L$ . The following theorem takes advantage of the latter property to describe  $\mathbf{W}_o$  and  $\mathbf{p}_o$  as explicit functions of one another.

*Theorem 3:* Let

$$\zeta_l(\mathbf{p}) \triangleq \sqrt{P^{(l)}} \left( \mathbf{f}_l^{(l)H} \mathbf{R}_l(\mathbf{p})^{-1} \Lambda(\mathbf{p}) \mathbf{R}_l(\mathbf{p})^{-1} \mathbf{f}_l^{(l)} \right)^{-1/2} \quad (29)$$

where

$$\mathbf{R}_l(\mathbf{p}) \triangleq P^{(l)} \left( \mathbf{F}_{\bullet l}^{(l)} \mathbf{D}(\mathbf{p}_l) \mathbf{F}_{\bullet l}^{(l)H} + \Gamma_v^{(l)} \right) + \sigma_{n_l}^2 \Lambda(\mathbf{p}) \quad (30)$$

with  $\mathbf{F}^{(l)} \triangleq [\mathbf{f}_1^{(l)} \dots \mathbf{f}_L^{(l)}]$ . Then,  $\mathbf{W}_o$  and  $\mathbf{p}_o$  are jointly optimal if and only if [8]

$$\mathbf{w}_o^{(l)} = \zeta_l(\mathbf{p}_o) \mathbf{R}_l(\mathbf{p}_o)^{-1} \mathbf{f}_l^{(l)} \quad l = 1, \dots, L \quad (31a)$$

$$\begin{bmatrix} \mathbf{p}_o \\ 1 \end{bmatrix} = \Omega(\Lambda_{l^*}(\mathbf{W}_o)) \quad l^* = \underset{1 \leq l \leq L+1}{\operatorname{argmax}} \lambda_{\max}(\Lambda_l(\mathbf{W}_o)). \quad (31b)$$

The fact that  $\mathbf{W}_o$  and  $\mathbf{p}_o$  are explicit functions of one another in (31a) and (31b) gives rise to Algorithm I that obtains  $\mathbf{W}_o$  and  $\mathbf{p}_o$  by alternately optimizing  $\mathbf{W}_{[n]}$  for the given  $\mathbf{p}_{[n-1]}$  and then  $\mathbf{p}_{[n]}$  for the given  $\mathbf{W}_{[n]}$ . The convergence of  $\mathbf{W}_{[n]}$  and  $\mathbf{p}_{[n]}$  to  $\mathbf{W}_o$  and  $\mathbf{p}_o$  is guaranteed and is shown in [8].

---

### Algorithm I: Algorithmic solution to joint power control and relaying matrices design

---

```

1: Initialize:  $n = 0$ ;  $\mathbf{p}_{[0]} \geq 0$ 
2: Repeat
3:  $n = n + 1$ 
4:  $\mathbf{w}_{[n]}^{(l)} = \zeta_l(\mathbf{p}_{[n-1]}) \mathbf{R}_l(\mathbf{p}_{[n-1]})^{-1} \mathbf{f}_l^{(l)} \quad l = 1, \dots, L$ 
5:  $\mathbf{W}_{[n]} = [\mathbf{W}_{[n]}^{(1)} \dots \mathbf{W}_{[n]}^{(L)}]$ 
6:  $l_{[n]}^* = \underset{1 \leq l \leq L+1}{\operatorname{argmax}} \lambda_{\max}(\Lambda_l(\mathbf{W}_{[n]}))$ 
7:  $\begin{bmatrix} \mathbf{p}_{[n]} \\ 1 \end{bmatrix} = \Omega(\Lambda_{l_{[n]}^*}(\mathbf{W}_{[n]}))$ 
8: until  $\|\mathbf{p}_{[n+1]} - \mathbf{p}_{[n]}\| \leq \epsilon$  where  $\epsilon$  is a small number.
```

---

Simulation results in Section V show that the iterative algorithm proposed in Algorithm I enjoys a high convergence rate.

## V. SIMULATIONS

Numerical experiments are carried out to validate the analytical results. In the first two examples, we consider  $K = 8$  relay antennas and  $L = (K/2) + 1 = 5$  source-destinations pairs and assume that the noise power at all relay antennas and all destinations is  $\sigma^2$  and set  $P^{(l)} = \gamma_l = 10\sigma^2$  for  $l = 1, \dots, L$ .

Fig. 1 shows  $\bar{\eta}_l(\mathbf{w}_{[n]}^{(l)}, \mathbf{p}_{[n-1]})$  from Algorithm I for  $l = 1, \dots, L$  versus the iteration index  $n$  in the case that the upper-bounds of the source powers are equal and are given by  $P_l = 1.1 \cdot \gamma_l = 11\sigma^2$ ,  $l = 1, \dots, L$  and the upper-bound on the sources total transmit power is  $P_{L+1} = L\gamma_l = 50\sigma^2$ . As can be observed from Fig. 1, all normalized SINRs converge to the same value in  $n = 4$  iterations. This verifies the efficiency of Algorithm I to obtain  $\mathbf{W}_o$  and  $\mathbf{p}_o$ .

In the next numerical example, it is assumed that  $P_l = \varphi \cdot p$  for  $l = 1, \dots, L$  and  $P_{L+1} = Lp = 5p$ . Then, Algorithm I is used to obtain the optimal balanced normalized SINRs  $\bar{\eta}_l(\mathbf{w}_o^{(l)}, \mathbf{p}_o)$ . Fig. 2 shows  $\bar{\eta}_l(\mathbf{w}_o^{(l)}, \mathbf{p}_o)$  versus  $p/\sigma^2$  for several  $\varphi$ . For the sake of comparison, we have also shown with the dashed line the minimum of the normalized SINRs when only the relaying matrices are optimized and the transmit powers of all sources are equal to  $p$ . Note that in the latter case the sources total transmit power is equal to  $P_{L+1}$ , and, hence, is always larger than or equal to the sources total transmit power in the case when the sources' powers and the relaying matrices are jointly optimized. Despite the above fact, Fig. 2 shows that our joint optimization approach always performs better than the case that the sources transmit with equal powers. Note that as  $\varphi$  increases, the upper-bound on the sources' individual power becomes larger and the upper-bound on the sources total transmit power should become the only active constraint in (1). This explains why the curves corresponding to  $\varphi = 1.6$  and  $\varphi = 1.8$  in Fig. 2 are indistinguishable from each other: The optimization problems corresponding to the latter two curves share exactly the same set of active constraints. Finally, note that as the total transmit power constraint is active, the sum of the sources' transit powers when  $\varphi = 1.6$  and  $\varphi = 1.8$  is equal to that in

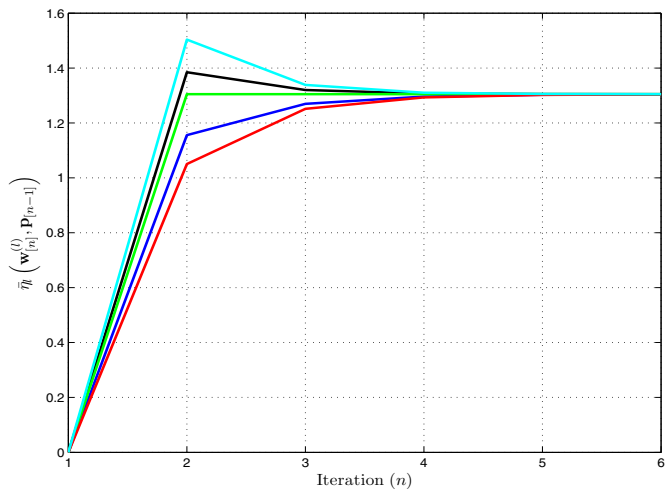


Fig. 1.  $\bar{\eta}_l(\mathbf{w}_{[n]}^{(l)}, \mathbf{P}_{[n-1]})$  v.s. the iteration index  $n$  for  $K = 8$  and  $L = 5$ .

the simulated equal-power case shown with the dashed line. Compared to the latter curve, the curves corresponding to  $\varphi = 1.6$  and  $\varphi = 1.8$  show more than 20% increase in the minimum of the normalized SINRs.

Fig. 3 shows  $\bar{\eta}_l(\mathbf{w}_{[n]}^{(l)}, \mathbf{P}_{[n-1]})$  versus the iteration index  $n$  for  $K = 4$  and  $L = (K/2) + 1 = 3$ . All other parameters are the same as in Fig. 1. Again, a rapid convergence to the optimal normalized SINR value can be observed in this figure.

## VI. CONCLUSIONS

We considered a cooperative communication network wherein multiple sources aim to communicate with their intended destinations through the help of a multiple-antenna relay. Assuming that the sources-relay communication is performed in a shared channel and the relay-destinations communications are carried out in dedicated channels, the jointly optimal sources' powers and the relaying matrices were obtained that maximize the minimum of the normalized signal-to-interference-plus-noise ratios at the destinations while satisfying constraints on individual and total sources' powers as well as relay's transmit power at every dedicated channel. An interesting research direction currently under investigation is to solve a similar problem in the case that the relay-destinations communication is also performed in a shared channel. The solution to the latter problem will be disclosed in a future publication.

## REFERENCES

- [1] Ö. Oyman and A. J. Paulraj, "Power-bandwidth tradeoff in dense multi-antenna relay networks," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2282-2293, June. 2007.
- [2] S. Fazeli-Dehkordy, S. Shahbazpanahi, and S. Gazor, "Multiple peer-to-peer communications using a network of relays," *IEEE Trans. Signal Process.*, vol. 57, pp. 3053-3062, Aug. 2009.
- [3] J. Joung and A. H. Sayed, "Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems," *IEEE Trans. Signal Process.*, vol. 58, pp. 1833-1846, Mar. 2010.
- [4] K. T. Phan, T. Le-Ngoc, S. A. Vorobyov, and C. Tellambura, "Power allocation in wireless multi-user relay networks," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 2535-2545, May 2009.

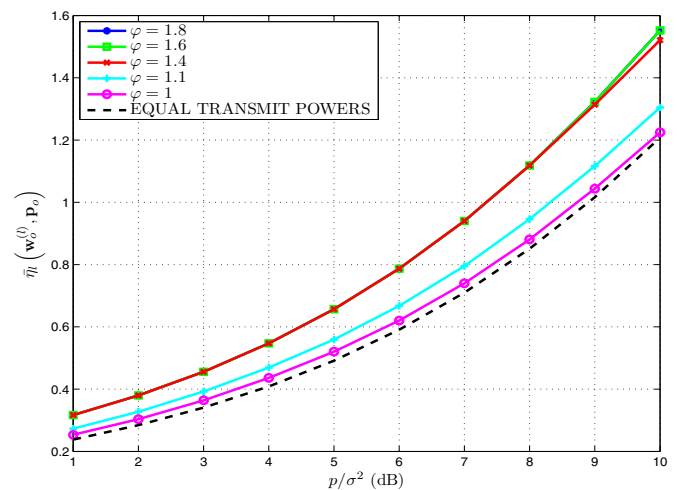


Fig. 2.  $\bar{\eta}_l(\mathbf{w}_o^{(l)}, \mathbf{p}_o)$  v.s.  $p/\sigma^2$  for several  $\varphi$  for  $K = 8$  and  $L = 5$ .

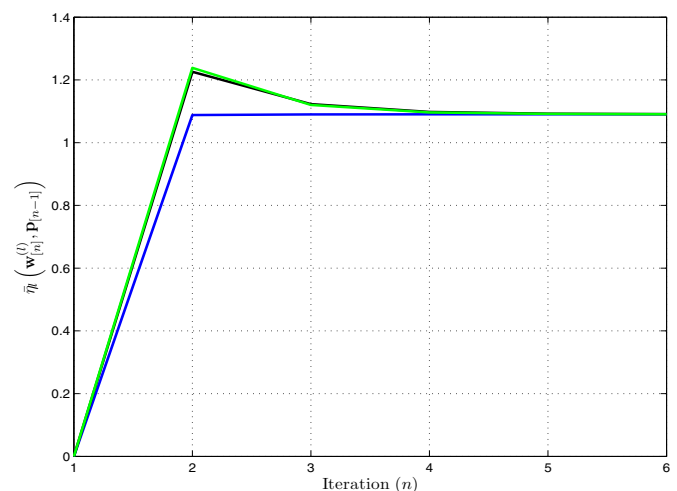


Fig. 3.  $\bar{\eta}_l(\mathbf{w}_{[n]}^{(l)}, \mathbf{P}_{[n-1]})$  v.s. the iteration index  $n$  for  $K = 4$  and  $L = 3$ .

- [5] J. N. Laneman and G. W. Wornell, "Distributed spacetime-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2415-2425, Oct. 2003.
- [6] Y. Zhao, R. Adve, and T. J. Lim, "Improving amplify-and-forward relay networks: Optimal power allocation versus selection," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 3114-3123, Aug. 2007.
- [7] K. Jitvanichpaibool, R. Zhang, and Y.-C. Liang, "Optimal resource allocation for two-way relay-assisted OFDMA," *IEEE Trans. Veh. Technol.*, vol. 58, pp. 3311-3321, Sep. 2009.
- [8] K. Zarifi, A. Ghrayeb, and S. Affes, "Jointly optimal source power control and relay matrix design in multipoint-to-multipoint cooperative communication networks," *IEEE Trans. Signal Process.*, submitted.
- [9] M. Schubert and H. Boche, "Solution of the multi-user downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, pp. 18-28, Jan. 2004.
- [10] E. Karipidis, N. D. Sidiropoulos, Z.-Q. Luo, "Quality of service and max-min fair transmit beamforming to multiple co-channel multicast groups," *IEEE Trans. Signal Process.*, vol. 56, pp. 1268-1279, Mar. 2008.
- [11] F. Rashid-Farrokhi, L. Tassiulas, and K. J. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Commun.*, vol. 46, pp. 1313-1324, Oct. 1998.
- [12] M. Schubert and H. Boche, "A generic approach to QoS-based transceiver optimization," *IEEE Trans. Commun.*, vol. 55, pp. 1557-1566, Aug. 2007.
- [13] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, UK: Cambridge Univ. Press, 1990.